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Thoughts on different types of mathematical enculturation at the secondary-tertiary transition

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In this paper, different courses of mathematical enculturation during the transition from school- to university-level mathematics are identified and illustrated. To achieve this, we reinterpret mathematical enculturation within a socio-cultural perspective of mathematics, viewing the learning of mathematics as a process of transformative education. We consider experiences of foreignness and crisis that arise due to cultural differences between educational institutions during the secondary-tertiary transition. On this basis, different responses to these experiences can be distinguished: (a) rejective, (b) assimilative or (c) productive.

Keywords: Secondary-tertiary transition, mathematical enculturation, transformative education.

Introduction

It has been extensively demonstrated and discussed that the transition from secondary to tertiary mathematics poses a major hurdle for students who enter university (Di Martino et al., 2022a). One way of describing this secondary-tertiary transition (STT) is the concept of mathematical enculturation (e.g. Pepin, 2014), where STT is understood as a type of socialisation into the culture of university mathematics with its own language and society. Learning mathematics is therefore not just about acquiring knowledge but is also a fundamentally social process (Schoenfeld, 1992/2016). Learners acquire the language, skills, and norms of mathematics by interacting with the specific mathematical community, both consciously and unconsciously. Following this line of thinking, students—who have been socialised through school mathematics at the beginning of their studies and who are now transitioning to tertiary mathematics—may experience a cultural shock or break due to the introduction of a new mathematical culture. To overcome this, they must enculturate themselves into this new environment.

Based on this understanding of mathematical enculturation, an ongoing research project at Leibniz University Hannover is investigating the transition from secondary to tertiary mathematics. The project is focused on the individual socialisation processes that students undergo as they enter the culture of university mathematics. It looks for certain breakages and institutionally predetermined breaking points as well as individual coping mechanisms within the first semester.

To empirically examine the enculturation process, the project reinterprets it as a transformative education process (translated by author from German 'Transformative Bildung'; Koller, 2018), in which an individual transforms his or her relationship to self, others and world relation as well as his or her frame of mind. The theory of transformative education not only describes the triggers of educational processes, so-called *experiences of foreignness* (Koller, 2018, p. 79 ff, transl. by author), but also various ways in which those experiences are dealt with, resulting in successful or unsuccessful education.

Based on the notion of experiences of foreignness and the respective personal responses, the ongoing research project is partially examining the diverse approaches learners employ to confront enculturation and cultural discontinuity within the STT. This paper will outline the deliberations of this part of the study, following the research question how students—from a theoretical point of view—handle the transition from secondary to tertiary mathematics, with particular attention to how they respond to cultural breaks throughout their enculturation process. To begin, this text will provide a brief outline of the reinterpretation of mathematical enculturation as a process of transformative education. This will be followed by a description of possible types of enculturation based on the concept of experiences of foreignness and transformative education. Prior to the conclusion, those types will be exemplified using interview transcriptions from the study.

Mathematical Enculturation as Transformative Education

Following the influential review by Gueudet (2008), Di Martino et al. (2022a) describe the international research on STT in their recent literature review for the timeframe from 2008 to 2021. They provide a summary of four main theoretical lenses used during this period: epistemological, cognitive, socio-cultural and affective. Thereby the authors demonstrate both the diversity of approaches and “the growing awareness of the complexity of phenomena at play in STT” (Di Martino et al., 2022a, p. 18). However, Di Martino et al. (2022b) also point out that even though the socio-cultural, and affective aspects of the students’ crisis in STT are being recognised for decades and are appearing more frequently in research, they are still underrepresented. Nevertheless, the authors highlight existing approaches, such as Clark and Lovric (2008, 2009).

In their papers, Clark and Lovric outline a theoretical framework of STT that is based on the anthropological concept of a rite of passage. This socio-cultural and partly affective approach views the transition from school to university mathematics as leaving an old, established culture of mathematics and entering a new one. This often causes ‘life crises’ for the individual. The learner must adapt to established routines and become integrated into the new mathematical culture through various phases. Especially they highlight that a transition, in line of their approach, is emergent and significantly influenced by intrinsic factors of the individual. Clark and Lovric portray STT as a form of necessary socialisation—particularly enculturation—into a new mathematical culture where the learner must adapt to the new values, norms, and conventions university mathematics requires to be able to further engage with mathematics.

Within such socio-cultural and affective frameworks mathematics is viewed as a “cultural phenomenon” (Bishop, 1988, p. 3) that is socially constructed, utilised, and shaped. Hereby learning mathematics is not only an acquisition of knowledge, facts, and procedures, but also a process of social interaction between the learner and the educational environment (Schoenfeld, 1992/2016). This involves internalising and practicing social, cultural, and linguistic aspects of mathematics, mathematical practices as well as structures of perception, thinking, and action (Loch, 1968).

When entering an unfamiliar mathematical community and its culture, an individual lacking socialisation may be presented with a distinct perspective of the (mathematical) world. This perspective includes particular mathematical knowledge, strategies of problem-solving, self-regulation, and practices within and shaped by the culture and its mathematical worldview. This way

of thinking about mathematics is foreign to the individual and may lead to the crisis described by Clark and Lovric (2008).

During an individual, emergent, and performative process (Butler, 2001; Koller, 2018) the learner modifies and broadens their mathematical worldview to meet the new cultural challenges and demands while assimilating the specific values, norms, and conventions. This marks a significant transformation of its established self-other-world-relation and structures of thinking, likely triggered by a (unavoidable) break and a subsequent crisis between the previous and the new cultural framings (Bachelard, 2002; Clark & Lovric, 2009; Pepin, 2014).

Even though an increasing amount of research focuses on the social-cultural and affective aspects of STT and the rite of passage approach describes crisis and adaption phases for learners, there is comparatively less theoretical or empirical research on the specific process, how the individual learner is coping with STT and related hurdles. The main issue here seems to be the above-mentioned individuality, emergence, and performativity of the transition, as these render observation and analysis of these processes more difficult and complex.

As it deals with such processes, the theory of transformative education, as outlined by Koller (2018), presents a plausible theoretical framework for further comprehending and studying mathematical enculturation, especially focusing on the passages and individual processes described by Clark and Lovric. Based on the classic concept of education by Humboldt (1960–1981), Koller's theory elaborates on the structure of the individual world-, others- and self-relations, the starting points of educational processes and the shifting of these relations as well as perspectives on the empirical investigation of these processes. The following description of mathematical enculturation as a process of transformative education is based in an abbreviated form on Günther (2023), which presents this reinterpretation in more detail.

Koller defines education and learning in terms of a linguistic approach: Learning describes interacting with subjectively new problems within an established framing of perception, interpretation, and processing. Education, however, only takes place as “people dealing with new problem situations develop new dispositions of perceiving, interpreting and dealing with problems that enable them to deal with these problems better than before” (Koller, 2023, p. 16; transl. by author). It is assumed that the disposition system of thinking and acting is relatively stable and sluggish, culturally shaped within a society and represented collectively within it.

To trigger a change in this disposition system, experiences are necessary that “challenge the relative stability of an established understanding of the world and oneself” (Koller, 2018, p. 71; transl. by author). These are referred to as *experiences of crisis* (Koller, 2018, p. 71) based on Waldenfels' concept of *experiences of foreignness* (1987). Waldenfels assumes that the individual perceives, thinks, and acts within and according to a specific social order. Experiences made by the individual refer to particular orders. Experiences of foreignness are those that challenge the inherent order of the individual, necessitating an alternate order that is initially unfamiliar. Such experiences of foreignness may be interpersonal, intrapersonal, intercultural, or intracultural.

If mathematical enculturation and thereby STT is understood as a process of transformative educational, then both school and university mathematics can be seen as culturally shaped and

individually represented disposition systems of mathematical perception, thinking, and acting. They imply corresponding self-, other- and world-relations and are shaped and consolidated by the institutional social environment and its mathematical community. In STT, the learner's established school mathematical self-, other- and world-relation challenged with discontinuities between school and university mathematics (cf. e.g. Bauer et al., 2020). Consequently, the assumptions of first-year students, which stem from school experiences, are contradicted by university mathematics and lead to a questioning of their own knowledge, thinking, and acting (Buck, 1981; Koller, 2018). Consequently, these new experiences or problem situations may in part elude the established mathematical world-relation and its order and are thus considered as something foreign (Koller, 2018, p. 71 ff).

Ways of Reacting

According to Koller, the questioning or overruling of an established world-relation and its order necessitates a reaction on the part of the learner. The theory of transformative education describes three possibilities for this: (i) the "equating of the foreign as an enemy", (ii) the "acquisition of the foreign" and (iii) the productive response to the foreign (Waldenfels, 1997).

The (i) equation of experiences of foreignness with a potential enemy can lead to a deconstructive response, resulting in the demarcation from or an exclusion of the foreign. In extreme cases, "physical annihilation" (Waldenfels, 1997, p. 48; transl. by author) might also be a necessary consequence. By contrast, (ii) acquisition represents a much more peaceful, almost ignorant response to the foreign. The learner presupposes the order challenged by the foreign as an unavoidable norm and considers the incorporating and subsuming of the experience of foreignness. This leads to an assimilation of the foreign into the established order while depriving it of its foreignness. Lastly, (iii), the only productive reaction takes the "disquiet by the foreign as a challenge" (Koller, 2018, p. 84; transl. by author). It proceeds less from one's order, more from its questioning, and consequently makes a response to the foreign necessary. Hereby two responses are possible: a reproductive response reproduces pre-existing meaning. A productive response brings forth something new from the difference between the subject and the foreign, and in that sense can be called creative.

Transferred to mathematical enculturation in STT reinterpreted in the sense of transformative education, three possible responses and consequent courses of enculturation processes can be assumed: When first-year students are faced with problems that they are unable to solve and that challenge not only their knowledge, but also their underlying dispositional systems of mathematical perception, thinking, and acting, they can react (a) rejective, (b) assimilative, or (c) productive. As (a) rejecting those reactions of students can be described which—in the sense of (i)—reject a reorientation of their own mathematical self- and world-relation and declare any problem as superfluous, unnecessary, or even wrong. Thinking further about the annihilation of the foreign described above, it can be hypothesized here that these students are more likely to fail at the transition and represent at least part of the dropouts in first-year mathematics. Students, on the other hand, who—following (ii)—assimilate experiences of foreignness and questioning of their world-relations, may as well successfully complete their studies without ever transforming their world-relation. They react (b) assimilative insofar as they regard their school-mathematical thinking and acting as the norm and

consequently try to solve problems beyond it within this order. Such an attitude may, for example, give rise to misconceptions which create bridges between the order and the problem persisting and consolidating beyond the study of university mathematics. Lastly, (c) productive responses can be understood as those which, in the sense of (iii), take the questioning of the mathematical dispositional system as a challenge, i.e., trying to critically question, adapt, and extend their mathematical perception, thinking, and acting. In this way, the shift of one's perspective towards a university mathematical point of view is possible. For this purpose, pre-existing meaning can be repeated or the difference can be overcome with something new (Butler, 1998). Thus, this last response leads to a reordering of the mathematical self-, other- and world-relation.

Examples from a Interview Study

The reinterpretation of mathematical enculturation described here is investigated within the framework of a qualitative interview series at Leibniz University Hannover in Germany. In the winter term 2022/2023, a total of nine first-year students from different study programs (mathematics, physics, teacher education) were accompanied by five one-hour narrative interviews each during their first semester (if not dropped out of university) (Heinze, 2001). The objective is to closely observe the (individual) process of enculturation into mathematics (specifically during the first semester) to identify enculturation stimuli and media. Additionally, the study seeks to trace learners' reactions and coping mechanisms of the learners and describe possible support measures. The study design is based on the approach proposed by Koller for the study of transformative education (Koller, 2018, p. 137 ff). The data analysis is still ongoing and final results of the study are not yet available. However, the responses discussed in this paper can already be seen through student behavior.

Firstly, the following transcript shows a student struggling with seeing connections between calculus and algebra when pointed out by a lecturer:

Student A: So it [linear algebra] is still like that: yes, you also do it somewhat like that in calculus. Or they say in calculus: you know something like this from linear algebra, and it somehow blurs like that, but it also not really. I also noticed that the other day in calculus our professor said: yes, you work with matrices in linear algebra. Tell me what linear function translates into a matrix. I look at him like this, I am like: what are you talking about? what translates into what? So that has, that has what to do with it?

In this passage, an experience of foreignness can be detected. The student understands calculus and linear algebra as two separate mathematical fields. His mathematical self-world-relation is challenged by the lecturer's questions and insinuations, forcing the student to somehow react to this questioning of his disposition system. The student follows up with:

Student A: That's when I realised a little bit: At the moment in math, I'm not looking at how much I enjoy this subject, but how good the professor is. It's actually: OK, I'm doing something with numbers, that's always about the same, but how good is the professor? And then I hope that at some point it will become so that I can actually look at the subject again and say: Yes, the subject is what I'm studying at the moment, and I'm not studying with a certain professor.

Instead of taking the insinuations of the lecturer seriously and redefining the understanding of mathematics by seeing connections between different areas, the student chooses to label the insinuations as a bad way of teaching. Thereby the learner rejects the possible educational step, the expansion of the disposition system and therefore a step towards mathematical enculturation.

Another participant was confronted with an experience of strangeness in the context of an exam as it went much worse than he had expected. His response can be described as assimilative:

Student B: I think I passed, yes, it's more frustration than ego. The frustration was very high after the exam because I felt it was something out of my control. Um, which is not necessarily my fault, I would say it was. I think I prepared adequately, um, and there was frustration that it was out of my control, that's how I would put it.

In the interview, student B describes that he was very frustrated after the exam, but does not think that his preparation was inadequate. Rather, he sees the exam itself as the problem; it was too difficult. When asked, he sees no reason to change his learning strategy after the current experience. The experience of foreignness is thus deprived of its foreignness by a justification from his own order, an assimilative response.

Lastly, in the following transcript a described experience of foreignness is followed by a seemingly productive attitude:

Student C: I found it very abstract at first, perhaps because I hadn't been confronted with it before and perhaps because the idea I had of linear algebra was always like that. You always imagine it rather to the power of three and then try to attach everything to it somehow, and that's where you first had to fully detach yourself, somehow. And because that's also somehow what you do at school in the vector geometry part, everything actually works more or less on the basis of being able to imagine it, and then it was difficult to get away from it at first.

Student C initially describes the abstractness she faces in linear algebra. However, she is willing and open to changing her view of mathematics when her school ideas no longer seem applicable. On the one hand, this is a productive response; at the same time, it reveals a progressive, unfinished shift in her relation to the mathematical world.

These first impressions from the interview study, chosen as examples, illustrate different experiences of foreignness and the possible responses. On the one hand, they show responses to small and local, but also to more fundamental crisis-like experiences during the entry into the university. It is important to note that only excerpts of the interviews could be shown here as illustrations. At the same time, these embed themselves in a larger, more complex context of an individual.

Conclusion

Based on a socio-cultural and affective understanding and learning of mathematics as well as a reinterpretation of mathematical enculturation in STT as a process of transformative education, three possible courses of enculturation were presented in this paper. They can be understood as responses to experiences of foreignness and as a questioning of the underlying disposition system of

mathematical perception, thinking, and acting: (a) a rejective response, which views the foreign as an enemy and tries to fight it and defend one's mathematical world-relation; (b) an assimilative response, which assimilates the foreign under one's world-relation; (c) a productive response, which accepts the difference to the foreign in a productive manner, to expand one's mathematical perception, thinking, and acting. First impressions from the affiliated study show positive results regarding this understanding and allow to illustrate distinctions emerging from it.

Following the argumentation of this paper, the approach not only allows to describe more precisely how mathematical enculturation takes place and what individual reactions in this process look like. Rather, the short excerpts into the interviews already point to the multi-layered nature of enculturation in STT and the complex interactions between the learning individual, the institution, and the community within it. These can be meaningfully elaborated through the described perspective. In this context, the responses to experiences of foreignness described above represent ideal types of enculturation processes that can serve as reference points in that analysis. Nevertheless, in the discussion of the theoretical framework presented here, the links to the rites of passage as described by Clark and Lovric (2008) will also have to be further explored, since significant similarities are visible.

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