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Posing and solving problems about the functions of two variables

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This study explored the problems posed by students on the functions of two variables. The participants of the study consisted of 70 university students. All students engaged with the same four problem-posing tasks, with half of the students also being asked to solve the problems they posed. Findings revealed that the students who posed and solved the problems had fewer errors with the problems compared to the students who only posed problems. The students who solved the problems while generating could pose solvable problems more than the other students. The errors students made during the problem-posing process caused them to pose unsolvable and unsuitable problems. Findings also revealed that the content of the tasks affected the context of the posed problems.

Keywords: Problem-posing, problem-solving, functions of two variables, tertiary level.

Introduction

Problem-posing has been identified as a significant learning goal for students in most countries (Li et al., 2022), and there is an attempt to integrate problem-posing in school mathematics at various levels of education (Zhang et al., 2022). Problem posing can help the development of mathematical and conceptual understanding (Brown & Walter, 2005; English, 1998) and mathematical creativity (Bonotto & Dal Santo, 2015). Even though problem-posing is crucial and valuable in the learning and teaching process, the number of studies conducted at the tertiary level is very limited compared to studies conducted at the primary and high school levels (Ergene, 2022; Nedaei et al., 2022). In addition, the number of studies on the integration of problem posing into calculus courses is relatively low (Ergene & Çaylan Ergene, 2023; Nedaei et al., 2022; Perrin, 2007). Problem-posing tasks were powerful in uncovering students' knowledge about calculus concepts such as function, series and integrals (Ergene & Çaylan Ergene, 2023; Nedaei et al., 2022; Parhizgar et al., 2022).

Function as an essential concept of calculus has a vital role in the development of mathematical concepts (NCTM, 2000). The idea of function is closely related to the other calculus concepts, including limit, derivative and integral, and helps to understand areas such as algebra and geometry better. Despite the importance of function, many students do not understand this concept clearly. In particular, students' understanding of the function of two variables is related to their existing schemas and flexibility to use different representations (Trigueros & Martinez-Planell, 2010). For instance, most students encounter problems while identifying the domain and range of a function of two variables (Martínez-Planell & Trigueros Gaisman, 2012). Even though there is an interest in integrating problem-posing activities in lessons, there needs to be more understanding of problem-posing processes at the tertiary level, particularly in calculus courses. Furthermore, there is a close relationship between problem-posing and problem-solving. That is, problem-solving supports the problem-posing process (Xie & Masingila, 2017). Thus, this study examined students' problem-posing and solving about functions of two variables at the tertiary level. In addition, it also explored how thinking about the solution of problems influenced posed problems. The following research question guided the study: What are the characteristics of the problems posed by the posing-only

group and posing-and-solving group students regarding the functions of two variables? The findings will help researchers and instructors by providing insight into students' understanding.

Problem posing as a conceptual framework

There are several frameworks for problem-posing design (e.g., Christou et al., 2005; Stoyanova, 1998) or analysis of problem-posing skills (e.g., Cankoy & Özder, 2017; Silver & Cai, 1996). In this study, regarding the nature of the functions of two variables, the framework developed by Christou et al. (2005) was used to design the tasks. Encountering a problem-posing task leads to organising information based on the current situation. Therefore, a problem-posing situation was categorised into four cognitive processes: editing, selecting, comprehending and translating (Christou et al., 2005). In *editing quantitative information*, students are expected to generate a problem based on the information, prompts or stories in the given task without restriction (Mamona-Downs, 1993). *Selecting quantitative information* includes tasks in which students generate a problem for the specific answer given. Here, the answer acts as a constraint and makes the process challenging, as students must consider mainly the structural context (English, 1998). In *comprehending quantitative information*, students pose a problem using the mathematical equation or operation provided in the given task. Hence, it is necessary to understand the meaning of the calculations, and students often focus on the operational structure of the problem rather than the semantic structure (English, 1998; Silver & Cai, 1996). In *translating quantitative information*, students generate a problem using tables, graphs or diagrams provided in the given task. The present study used this framework because these four processes represent different modes of students' thinking while posing problems (Christou et al., 2005). However, problem-posing instructions can be carried out using one of the four approaches (editing, selecting, comprehending, translating), and one of these approaches can be the most suitable depending on the nature of the instruction.

While designing tasks based on this framework, each of the mentioned four cognitive processes was associated with one of the following in this study: a domain and range of functions of two variables, algebraic representation of functions, a result of functions, and real-life situations. The framework proposed by Cankoy and Özder (2017) was utilised to analyse the problems posed by students, and the problems were analysed based on suitability, solvability and contexts of the problems. Suitability refers to students' understanding of the requirements (explicit and implicit) of a problem-posing task in a given context. Solvability means that posed problems have solutions (Cankoy & Özder, 2017). Context is "a situation or event in the task" (Vos, 2020, p. 36). Detailed information will be provided in the methods section.

Methods

The qualitative research method was used to explore the posed problems on the functions of two variables, and the case study was the research design (Yin, 1994). The units of analysis were the problems with solutions and problems without solutions.

Participants

Seventy second-year students in a four-year bachelor's degree mathematics education program in one of the state universities in Turkey were the participants of the study. These participants were aged 19-

20 years and consisted of 28 males and 42 females. Before the study, they had already completed the Calculus course and learned the functions of two variables as a part of the course. Problem-posing tasks were intended to support the students' learning of university mathematics rather than to prepare them for their future teaching. At the time of the study, the students had not yet taken any mathematics education courses, as such courses are positioned in the latter half of the program. They did not take a course related to problem-posing in the program and did not have much experience with problem-posing prior to the study. They took part in the study voluntarily. The students were randomly divided into two groups, and the implementation was conducted in two groups of 35 participants.

Data collection procedure

Four tasks about the functions of two variables were designed: one for each cognitive process in Christou et al.'s (2005) framework. Four different domains and ranges were given for the translating process, and the students were asked to pose a problem for any of them. An algebraic expression was given for the comprehending process, and the students were asked to pose a problem by using it. For the selecting process, the students were asked to pose a problem using the given result. For the editing process, a scenario was presented, and the students were asked to pose a problem based on this scenario. Table 1 shows the content of the tasks designed for each process.

Table 1: The content of the tasks

Translating Process			Comprehending Process	Selecting Process
Pose a problem by choosing one of the following functions.			Pose a problem by using the algebraic expression:	Pose a problem about a function of two variables whose result is 10.
Function	Domain	Range	$3x + 4y - xy$	
$A(x, y)$	Entire Plane	\mathbb{R}		
$B(x, y)$	$y \geq x^2$	$[0, \infty]$		
$C(x, y)$	Entire Plane	$[-1, 1]$		
$D(x, y)$	$xy \neq 0$	$(-\infty, 0) \cup (0, \infty)$		
Editing Process				
Pose a problem for functions of two variables using the following scenario: "A carpenter produces x products A and y products B in a month. The selling price of product A is 500 TL, and the selling price of product B is 200 TL.				

The researchers and the students came together to implement the tasks outside of class at the university. The students were all in a room together, but they worked individually. The students were presented with the designed tasks at the same time and asked to pose a problem for each of the four tasks, i.e., four independent problems. In this way, the students had an opportunity to start from the task of their choice and move back. While one of the groups was asked to pose a problem and solve the problem they posed for each task, the other group was asked to pose a problem. A specific time was not provided for posing problems, but the posing-only group completed about one hour, whereas the posing-and-solving group completed 90 minutes.

Data analysis

The problems posed by the students were analysed in three stages. In the first stage, the problems were identified as either "suitable and solvable" or not. The problems that included proper variables and were based on the tasks were categorised as suitable. The problems that did not meet at least one of these requirements were categorised as unsuitable. For solvability, solutions to posed problems were examined. In the second stage, the context of the "suitable and solvable" problems was

categorised. Context is a classification of suitable and solvable problems based on Vos's (2020) categorisation: a bare context [BC], a mathematical context [MC], a dressed-up context [DC], a realistic context [RC] and an authentic context [AC]. In this study, since the students did not pose any problems in an authentic context, the contexts of the problems were identified as BC, MC, DC and RC. Examples of posed problems are provided below.

$$D(x, y) = \frac{x^2 + 3xy + y^2}{x^2 + y^2}$$

Where is D(x,y) continuous? A technology company sold x chargers for 3 euros and y headphones for 4 euros in one day. This company also spent a total of xy euros on this day. What is the maximum amount of money this company receives daily? (An example of a posed problem in BC) (An example of a posed problem in RC)

In the third stage, using an open coding method, the reasons for the unsolvable and unsuitable problems were identified, and the number of errors was determined. The posed problems were analysed by the researchers individually, and then the codes were compared, and discrepancies were discussed to arrive at a consensus. The inter-coder agreement between the researchers was 93.5%.

Findings

The findings are presented in two parts: findings for the posed problems for the cognitive processes with the contexts of the problems and findings for unsolvable and unsuitable problems and students' errors.

Problems posed for the cognitive processes

While 100 of 140 problems posed by the posing-only group for all processes were solvable, 113 of 140 problems were solvable in the posing-and-solving group in total. The findings for the problems posed by the students for the translating process are given in Table 2.

Table 2: Findings related to the problems posed for the translating process

Suitability	Solvability		posing-only	posing-and-solving	Total
Suitable	Solvable	BC	8 (23%)	17 (49%)	25 (36%)
		MC	4 (11%)	5 (14%)	9 (13%)
		DC	5 (14%)	5 (14%)	10 (14%)
		RC	6 (17%)	3 (9%)	9 (13%)
	Unsolvable		1 (3%)	0 (0%)	1 (1%)
Unsuitable			11 (32%)	5 (14%)	16 (23%)
Total			35 (100%)	35 (100%)	70 (100%)

In the translating process, the number of suitable problems posed by the posing-and-solving group (n = 30) was higher than that of the suitable problems posed by the posing-only group (n = 24) (Table 2). Furthermore, problems with a bare context were more common in the posing-and-solving group compared to the posing-only group.

Table 3: Findings related to the problems posed for the comprehending process

Suitability	Solvability		posing-only	posing-and-solving	Total
Suitable	Solvable	BC	7 (20%)	11 (32%)	18 (26%)
		MC	1 (3%)	4 (11%)	5 (7%)
		DC	7 (20%)	5 (14%)	12 (17%)
		RC	11 (31%)	7 (20%)	18 (26%)

	Unsolvable	1 (3%)	0 (0%)	1 (1%)
Unsuitable		8 (23%)	8 (23%)	16 (23%)
Total		35 (100%)	35 (100%)	70 (100%)

An equal number of suitable problems ($n = 27$) was posed in both groups in the comprehending process (Table 3). In addition, as in the translating process, the number of problems with a bare context generated by the posing-and-solving group ($n = 11$) was higher than that of the problems posed by the posing-only group ($n = 7$). On the other hand, the posing-only group posed more problems with a realistic context ($n = 11$) compared to the posing-and-solving group ($n = 7$).

Table 4: Findings related to the problems posed for the editing process

Suitability	Solvability		posing-only	posing-and-solving	Total
Suitable	Solvable	BC	0 (0%)	1 (3%)	1 (1%)
		MC	0 (0%)	0 (0%)	0 (0%)
		DC	6 (17%)	5 (14%)	11 (16%)
		RC	18 (52%)	21 (60%)	39 (56%)
	Unsolvable	4 (11%)	0 (0%)	4 (6%)	
Unsuitable		7 (20%)	8 (23%)	15 (21%)	
Total		35 (100%)	35 (100%)	70 (100%)	

The number of suitable problems posed in both groups was close in the editing process (Table 4). Moreover, the number of problems with a realistic context in the editing process was higher in both groups compared to the other processes.

Table 5: Findings related to the problems posed for the selecting process

Suitability	Solvability		posing-only	posing-and-solving	Total
Suitable	Solvable	BC	7 (20%)	7 (20%)	14 (20%)
		MC	4 (11%)	13 (37%)	17 (24%)
		DC	7 (20%)	1 (3%)	8 (12%)
		RC	9 (26%)	8 (23%)	17 (24%)
	Unsolvable	1 (3%)	0 (0%)	1 (1%)	
Unsuitable		7 (20%)	6 (17%)	13 (19%)	
Total		35 (100%)	35 (100%)	70 (100%)	

In the selecting process, the number of suitable problems in both groups was close, as in the editing process (Table 5). Additionally, the number of mathematical context problems posed by the posing-and-solving group ($n = 13$) was higher than that of the problems with the same context posed by the posing-only group ($n = 4$).

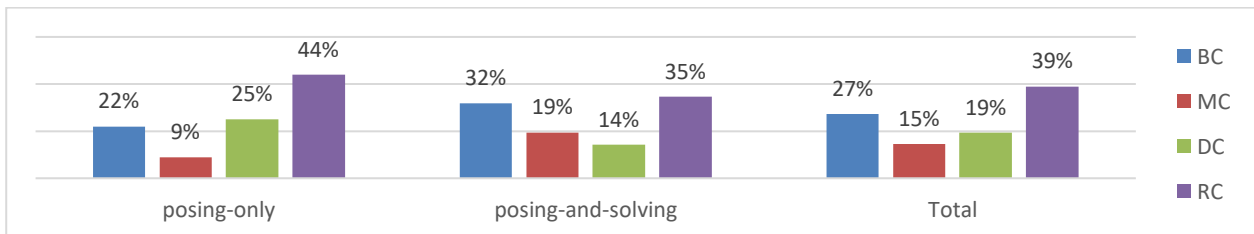


Figure 1: The contexts of the solvable problems

Overall, the number of RC problems was the highest in both groups (Figure 1). A higher number of problems were posed with BC and MC in the posing-and-solving group compared to the posing-only

group. On the other hand, the number of the posed problems with RC and DC was higher in the posing-only group than that of problems in the posing-and-solving group.

Unsolvable and unsuitable problems

Among the unsolvable and unsuitable problems, there were some common errors (Table 6).

Table 6: The errors with frequencies

	Errors	posing-only	posing-and-solving	Total
Based on the functions of two variables	Ignoring domain	7	2	9
	Incorrect use of x-y variable	11	2	13
Based on the way of problem posing	Missing information	12	3	15
No response		20	10	30
	Total	50	17	67

The errors were categorised into two themes: errors that arose from the functions of two variables and errors that emerged from the way of problem posing. While 30 of these errors were identified in the posing-only group, only seven errors were observed in the posing-and-solving group. All the unsolvable problems in the posing-only group were due to needing more information, i.e., missing information. Among 280 problems that are expected to be posed by the students, 30 problems could not be posed (10 of these came from the posing-and-solving group, and 20 came from the posing-only group).

Errors related to the functions involved posing a problem using one (only x) or three (x-y-z) variables or posing a problem without including a function. Moreover, the students posed problems in which functions were unsuitable for the given domain. Errors related to the way of problem posing included failing to use the information provided in the task, such as posing a problem whose answer is not 10 for the selecting process or posing a problem without using the function $3x+4y-xy$ for the comprehending process.

Discussion and conclusion

The findings of the study revealed that the posing-and-solving group students posed more solvable problems than the posing-only group students. At this point, solving the problems during the problem-posing process might have contributed to problem-posing (Xie & Masingila, 2017). Fewer students in the posing-and-solving group did not respond to the tasks compared to the posing-only group, which may indicate that thinking about the solution to the problem deepens the understanding of mathematical problems (Erkan & Kar, 2022). For an in-depth exploration of this situation, interviews with students can be conducted. In addition, considering the solution during the problem-posing process may have encouraged students to pose problems with BC or MC. Adding the idea of posing a problem that can be solved may have directed the students to pose routine problems using mathematical formulas and rules found in mathematics textbooks (Nedaei et al., 2022).

Posing problems with DC and RC mostly in the editing process and posing problems with BC and MC in the translating and selecting processes can be related to the content of the tasks provided for these processes (Christou et al., 2005; Ergene, 2022; Ergene & Çaylan Ergene, 2023). Furthermore, students experience difficulties in understanding the functions of two variables and feel challenged

while posing a problem. In the present study, the students made errors, such as the incorrect use of the variables, which was consistent with several research studies (Trigueros & Martínez-Planell, 2010). When the errors that emerged in the problem-posing process were considered, the errors made by the students in identifying the domain and range of a function of two variables (Martínez-Planell & Trigueros Gaisman, 2012) reflected on the posed problems. In this regard, problem posing can be used as a method both in calculus courses and in the teaching process of functions of two variables. For further research studies, a comparison of the patterns of errors across two groups according to the processes is recommended. Moreover, specific tasks can be designed to reveal students' errors on different concepts, and interventions can be implemented to eliminate these errors.

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