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# The relevance of advanced mathematics courses for secondary school mathematics teachers: Voices of Tanzanian teachers

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*In this paper, we analyse how mathematics teachers rate the relevance of their university mathematics training in their careers as secondary teachers, focusing on a location usually absent in the international literature, Africa, where pre-service training programmes usually include a significant number of advanced mathematics courses. We map the secondary mathematics content with university course content and validate our model with questionnaires and interviews. Our results indicate that only a small portion of the secondary mathematics content is explicitly addressed in the university courses and that Tanzanian secondary mathematics teachers see their mathematical training as largely unrelated to their practice. This may explain why their perception of what mathematics is seems to be less positive than in other studies.*

*Keywords: Mathematics for teachers, pre-service teacher training, advanced mathematics courses.*

## Introduction

The last ten years have seen an increasing number of studies examining the relevance of advanced mathematics courses in the training of secondary school teachers. Rationales for including these courses in teacher training programmes usually assume that their presence helps to build advanced content knowledge that can scaffold the work of teaching mathematics, through the development of expertise and proficiency pertinent to this teaching (e.g., Biza et al., 2022). However, as Biza et al. (2022) point out, there is little empirical evidence that supports the above rationales. These authors point to several recent studies suggesting that the assumed benefit for secondary teachers may not be realised in the classroom (e.g., Wasserman et al., 2018) and that “pre-service teachers taking such courses find them unproductive and irrelevant for their future profession while practicing teachers find it difficult to cite specific examples in which their learning experiences in tertiary mathematics courses were applicable in their teaching” (p. 405).

On the other hand, some studies indicate that practitioners find these advanced mathematics courses relevant to their profession, although this may be the case more so when teacher training programmes organise these courses in a specific way (e.g., Even, 2011). In particular, the benefit of these courses lies in their contribution to teachers’ knowledge of what mathematics is, rather than to specific content; for instance, in the study by Hoffmann and Even (2021), most participants provided answers connected to these topics: *Wide and varied, Lively and developing, Asking questions and explaining why, Using intuition and formalism, and Practical worth* (p. 59 – see also Figure 1). Mytlis and Even (2021) have also posited that the scarcity of schoolteachers’ reports on the contribution of tertiary mathematics to their practices could be related not to the absence of such contribution, but rather to teachers’ restricted capabilities to recognise such contribution. In their study involving five

experienced secondary teachers, all participants provided specific examples of how their academic mathematics contributed to their teaching of various topics, citing analysis as particularly helpful for presenting mathematics concepts more accurately and in justifying procedures used in class. We note, however, that the need for advanced mathematics content may be different for college (where content is closer to university) than for secondary school. Mytlis and Even (2021) add that it is important to investigate the nature of the mathematics courses included in teacher training programmes.

With respect to this last point, we note that most research on these issues has been conducted in Europe (or countries neighbouring Europe) and North America. We believe that examining teacher training systems in other countries may help build on existing studies. In what follows, we provide details about the school system and mathematics teacher training in Tanzania.

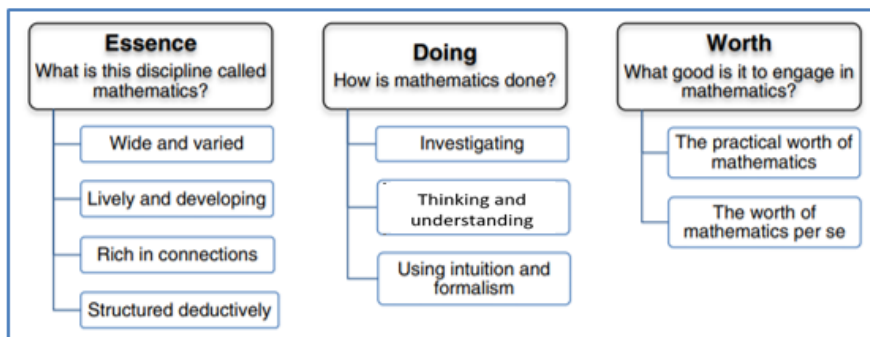
The current basic education system in Tanzania comprises two years of pre-primary school, seven years of primary education (ages six to 12), four years of Ordinary level (O-level) secondary education (Form 1 to Form 4, ages 13 to 16), and two years of Advanced level (A-level) secondary education (Form 5 and Form 6, ages 17 and 18) (URT, 2000). Compulsory education continues until the end of the O-level. Tanzania follows a centralised system of curriculum development for teacher education from primary to the Advanced level. Curricula for O-level mathematics are centrally developed by the Tanzania Institute of Education under the Ministry of Education, Science, and Technology. Universities develop teacher training programmes for this level, which are accredited by the Tanzania Commission for Universities. Currently, secondary teacher training programmes offered by universities have a duration of three years, and most students are trained to teach two subjects. Courses are structured around the following blocs of content: 1) Advanced mathematics courses (generally  $\approx 30\%$  of the programme); 2) general education courses (generally  $\approx 40\%$ ); 3) courses related to the second science subject and other supporting courses (generally  $\approx 30\%$ ). We can observe that advanced mathematics courses have an important place in these programmes, and secondary teachers therefore are given a strong foundation in advanced mathematics. Note, however, that specific courses on mathematics education are not generally a part of these programmes; instead, notions related to the teaching of mathematics are covered in general education courses.

This contextual information helps us state the aim of the study, and we note that the contribution of advanced mathematics courses to actual teaching practices (and not simply to teachers' overall knowledge development) has rarely been examined (e.g., Hoffman & Even, 2021). Therefore, we seek to analyse the relationship between secondary school content and university mathematics courses in teacher training programmes, as well as examine how secondary teachers value their advanced mathematical training in their teaching, particularly outside the European (or Europe-adjacent) and North American contexts, which tend to dominate the international literature on these issues. We chose the Tanzanian context for convenience, but we note that other countries in this region of Africa have similar school systems and teacher training curricula.

## **Theoretical framework**

Like Zazkis and Leikin (2010), we consider advanced mathematical knowledge (AMK) as knowledge related to tertiary mathematics. Hoffmann and Even (2018) proposed a framework for analysing teachers' knowledge of what mathematics is. It comprises three main dimensions (the *essence* of

mathematics; *doing* mathematics; the *worth* of mathematics), which are further divided into a total of nine topics (Figure 1).



**Figure 1: Framework for teacher knowledge about the discipline of mathematics (Hoffman & Even, 2021, p. 52)**

We considered this framework in the third stage of our study. In the first two stages, we constructed a model to illustrate the relevance of AMK in teacher training programmes to the content of the O-level mathematics curriculum in Tanzania (see Methods). This model was tested with teacher questionnaires, which helped us to validate our findings and pinpoint any needs for adjustments to the model. We then conducted focus group interviews with Tanzanian teachers, which revealed our model’s level of accuracy in predicting content the teachers perceive as useful. We also looked for insights into how the AMK taught at university contributed to the teachers’ views on mathematics.

## Methods

### First phase

For the first phase, we compiled the mathematical content present in the O-level (Form 1 to Form 4) curriculum in Tanzania. This content is distributed across the four years of training, in 38 topics subdivided into a total of 339 content units (Ordinary level secondary basic mathematics, or OBM units). Figure 2 provides an example of topics and some content units.

We then chose a major university in the country and examined the advanced mathematics courses offered in its teacher training programme. Courses in other universities were previously screened and it was concluded that their structure is quite similar. The programme requires pre-service teachers to complete 12 advanced mathematics courses. Using the syllabus of each course, we organised its content into modules, for a total of 48 modules (AMK modules). We then constructed a grid, with the 339 OBM units as rows and the 48 AMK modules as columns, mapping OBM content covered in any of the AMK modules. The codes were: 0 – Not aligned at all (OBM content that is not present in the corresponding AMK module); 1 – Implicitly aligned (OBM content that is used in the AMK module, but is not the subject being taught; e.g., using fractions when integrating); 2 – Explicitly aligned in a different way (OBM content that is explicitly taught in the AMK module, but from a different perspective; e.g., rational numbers in an abstract algebra course where fields are presented); 3 – Explicitly aligned in the same way (OBM content that is explicitly taught in the AMK module, from the same perspective; e.g., the definition of function in an analysis course).

The third author proceeded with an initial mapping of content. The whole team tested the coding with a data sample (one AMK course) and then the third author completed the mapping. The rest of the Tanzanian team validated the mapping and any discordances were resolved. Finally, the entire team met and validated the mapping.

OBM Topic	Content Units	Advanced mathematics Course 1	Advanced mathematics Course 2	...	Advanced mathematics Course 12
Algebra	Algebraic operations-Simplify algebraic expression				
	Equation in one unknown-Solve equation of one unknown				
	Equation in one unknown-Form & solve equation from word problems				
	...				
Circles	Tangent properties-Prove tangent theorems				
	Definition of terms-Define different terms of circles				
...	...				

**Figure 2: Example of OBM topics and content units and the grid used to map contents. For this paper, instead of 48 columns of AMK modules, we use 12 columns of advanced mathematics courses.**

### Second phase

In this phase, questionnaires were distributed to O-level teachers to validate the mapping model. Data were collected from 10 purposively sampled public secondary schools<sup>1</sup> in the Dar es Salaam region. In each of the sampled schools, all graduate mathematics teachers teaching in the O-level were invited to voluntarily participate in the study. In total, 31 mathematics teachers volunteered for this phase, having graduated from eight different universities in Tanzania. They had an average of almost nine years of mathematics teaching experience. We note that this is not at all a statistically significant sample; however, the demographic data, training, and professional experience of the participants can be considered as representative of regular Tanzanian teachers.

The questionnaire had three sections. The first section collected some demographic data; the second one presented the 48 AMK modules and teachers were asked to rate them as follows: 1 – Not at all useful; 2 – Somewhat useful; 3 – Useful; 4 – Very useful. For the modules rated 3 or 4, participants were asked to identify the OBM content for which they found the AMK content useful. Finally, the third section consisted of 16 items measuring the participants’ perceived usefulness of AMK for the teaching of OBM content, as well as their knowledge of what mathematics is. These items were adopted from Even (2011), with modifications, and measured on a four-point scale ranging from 1 – Strongly disagree to 4 – Strongly agree. The questionnaires were first analysed descriptively by the second author with the aid of SPSS version 25. The whole team then convened to discuss the analysis and the results.

### Third phase

The Tanzanian team visited ten schools in Dar es Salaam and conducted focus group interviews. Teachers in nine schools volunteered to participate. In schools with just one volunteer, an individual

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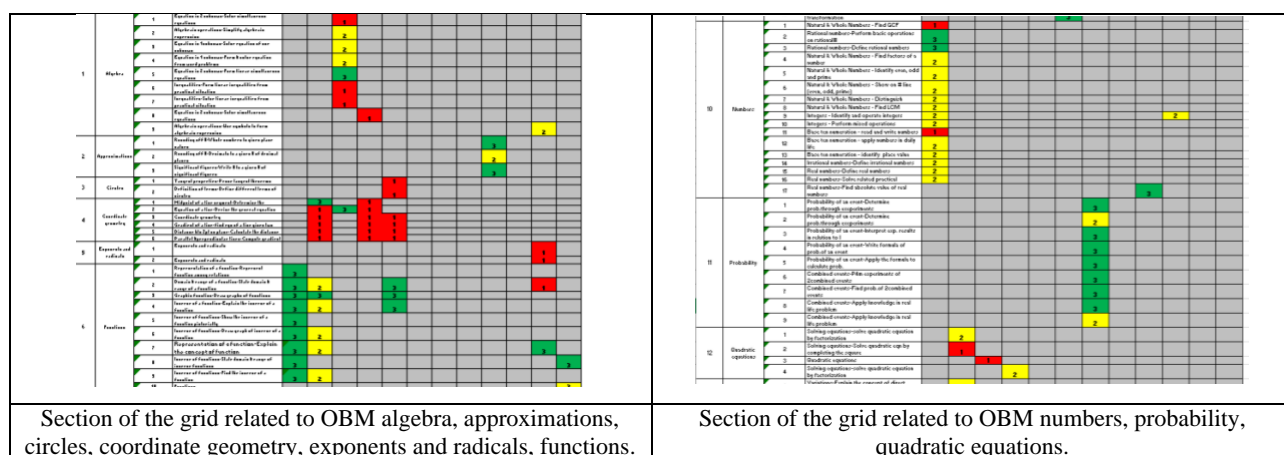
<sup>1</sup> Schools were sampled that ranked highest in terms of school GPA, based on students’ performance in Form 4 according to national examinations statistics (NECTA, 2017), with the assumption that their teachers would be keener to participate in the study. Only public schools were considered, given that they represent the majority of schools in the country.

interview was conducted. Five schools had three participants, two schools had two participants, and two schools had one participant, for a total of 21 participants. The interviews and focus group discussions lasted between one hour and 90 minutes and were semi-structured. The protocols included asking questions about the perceived usefulness of AMK content in teaching OBM content, asking for examples of OBM content for which AMK content was deemed useful, discussing the teachers' perception of what doing mathematics is, and asking their opinion on their mathematical training.

The interviews and focus group discussions were transcribed and the first author proceeded with an initial qualitative coding, considering both the results of the two previous phases and the framework for teacher knowledge about the discipline of mathematics (Figure 1). Then, the whole team convened to discuss the results. The analysis is ongoing and we provide some preliminary data here.

## Results

The mapping in the first phase revealed some striking results. To facilitate visualisation, we used colours in our large grid: grey for Category 1 (content not mapping), red for Category 2 (implicitly aligned), yellow for Category 3 (explicitly aligned, but differently), and green for Category 4 (explicitly aligned, in the same way). Figure 3 presents a glimpse of two sections of the grid:



**Figure 3: Sections of the grid used to map OBM and AMK content**

The analysis shows that out of the 339 OBM units, only  $\approx 18\%$  are explicitly addressed in the AMK courses. Moreover, of the 38 OBM topics, 11 ( $\approx 29\%$ ) are not at all addressed in the AMK courses. If we have a look at the AMK courses, only five out of the 12 AMK courses address OBM content explicitly and in the same way.

We sought to validate these impressions with practitioners in the field (second phase). The analysis of the questionnaires seemed to confirm our first impressions. Let us recall that the third part of the questionnaire used a scale from 1 (Strongly disagree) to 4 (Strongly agree). The participants seem to agree with the statement that “Secondary school teachers need a specific [AMK] content that is different from other professions that use mathematics” (average of 3.06) and seem to imply that their AMK training does not cover all their needs; their responses are broadly distributed around the statements: “Secondary school mathematics teachers often do not have a sound understanding of the mathematics that is needed in teaching secondary school mathematics” (average of 2.26) and “University mathematics courses do not adequately support the development of mathematical

knowledge required for quality teaching of secondary school mathematics” (average of 2.29). Additionally, the statement “Some of my [AMK] courses are helpful in my practice as a teacher, but I believe that many of these courses could be replaced by courses that prepare me better to teach secondary school content” has an average of 3.23. This is connected with the average of 3.32 for the statement “Secondary school mathematics teachers need to be provided with a body of mathematical knowledge that does not stray very far away from the material they teach in school.” Despite this criticism of the received AMK content, the participants also seem to see some value in mathematics, and it helps them to solve problems (*Thinking and understanding* and *Using intuition and formalism* in Figure 1) and add information to their lessons (*Rich in connections*). When asked to rate the 48 AMK modules, only six ( $\approx 12.50\%$ ) have averages above 3 (somewhat useful). These modules are as follows: Sets (3.8), Functions of one variable (3.3), Introduction to probability (3.3), Number system (3.3), Relations (3.2), Introduction and graphical method for linear programming (3.1). Three other modules have averages between 2 and 3: Linear algebra (2.7), Sequences (2.4), Simplex method for linear programming (2.2). It seems the participants only see a portion of Logic (1.7) and Proofs (1.7) as useful. Fourteen modules have an average rounded to 1.1 (e.g., Topological spaces, Complex integration, Numerical analysis, Numerical solution of ODE, Inner product). We note that these results fit very well with the results of our OBM-AMK mapping, providing us with a model of advanced mathematics content that does not seem to have direct, explicit connections to the teaching of O-level mathematics.

The ongoing analysis of the interviews leans in the same direction. For instance, when asked to estimate the percentage of their AMK training that they see as useful for their practice, participants provided estimations of around 20%, sometimes up to 30%. They complained that certain topics are necessary to some degree, but not to the extent they are covered at university. For instance, a participant (P<sub>6</sub>) in the second focus group (I<sub>2</sub>) stated:

I<sub>2</sub>P<sub>6</sub>: I can say for extent is just a small extent that what I studied there and here to say that it has helped me to a very small extent. For example, topics like probability are just useful university introduction but the rest of the content does not relate, even sets, it is just an introduction, other things are tough.

However, we note that the teachers see some value in what mathematics is. For instance, when talking about linear programming, some participants highlighted terminology and ways of arranging information to solve a problem (*Using intuition and formalism* in Figure 1). The participants also mentioned the deductive structure of mathematics (*Structured deductively* in Figure 1), but it would appear that if this is overemphasised, it can become a barrier in the training of O-level teachers (for instance, Logic is one of the modules cited as not being useful, or with “only an introduction” being necessary). In connection to one conjecture we formulated in the Introduction, some participants see the value of certain AMK content for teaching in the A-level, but not in the O-level:

I<sub>2</sub>P<sub>2</sub>: Yes, there are some topics like the concept of differentiation of functions; in O-level, not applicable. Maybe in Advanced level mathematics, you can use it. A concept like [...] integration of a function of a single variable you cannot use it in O-level but you can apply it in Advanced [level].

Finally, we note that the participants also complained about the content they have to teach for which they were not adequately prepared:

- I<sub>2</sub>P<sub>5</sub>: Let me begin with Form 2 topics. There is a topic like radical and exponent, but it is not in [AMK courses]; logarithm, quadratic equation of university and O-level is completely different [...] Congruence and similarity also are not in [AMK courses].
- I<sub>3</sub>P<sub>1</sub>: One of the topics is about congruence and similarity. That one is in Form 2, this topic is not taught in the university but we have to face them in O-level. Some topics about circles and many topics about geometry, like area, volumes, three dimensions... Three dimensions, you find it in vectors but not to such extent.

We plan to finalise the analysis of the interviews; however, the three interviews we have analysed indicate that participants value a part of their AMK training, while being very critical of large portions of it, to the point that they consider these parts as completely useless. We believe that the dissatisfaction with the received mathematical training may explain the fact that our participants seem to be more critical about what mathematics is, compared to participants in other studies.

## Discussion

The study presented here aims to add layers to the existing literature on the contribution of advanced mathematics courses to the training of secondary teachers, by considering contexts less present in this literature, such as the African context. In particular, in the Tanzanian context, secondary teachers (both for O- and A-levels) complete a significant number of such courses (approximately 30% of their courses).

In considering the O-level, our mapping of OBM content and AMK modules shows that only  $\approx 18\%$  of OBM units are explicitly addressed in the AMK courses; additionally, 11 out of the 38 OBM topics ( $\approx 29\%$ ) are not addressed at all. This result is coherent with the data obtained from the questionnaires: of the 48 AMK modules, only nine (18.75%) received an average rating above 2. The participants seem to agree that their profession requires specific mathematical training but that the received AMK content does not address all their needs. We note that, in spite of all the advanced content they have been exposed to, some teachers seem to believe that this does not give them a firm grasp of the content they have to teach. Finally, the data from the interviews also seem to confirm our model of OBM-AMK mapping; teachers estimate that approximately 20% to 30% of the AMK content they received in their training is useful for their practice. As predicted by our model, and as seen in the questionnaires, interview participants pinpointed a number of OBM topics that they have to teach and for which they do not feel adequately prepared. We believe that this overall dissatisfaction may explain why we are seeing fewer indicators of teachers' knowledge of what mathematics is. So far, *Structured deductively*, *Thinking and understanding*, and *Using intuition and formalism* are the mentioned topics in our data, albeit cited with some precaution from the participants.

We believe that our results help us to identify significant gaps in teacher training in Tanzania. This could be valid for other regions of Africa, which take similar approaches to training secondary mathematics teachers. It is possible that, on other continents, similar results can be found in countries with a high number of AMK courses in their teacher training programmes. We plan to continue with our analysis in order to establish a complete map of the content identified by teachers as useful and not useful, and incorporate it into our OBM-AMK grid. We also plan to continue identifying which elements of the content taught to teachers develop their knowledge of what mathematics is, despite the fact that they consider much of the content as not useful. Our conjecture is that teachers' appreciation of what mathematics is can be connected to their perceived value of their mathematical



training. This will be the focus of future publications, and we hope that this work will help improve the teacher training curriculum in Tanzania. The results call for AMK curriculum reforms in Tanzania for improved OBM teaching.

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