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# University students' understanding of exponential and logarithmic concepts: in case of real-world situations

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*Understanding a mathematical concept in real-world situations is of practical importance. One potential way to help students learn mathematical concepts in real-world situations would be to engage them with contextual problems. Most students have a procedural understanding of exponential and logarithmic concepts and their interpretation in real-world situations has less attention. We examined students' understanding in designing contextual problems for exponential and logarithmic statements. The process included an initial interview, implementing a set of activities, and an exit interview. The results of the initial interview showed that the students did not have any understanding of these concepts in real-world situations. However, the class activities helped them to develop their knowledge. We report characteristics of students' understanding that would be productive in teaching and learning exponential and logarithmic topics through contextual problems.*

*Keywords: Exponent, logarithm, real-world situation, conceptual understanding.*

## Introduction

Teaching focused on exponential and logarithmic concepts has not been widely effective in supporting students to conceptually understand these ideas (Kuper & Carlson, 2020). In teaching the concept of logarithm, many instructors focus only on Euler's definition: If  $x > 0$ , the logarithm of  $x$  to base  $a$  ( $a > 0$ ,  $a \neq 1$ ), is the real number  $y$  such that  $a^y = x$  and is symbolized with  $y = \log_a(x)$ . However, developing students' understanding of logarithm goes far beyond introducing Euler's definition and applying it to algebraic calculations. Students' difficulties with developing a conceptual understanding of the idea of logarithm are probably multidimensional. Logarithmic functions are the first family of functions that students encounter that are not explicitly expressed as an algebraic formula. In fact, it leaves students with no direction on how to compute  $\log_b a$  with the given values  $a$  and  $b$ . Instead, students are expected to use their understanding of the idea of powers and exponents to approximate the value of a logarithm for some input values, or use technology to calculate it. Logarithmic functions are also the first family of functions that students encounter where the function notation is not a single letter. Students who develop the concept of exponential growth only as repeated multiplication are likely to be limited to interpreting natural exponents. In situations where the exponent is not a natural number, interpreting the exponential concept as repeated multiplication is ineffective. Weber (2002) used APOS theory (Arnon et al., 2014) to study students' understanding of the exponents and logarithms. He argued that students first understand the concept of exponential growth as a process (a mental conception in APOS theory) before developing exponential and logarithmic expressions as a result of applying the process. Having a process conception, a student can generalize his understanding of the concept of exponent to situations where the exponent is not a natural number. Weber designed a series of activities for his students using the

idea “ $a^x$  represents the number that is the product of  $x$  many factors of  $a$ ”. Using this definition one can describe  $4^{3.5}$  as the number that is the product of three and a half factors of 4. Díaz-Berrios and Martínez-Planell (2022) used the same framework (APOS) like Weber to further study high school student understanding of the construction of exponential and logarithmic functions. They designed didactic materials to include exponents on the rational numbers and to help students construct logarithms as numbers. In their suggested learning framework, a process conception of unitary fractional powers was needed to allow students to think that for  $a > 0$  and positive  $n$  ( $n \in \mathbb{N}$ ),  $a^{\frac{1}{n}}$  is a number that multiplied by itself  $n$  times results in  $a$ . Their results from the problem-solving activities of students during a series of teaching episodes suggested that some students can use these materials successfully. Ellis et al. (2015) used a designed GeoGebra applet about a cactus that grows exponentially. The students had to express the growth rate of this cactus numerically at several times and express the relationship between the available variables. The activities focusing on the exponential growth of the cactus provide an opportunity for students to explore continuous growth.

### **Learning the concepts of exponent and logarithm in real-world situations**

Students at elementary schools are often instructed to interpret numerical statements such as  $3 + 4 = 7$  or  $2 \times 3 = 6$  based on their everyday experiences. However, are students at the tertiary level able to interpret exponential and logarithmic statements (i.e., equations with exponents or logarithms in them, e.g.,  $\log_2 3 + \log_2 5 = \log_2 15$ ) in real-life contexts by designing story problems? Studies document the increasing importance of mathematical literacy in the modern work environment and the need for learners to understand the meaning of their calculations in the context of work (Chapman, 2006). However, many students fail to see the application of mathematics (Gainsburg, 2008). The K-12 mathematics curriculum takes a special look at the importance of connecting classroom mathematics to the real world (e.g., NCTM, 2000). If students are not familiar with the contexts, doing mathematics in those contexts will not be productive for them. The effective use of real-world connections in teaching mathematics requires the use of examples that are related to students’ real and everyday experiences (Chapman, 2006). Despite the importance of understanding the concept of exponent and logarithm and their properties in real-world applications, little research has been done in this area, and many previous studies have encouraged such research (Kuper & Carlson, 2020). Our study seeks to fill this research gap. The research questions that we seek to answer in this study are: Are university students able to interpret exponential and logarithmic statements in real-world situations? How does a set of activities, involving story problems, effectively help students in acquiring a meaning for the concepts of exponent and logarithm and their properties in real-world situations?

### **Conceptual framework for exponent and logarithm in contextual situations**

Our conceptual framework describes the main constructions students might need to understand exponential and logarithmic concepts in real-life contexts. We used “tupling language” developed by Kuper and Carlson (2020) as a productive construction to help students understand non-natural number exponents and generalize their understanding of exponents and logarithms from numerical and algebraic representations to contextual situations. We consider a story that focuses on a population of city rats growing in such a way that the population triples in size each month. Using

this story context, we describe the constructions of the conceptual framework of exponent and logarithm. *Exponential Relationship*: when relating two continuous quantities,  $Q_A$  and  $Q_B$ , if for equal changes in  $Q_A$ ,  $Q_B$  grows by a constant factor, then the two quantities have an exponential relationship. In the population context, *the number of months* and *the population of rats* are two quantities and there is an exponential relationship between them because for any one month change in time, the population grows by a factor of three. The terminologies of tupling language involved in the framework are: To *b-tuple* is to become  $b$  times as large, e.g., the population *3-tuples* means the population becomes 3 times as large. A *b-tupling* is the event in which a quantity's value becomes  $b$  times as large, e.g., a *5-tupling* for the population is the event in which the population becomes 5 times as large. A *b-tupling period* is a change in one quantity (usually time) necessary for a second quantity to become  $b$  times as large, e.g., a *27-tupling period* (three months) is a change in the number of months necessary for the population to become 27 times as large. Now we describe the concepts of exponent and logarithm in terms of tupling language. An exponent is the number of elapsed *b-tupling* periods; written  $b^x$  where  $x$  is the number of elapsed *b-tupling* periods. For example, in the statement  $3^{2.5}$  in the given context, the exponent 2.5 is the number of elapsed 3-tupling periods or months (i.e., 2.5 months elapsed).  $\log_b X$  represents the number of *b-tupling* periods it takes the initial value of a quantity (for example population) to result in an  $X$ -tupling. For example,  $\log_3 15$  is the number of 3-tupling periods (months) it takes the population of city rats to become 15 times as large. One can represent the number of 3-tupling periods needed to  $\frac{1}{3}$ -tuple as  $\log_3 \frac{1}{3} = -1$ . Based on the tupling language, the product property of logarithm (i.e.,  $\log_b XY = \log_b X + \log_b Y$ ), can be described as: the number of *b-tupling* periods needed to result in an *XY-tupling* (i.e.,  $\log_b XY$ ) is the same as the number of *b-tupling* periods needed to result in an *X-tupling* (i.e.,  $\log_b X$ ) plus the number of *b-tupling* periods needed to result in a *Y-tupling* (i.e.,  $\log_b Y$ ). For example, to describe the logarithmic equation  $\log_3 729 = \log_3 81 + \log_3 9$  one can say the number of *3-tupling periods* (months) needed for the population result in an *729-tupling* (i.e.,  $\log_3 729$  or 6 months) is the same as the number of *3-tupling periods* needed to result in a *81-tupling* (i.e.,  $\log_3 81$  or 4 months) plus the number of *3-tupling periods* needed to result in a *9-tupling* (i.e.,  $\log_3 9$  or 2 months). To develop a conceptual understanding of this logarithmic property, it is helpful if students understand that an  $X$ -tupling followed by a  $Y$ -tupling results in an  $XY$ -tupling (Kuper & Carlson, 2020).

## Method

We conducted the study separately in two countries: Czech Republic and Iran. Six Czech preservice secondary teachers in their third year of an undergraduate mathematics field from a teacher training department in Czech Republic, and seven Iranian preservice secondary teachers in their second year of an undergraduate mathematics field from a teacher training department in Iran voluntarily participated in this study. All the students (in both countries) were chosen so that they showed above-average performance as determined by their course grades. We did the study in these two countries because they were available to us and we did not want to miss this opportunity. We were also interested to see how students from different cultures and backgrounds react on the interview questions and the set of class activities. However, we did not aim to compare students' understanding between the two countries because the amount of data was not enough to make any meaningful comparison. Furthermore, the students were from two different communities, and in order to make

such a comparison, many other factors and variables had to be examined, which were out of the scope of this study. The students in both countries learned exponential and logarithmic concepts in school and university courses, and applied these concepts to other advanced mathematical concepts (e.g., limit, derivative, and integral). This study consisted of an initial interview, three sessions of instruction, and an exit interview. For the instruction, we designed a set of activities with the aim of building a conceptual understanding of exponential and logarithmic concepts and their real-world significance. The activities were implemented in the classes by an experienced instructor in each country. Three class sessions (each 90 minutes) were held to familiarize students with describing the concepts of exponent and logarithm using the tupling language and interpreting them in real-world situations. The purpose of the activities was to help students interpret the ideas described in the conceptual framework and to promote the meaning of exponential and logarithmic ideas in real-world situations rather than to do procedural/algebraic computations. Students worked with situations where the exponent in an exponential statement or the answer of a logarithm was a real number and not necessarily a natural number. For example, a story problem that students worked on featured a cactus growing in such a way that its height 2-tuples in size each week. Then students answered questions like: by what factor of 8 does the cactus grow every one (two, six) week(s) [Answer:  $8^{\frac{1}{3}}$  ( $8^{\frac{2}{3}}$ ,  $8^2$ )] or every 1 year (52 weeks)? [Answer:  $8^{\frac{52}{3}}$ ]. Regarding the concept of logarithm, students worked on questions like: how many 2-tupling periods (1-week period) does it take for the cactus's height to 5-tuple? [Answer:  $\log_2 5$ ].

The initial and exit interviews were audio recorded. The interviews in both groups were done by one of the authors of this paper who separately gathered the data in each country. The initial interview questions were: **1.** Design a story problem using numbers 28 and 2 in the story, such that the answer of the story problem is  $\log_2 28$ . **2.** Design a story problem using numbers 2 and  $\frac{1}{7}$ , such that the answer of the story problem is  $2^{\frac{1}{7}}$ . **3.** Emily purchased a cactus and decided to record the displayed time-lapse video of the cactus's growth and noticed it was growing in a peculiar way: the cactus's height becomes 2 times as large each week. **a)** How many weeks does it take for the cactus's height to become 12 times as large? **b)** Using this scenario,  $\log_2 28$  would be the answer to what question? **c)** Using this scenario,  $2^{\frac{1}{7}}$  would be the answer to what question? **4.** Complete the following logarithmic properties:  $\log_c a + \log_c b = ?$ ,  $\log_c a^n = ?$ ,  $\log_c a - \log_c b = ?$ ,  $\frac{\log_c a}{\log_c b} = ?$  **5.** Find the value of  $x$  in  $\frac{\log x}{\log x+1} = -1$ .

After the initial interview, we started the instruction phase in both groups (one in the Czech Republic and the other one in Iran), and one week after the instruction we conducted the exit interview. The questions of the exit interview were: **1.** Design a story problem using numbers 2.5: **a)** and  $-2$  in the story such that the answer is  $(2.5)^{-2}$ , **b)** and 0 such that the answer is  $(2.5)^0$ , **c)** and  $\frac{3}{7}$  such that the answer is  $(2.5)^{\frac{3}{7}}$ . **2.** Design a story problem using numbers 45, 3, and 5.5 in the story, such that the answer of the story problem is  $45 \times 3^{5.5}$ . **3.** A population of city rats grows exponentially, doubling each month. Find the number of years it will take for the population to grow by a factor of 17? **4.** Design a story problem: **a)** using the numbers 2, 3, and 18 such that the answer of the question can be found as  $\log_2(18) - \log_2(3)$  or  $\log_2(6)$ . why are  $\log_2(18) - \log_2(3)$  and  $\log_2(6)$  equal based

on your story? **b)** what do  $\frac{\log_2(15)}{\log_2(5)}$  and  $\frac{\log_4(15)}{\log_4(5)}$  mean in your designed story in part (a)? why are  $\frac{\log_2(15)}{\log_2(5)}$  and  $\frac{\log_4(15)}{\log_4(5)}$  equal based on your story? and how are they related to  $\log_5(15)$  and why?

Students' responses in the interviews were translated and transcribed and depending on the correctness of a response, we assigned a score from 0 to 1 to each response. This helped us to see patterns in students' responses. Then, we made a descriptive narrative for each of the tasks for all the participants. In the next step, we compared the descriptive narratives of the interviews and created codes using open coding (Strauss & Corbin, 1998). In the coding, we tried to categorize the meaningful insights that the students showed as well as the challenges they faced. For example, some codes were: using the tupling language (Yes/No); using algebraic computations; expressing non-natural answers as logarithm; interpreting the exponent and base of an exponential statement (or the argument and base of a logarithm) inside the context. Although the codes emerged from the data, we used the conceptual framework presented earlier as a lens through which to orient our view of the data. After fracturing the data, using open coding, we performed axial coding to identify relationships between categories and selective coding to identify main categories and create hierarchies between categories (Strauss & Corbin, 1998). To check the reliability and validity of the codes, 25% of the data was re-coded by one mathematics education researcher and 85% agreement was reached.

## Results

Recall, that we assigned a score from 0 to 1 to a response depending how much the explanation and reasoning was mathematically correct (for a partially correct answer we assigned a 0.5). Also, we set mean scores of 0.80 and above as representing strengths in students' understanding ( $\bar{x}_{\text{strength}} \geq 0.80$ ), below 0.60 as representing limitations in students' reasoning ( $\bar{x}_{\text{limitation}} < 0.60$ ), and 0.60 and above and less than 0.80 as representing neither strengths or limitations of students' reasoning ( $0.60 \leq \bar{x}_{\text{not a strength or limitation}} < 0.80$ ).

*Initial interview results.* We coded 91 responses for the initial interviews (i.e., 13 students  $\times$  7 problem parts for the initial interview). The mean score of students' responses to questions were  $\bar{x}_{Q1} = 0$ ,  $\bar{x}_{Q2} = 0$ ,  $\bar{x}_{Q3a} = 0.5$ ,  $\bar{x}_{Q3b} = 0.625$ ,  $\bar{x}_{Q3c} = 0.5$ ,  $\bar{x}_{Q4} = 0.875$ ,  $\bar{x}_{Q5} = 0.82$ . The overall mean score for the initial interview,  $\bar{x} = 0.416$  and  $SD = 1.2$ , shows the limited understanding in the students. Here we bring some sample answers of students to questions 1, 2, and 3a. None of the students could answer Q1 and Q2 of the initial interview correctly. Indeed, of the seven problem parts in the initial interview, Q1 and Q2, which were about designing story problems for exponential and logarithmic statements, were associated with the most limited understanding.

Veronika: [from Czech; Q1] A car moves with a speed of 28 km/h, find the total distance covered by the car for 2 hours?

Hamid: [from Iran; Q2] A house has 7 rooms. Two painters want to paint these seven rooms. How many days does it take to paint a room?

Veronika's story for Q1 is about the speed of a car and she asked the total distance which its correct answer is  $28 \times 2 \text{ km}$  and not  $\log_2 28$ . For Q2, Hamid tried to make  $\frac{1}{7}$  in his story when he said "to paint a room [of seven rooms]?", but his story problem is incomplete and there is not an exact answer for the story. In relation to Q3a, based on our conceptual framework, a student with a comprehensive understanding of the meaning of logarithm in real world situations is able to determine the answer of

this question [Q3a] as a *log* without doing algebraic computations. However, none of the students in our study had such understanding in the initial interview. Indeed, 8 students (3 of Czech and 5 of Iran) used mechanical computations to find  $\log_2 12$  as the answer to Q3a, and the other students had incorrect answers. To answer Q3a, some students considered the number of weeks as  $x$ , then they made an exponential equation [ $2^x = 12$ ] for the given story and finally tried to solve the equation for the exponent  $x$  algebraically [ $2^x = 12 \rightarrow \log 2^x = \log 12 \rightarrow x \log 2 = \log 12 \rightarrow x = \frac{\log 12}{\log 2} = \log_2 12$ ]. The answer of Jan to Q3a was interesting. It seemed that Jan was able to find the number of weeks for the situations that the cactus's height becomes  $2^n$  ( $n \in \mathbb{N}$ ) times as large. However, when the height is not in the form  $2^n$ , he was not able to find the exact number of weeks and only computed the number of weeks approximately between two consecutive natural powers of 2.

Jan: [from Czech] After three weeks its height will be 8 times as large and after four weeks it will be 16 times as large, umm so approximately something between 3 weeks and 4 weeks it takes to become 12 times as large, but I can't find the exact time.

The mean scores to questions 4 and 5 ( $\bar{x}_{Q4} = 0.875$  and  $\bar{x}_{Q5} = 0.82$ ) give evidences that the students perform better with algebraic questions of exponents and logarithms than contextual problems. Overall, 11 students (5 of Czech and 6 of Iran) had a mean score less than 0.6 in the initial interview showing a limited understanding of the concepts of exponential and logarithm in terms of real-world situations. The results of the initial interview show that although students are able to do algebraic computations with exponential and logarithm formulas (Q4 and Q5), they are not able to interpret these concepts in real-world situations (Q1, Q2, and Q3).

*Exit interview results.* One week after the instruction in each group an exit interview was taken from the students in each country. We coded 91 responses for the exit interview (i.e., 13 students  $\times$  7 problem parts for the exit interview). The mean score of students' responses to exit interview questions were:  $\bar{x}_{Q1a} = 0.85$ ,  $\bar{x}_{Q1b} = 0.82$ ,  $\bar{x}_{Q1c} = 0.82$ ,  $\bar{x}_{Q2} = 0.812$ ,  $\bar{x}_{Q3} = 0.75$ ,  $\bar{x}_{Q4a} = 0.75$  and  $\bar{x}_{Q4b} = 0.625$ . The overall mean score for the exit interview,  $\bar{x} = 0.8$  and  $SD = 1.25$ , shows strength. As a summary, we started with 13 preservice teachers who showed no understanding of the concepts of exponent and logarithm in real-world contexts as evidenced by the results of the initial interview and after an instructional intervention, most of them showed an improvement in understanding of these concepts in the contextual situations. Here we bring a typical answer of students to Q1a, Q1c and Q4a. In Q1a we asked students to design a story problem for  $(2.5)^{-2}$ . Nine students (4 of Czech and 5 of Iran) solved this question correctly. An exemplary response to Q1a comes from Hanane.

Hanane: [from Iran] We have a tree and the height of the tree 2.5-tuple per week, umm by what factor of 2.5 does the tree grow every  $-2$  weeks?

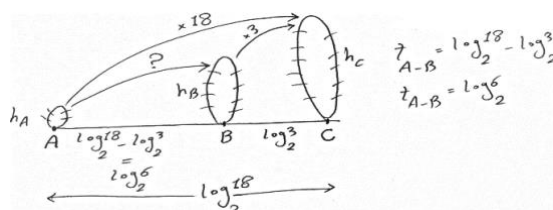
Interviewer: What do you mean by " $-2$  weeks"?

Hanane: I mean 2 weeks ago from now. It means the height of the tree two weeks ago from now was  $(2.5)^{-2}$  times as large as its current height or  $\frac{\text{the height of the tree two weeks ago}}{\text{its current height}} = (2.5)^{-2}$ .

Hanane correctly considered 2.5 as the growth factor of the tree's height per week (i.e., tree's height becomes 2.5 times as large per week) and the exponent  $-2$  as the period of time for two weeks ago (i.e.,  $-2$ -tupling period) from now. She interpreted the exponential statement  $(2.5)^{-2}$  as the factor the tree grows every  $-2$  weeks. Hanane also interpreted the exponential statement  $(2.5)^{\frac{3}{7}}$  in Q1c as

“the factor the tree grows every 3 days”. In Q4a students were asked to interpret the logarithm property  $\log_2(18) - \log_2(3) = \log_2(6)$  by designing a story problem using numbers 2, 3, and 18 in the story. Nine students (4 of Czech and 5 of Iran) answered this question correctly. We consider Eva’s answer (from Czech) as an exemplary response for Q4a.

- Eva: I had a cactus and it 2-tupled per week, I measured the cactus’ height at three different points of time, respectively time A, time B, and time C [Figure 1]. At time A, the cactus had a particular height,  $h_A$ . From time A to time B, the cactus grew by some factor and its height became  $h_B$  at time B. From time B to time C, the cactus’ height 3-tupled. In total, the cactus’s height 18-tupled in size from time A to time C. How many weeks did it elapse to cactus grows from height  $h_A$  to height  $h_B$ ?
- Interviewer: Can you explain both  $\log_2(18) - \log_2(3)$  and  $\log_2(6)$  in terms of your story?
- Eva: To find the time from A to B, I can compute the time from A to C, and subtract from it the time from B to C, umm to 18-tuple from A to C it elapses  $\log_2 18$  weeks, and to 3-tuple from B to C it elapses  $\log_2 3$  weeks umm and now if I subtract them from each other I will have the time from A to B, I mean the duration from A to B [ $t_{A-B}$  in Figure 1] is  $\log_2(18) - \log_2(3)$  weeks.
- Interviewer: And  $\log_2(6)$ ?
- Eva: I can find the duration from A to B [ $t_{A-B}$ ] in another way, we know from A to C the cactus 18-tupled and from B to C it 3-tupled, so we can conclude that from A to B it 6-tuples. Therefore, to 6-tuple from A to B it elapses  $\log_2 6$  weeks, it’s like to say the number of 2-tupling periods [weeks] to 6-tuple is equal to  $\log_2(6)$ .



**Figure 1: Eva’s drawing to interpret  $\log_2(18) - \log_2(3) = \log_2(6)$  in a contextual situation**

## Discussion and conclusion

This study was designed to examine aspects of university students’ understanding of exponential and logarithmic statements in real-world situations and the capacity of a set of activities (involving story problems) accompanied by tupling language, to effectively be helpful in understanding these concepts in contextual problems. To briefly summarize the results, the preservice teachers did not have any understanding of the concepts of exponent and logarithm in real-world situations in the initial interview. Since traditional teaching approaches are applied worldwide, the results are very similar across different countries (Ellis et al., 2015; Kuper & Carlson, 2020). This suggests that if preservice teachers are to learn about the concepts of exponent and logarithm in a mathematics course as preparation for their future teaching, they need help to construct the meanings of these concepts in real-world situations as a conceptual base. Indeed, in answering the research questions our study contributes to a better understanding of some issues dealing with the teaching and learning of the concepts of exponent and logarithm in meaningful contextual situations. The study shows that students of a traditional mathematics section can be expected to be constrained to a procedural understanding of the concepts of exponents and logarithms without any relation to real-world situations and that this will consequently limit their understanding of these concepts in a mathematics course. We did not discriminate effects from using the real-world context from those emanating from the use of tupling language. Indeed, it’s their combined use that has been effective. Our research is in



line with the studies carried out by Ellis et al. (2015) and Kuper and Carlson (2020) where they tried to bring contextual situations into the teaching of the concepts of exponent and logarithm. However, they focused on very basic notions of these concepts (e.g. their early definitions), but we further developed their work to describe exponential and logarithmic statements and rules in real-world situations. Like Díaz-Berrios and Martínez-Planell (2022) and Weber (2002), we considered situations where the exponent of an exponential statement or the answer of a logarithmic statement was a real number (and not necessarily a natural number) and also extended their work on numerical and algebraic representations to contextual problem situations. In this study, we presented concrete and practice-focused conceptualizations of mathematical knowledge related to real life. This is especially important for concepts such as exponent and logarithm, where curricula, standards, textbooks, and other learning resources traditionally focus on procedural and computational representations in the absence of real-world contexts (Kuper & Carlson, 2020).

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