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► **To cite this version:**

Zelha Tunç-Pekkan, Özlem Kayıtmaz. Spontaneous noticing and responding to students' non-conventional fractional thinking. Thirteenth Congress of the European Society for Research in Mathematics Education (CERME13), Alfréd Rényi Institute of Mathematics; Eötvös Loránd University of Budapest, Jul 2023, Budapest, Hungary. hal-04401577

**HAL Id: hal-04401577**

**<https://hal.science/hal-04401577>**

Submitted on 17 Jan 2024

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# Spontaneous noticing and responding to students' non-conventional fractional thinking

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*Noticing and responding to children's thinking are necessary skills for teachers and important part of professional awareness. This study investigated preservice teachers (PSTs)' noticing of students' unconventional fractional thinking and following orchestration of online classroom discussions. There were 10 PSTs who taught 4th-5th-6th grade mathematics in 8-week Online Laboratory School (OLS) and 24 fraction related lessons (video-recordings) were analysed. Findings revealed that most common unconventional student thinking were in "operations with fractions" and "representation of fractions on the number line" categories. PSTs often realized those instances in the moment. They typically asked students who had shared their thinking closed or open-ended questions or asking a general question to the other students. Some responses stimulated misconceptions in students, and analysis of lesson plans revealed that some problem statements had poor construction.*

*Keywords: Mathematics instruction, preservice teachers, fractions.*

## Introduction

Fractions as a topic is one of the most difficult subjects in mathematics for students especially in middle schools (Siegler et al., 2013). Knowledge of fractions is correlated with especially algebra and learning fractions is so important to understand related future subjects and to support mathematical competence (National Mathematics Advisory Panel, 2008).

Fractions in Turkey constitutes 30 percent of middle school mathematics curriculum (Ministry of National Education [MEB], 2013). If teachers determine difficulties and unconventional student thinking in fractions and define a teaching practice based on these, learning of fractions may take place on a conceptual level (Lestiana et al., 2016). Therefore, it is important that teachers or future teachers identify unconventional student thinking in the moment and gain skills of how to handle it.

In this study, we first aimed to determine 4th-5th-6th students' unconventional thinking in fractions that they demonstrated in video recorded online classes. We examined how PSTs perceived and orchestrated these situations during the lesson implementations in their practicum. In relation, we investigated possible causes of these instances. In this research, the main research question is: How do preservice teachers attend, respond, and orchestrate the non-conventional thinking in fractions during lesson implementations?

## Noticing, responding skills of preservice teachers

For effective teaching, it is necessary to keep students' thinking central to the instruction, attending to their thinking, interpreting, and responding accordingly when in the moment teaching takes place (Stocker et al., 2017). Jacobs et al. (2010) introduced professional noticing as: attending to children's thinking (i.e., identifying mathematically significant instances in children's thinking), interpreting (i.e., making sense of students' mathematical thinking), and responding (i.e., deciding how to respond students and what to do as a teacher). They claim that all these skills are interrelated.

Teachers' noticing students' mathematical thinking and making in the moment decision for whole class discussion are complex processes for researchers to frame and investigate (Kooloos et al., 2022). Historically, we observe changes in research focuses for unpacking this complex process. For example, patterns of teachers move, such as Initiation, Response and Evaluation (IRE), (Mehan, 1979) were introduced and these interactions are still commonly observed in mathematics classes. Van Zee and Minstrell's (1997) exploration of 'reflective toss' was an addition to IRE interaction—a pattern that consists of a student statement, teacher question, and additional student statements — and it changes the purpose of interaction which may not be evaluative anymore. Bishop et al. (2016) described teachers moves through the lens of 'responsiveness' where students and teachers "mutually acknowledge, take up, and reflect an awareness of student thinking" (p. 1173). Regardless of the opportunities provided with the teachers move for the students learning, we as researchers look for patterns to understand the complexity of teaching and interaction that we observe in classrooms.

Besides the investigations of patterns related to teachers moves centred around children's thinking, a specific methodology outside of the immediate classroom environment help us understand this complex phenomenon. Stockero et al.'s (2017) introduced 'high-potential instances of student thinking' which are used to unpack how teachers shape meaningful discourse in their practice. Researchers themselves conceptualized these high potential instances and they were proved to be useful contexts for Scenario Interviews (Stockero et al., 2017). Our conceptualization of unconventional student thinking serves similar purpose to 'high potential instances of student thinking' that they need to be addressed as an important asset for teaching and learning. While we agree the benefits of the Scenario Interviews especially for understanding how teachers 'interpret', we also think that analysing instances of teachers moves recorded during their teaching is closer to the reality. Therefore, analysing videotapes of real classroom teaching was a necessity in our work.

Llinares (2013) also adds to this discussion that pedagogical knowledge plays an important role not only noticing children's mathematical thinking but way before for example in lesson planning, when choosing and forming mathematical tasks and identifying students mathematical thinking and using that information when conducting the lessons. Hawthorne (2016) similarly concluded that one of the two teachers in the study had 'differing degrees of professional noticing correlated with the teachers' respective lesson-planning practices.' How noticing students' thinking can be improved as a skill is a new area. Especially in teacher education that is also a central question. Tekin-Sitrava et al. (2021) investigated different levels of noticing skills of a group of preservice teachers selected to study at a top achieving university with a pre-and-post design assessment. They indicated that as they gained teaching practice experience over a year, PSTs' responding skills to students' strategies improved. The result is interesting that the task used in the pre-and-post assessment was related to fraction division and during this time the PSTs did not take any course specific to fraction knowledge.

Research on noticing skills usually focus on phases of noticing and it is mostly independent of the topic such as fractions. There is few research focusing on teachers moves on students' mathematics such as mathematical argumentation (Conner et al., 2014). As Ivars et al. (2020) suggested research designs that use specific mathematics domains may contribute to the field and how pre-service teachers develop or not develop noticing, interpreting, and responding skills in relation to specific domains should be investigated.

## **Methods**

### **Context**

Online Laboratory School (OLS) was established during COVID to provide internship experiences for pre-service teachers (Tunç-Pekkan & Taylan, 2022). OLS functions as a virtual school with classes and PSTs acted as fully responsible classroom teachers. Supervisors supported PSTs during planning by giving feedback on lesson plans, observed and gave support/feedback during lesson implementations and discussed the teaching practices and student thinking after lesson implementations. OLS gave PSTs the opportunity to teach online mathematics lessons to public school students (usually with low socio-economic status) from all over Turkey. The OLS lasted for 8 weeks in the Spring 2021 semester. In this process, 10 PSTs worked as a team and planned and implemented mathematics lessons to 4th, 5th, and 6th grade students.

In this process, every week PSTs as a team prepared lesson plans suitable for the learning goals of that week. Weekly planning meetings were held for each grade level and the lesson plans were examined and supervisors gave three sets of feedbacks for the improvement of the lessons.

### **Data**

The data is archived 24 video recordings of the 4th (2 sections, 3 weeks, 12 lessons), 5th (2 sections, 2 weeks, 8 lessons), and 6th (1 section, 2 weeks, 4 lessons) grade fraction lessons. “Blackboard Collaborate Ultra” was used as the learning management system. Each lesson lasted 40 minutes and they were video recorded.

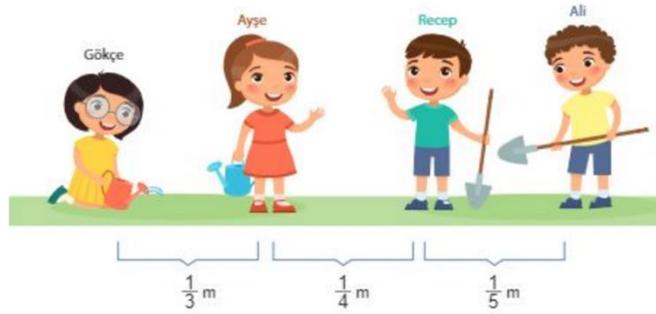
### **Data analysis and frameworks**

Content analysis and grounded theory were used in this study. To analyse the data, the video recordings of the 4th, 5th and 6th grade fractions lesson were watched by researchers and transcribed specifically decided video clips. The following procedure in Table 1 has been applied while analysing data. Grounded theory, which is used for this research, is a general methodology for developing a theory that is grounded in data and data is systematically gathered and analysed (Cohen et al., 2007). First author was the founder and coordinator of OLS, and second author was one of the PSTs benefited from OLS.

### **Data analysis procedure**

Step 1. All the video recordings of 24 lessons were watched by the researchers. The cases (mathematical incidents) where students had unconventional mathematical thinking were noted by using the literature and researcher created framework.

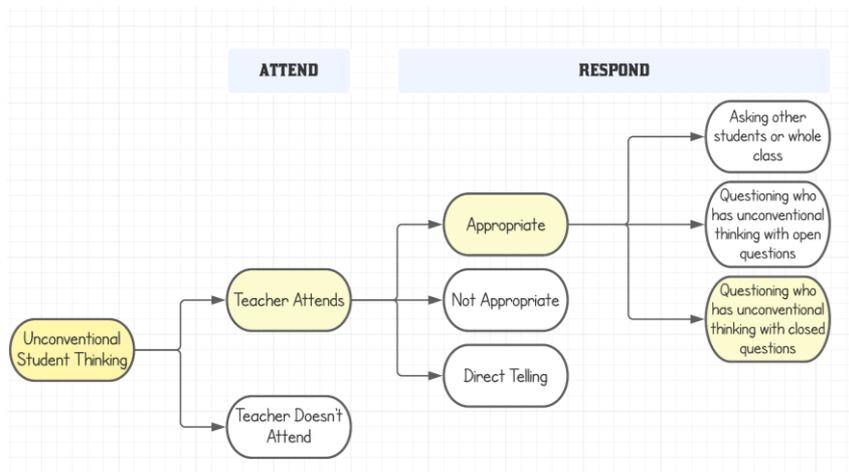
Step 2. After determining the unconventional student thinking, the video recordings focusing on those student thinking were transcribed for understanding how the PSTs responded to these situations (as an example see Figure 1 and Example Case below). Then a framework is created in the analysis process to be used for categorizations of all PSTs in the moment responses and it is applied to all the mathematical instances derived in Step 1 and revised, if necessary, see Figure 2.



**Figure 1: A problem for fraction addition**

Example Case: In Figure 1, the distances of Gökçe, Ayşe, Recep and Ali standing side by side to each other are given. Calculate the distance between Gökçe and Recep.

- 1 PST: Okay, who would like to help me with this question? Would you like to help A.?
- 2 Student: Ok.
- 3 PST: Now, how did you calculate the distance between Gökçe and Recep?
- 4 Student: Teacher, I added them together and found the result as 2 over 7 ( $2/7$ ).
- 5 PST: Hmm...now where is my denominator my dear?
- 6 Student: Teacher...eee...in the denominator for Gökçe and Ayşe 3 and for Ayşe and Recep, it is 4.
- 7 PST: Yes...for me to add them, what should be the denominators?



**Figure 2: Analysis of PST's actions for Example Case (yellow highlighted)**

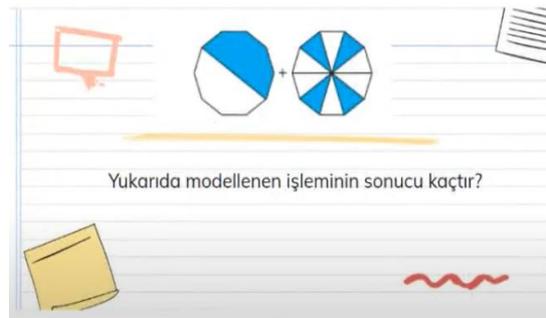
The unconventional thinking was adding numerator and denominators of two fractions instead of finding the common denominator and then adding. The PST attended the unconventional thinking (see Line 3 in Excerpt). She questioned the student with closed ended question (Line 5 & 7 in Excerpt): she focused on having the denominators being same for addition of two fractions. However, PST did not check the students what  $1/3$  m and  $1/4$  m means quantitatively (Steffe & Olive, 2010) as a fraction and when two fractions are added the result is close to a half meter but  $2/7$  was a lot smaller and it is almost  $1/3$ .

## Results

There were 29 unconventional thinking instances identified within the 24 fraction lesson videos (see Table 1). We speculated the possible causes of students' thinking by investigating videos and using fraction literature to categorize students' conceptions and investigating written lesson plans used for those 24 lessons.

**Table 1: The first four frequent unconventional thinking categories out of 29 instances**

Students' Unconventional Thinking	Frequency			
	4 <sup>th</sup>	5 <sup>th</sup>	6 <sup>th</sup>	Total
Grade Category	4 <sup>th</sup>	5 <sup>th</sup>	6 <sup>th</sup>	Total
Operations with fractions	0	8	2	10
Representation on the number line	5	0	0	5
Reference unit confusion	5	0	0	5
Fraction reading & writing (Turkish)	3	1	0	4



**Figure 3: Nearpod Activity**

Example for 'Teacher Does not Attend' (see Figure 2) Category and 'Reference Unit Confusion' (see Table 1): This was a Nearpod activity (see Figure 3) where students' individual drawings and answers could be seen by the teacher. Problem statement was: 'What is the result of the operation represented with the drawings?' (Yukarıda modellenen işlemin sonucu kaçtır?) Students were asked to provide a result for the sum of two fractions represented with the blue-coloured parts of two decagons. The PST's expected result was  $1/2 + 4/10 = 9/10$ . But one student found  $5/7$ . Only counting blue unequal parts and writing 5 to numerator and 7 to the denominator. PST did not attend to this answer or respond. She followed up a student who produced a correct answer. Reason for the student's unconventional thinking was student's different conception in relation to unit (Steffe & Olive, 2010). Child was counting with whole numbers. Units were not equal, but student considered only counting blue parts (Reference unit confusion category, in Table 1 and 2).

According to the results, in case unconventional thinking is realized and attended, PSTs follow three practices: appropriate responding, not appropriate responding, or direct telling (see Figure 2). The first three frequent responding methods of PSTs for 29 unconventional thinking instances are given in Table 2.

**Table 2: The first three frequent responding methods of PSTs for 29 unconventional thinking instances**

Unconventional student thinking category (total 29 instances)	Attend	PSTs' responding method.
Operations with fractions (10)	Attend (6) Does not Attend (4)	<b>-Asking other students (2)</b> -Questioning the student's thinking-asks closed-ended questions (2) and asking open questions (1) -Direct Telling (1) -Does not address unconventional thinking (4)
Representation on the number line (5)	Attend (5)	<b>-Asking other students (3)</b> -Doesn't guide the students appropriately (1) - Questioning the student's thinking -asking open-ended questions (1)
Reference unit confusion (5)	Attend (5)	<b>-Asking other students (3)</b> -Questioning the student's thinking-asks closed-ended questions (1) and asking open-ended questions (1)

## Discussion and conclusion

The results indicated that students most frequently have unconventional thinking in “operations with fractions” (10/29) and pre-service teachers have difficulty in noticing these instances, they noticed only 6 of them. The students may have unconventional thinking in operations with fractions because they learned it procedurally. Likewise, pre-service teachers may also have procedural knowledge so that they have difficulty in noticing these instances. The investigation of lesson plans showed that there are instances of poor problem construction which are related to PSTs pedagogical content knowledge of fractions. Students might have difficulty due to these errors in the problem statements. When we investigated the lesson plans, we also observed that the instructions were not clear in the problems, especially ‘unit’ of fraction and ‘whole’ were implicit for the students but not for PSTs (see Figure 3). Therefore, implementing those problems in the lessons resulted in more problematic situations with which PSTs had more and more difficulty first to notice and secondly to guide. We can conclude that PSTs' limited pedagogical knowledge in fractions prevented them from preparing problem statements for their intended learning goals. Fraction scheme theory and educating PSTs for the developmental stages of fraction knowledge might help this situation to improve. We can suggest

that even before practicum, there should be focused programs to enhance PSTs' pedagogical knowledge in specific and important areas such as fractions (Llinares, 2013; Hawthorne, 2016; Bishop et al., 2016). Such programs will help them to gain more experience so developing better noticing and responding skills.

The results indicated that when a PST encounters and notices an unconventional student thinking, he/she either asks the other students' opinions or examines that student's thinking and tries to eliminate the unconventional thinking. This might be understandable since in our method courses and lesson planning meetings in OLS, PSTs were encouraged to model accountable talk moves in their teaching (Michaels et al., 2010), so including other students which could be conceptualized as 'reflective toss' (Van Zee & Minstrell, 1997) were expected. On the other hand, asking other students was the effortless move a PST could choose. It could be interesting to investigate how PSTs or teachers can use that information to orchestrate a discussion centering around the child's thinking and learning goal of the lesson. For this study, we only investigated the instances which were relatively short in time when they occurred and our analysis was not projected for the whole class period.

We speculate that it is more likely that the unconventional student thinking will be eliminated, and new unconventional thinking will not occur if the PSTs directly ask questions to the student with the unconventional thinking. Helping students to think more deeply about their answers and asking probing questions to find out more may be a good strategy for teachers as a responding action.

This study was conducted during COVID pandemic. While having OLS and providing internship experiences for PST were abundant, COVID slowed the teaching and learning processes of all PSTS. This might be a limitation of the study that unlike normal classrooms, our observations were also based on online teaching. For future studies, Scenario Interviews (Stockero et al., 2017) could be used for designing studies focusing on fractions. In addition, studies focusing on enhancing PSTs noticing, interpreting, and responding skills particular to fractions can be designed such as Scenario Interviews (Stockero et al., 2017).

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