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Designing tasks for prospective teachers: specialised knowledge on the inclusive teaching of problem-solving

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This study describes the design of a training session aimed at developing prospective teachers' specialized knowledge by means of a series of video-based tasks. Based on a problem-solving workshop for students diagnosed with Autism Spectrum Disorder-Asperger Syndrome, the MTSK model is used to discuss the teaching-learning results with and to subsequently develop a training proposal for prospective teachers. Video excerpts of the workshop, along with the work produced by the children, have been edited into a series of guided tasks with the aim of mobilising knowledge in terms of both mathematical content, and inclusive teaching and learning methodologies for developing heuristics for Asperger Syndrome children.

Keywords: Mathematics education, Mathematics teachers, Inclusive Education, Problem-solving.

This paper delves into the design of problem-solving teacher training tasks. To do so, a real teaching situation obtained from a problem-solving workshop for students with Asperger Syndrome will be the base for the design. The tasks design aims at fostering prospective teachers' (PTs') understanding of heuristic strategies, and to encourage them to reflect on the potential of these tools for developing inclusive methodologies. As different aspects of specialised teacher knowledge will be involved, MTSK subdomains (Carrillo et al., 2018) will structure the design.

The tasks will be implemented in the Didactics of Mathematics course in the first year of the Primary Education degree at the University of Huelva. The two-months course, usually taken by around a hundred PTs, is appropriated as it includes work on heuristics and problem-solving.

Theoretical framework

In order to work on problem-solving in the classroom, teachers need not only to develop their own problem-solving skills, but also to conceptualise potential difficulties, methodologies and their students' abilities as problem-solvers (Chapman, 2015). Given that diversity is a key feature of the primary education classroom, the teaching-learning process should incorporate different strategies to meet students' needs, such as those associated with the Autism Spectrum Disorder-Asperger Syndrome (henceforth, Asperger Syndrome -AS). This underlines the need for teachers to amplify their knowledge of specific strategies and resources linked to how Asperger Syndrome students learn problem-solving, with regards to potential difficulties and benefits.

This study concerns the design of training tasks, taking into account the mathematical and pedagogical content knowledge required by PTs. Previous research (Carrillo et al., 2019) has enquired into the knowledge mathematics teachers need to design and manage problem-solving tasks in the light of the Mathematics Teacher's Specialised Knowledge Model (MTSK) (Carrillo et al., 2018). This model consists of a system of subdomains and categories that can help to identify the knowledge involved in the development of training tasks (Giberti, 2022). By using the model to guide their analysis of children's work and their problem-solving strategies, PTs' knowledge is enhanced.

In the sphere of strategies, resources, and learning theories offering an inclusive perspective on problem-solving, research on autism and problem-solving (Root et al., 2021) underlines the phase structuration of the problem statement and solving procedure, alongside representation as prime strategies (teachers' knowledge on these strategies would be KMT). Applying teaching strategies also requires knowledge of those features associated with AS that may influence the solving process (KFLM–difficulties and abilities). Representational approaches include different communication registers to mitigate problems associated with verbal understanding (Delisio, et al., 2018). It is essential to stage the problem-solving process into a clear learning sequence and to help pupils overcome difficulties in the identification of general structures (Klaren et al., 2017). Heuristics such as using tables provide a visual representation of the problem statement and improve the handling of the data (Novotná et al., 2014); other heuristics, such as going backwards and generalisation, increase the sequencing, which becomes a key dynamic for finding a solution (Chico et al., 2022).

All these methodologies can be combined through transversal methods (KMT–teaching theories) as they share the principles of the Universal Design for Learning (hereafter UDL) (Rose & Gravel, 2010), in terms of presenting the problem-solving using different types of registers, and involving all students participating in the solving process through graded, structured instruction which allows them to progress by considering their abilities and difficulties.

At the same time, problem-solving and mathematical content must be considered. Clearly staged procedures and the heuristics described by Polya and refined by Schoenfeld (KPM–problem solving) can also be highlighted as a guide for visual comprehension and structuring the problem. Using heuristics opens the door to great diversity of intertwined solving methods which make the solver aware of the paths to take. The levels of sophistication devised by Blanton et al. (2015) can also help teachers to be aware of students' pre-algebraic comprehension and strategies when facing a generalization problem. In this scheme, counting and addition strategies will be related to level 1 (pre-structural), while recursive strategies are indicative of levels 2 and 3. As the student constructs more complex functional relationships and uses a variable-related expressions, a higher level of sophistication will be evidenced (4-functional-particular, 5-primitive functional-general, 6-emergent functional-general, 7-condensed functional-general and 8-function as an object).

This research will serve of the MTSK model in order to configure the design of training tasks for PTs in the context of problem-solving considering students with Asperger Syndrome. How to mobilise specific teacher knowledge on a generalising problem solving task? How to delve into specialised teacher knowledge on an inclusive approach to the problem-solving teaching?

Methods

A seven-month problem-solving workshop was run with a group of 4 children diagnosed with Asperger Syndrome. They studied in the third cycle of primary education and attended the ordinary classroom. The participants were selected according to their mathematical abilities and needs by the psychologist working with a local Asperger syndrome association, AOSA.

Over a series of 12 sessions, the participants were introduced to different problem-solving heuristics. In keeping with their needs, they were supported with graded teacher instruction and provided with TEACCH (Treatment and Education of Autistic and Related Communication Handicapped Children)

material, following the principles of UDL. This study is focused on “The archaeologist problem” (Figure 1), a generalisation problem adapted from previous research into the generalising strategies adopted by gifted primary pupils the same age as the workshop participants (Benedicto et al., 2015). The children used an organizer, pictograms, and EVA foam square pieces (Figure 2) to work on the relationship between the two variables, height and number of boxes.

An archaeologist wants to clean the museum dinosaur models. She doesn't have a ladder, but she has found a pile of boxes, all the same size, which she can stack to reach the dinosaurs. Each box is 1 metre high, and she can stack them as in the picture. How many boxes does she need to reach the height of the Velociraptor (2m), the Stegosaurus(4m), the Gigantoraptor(8m), and the T-Rex (10m)?



Figure 1: The archaeologist problem



Figure 2: TEACCH material: EVA foam pieces, organiser and Velcro stair pictograms

The analysis of the children’s work and the video-recording of the workshop enabled to identify key points that provide evidence for the use of heuristics and the manner in which the material and strategies may influence the solving process. Once the excerpts to be included in the training tasks were selected, the MTSK model was applied as the instrument of analysis for identifying the specific knowledge to be constructed and how to promote it. This stage of the analysis was first conducted by the three research authors working independently, and then jointly agreed.

Problem-solving with Asperger Syndrome students

Given that children were free to choose their preferred solving method, the four participants started the solving process by applying a counting strategy (Blanton et al., 2015) using the statement drawings for that purpose. The visual support reinforced their visual thinking (de Giambattista et al., 2019). However, executive functions conditioned the students’ progress. In the case of Ivan, he struggled to develop the correct strategy because of the rigidity of thinking and working memory (Klaren et al., 2017). Regardless of whatever new strategies he tried, his first (and mistaken) idea made him constantly resort to calculating the height of each stair via the total number of boxes, instead of focusing on the final stack. Javier resorted to drawing by himself the new boxes for each stair. The disorganized, unbalanced structure caused him to struggle with the counting (Figure 3).

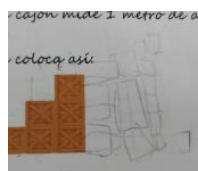


Figure 3: Javier’s disordered drawing of the fifth stair.

In both cases, the teachers suggested that they use the foam pieces, the Velcro stairs and organiser to minimise the difficulties observed. The EVA Foam squares allowed the children to easily modify the stairs by changing the number of stacks. However, the construction strategy became a distraction for

Alex, who struggled when counting the increasing number of squares, and for Javier, who focused on creating the correct structure rather than the problem in question.

The incorporation of the organiser supported recursive level to be evidenced (Blanton et al., 2015). Using the Velcro stairs and the tabular representation allowed to register and organise the information (Chico et al., 2022), and so visually manipulate the relationship between the height and number of boxes, discovering the pattern involved in the problem (Novotná et al., 2014). The pattern recognition ability usually associated with the Syndrome (de Giambattista et al., 2017) was reinforced as the children were challenged to solve the generalisation problem. It should be also underlined the potential of the organiser as a monitoring tool for checking the coherence of the students' hypothesis (Delisio et al., 2018). When the teacher pointed out the responses he had entered on his organiser, Isaak decided to draw some arrows to vertically visualise the progression of the stairs and so reduce calculation errors. With regards to Alex, following the sequence established by the organiser helped him to stick to the problem goal and minimise the attention issues (Klaren et al., 2017).

Furthermore, encouraging students to explore their own solving methods resulted in an opportunity to examine their mathematical understanding. Considering the comprehension of the close and far terms of the sequence (Goñi et al., 2022), the organiser led them to apply a recursive strategy for determining each following term, which was highly dependent on the previous term in the sequence (Alex and Isaak). With regards to heuristics, Javier developed his own visual scheme to represent the data obtained from the EVA foam construction, whereas Ivan worked by means of an analogy: he used the EVA squares he had previously employed to solve a parallel generalising problem.

In light of the mathematical knowledge and pedagogical content knowledge involved in the analysis of the archaeologist problem, we purpose a training task. The task aims, on the one hand, to provide PTs with an opportunity to observe a teaching situation which encouraged children to explain their own ideas and strategies in a problem-solving situation (Santagata & Guarino, 2011), and, on the other, to elicit PTs' ideas in relation to what teachers need to know to improve the teaching and learning process within inclusive contexts.

Task design

Excerpts from the video-recording of the workshop on the Archaeologist Problem will be played to the PTs. This material exemplifies will allow PTs to observe how the children struggled along the problem-solving process and to perceive key elements of problem-solving in terms of phases, representations, mathematical operations, and transformations involved (KMT, KFLM). The excerpts also exemplify methodologies and teaching strategies (KMT) aimed at reinforcing children's abilities and encouraging them to apply various heuristics for generalisation. To facilitate the identification of these children's generalisation strategies, the PTs will also be provided with some reading on Blanton et al.'s levels of sophistication (KFLM). The AS is a teaching-learning context that exemplifies an inclusive methodology, aiming PTs to understand some Syndrome features in terms of classroom diversity. The aim is not so much that they learn characteristics of AS but strategies and resources for dealing with classrooms with students with different needs.

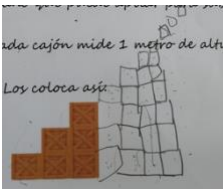

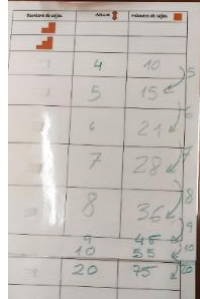
Preparation phase:

The PTs will be given the opportunity to familiarise themselves with the Archaeologist Problem, and to solve it by applying a variety of strategies. This first phase aims to delve into the mathematical knowledge involved in the problem, in terms of the use of heuristics and the difficulties associated with the solving process that might arise when using the problem in a primary education class.

Task phase:

A real teaching situation involving the Archaeologist Problem will be introduced: the problem-solving workshop with Asperger Syndrome students. The PTs will be invited to investigate the problem-solving possibilities using the TEACCH material. After that, two situations which arose in the workshop will be presented via the corresponding video excerpt (see Table 1)

Table 1: Teaching situations

 <p>Figure 4: Isaak's floating steps</p>	<p>Situation 1.1: Isaak started to solve the problem independently, by applying a counting strategy based on the problem statement drawings, and using mental arithmetic. When the teacher asked him to explain his strategy, he talked about “stacking the boxes”. In order to solve the Gigantoraptor case (8m), he decided to model the example, but drawing only the topmost box of each stack (Figure 4). As he developed this new strategy, he realized that he had made some calculation mistakes, which he managed to correct.</p>
 <p>Figure 5: Isaak's arrows strategy</p>	<p>Situation 1.2: The teacher next challenged Isaak to discover how many boxes the archaeologist would need to reach a 20-metre dinosaur, and gave him the organiser. After a period of silence, the teacher repeated the question, this time asking how many boxes would the archaeologist needed to climb 4, 5 and 6 metres. Isaak again resorted to counting in order to fill in the organiser, so the teacher suggested finding a faster strategy. Isaak answered: “jumping”. The teacher and Isaak then started to “jump”, using the organiser, and registering the information for each stair. After “jumping” to 9 metres, Isaak started to draw some arrows (Figure 5) to vertically connect the number of boxes on the organiser. Until that point, he had calculated the total from memory, leading him to forget the data.</p>
 <p>Figure 6: Isaak's incorrect sequence</p>	<p>Situation 2.1: Once Isaak had completed his organiser to solve all the questions in the problem statement, the teacher again asked him to calculate how many boxes the archaeologist needed to reach 20 metres. Isaak thus wrote “20” in the column marked “height” on his organiser and “20” alongside the corresponding arrow to calculate the total number of boxes following the sequence in his table (Figure 6).</p>
	<p>Situation 2.2: After being given a similar challenge to Isaak's, Ivan suggested finding out how many boxes the archaeologist would need to reach a new dinosaur (“What about a 30-metre high dinosaur? Let's do it in our heads!”). To do so, after consulting the organiser, Ivan settled on a multiplication, 210×30, instead of carrying out a lengthy addition.</p>

After watching the videos, PTs will work in groups, provided with the transcripts for each of the excerpts, in order to answer to the following questions:

1. What kind of heuristics can be observed during situations 1 and 2?

2. Describe Isaak's strategy to solve the problem in the cases of the Gigantoraptor and the 20-metre dinosaur (situations 1.1 and 1.2). Compare the two strategies to identify similarities and differences. What do these strategies mean for the generalisation?
3. Describe Isaak's different representations in terms of their potentialities and limitations for the problem-solving process.
4. Assess how the tabular organiser influences Isaak's problem-solving process (situations 1.2 and 1.3) in terms of learning potentialities and limitations.
5. Describe the strategies Isaak and Ivan developed to carry out the Gigantoraptor case (Situations 2.1 and 2.2). Compare the two strategies, identifying similarities and differences. What do these strategies mean from the perspective of generalisation?
6. Watch Isaak tackling the problem (situations 1.1, 1.2 and 1.3). Identify the possible problem-solving phases and the heuristics employed with respect to Polya's theory.

Though extra material and time might be required to delve into the specialised knowledge, we hope the following elements arise during the class discussion:

- Problem-solving phases and heuristics; generalisation in problem-solving (KPM).
- How students tackle the generalisation problem: generalising strategies (counting, recursive, etc.), first and last terms, representations (KFLM–modes of interaction).
- Features of Asperger Syndrome that may influence the generalisation process: tendency to sequence tasks, ability to perceive details, visual thinking, ability to discover patterns, literalness, oral comprehension difficulties, attention and working memory issues, rigidity of thinking (KFLM–abilities and difficulties).
- Inclusive teaching strategies in problem-solving: sequenced learning, use of representations and manipulatives (KMT–strategies).
- Inclusive resources in problem-solving: graphic organisers and tables (KMT–resources).
- Inclusive methods in problem-solving: Schema-Based Instruction, Concrete-Representational-Abstract and UDL (KMT–teaching theories).

Post-task phase:

PTs will be engaged in the design of a problem-solving teaching proposal for the last cycle of primary education from the neurodiversity perspective.

Conclusion

This study explores how solving situation and strategies drawn from a problem-solving workshop for children with Asperger Syndrome can be used to configure the design of training tasks for PTs. The Archaeologist Problem tackled in the workshop illustrates an example of teaching situation to structure the students' representation of the problem, and to develop their own solving strategies evidencing different levels of sophistication (Blanton et al., 2015; Goñi et al., 2022).

With regards to the generalisation tasks, though it delves into a workshop with students with AS, the task pursues the discussion on inclusive methodologies to manage students' features present in every primary education classroom: organization/planification issues, visual learning, self-regulation, etc. Besides, the role transition from solver to teacher aims to promote skills and knowledge related to the

representational and teaching possibilities of the generalisation problem (KMT) and to noticing students' strategies, difficulties and comprehension mechanisms (Callejo & Zapatera, 2017) (KFLM). This interpretation requires PTs to comprehend the learning implications of Asperger Syndrome features, and the cognitive demands involved in the task with regards to the functional relationships described in Blanton et al.'s (2015) model. Bearing in mind that PTs' didactical content knowledge of problem-solving is mainly theoretical (Piñeiro, 2019), the training tasks provide a reflective teaching experience closer to real classrooms to reinforce their professional knowledge.

This research will continue by implementing the tasks with PTs from the University of Huelva. Our intention is to contribute to the construction of a methodology focusing on the knowledge and skills teachers require for developing problem-solving in inclusive contexts.

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