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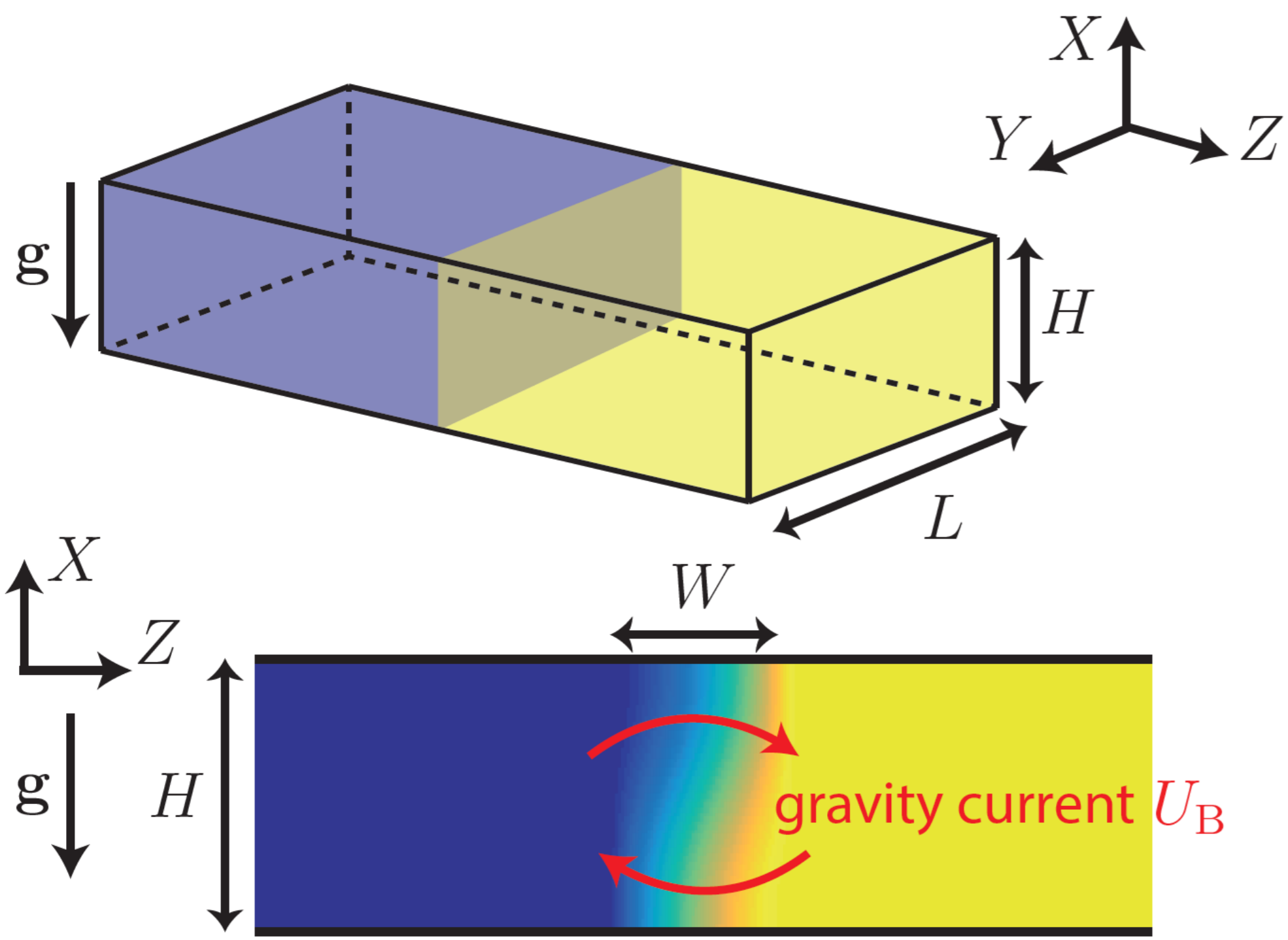
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Role of solutal free convection on interdiffusion in a horizontal microfluidic channel

Jean-Baptiste Salmon⁽¹⁾, Laurent Soucasse⁽²⁾, Frédéric Doumenc^(3,4)

(1) CNRS, Solvay, LOF, UMR 5258, Univ. Bordeaux, F-33600 Pessac, France, (2) EM2C, CNRS, CentraleSupélec, Université Paris-Saclay, 8-10 rue Joliot Curie, 91192 Gif-sur-Yvette, France
(3) Université Paris-Saclay, CNRS, FAST, 91405, Orsay, France, (4) Sorbonne Université, UFR 919, 4 Place Jussieu, F-75252, Paris Cedex 05, France

Problem statement



- Horizontal microfluidic channel of height 5–500 μm .
- One half of the channel filled with a binary solution and the other half with pure solvent.

Theoretical investigation of the role of solutal free convection on the transport of a buoyant solute at the microfluidic scales.

The buoyant forces generate a gravity current that couples to the diffusion of the solute.

Objectives

- Conditions leading to non-negligible gravity-induced advection ?
- Time evolution of W , width of the mixing zone ?
- 3D effects in a square channel ?

$$\nabla \cdot \mathbf{u} = 0$$

$$\frac{1}{Sc} \left(\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} \right) = \Delta \mathbf{u} - \nabla p - Ra \varphi \mathbf{e}_x$$

$$\frac{\partial \varphi}{\partial t} + \mathbf{u} \cdot \nabla \varphi = \Delta \varphi$$

- **Boundary conditions:**

$$\mathbf{u} = 0 \quad \text{and} \quad \mathbf{n} \cdot \nabla \varphi = 0 \quad \text{at solid walls}$$

$$\mathbf{u}(z \rightarrow \pm\infty) = 0 \quad \text{and} \quad \partial_z \varphi(z \rightarrow \pm\infty) = 0$$

- **Initial conditions:**

$$\mathbf{u}(x, y, z, t = 0) = \mathbf{0}$$

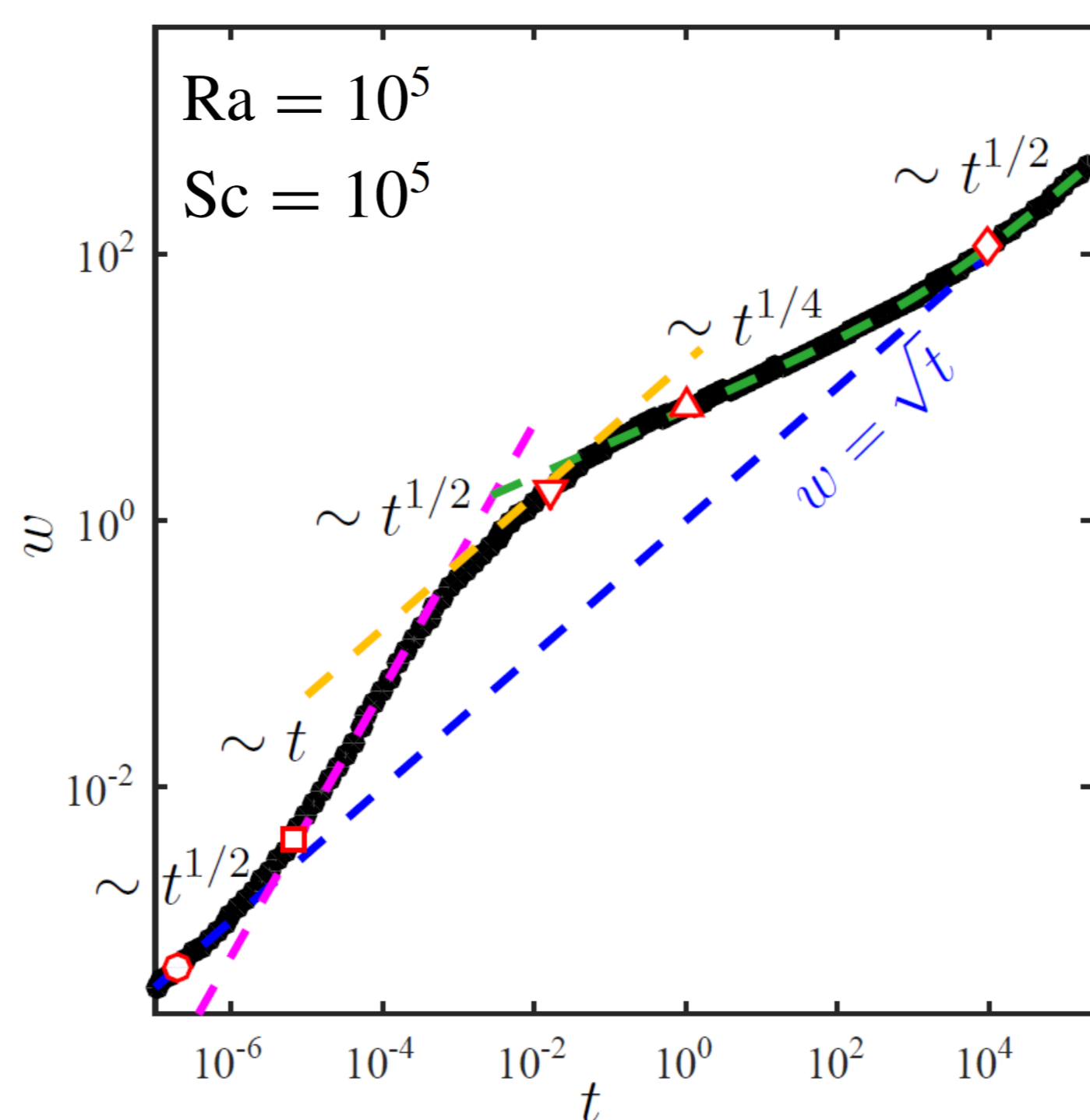
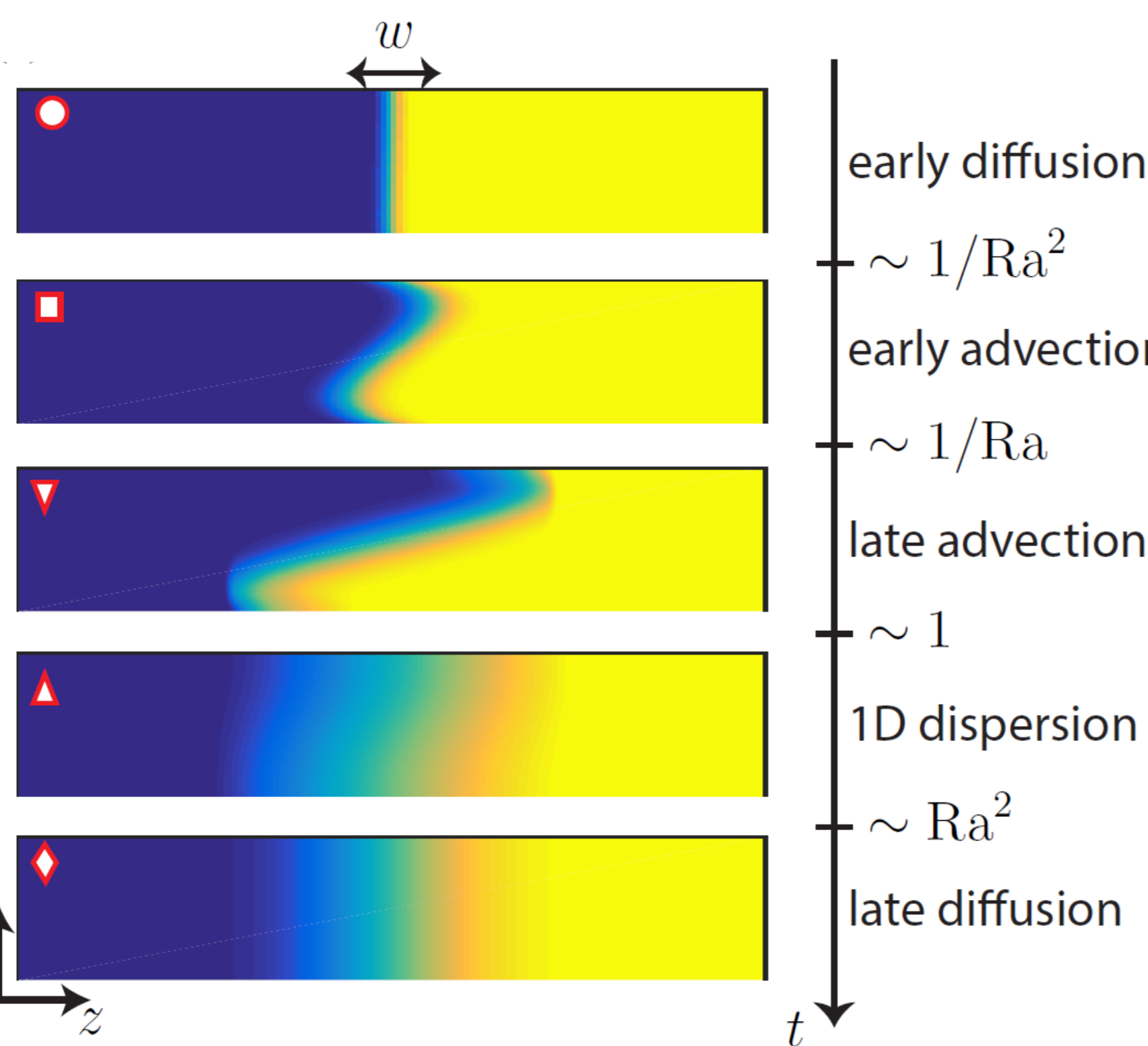
$$\varphi(x, y, z, t = 0) = \mathcal{H}(z)$$

Two dimensionless numbers:

$$Ra = \frac{\beta \Phi_i g H^3}{\nu D} \quad Sc = \nu / D$$

The case of a slit: 5 asymptotic regimes

$$\varphi(x, z, t) \longrightarrow \varphi_0(z, t) = \langle \varphi \rangle \longrightarrow w(t) = \sqrt{\frac{1}{2} \int_{-\infty}^{\infty} z^2 \frac{\partial \varphi_0}{\partial z} dz}$$



Early and late diffusion:

Negligible advection

$$w = \sqrt{t}$$

Early advection:

$$0 = \Delta \mathbf{u} - \nabla p - Ra \mathcal{H}(z) \mathbf{e}_x$$

$$\varphi_0(z, t) = \int_0^1 \mathcal{H}[z - u_z^p(x, z = 0)t] dx$$

$$w \simeq 0.00538 Ra t$$

Late advection:

Quasi-parallel flow
Negligible diffusion

Lock-exchange
(see ref.1)

$$w \simeq 0.04879 \sqrt{Ra t}$$

1D dispersion:

Quasi-parallel flow
Cross-section diffusion

$$\frac{\partial \varphi_0}{\partial t} = \frac{\partial}{\partial z} \left(D_{\text{eff}} \frac{\partial \varphi_0}{\partial z} \right)$$

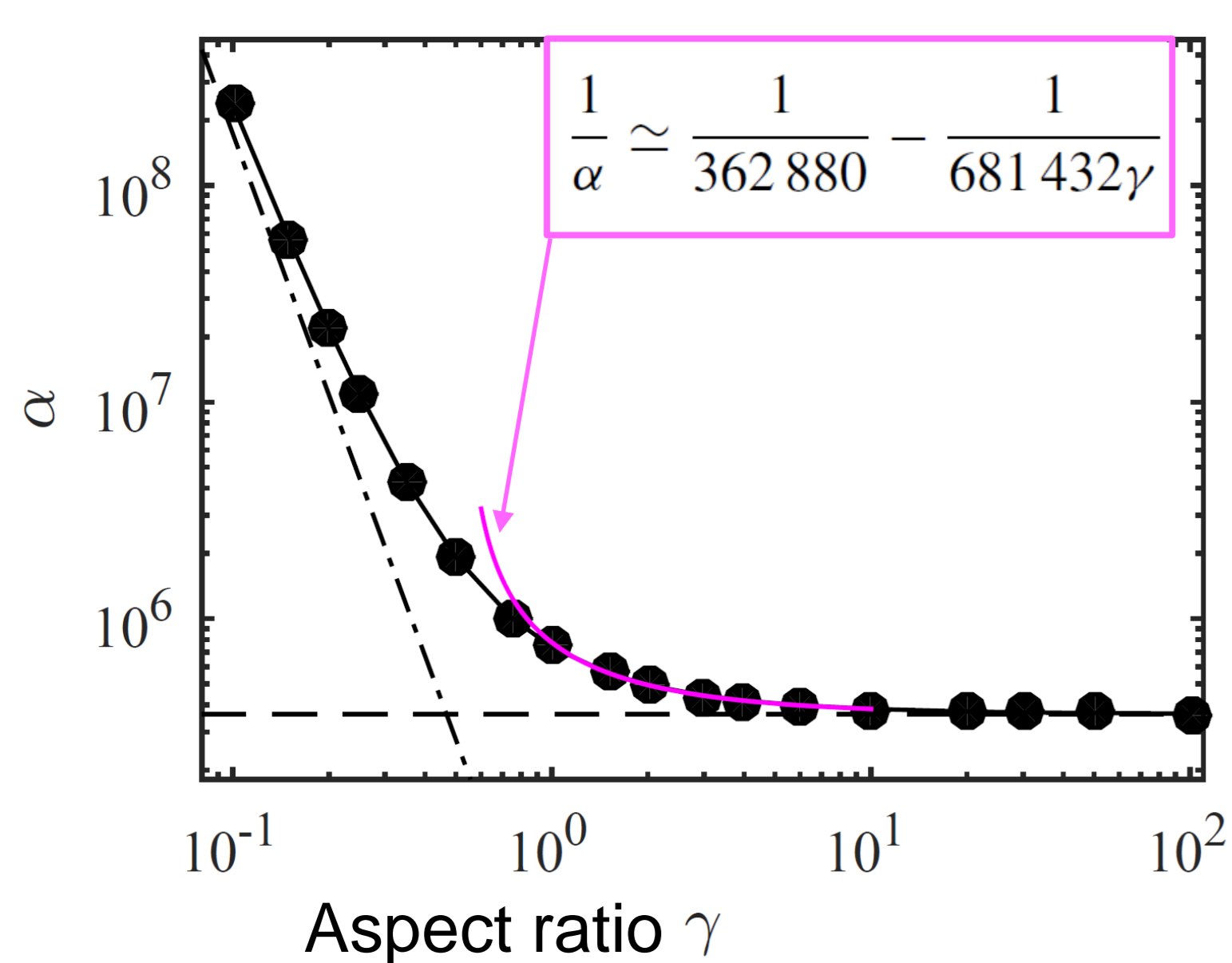
$$D_{\text{eff}} = 1 + \frac{1}{\alpha} \left(Ra \frac{\partial \varphi_0}{\partial z} \right)^2$$

$$w \simeq \sqrt{\frac{Ra}{2\pi}} \left(\frac{3t}{\alpha} \right)^{1/4}$$

1D dispersion regime in the 3D case (rectangular section)

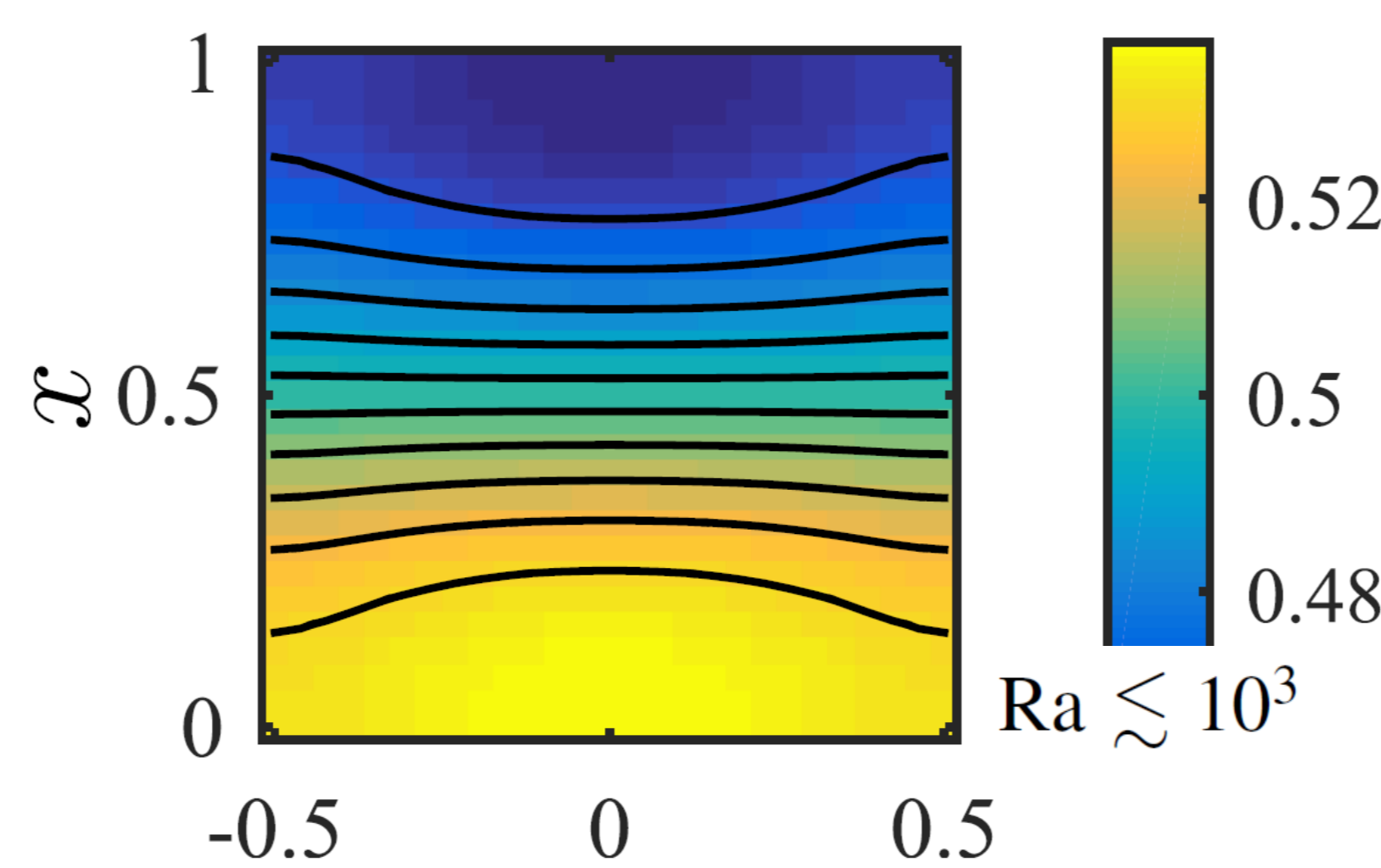
Dispersion coefficient

$$D_{\text{eff}} = 1 + \frac{1}{\alpha} \left(Ra \frac{\partial \varphi_0}{\partial z} \right)^2$$

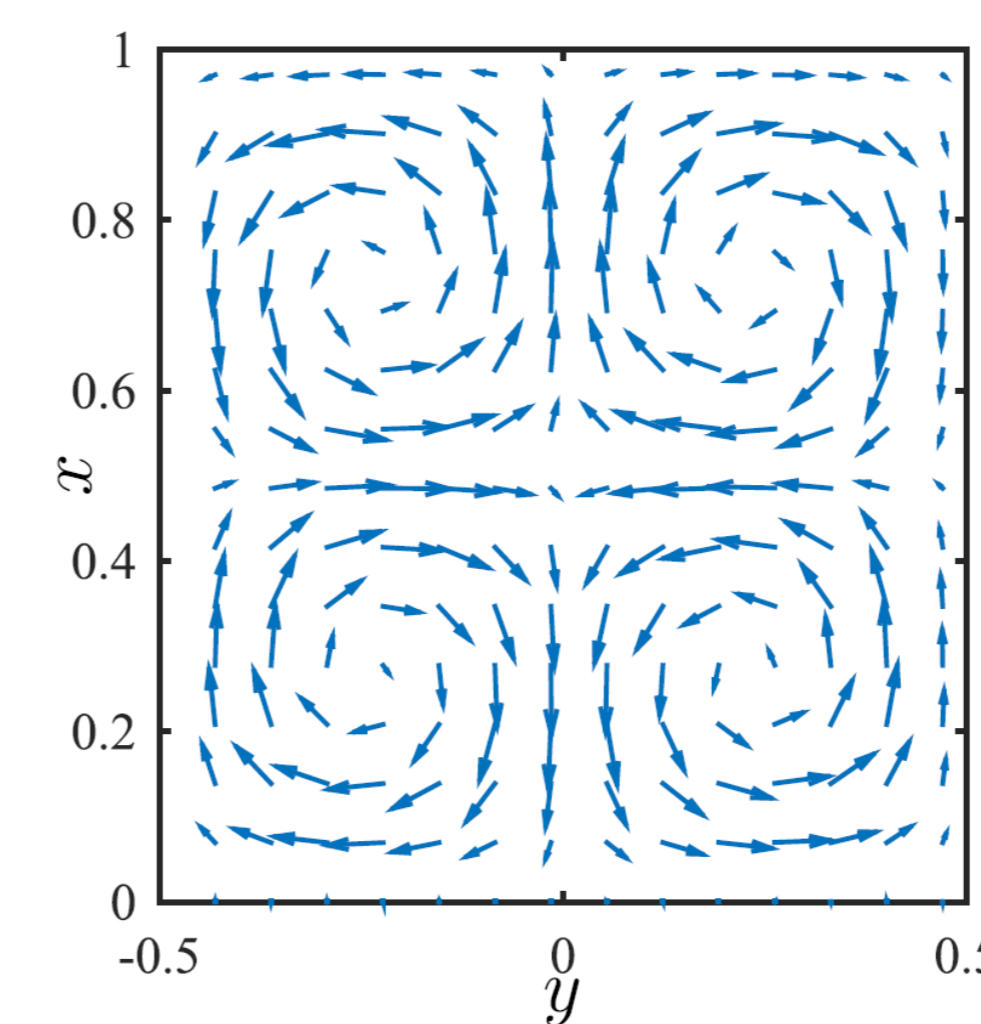


Presence of a secondary cell in the cross-section (transverse flow)

$\varphi(x, y, z = 0, t)$



Velocity field



Maximum velocity of the transverse flow:

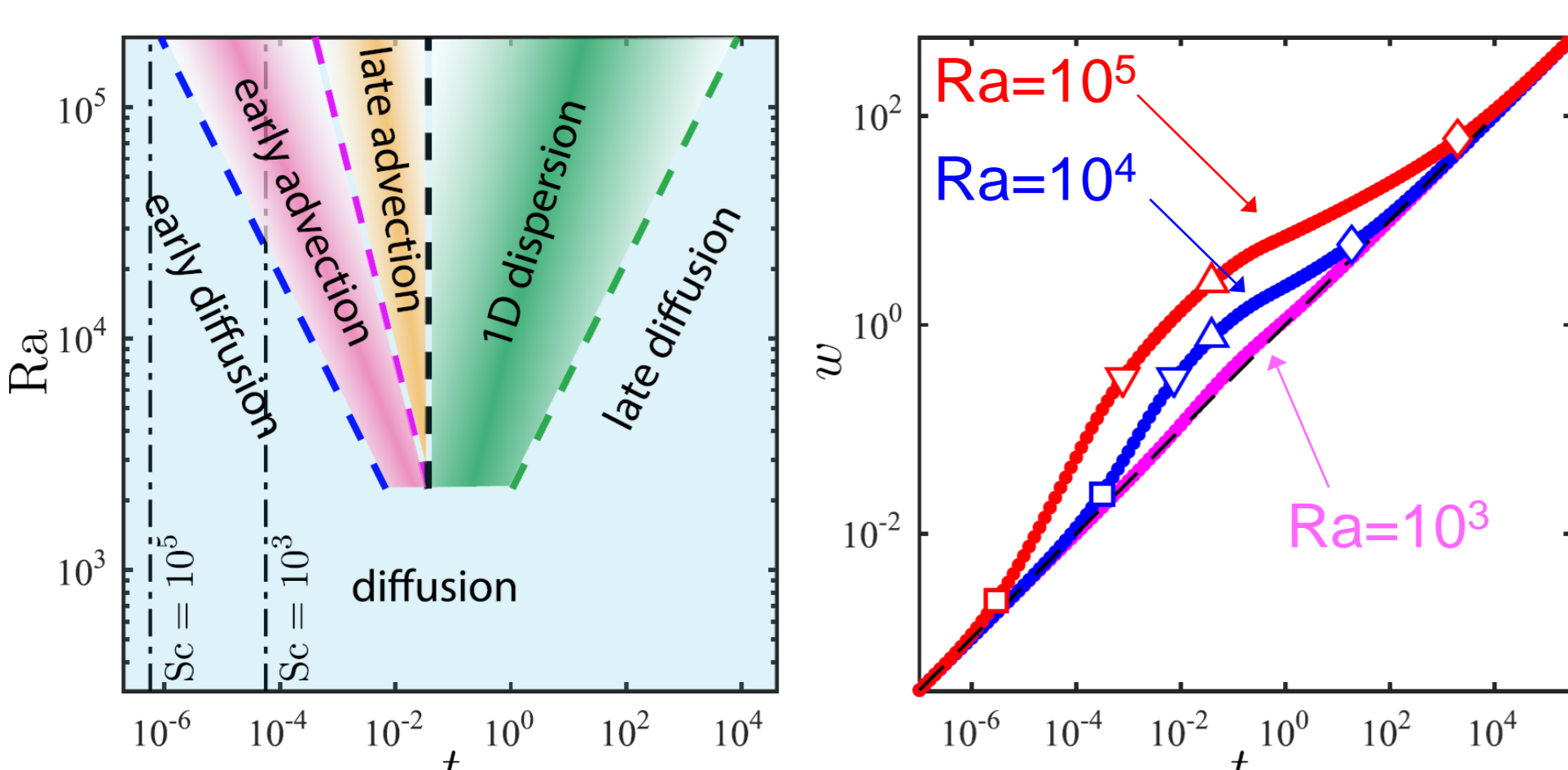
$$\max(u_x) \simeq \max(u_y) \simeq 2.2 \times 10^{-5} \frac{Ra}{\sqrt{t}}$$

Negligible transverse flow for

$$t \geq 4.9 \times 10^{-10} Ra^2$$

Conclusion

5 asymptotic regimes



- Buoyancy vanishes for $Ra \lesssim 10^3$
- Early diffusion and early advection visible for $Ra \geq 10^4$ and $t \leq 10^{-3}$
- The impact of transverse flow on solute transport remains moderate for $Ra \leq 10^5$
=> It can be neglected in most microfluidic experimental applications.