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Online search routines in undergraduate students' agentic participation in mathematical discourse

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Use of online resources is becoming a prominent part of mathematical activity for learners, teachers and researchers. Here, we focus on the role online resources play, or may play, in the mathematical activity of university students who are enrolled in Mathematics or Education programmes – and have opted for courses in Mathematics Education. We explore their participation in problem-solving activities of these courses through a commognitive lens, and with a particular focus on evidence in their coursework of exploratory, confirmatory and other routines in their use of online digital resources. We characterise the students' agentivity – the initiative they exercise as they decide what, when and how they use online digital resources – as ranging from various forms of embrace to plain denial. We conclude with proposing that pedagogical practice in university mathematics needs to adapt towards fostering digitally literate, agentic use of online resources.

Keywords: Commognition, mathematical discourse, agentivity, problem-solving, online search.

Use of online resources as part of mathematical activity

The use of online digital resources in higher education has been attracting more attention in recent years (McMartin et al., 2008). This discussion became even more urgent and prominent, especially in relation to mathematical studies, during the pandemic with the necessity of online assessment (Henley et al., 2022) and the rapid expansion of Artificial Intelligence (AI) online platforms, such as ChatGPT (<https://openai.com/blog/chatgpt/>). Here, we focus on the role online resources may play on undergraduate students' mathematical and pedagogical discourses. Such discourses are strongly related to the available environment and the dialectic relationship between *questions* and *answers* (Bosch & Winsløw, 2015) as well as to the affordances of digital resources (Santos-Trigo, 2019). The availability of digital resources may assist mathematical activities and even shift the nature of such activity. For example, in Biza (2023), a problem was given to students with the request to solve it and then reflect on their solutions. Analysis of students' responses indicated that digital resources provided useful information but also provided answers to the problem and facilitated hypothesis building or fast execution of time-consuming procedures. Although the problem had been proposed to trigger engagement with “explorative” routines (p. 589) – such as trial and improvement, identification of patterns, building conjectures etc. – students' online searches for similar problems shifted the nature of the activity towards engagement with, for example, “confirmatory” routines (p. 589), when students found a solution to the problem online and merely confirmed that this response was correct. Such observations motivated our interest to investigate further how the availability and use of online resources become part of students' engagement with mathematical activity. In the next section, we outline the theoretical underpinnings of our investigation – a commognitive lens (Sfard, 2008) – and we state the research question that this paper explores.

A commognitive lens on students' use of online resources in mathtask responses

The commognitive framework (Sfard, 2008) sees the learning of mathematics as a process of individualizing mathematical discourse. Mathematical discourse is defined by *word use* (such as mathematical terms), *visual mediators* (such as notation, diagrams or gestures), *endorsed narratives* (such as definitions of mathematical objects or theorems about their properties) and *routines* (such as defining, conjecturing, proving, etc.). Of particular interest in this paper is how *online search routines*, such as looking for definitions in search engines (e.g. Google) or asking questions in specialised platforms (e.g. [Mathematics StackExchange](#)), are integrated into learners' participation in mathematical discourse. Here, we build on Biza's (2023) identification of evidence of exploratory and confirmatory routines in students' problem-solving activity, especially in relation to the different levels of criticality identified in their responses. Specifically, we explore whether and how searching, finding and using online materials contribute to learners' "agentive participation" (Nachlieli & Tabach, 2022) in mathematical discourse – namely, "the freedom and initiative performers exercise in deciding what should be said and done" (p. 4). In our work, we identify "agentive participation" in mathematical discourse as evidence of the initiative students exercise as they decide what, when and how they use online digital resources in their mathematical activity. Agentive participation in an online search routine implies explorative work such as exercising criticality in selecting sources and checking the validity of materials or narratives found online. Less agentive participation in an online search routine may involve a ritualised find-and-copy routine that may produce an acceptable mark (e.g. in assessment) but shows no evidence of individualised mathematical discourse.

To explore how online search routines are integrated into undergraduate students' mathematical activity, we analyse responses of two cohorts of undergraduate students to tasks that invite them to solve a problem and then reflect on said solutions in the light of how others (other undergraduates or school children) have solved the problem. Our data originates in students' responses to tasks that they engage with in Mathematics Education undergraduate courses they have opted to enrol in. The tasks follow the design and principles of the [MathTASK](#) programme. In MathTASK, we engage participants with fictional but realistic classroom situations which we call "mathtasks" (Nardi & Biza, 2022). Mathtasks are presented as short narratives that comprise a classroom situation where a teacher and students deal with a mathematical problem and with a conundrum that may arise from the different responses to the problem put forward by different students. Respondents are invited to solve the mathematical problem in the mathtask first and, then, to interpret and respond to the classroom situation. The use of mathtasks in our undergraduate courses has generated opportunities for tracing students' routines and narratives about mathematics and its pedagogy (Nardi & Biza, 2022).

In this paper, we use mathtasks to trace routines in students' use of online resources as they solve mathematical problems and reflect on their solutions and the solutions of others. We see students' solutions to the mathematical problem as evidence of their mathematical discourse and we see reflections on – and reactions to – the solutions of others as evidence of their pedagogical discourse. In sum, the research question we investigate here is: *what is the role of online search routines in undergraduate students' mathematical and pedagogical discourses?*

Context, participants, data collection and data analysis

Here, we draw on data collected from two cohorts (Bachelor's in Mathematics, BMath; Bachelor's in Education, BEd) of final-year undergraduate students. The BMath course is offered as an option to final-year students of BSc Mathematics or Physics programmes, is entitled *The learning and teaching of mathematics* and has been led by the first author since 2016. The aim of the course is to introduce undergraduates to the study of the teaching and learning of mathematics typically included in the secondary and post compulsory curriculum (Nardi & Biza, 2022). Here, we focus on a sample of five responses (out of 21) to the mathtask in Figure 1 (from students who had filled in consent forms at the time of writing). The BEd course is offered as an option to final-year students of a BA Education programme, is entitled *Children, teachers and mathematics: Changing public discourses about mathematics* and has been led by the second author since 2012. As a significant proportion of the programme's graduates continue into training to become primary teachers, the course is designed "to address directly the widely reported reticence of those students towards mathematics and their generally low self-esteem in mathematics" (ibid., p. 320). In this paper, we draw on the responses of 19 students to the mathtask presented in Figure 2.

Mr Jones is teaching a module on problem-solving for final year undergraduate mathematics students. He gives to his students the following problem:

If possible, construct a 10-digit number, which is divisible by all natural numbers up to 18, including, by using ALL the 0, 1, 2, ..., 9 digits only ONCE.

Mr Jones observes David, Mary, Peter and Laura working on the problem:

David: We need to create a set of equations from the conditions of the problem. To start with ... in order for the number to be divisible by 10 the last digit should be 0, then in order to be divisible by 4 the last two digits should be divisible by 4, so the last two digits should be 20, 40, 60, 80 ...

Peter: [facing David] This does not go far. My suggestion is trial and improvement. I will find the Lower Common Multiple of all the natural numbers up to 18 and then I will keep multiplying until I will find the right number, if this number exists.

Mary: [facing David and Peter] Your suggestions take ages. I will write a computer programme that can give all the 10-digit numbers and check whether there is any 10-digit number that satisfies the conditions.

Laura: Come on guys! Let me check Google, somebody else should have done this problem before.

Questions (in each of the items below justify your response, also with references to the Mathematics Education research literature, where needed):

- Q1. Give your response to the problem.
 - Q2. How do you interpret David's approach? What would you suggest to David to do next?
 - Q3. How do you interpret Peter's approach? What would you suggest to Peter to do next?
 - Q4. How do you interpret Mary's approach? What would you suggest to Mary to do next?
 - Q5. How do you interpret Laura's approach? What would you suggest to Laura to do next?
-

Figure 1: The BMath mathtask A divisibility problem, based on the mathtask in (Biza, 2023)

Agentivity in embrace of online digital resources in BMath mathtask responses

The mathtask in Figure 1 was inspired by responses to the same divisibility problem given by undergraduate mathematics students in previous years. The problem was initially chosen because it can be approached with different methods, it requires simple divisibility rules, it does not rely on a known algebraic approach and it requires a level of exploration of what the target number might be, without knowing whether such number exists or not (Biza, 2023). The variation of students' use of online resources in their responses motivated the design of the mathtask in Figure 1 in which the same

problem is discussed by a fictional problem-solving class when students, “David”, “Peter”, “Mary” and “Laura”, propose approaches akin to those identified in the analysis of student data in Biza (ibid). We are interested to see not only how the respondents solve the problem (Q1), but also how they interpret and respond to what other students say (Q2-Q5). Here, we focus particularly on approaches to online searches in the responses to Q1 (own solution) and Q5 (reaction to the solution by “Laura”) in the responses of five students: Will, Terry, Simon, Sophie and Linda.

Two students acknowledge explicitly that they sought some assistance through online search on Google (Will, Terry) and Wolfram Alpha¹ (Will). Three students mentioned programming, most likely accessed online, such as Python (Simon, Sophie) or other unspecified software (Linda). In what follows, we examine how the five students’ responses to Q1 and Q5 are intertwined.

In terms of online search, Will, who does not propose any numbers in his response to Q1, sees Google search as his first course of action: “I would probably do similar to “Laura” and start by googling the problem to see if anyone has solved anything similar”. He then describes the multiplication of prime numbers up to 18 and the use of “wolfram alpha to check that the values I get are divisible by all natural numbers up to 18”. Later in Q5, Will writes about “Laura”:

Although Laura’s method is similar to the way in which I would go about this, I would probably advise her to attempt to come up with a potential method before googling to find the answer. This would then allow her to apply methods she sees online on websites such as stack overflow² to her own problems, enforcing her understanding and making sure that she does not simply write down answers she does not fully comprehend. This would be exactly what I would ask her to do and see if she can piece together some form of resolution of her own.

We observe inconsistencies between Will’s described routine in Q1 and his response to “Laura” in Q5. In the latter, he endorses narratives that suggest agentive participation in his engagement with the solution process. The reference to *Q&A platforms*, such as Stack Overflow that may offer solutions to similar problems, indicates that Will might have visited such platforms in his search for “something similar”. Again, although his immediate action involves an online search, in Q5, he endorses a more cautiously agentive approach attached to this search. “Laura”’s reliance on online search is a concern for Simon as well: “Laura’s approach is troublesome as it does not allow her to solve the problem herself, and it relies on Google having an answer”; thus, his suggestion to Laura is “to understand the reasoning behind the answer she finds on Google, so that she can apply those skills to similar questions”. Terry, the other student who also eventually sought assistance from Google, describes a set of steps he followed first to identify a set of conditions the target number should satisfy – e.g. the number should end with 0, be divisible by 16 and be a multiple of 272272 ($7 \times 11 \times 13 \times 16 \times 17$). However, he writes “there are thousands of multiples of 272272 that have 10 digits and finding one that satisfies the conditions manually is basically impossible” and then adds that:

¹ The Wolfram Alpha engine (<https://www.wolframalpha.com/>) computes answers from externally sourced data.

² Stack Overflow (<https://stackoverflow.com/>) is an online Q&A community part of the Stack Exchange Network (<https://stackexchange.com/>). Community exchanges include “puzzles” similar to the divisibility problem in Figure 1.

The ideal next step would be to formulate a program to check which satisfy the conditions; however, as I don't have a coding background, I instead turned to Google, to see if there was a number that satisfied the conditions. 2438195760 did, so I checked on a calculator that this number was divisible by all integers 1 through 18, which it was!

Terry's approach in Q1, starts with exploratory routines (finding the conditions), then continues with an online search (search for a number that satisfies the conditions) and concludes with confirmation routines (confirming that the number found online satisfies the conditions). Later, in his response to "Laura" (Q5), he is concerned about the fact that she ignores other students' ideas: "she was not facing any of her peers or participating in the discussion and then instead proceeded to disregard all their approaches and resort to Google". Also, he doubts the accuracy of Google – "not everything on Google is accurate or reliable by any means so the answer she gets to may not even be correct" – and he would advise "Laura" to "build on her peers' ideas and at best use Google to check if the answer she comes up with is validated by others' work". Terry's response is consistent in relation to his appreciation of exploration routines and the supportive role of a Google search.

In Terry's response, however, we also see caution against online search that is disconnected from class norms according to which interaction with *more knowledgeable others* is of primary value. We see a similar tendency in two other responses as well. Linda and Sophie, who do not mention online search in their responses to Q1, acknowledge that "Laura" has "given up" (Linda) or is "not knowing how to start the question" (Sophie). Linda also acknowledges that "classroom norms do not usually involve googling an answer". However, Linda and Sophie see online search as a potentially legitimate and potent approach: "Laura seeking help online from the mathematics community is not necessarily 'cheating' [her use of quotations]", says Linda in Q5. Sophie too highlights that online search is an opportunity to learn from the community where Google is a "shared resource". Overall, across the five students' responses, we see evidence of agentive – thoughtful, critical, cautious – participation in their embrace of online search as potent support towards solving a mathematical problem. Our analysis of student responses to the mathtask in Figure 2 however revealed a different form of agentivity in students' mathematical and pedagogical discourses: that denying, steering away from – or even ignoring the support of – online digital resources may be evidence of learner autonomy.

Agentivity in denial of online digital resources in BEd mathtask responses

The mathtask in Figure 2 was intended to elicit reactions to the use of online resources (Calendar, search engines such as Google) by "Anna" and "Barack": both use their digital calendar to identify quickly which day of the week is "Neil"'s birthday on for various years. "Anna" extrapolates from checking two cases, 2021 and 2023, that "Neil"'s birthday moves one day per year and therefore it will take seven years for his birthday to be on a Friday again. "Barack" uses his digital calendar to check the accuracy of the answer by "Anna" and produces a counterexample to her claim. The mathtask is intended to generate reactions to primary pupils' acts of mathematical reasoning (rushed extrapolation for "Anna"; counterexample generation for "Barack") and to their use of online resources to support these acts. Six students wrote explicitly – albeit mostly in a fleeting fashion – about the use of online resources by "Anna" and "Barack" or the lesson at large; and, only one (other) student commented in their response on the different – and variably productive – forms of

mathematical reasoning manifested in the responses by “Anna” and “Barack”. Furthermore, 11 of the 19 students demonstrated their handwritten work on the problem through inserting word-processed or photographed snips of said work.

Today is Friday 16 December 2022. Ms Harris is about to start the last Year 6 maths lesson before school breaks for the Christmas and New Year holidays. As she enters the classroom, she finds the children quite excited and gathered around a surprise birthday cake made for Neil’s birthday.

Neil: Ms, it’s my birthday today! Let’s do no maths. I only want to do fun things on my birthday! And, today is the last day of school and everyone is coming to my party right after...and my mum says I’m so lucky it’s on a Friday and everybody will be able to come...and having my birthday on a Friday won’t happen again for God knows how many years! I won’t even be in school, when this happens again, she says! Please, Ms, no maths today!

Ms Harris: Well, well, who says we cannot do maths with what you just said, Neil? So, everyone, do you think Neil’s mum is right? That he won’t have his birthday again on a Friday while he’s still in school? What do you think?

The class is starting to chat about this. Anna raises her hand.

Anna: Ms, I checked the calendar on my laptop and Neil’s mum is not right. His birthday was on Thursday last year, in 2021, and it’s today, Friday, this year, 2022, and it is on Saturday in 2023! It’s one day plus every year. So, it will be on Friday again in seven years, when he is in year 13, in 2029!

Barack: Hold on, I just checked the calendar for 2029 and it says that December 16 is on a Sunday! What’s happening here? This is odd. I kinda thought Anna made sense to be honest: a year has 365 days and a week has 7 days...if I divide by 7... this gives us 52 weeks plus 1 day leftover. So, yes! Neil’s birthday will move forward a day every year [*pauses briefly*] Ah, hold on. Not all years have 365 days though, right? What’s it called... a leap year? Therefore... [*mutters to himself*] ...That’s it! OK. Problem sorted. I know what’s happening now!

Clive: Well, do you, Barack!?! You are probably the only one. I have no idea about what you just said. Gibberish, I say. If you are right, then 365 days is divided by 5 just fine, nothing left. And we only go to school Monday to Friday, which is five days and “therefore” as you like to say [*chuckles*], every year Neil’s birthday will be on the same day. Huh! Anyway, what’s the point of all this? I’m really starving and I want a slice of cake!

You are the teacher and you just heard what Neil, Anna, Barack and Clive said....

Questions (We advise that, in your analysis, you attend to mathematical, social, affective and metamathematical issues and that you do so in consultation with relevant Mathematics Education research literature):

Q1. Do you agree with Neil’s mum? Justify your answer using school mathematics.

Q2. How would you respond to Anna?

Q3. How would you respond to Barack?

Q4. How would you respond to Clive?

Q5. How would you respond to the whole class – also in the light of Neil’s initial comment – and conclude the lesson?

Figure 2: The BEd mathtask *Neil’s birthday on a Friday!* inspired by (Biza, 2023)

We note that all but one³ students concluded that the next time “Neil” will have his birthday on a Friday is in 2033: the students noticed that, in leap years, the day of a birthday celebration will move forward by two days. The students’ handwritten diagrammatic and tabulating realisations offered overall solid, transparent and confident evidence of their grasp of the problem and its solution.

³ This student acknowledged the impact of leap years but failed to identify 2028 as a leap year and thus concluded that in 2028 “Neil” will celebrate his birthday on a Friday.

Only three students – Melanie, Lucio and Clare – addressed the use of online resources explicitly. Melanie does so at the end of her response to Q5:

...and I would finish with, “Now that we know that Neil’s birthday won’t happen again [on a Friday] until 2033, we can see that his mother was right. It may seem easier to just check the calendar, but now we have all learned how to calculate it by ourselves, without any external help, isn’t that amazing?”

In the Feed-Forward session – optional, brief, one-to-one tutorials with each student to discuss their mark and markers’ feedback – we explored the absence of evidence in the students’ scripts of noticing the use of online resources by “Anna” and “Barack”. With the caveats that only about a third of students came to the Feed-Forward session, that this issue was pursued only briefly and that the only evidence of student utterances about this is our own notes of recollection of these discussions, we note that every one of these student utterances was identical or a variation of Melanie’s comment above. In a nutshell, what Melanie and her peers articulate is that “how to calculate [...] by ourselves, without any external help” is worthy of celebration and elevates our “amazing” efforts to a higher standard than those of “Anna” and “Barack” who relied on “just check[ing] the calendar”. For Melanie, liberation from “external help” in mathematical activity is evidence of learner autonomy, a form of agentivity that, in contrast to the examples we saw of student responses to the BMath mathtask in Figure 1, emerges as empowering in its denial of support from online resources.

Lucio’s response is more akin to the examples of agentivity in embrace of online resources we presented in the BMath section (“a fast verification with a calendar with the following years (an online resource can be used to facilitate the teacher’s explanation”) as is Clare’s proposition which is also more specific to the one made by Lucio and also more explicitly triggering class interaction:

I think it would be a good idea to have a whole class demonstration where we display a calendar on the classroom interactive whiteboard, and go through the years, identifying which ones are leap years and why. Then the students can visually see when Neil’s birthday skips a day, and...

Finally, we note that one student makes a fleeting reference to projecting a calendar (for “visual cues”, Gina) and two students turn to the use of online videos (“Using the website ‘Nasa’s space place’ [...] I would show Anna and her peers the educational video demonstrating the earth’s orbit and the reason for leap years”, Joy) to support their exposition in the classroom for more “visual learning” (Joy).

Concluding remark: Agentivity in embrace and denial of online search routines

In the students’ mathtask responses, we saw evidence of variable appreciation for the role online resources may play in mathematical activity. Of particular interest to us in this paper is what use (or not) of online resources students see as promoting agentive participation in mathematical discourse. We saw in the student responses acknowledgement that uncritical endorsement of what is found online is risky (BMath) or of little educational value (BEd responses that located empowerment in mathematical activity as independent of reliance on online resources). We saw nuanced embrace of the potential of online resources (e.g. in BMath statements that valued critical exploration before and after a response is found online) that favoured agentive participation in mathematical activity which optimises the use of online resources. Across the BMath and BEd responses, we detected efforts to

reconcile between established classroom norms – where online search routines are, as yet, neither mastered nor fully legitimised – and embrace of the breath-taking, if overwhelming, new world of online access to information and to communities with similar interests (such as Q&A and AI platforms). We see therefore the need for adapting pedagogical practice in university mathematics – in designing tasks for the classroom and for assessment that explicitly acknowledge, problematise and empower use of online resources – towards fostering digitally literate, agentic participation in mathematical discourse.

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