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# Evolution of mathematics courses in engineering: new syllabus, same activity?

Ignasi Florensa<sup>1</sup>, Iria Fraga<sup>1</sup>, Kristina Markulin<sup>1</sup>, Víctor Martínez-Junza<sup>1</sup> and Noemí Ruiz-Munzón<sup>2</sup>

<sup>1</sup>Escola Universitària Salesiana de Sarrià-Uni. Autònoma de Barcelona, Spain; [kmarkulin@euss.cat](mailto:kmarkulin@euss.cat)

<sup>2</sup>TecnoCampus Mataró-Maresme, Univ. Pompeu Fabra, Barcelona, Spain

*The Bologna Process and in particular the establishment of the European Higher Education Area (EHEA) in 2008 represented a turning point in the design of the syllabus in engineering programs. One of the major changes was the incorporation of the official conception of "knowledge to be taught" in terms of competences. However, these changes in the way how engineering programs are conceived clash with the results of several analyses which seem to indicate that the existing mathematical activity in engineering mathematics education has changed very little in recent decades. We present a study of mathematical activity in the evaluation exams in four different engineering syllabi from 1994 to 2020 in a Spanish engineering school. The results indicate that even if the way knowledge to be taught is presented evolves significantly, the actual mathematical activity remains very stable.*

*Keywords: Anthropological theory of the didactic, program design, praxeological analysis.*

## Introduction

The Bologna Process initiated in 1999 culminated in 2010 with the launch of the European Higher Education Area (European Higher Education Area, 2023). One of the major changes that the EHEA brought to the higher education institutions in Europe was the adoption of competence-based programs, even if there was not a clear consensus on the competence definition (Davies, 2017).

The engineering programs in Spain are not an exception and the programs experienced a major evolution in 2009. The engineering degrees that used to be 3- or 5-year programs adopted a 4+2 model (bachelor's and master's degree) and all the programs adopted a competence-based syllabus, in which the knowledge to be taught was not anymore conceived in terms of works to be taught but in a set of competences and learning outcomes.

In contrast to these changes, several research works in the field of the mathematical education in engineering degrees highlight the invariance of the mathematical activity existing in higher education institutions. For example, Bosch et al. (2021) point out the extreme invariability of the undergraduate programs in pure mathematics in different countries, highlighting the difficulty to modify the way knowledge is conceived. Another recent work on the field was presented in the last INDRUM4 conference by Cuenca et al. (2022). The authors present the analysis of a major curricular reform in engineering programs in a higher education institution in Ecuador. The results indicate that mathematical knowledge to be taught remained unchanged even if several courses were created and some others disappeared: the specific contents moved from one course to another but with no significant changes in the overall program.

Chevallard (2015) and other researchers in mathematics education have described a didactic paradigm shift: from the old paradigm of "visiting works" based on the study of a set of well-established works

to be studied towards a new counter-paradigm of “questioning the world” where the study of relevant questions becomes the core activity. Even if the implementation of competence-based programs would seem to promote this paradigm shift, the results of previous studies indicate the difficulty to make changes on the prevailing epistemological conception of knowledge at school institutions.

We present a study of the evolution of the knowledge conception in the mathematical courses in an engineering school in Spain to evaluate to what extent the changes promoted by the EHEA have provoked an evolution of knowledge to be taught in the sense of the aforementioned paradigm shift. We analyse the syllabi from programs of 1994, 1999 (both pre-EHEA) and from 2009 and 2020. As our empirical material, we use both the official study guides and the exams of each.

## **Theoretical framework**

Our research is framed within the Anthropological Theory of the Didactic (ATD). Its institutional and epistemological framework is widely used in research on mathematics education for non-specialists, as evidenced by numerous presentations at the TWG14 of the CERME12 and the majority of research papers presented at the TWG4 of the recent INDRUM4 conference on the teaching and learning of mathematics for engineers and other disciplines.

The proposal of the ATD to model knowledge is the so-called praxeological model. It argues that all human activities, particularly those related to knowledge generation, transmission, and dissemination, can be modelled in terms of praxeologies (Chevallard, 1994). A praxeology is comprised of two blocks – praxis and logos – each with two elements, and the quadruplet  $[T/\tau/\theta/\Theta]$  provides a fundamental framework for modelling knowledge. The ATD views all human activities as a series of tasks, where the definition of a task is broad. For example, solving a linear equation is one type of task, but so is cleaning a car or doing the dishes. Each type of task (T) is conducted through a specific technique ( $\tau$ ), which is institutionally dependent. The discourse surrounding the adequacy and correctness of a technique is referred to as the technology ( $\theta$ ), and the fourth component, theory ( $\Theta$ ), consists of more abstract and general statements with strong justifying and generating power. Consequently, in order to fully analyse the evolution of mathematical courses in engineering degrees within the ATD framework, it is necessary to characterise the programs in terms of praxeologies.

A second theoretical development of the ATD that is relevant in our study is the scale of levels of didactic co-determinacy (Chevallard, 2002). The higher levels of the scale correspond to the conditions and constraints related to the general way of organising teaching and learning processes. In other words, the higher levels do not consider changes in the content at stake, but only in the pedagogical level. In contrast, the lower levels of the scale involve explicitly the way how the content at stake is conceived in school institutions. This tool allows us to study at what levels are placed the changes promoted by the evolution of the different programs. In Cuenca et al. (2022), the changes promoted by a program reform remained only in the upper levels of the scale.

## **Context and methods**

Our institutional setting is an engineering school in Spain that offers a variety of engineering degrees, including Automation and Industrial Electronic Engineering, Electrical Engineering, Mechanical Engineering, Industrial Organisation Engineering, Engineering in Renewable Energies and Energy

Efficiency, and Automotive Engineering. All these degrees share common mathematics courses taught during the first two years of the program. Over the 27-year history of the school, four different syllabi have been implemented (1994, 1999, 2009 and 2020), with the last two being competence-based.

To understand the evolution of mathematical knowledge and the impact of competence-based programs, we have analysed two main sources of empirical data. Firstly, we have analysed the course guides of each mathematics course across the different syllabi. Secondly, we have performed a praxeological analysis of exams for each course. The courses we have studied are Mathematics, Calculus, Linear Algebra, and Mathematical Methods (Table 1). Though it appears on all syllabi, the statistics course has been left out of our analysis since it is not the focus of our current research.

**Table 1: Organisation of mathematical courses in different syllabi**

Syllabus year	Course	Degree year	Semester	Course hours/week	total	Academic year of the exam
1994	Linear Algebra	1 <sup>st</sup>	1	3	135	1995-96
	Calculus	1 <sup>st</sup>	2	3		1994-95
	Mathematics	2 <sup>nd</sup>	1	3		1995-96
1999	Mathematics	1 <sup>st</sup>	1	4	180	2001-02
	Calculus	1 <sup>st</sup>	2	4		1999-00
	Mathematical Methods	2 <sup>nd</sup>	1	4		2001-02
2009	Mathematics	1 <sup>st</sup>	1	4	150	2009-10
	Calculus	1 <sup>st</sup>	2	6		2009-10
2020	Mathematics	1 <sup>st</sup>	1	5	150	2022-23
	Calculus	1 <sup>st</sup>	2	5		2021-22

The 1994 and 1999 programs described the syllabi using "knowledge labels", such as "Rolle Theorem" or "Homogeneous Differential First Order Equations". In contrast, the 2009 and 2020 programs describe it in terms of competencies, such as "performing operations with complex numbers and solving algebraic equations". According to the didactic transposition theory (Chevallard, 2002) our aim is to characterise the "knowledge to be taught" and its evolution. Moreover, using the didactic co-determinacy we can detect on what levels of the scale did the changes in the "knowledge to be taught" occur, and whether those changes coincided with the introduction of competence-based programs.

We conducted a praxeological analysis of exams from each course to examine the types and subtypes of tasks. The analysis did not consider the techniques or the logos block as their identification required teacher interviews to determine the expected or accepted techniques in each exam. We generated a dataset and coded blocks of contents, types of tasks (e.g., "To calculate eigenvalues of a matrix") and types of subtasks (e.g., "To factorise the characteristic polynomial"). We used the statistical program R for quantitative analysis, and official study guides for qualitative analysis.

### **Evolution of syllabus: describing knowledge to be taught**

The main difference among the description of the different course contents in the different programs is, as previously said, the competence-based model adopted in the syllabi from 2009 and 2020. In the programs from 1994 and 1999 the contents of each course were just a list of notions (see an example for differential equations in Table 2).

**Table 2: Content of the differential equations in the 1999 program (in the Mathematics course)**

<b>4. Differential equations.</b>
<i>4.1 Derivatives. Notion of derivative of a real function, geometric interpretation. Calculation of derivatives.</i>
<i>4.2 Notion of primitive. Calculation of primitives.</i>
<i>4.3 First-order differential equations.</i>
<i>4.4 Linear differential equations. Determination of the solutions.</i>
<i>4.5 Linear differential equations with constant coefficients, resolution of homogeneous cases and particular solution.</i>

In contrast, in the 2009 program the knowledge related to differential equations was described with the competences and learning outcomes that can be seen in Table 3.

**Table 3: Competence and learning outcome related to differential equations in the 2009 program (included in the Mathematics course)**

<b>Competence 01:</b> Ability to solve mathematical problems in engineering.
<i>Learning outcome 01.03 Apply some methods of resolution of ordinary differential equations and equations with partial derivatives.</i>

Another relevant example of this difference on the official description of content to be taught is that involving the domain of functions of one real variable (see Table 4 and 5).

**Table 4: Content description of the derivatives in the 1999 program (included in the Calculus course)**

<b>1. Functions of one real variable</b>
<i>1.1 Continuity, Bolzano's theorem, boundedness theorem and maximum principle</i>
<i>1.2 Derivability: Rolle's theorem, mean value theorem</i>
<i>1.2.1 L'Hôpital rule</i>
<i>1.2.2 Determination of relative extrema</i>
<i>1.2.3 Concavity, convexity, and graph of a function</i>
<i>1.2.4 Approximating Roots of a Function: Newton's Method</i>

**Table 5: Competences and learning outcomes related to derivatives and limits in the 2009 program (included in the Calculus course)**

<b>Competence 01:</b> Ability to solve mathematical problems in engineering.
<i>Learning outcome 01.03 Calculate extrema of functions and integral functions of a variable and be able to solve problems that involve the use of integrals (lengths, areas, volumes, etc.).</i>
<b>Competence 02:</b> Ability to apply an acquired knowledge of: linear algebra, geometry, differential geometry, differential and integral calculus, differential equations and partial derivatives, numerical methods, numerical algorithms, statistical and optimisation methods.
<i>Learning outcome 02.04 To be able to analyse and sketch the graph of functions of one single variable.</i>
<i>Learning outcome 02.05 To be able to deduce properties of a function of a single variable from its graph.</i>
<i>Learning outcome 02.06 To be able to work intuitively, geometrically, and formally with the notions of limit, derivative and integral.</i>

These examples illustrate to different extents how the incorporation of competences changed the official description of the knowledge to be taught. In both competence-based syllabus, the same two competences describe the mathematical courses: the competences conceive mathematical knowledge

as a tool to be applied to engineering problems. The full set of contents and learning outcomes can be found in the complementary material available at <https://rb.gy/mflfnu>.

In contrast to the applicationist conception of mathematical knowledge that is very explicit in the two competences, the conception of mathematical knowledge at the level of the learning outcomes (see Table 3 and 5) is more diverse. Regarding the evolution in the field of differential equations (Tables 3 and 4), the change is very important. It is evolving from a detailed succession of contents and tasks to a single learning outcome concerning the resolution of differential equations under the general competence “solve mathematical problems in engineering”. In the field of derivatives and limits the change is still relevant, especially regarding the learning outcome 01.03. However, the learning outcomes 02.04, 02.05 and 02.06 keep the sketch of a graph of a function as one of the main *raison d’être* of differential calculus in the undergraduate mathematics for non-specialists as already indicated by Fonseca et al. (2014). Another aspect that emerges from the syllabi analysis is the progressive disappearance of specific domains: Linear Operators (LO) disappear in the 1999 reform while Base of Vector Spaces (BV) and Analytic Geometry (AG) disappear in the change of 2009.

### Analysis of the mathematical activity in the exams

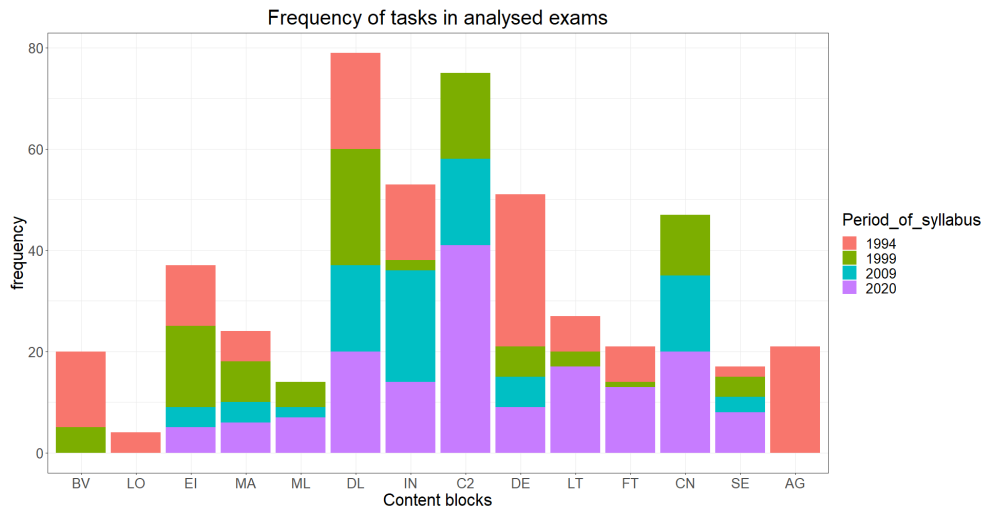
The mathematical activity is extracted from the sample of randomly chosen final exams from each subject at stake in different syllabi. Having coded all the types of tasks and deconstructing them into implied subtasks, not going into the expected techniques and the supporting logos, we grouped the tasks in bigger content groups in order to track the evolution of their implementations in different syllabi. Table 6 summarises the content groups and their rough aggregation in the most represented mathematical domains.

**Table 6: Overview of the content codification used in the analysis of the exams**

Domain	Linear Algebra					Calculus						Complex Numbers	Series	Geometry
Content code	BV	LO	EI	MA	ML	DL	IN	C2	DE	FT	LT	CN	SE	AG
Content	Base of vector spaces	Linear operator	Eigenvalues & Eigenvectors	Matrices	Matrices & Linear Equations	Derivatives & Limits	Integrals	Calculus in two variables	Differential Equations	Fourier Transform	Laplace Transform	Complex Numbers	Series	Analytic Geometry

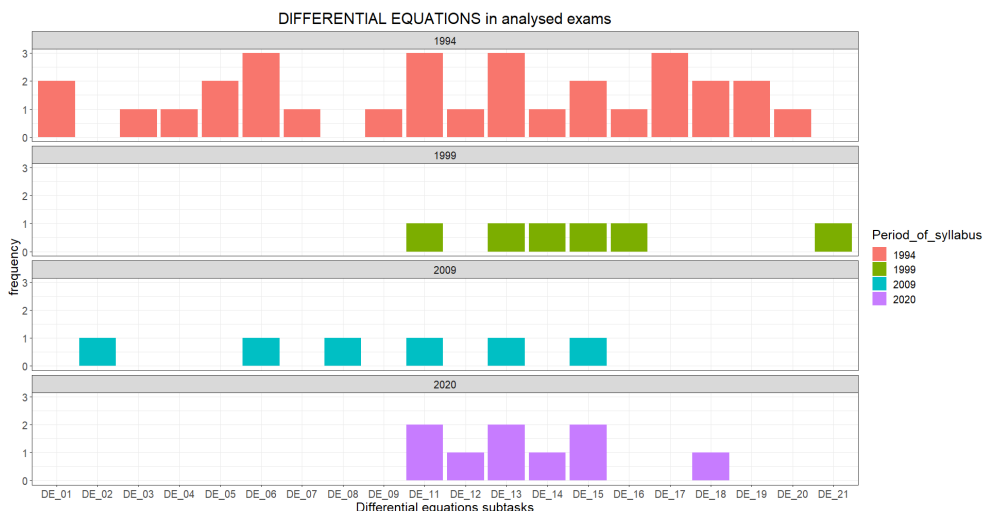
The first overview of the frequencies of detected subtasks per content groups (Figure 2) shows the predominance of Calculus related types of tasks, especially limits, derivatives, integrals and their specificities in two-variable calculus.

The results of this first analysis validate that in the institution of study, Calculus related competences are dominant which is coherent with the prevailing epistemology of other engineering schools (Romo-Vasquez, 2009). The graph also reveals the disappearance of BV, AG and LO as a consequence of its elimination in the 1999 and 2009 reforms.



**Figure 2: Overview of the frequencies of types of subtasks grouped per content blocks and their proportion over the syllabi**

Regarding the analysis of the evolution of content blocks, Figure 3 shows differential equations' subtasks that remain in the current syllabus. There is a clear reduction in the diversity of types of tasks involving differential equations. The remaining subtasks in the exams are to identify the characteristic polynomial; to solve a characteristic polynomial; to determine the solution; to apply the initial conditions; to determine a particular solution; and to solve by separation of variables. These types of tasks are the ones related to differential equations that can be more "algorithmised".

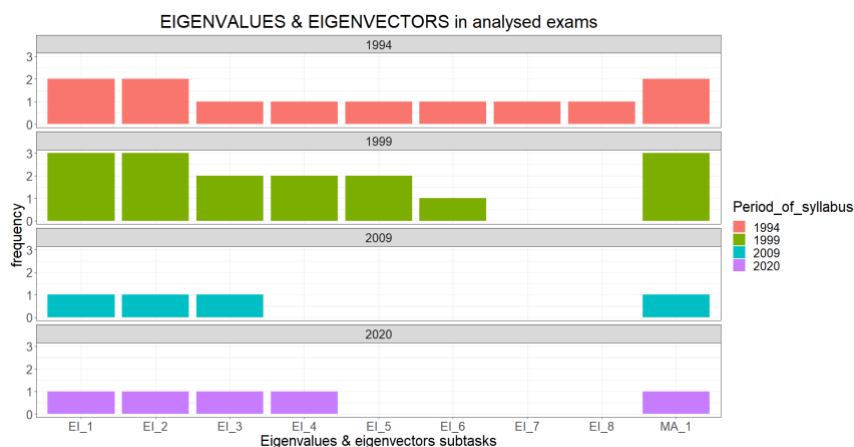


**Figure 3: Overview of the types of subtasks related to differential equations**

Another symptom of evolution of the types of tasks is the one related to linear algebra. In Figure 2, the blocks of bases of vector space and linear operators completely disappear from the competence-based syllabi from 2009 and 2020, while tasks that explicitly require calculation of eigenvalues and eigenvectors, and essential operations with matrices and their equivalence to linear equations gain more prominence. A symptom of the change can be appreciated in Figure 4.

The remaining types of subtasks are the most "algorithmisable" ones that can be exercised based on the minimum of theoretical knowledge about linear operators and eigentheory. Consequently, a shift

from insistence on the theoretical part of the pre-Bologna exams into almost complete vanishment of the theory on vector spaces and linear operators is one of the most important results of our analysis.



**Figure 4: Overview of the types of subtasks related to eigenvalues and eigenvectors**

Finally, the domain of complex numbers is the only one where new subtasks appear in contrast to the disappearance or reduction observed in the other domains. As shown in Figure 2, in the exams corresponding to the first syllabi from 1994, there were no tasks about complex numbers. Nevertheless, since 1999, they are steadily present in the exams and with each syllabi period, the variety of subtasks they comprise increases. This is due to the progressive movement of the complex numbers from upper-secondary institutions to universities. More information about our analysis of the evolution of mathematical content at stake is available at <https://rb.gy/mflfnu>.

## Discussion, limitations and future work

Our analysis of the evolution of the prevailing epistemology in an engineering school and to what extent the launch of the EHEA aligns with the transition to the paradigm of questioning the world reveals that the conception of the mathematical activity remains very stable. The important changes in the description of the syllabus that can be placed at the level of pedagogy-school has little or no significant impact on the existing mathematical activity, placed in the lower levels of the scale of levels of didactic codetermination. In other words, the design and implementation of new study programs intending to modify the prevailing epistemology should, at least, consider an explicit and structured description of the mathematical activity involving the lower levels of the scale.

More specifically, we observe a reduction in the number of types of tasks involving aspects such as proof and justification of mathematical properties and an increase of the type of tasks related to the training of algorithmisable techniques, such as determining the eigenvalues/vectors of a matrix or sketching the graph of a function. This illustrates that the teaching model in the institution has moved from a theoretical to a more technical approach in the sense of Gascón (2001). However, the introduction of the competence-base model does not promote, in the institution of study, new types of tasks related to inquiry, modelling and engineering problem solving.

Multiple limitations exist in the presented research. Firstly, the analysis of only one exam from each syllabus may not be sufficiently representative of activity involved in the course. Secondly, the



definition of the content blocks could have hindered the analysis from identifying types of tasks that appear in different domains. Finally, our work should be enriched with data from other institutions, to identify to what extent this is a generalisable phenomenon.

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