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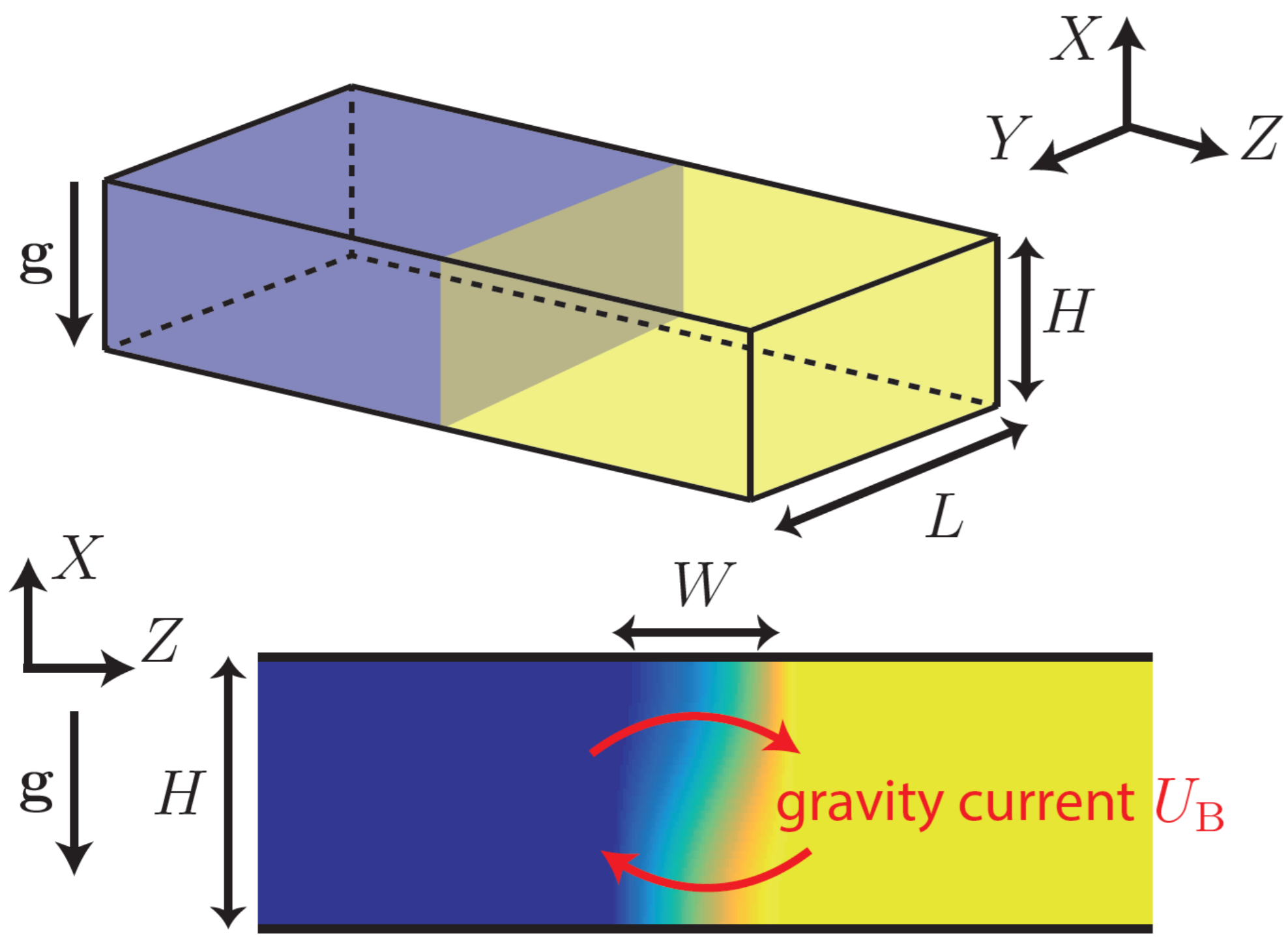
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# Role of solutal free convection on interdiffusion in a horizontal microfluidic channel

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## Problem statement



- Horizontal microfluidic channel of height 5–500  $\mu\text{m}$ .
- One half of the channel filled with a binary solution and the other half with pure solvent.

**Theoretical investigation of the role of solutal free convection on the transport of a buoyant solute at the microfluidic scales.**

The buoyant forces generate a gravity current that couples to the diffusion of the solute.

### Objectives

- Conditions leading to non-negligible gravity-induced advection ?
- Time evolution of  $W$ , width of the mixing zone ?
- 3D effects in a square channel ?

$$\nabla \cdot \mathbf{u} = 0$$

$$\frac{1}{\text{Sc}} \left( \frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} \right) = \Delta \mathbf{u} - \nabla p - \text{Ra} \varphi \mathbf{e}_x$$

$$\frac{\partial \varphi}{\partial t} + \mathbf{u} \cdot \nabla \varphi = \Delta \varphi$$

- **Boundary conditions:**

$$\mathbf{u} = 0 \quad \text{and} \quad \mathbf{n} \cdot \nabla \varphi = 0 \quad \text{at solid walls}$$

$$\mathbf{u}(z \rightarrow \pm\infty) = 0 \quad \text{and} \quad \partial_z \varphi(z \rightarrow \pm\infty) = 0$$

- **Initial conditions:**

$$\mathbf{u}(x, y, z, t = 0) = \mathbf{0}$$

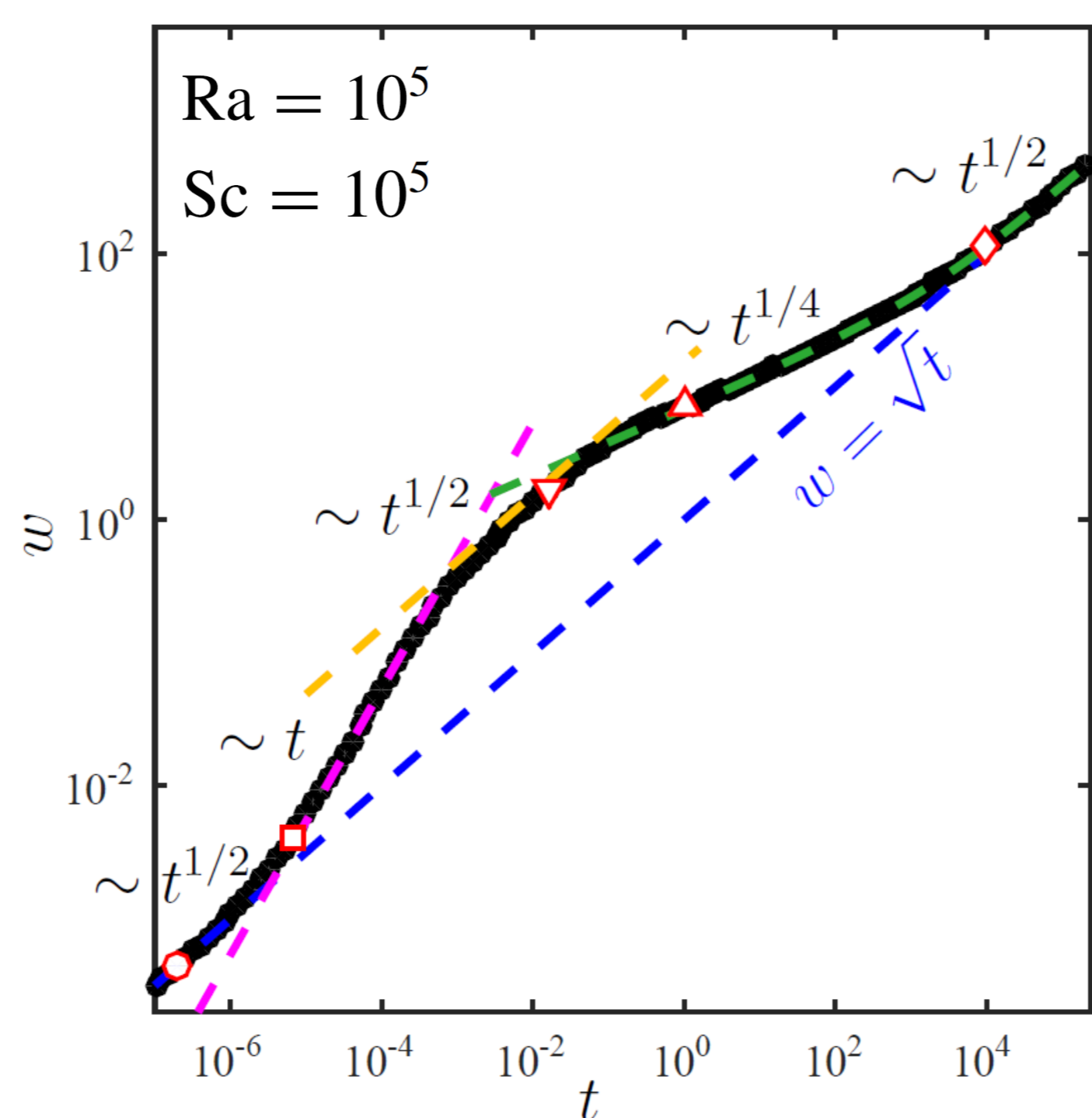
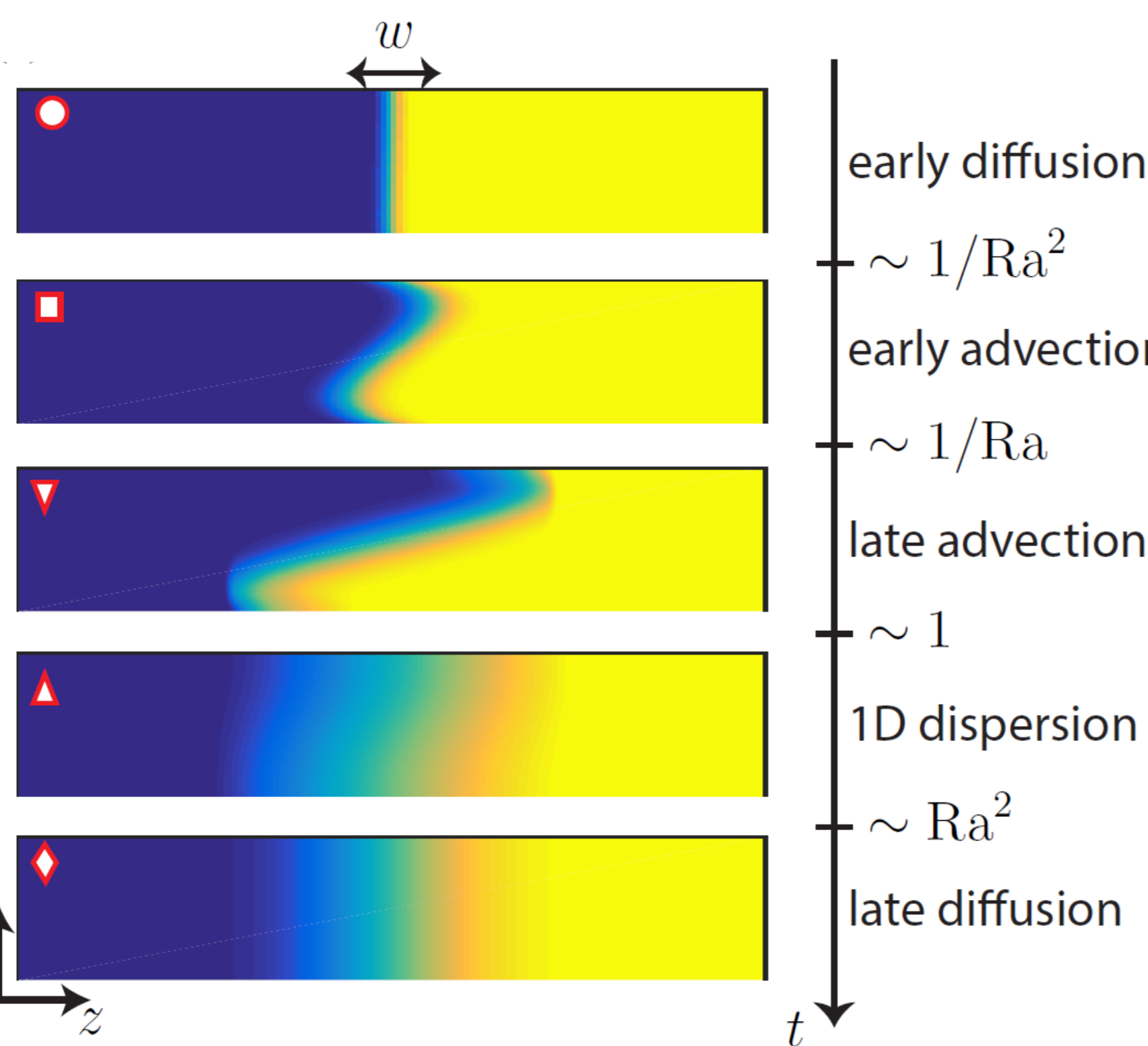
$$\varphi(x, y, z, t = 0) = \mathcal{H}(z)$$

**Two dimensionless numbers:**

$$\text{Ra} = \frac{\beta \Phi_i g H^3}{\nu D} \quad \text{Sc} = \nu / D$$

## The case of a slit: 5 asymptotic regimes

$$\varphi(x, z, t) \longrightarrow \varphi_0(z, t) = \langle \varphi \rangle \longrightarrow w(t) = \sqrt{\frac{1}{2} \int_{-\infty}^{\infty} z^2 \frac{\partial \varphi_0}{\partial z} dz}$$



### Early and late diffusion:

Negligible advection  $\longrightarrow w = \sqrt{t}$

### Early advection:

$$0 = \Delta \mathbf{u} - \nabla p - \text{Ra} \mathcal{H}(z) \mathbf{e}_x$$

$$\varphi_0(z, t) = \int_0^1 \mathcal{H}[z - u_z^p(x, z = 0) t] dx \longrightarrow w \simeq 0.00538 \text{Ra} t$$

### Late advection:

Quasi-parallel flow  
Negligible diffusion  $\xrightarrow{\text{Lock-exchange (see ref.1)}} w \simeq 0.04879 \sqrt{\text{Ra} t}$

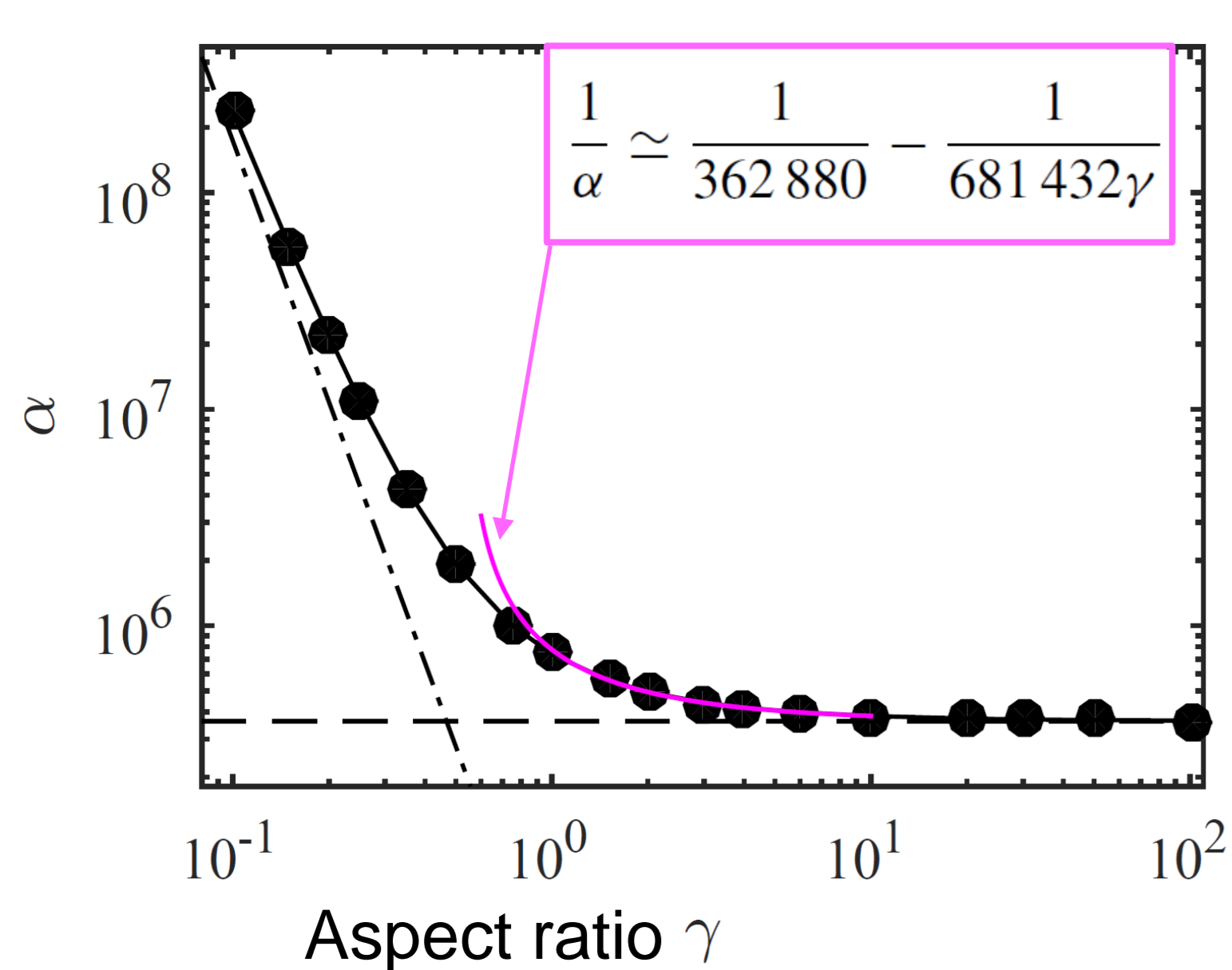
### 1D dispersion:

Quasi-parallel flow  
Cross-section diffusion  $\xrightarrow{D_{\text{eff}} = 1 + \frac{1}{\alpha} \left( \text{Ra} \frac{\partial \varphi_0}{\partial z} \right)^2} w \simeq \sqrt{\frac{\text{Ra}}{2\pi}} \left( \frac{3t}{\alpha} \right)^{1/4}$

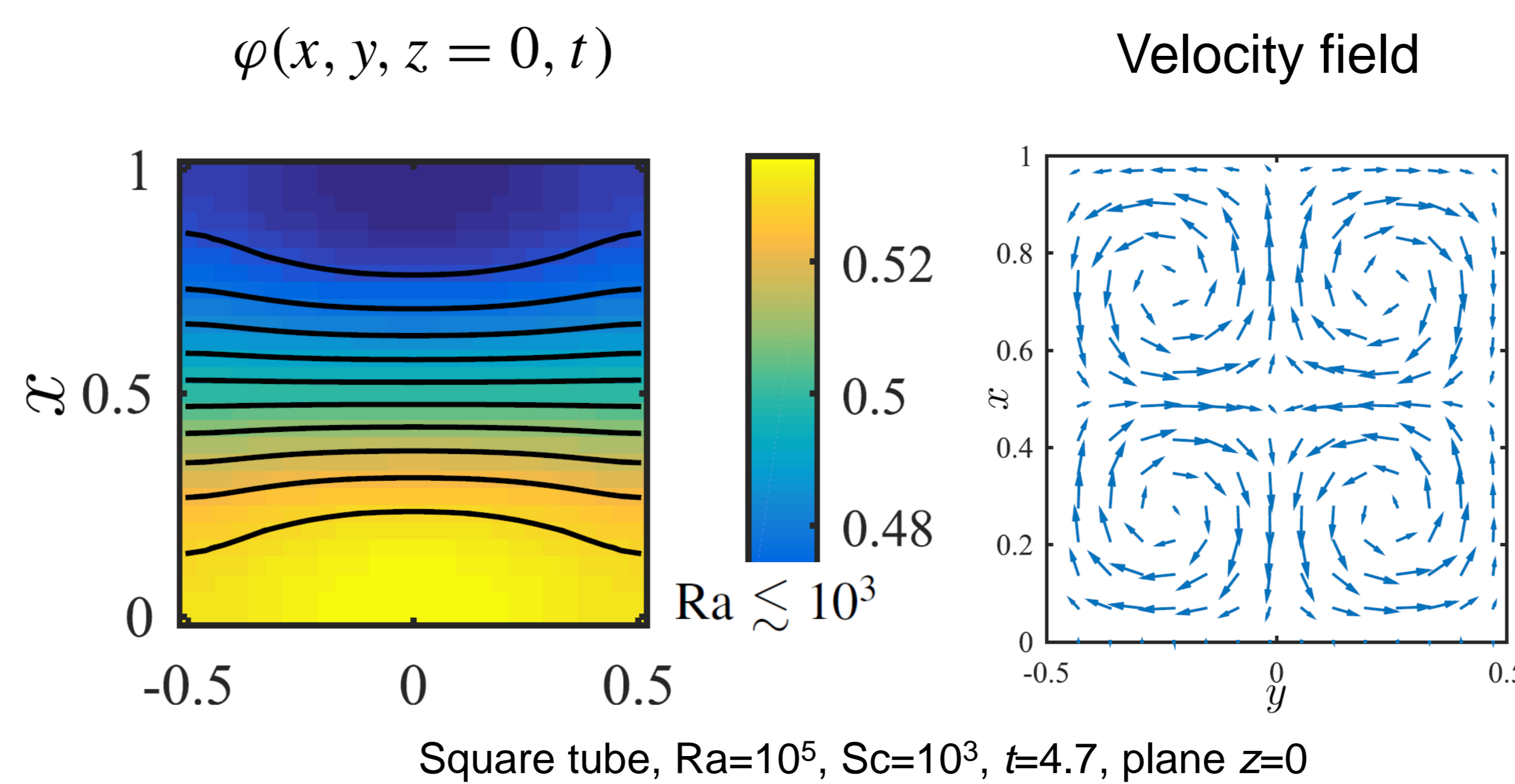
## 1D dispersion regime in the 3D case (rectangular section)

### Dispersion coefficient

$$D_{\text{eff}} = 1 + \frac{1}{\alpha} \left( \text{Ra} \frac{\partial \varphi_0}{\partial z} \right)^2$$



### Presence of a secondary cell in the cross-section (transverse flow)



Maximum velocity of the transverse flow:

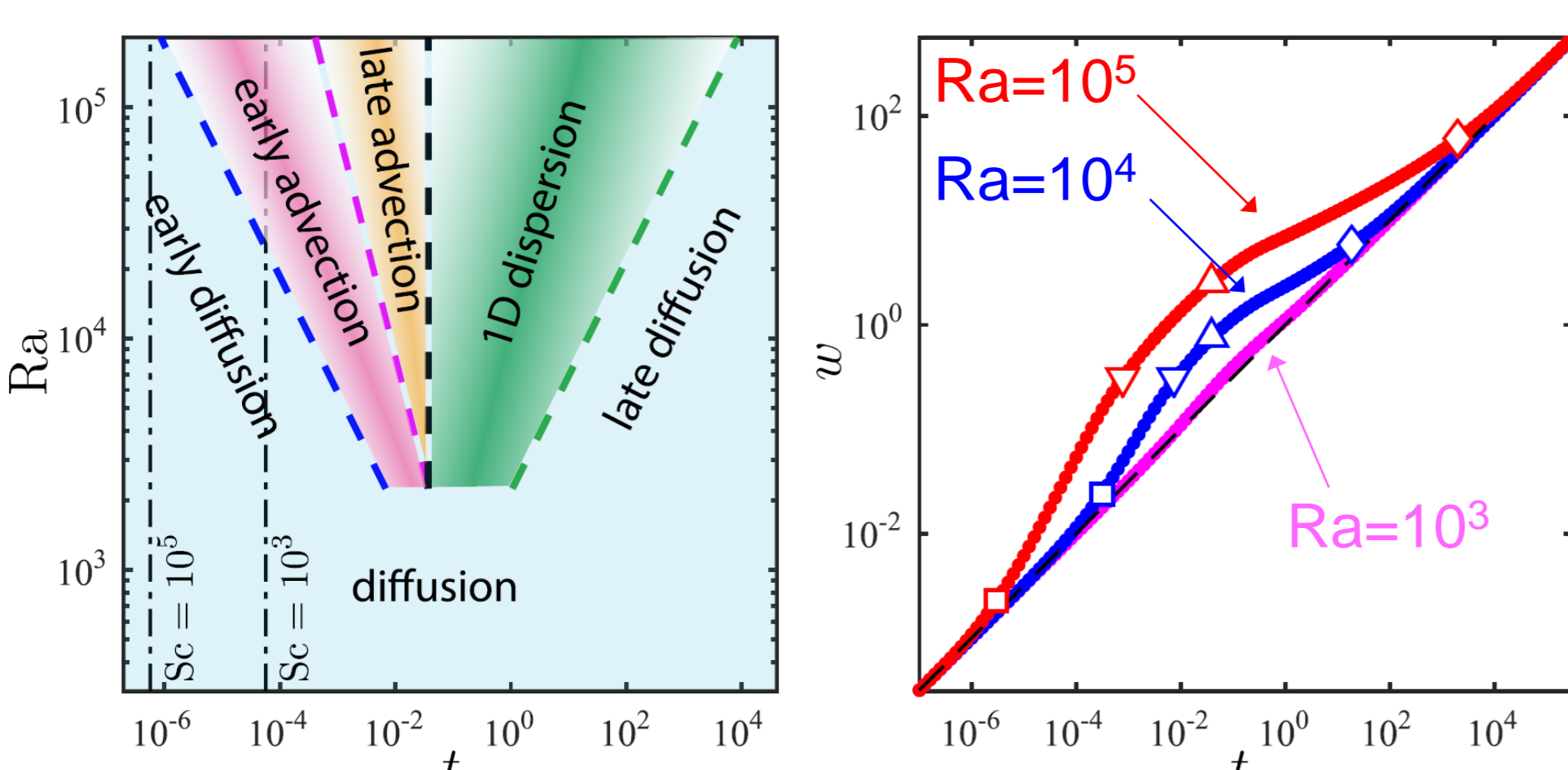
$$\max(u_x) \simeq \max(u_y) \simeq 2.2 \times 10^{-5} \frac{\text{Ra}}{\sqrt{t}}$$

Negligible transverse flow for

$$t \geq 4.9 \times 10^{-10} \text{Ra}^2$$

## Conclusion

### 5 asymptotic regimes



- Buoyancy vanishes for  $\text{Ra} \lesssim 10^3$
- Early diffusion and early advection visible for  $\text{Ra} \geq 10^4$  and  $t \leq 10^{-3}$
- The impact of transverse flow on solute transport remains moderate for  $\text{Ra} \leq 10^5$   
 $\Rightarrow$  It can be neglected in most microfluidic experimental applications.