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***Abstract*—Kalman filter (KF) is a priori unsuitable for the estimation from sample covariance matrices as they cannot be formulated analytically as a function of state parameters to be estimated. In this work, we propose a novel KF adapted to sample covariance matrices under the unconditional signal model. It is evaluated on simulated data representative of a dynamic radio astronomy framework, considering multiple uncorrelated sources and Gaussian noise. The results show that our method is capable of effectively tracking moving sources in complex scenes with greater accuracy than a KF regularized in a standard way, i.e., without proper formalization of the noise model.**

***Index Terms*—Estimation, Kalman filter, Visibility matrix, Radio astronomy**

I. Introduction

The design and use of techniques for estimating a dynamic state from multiple observations is fundamental in a plethora of applications, such as robotics, tracking systems, guidance, and navigation [1] [2] [3] [4]. For a linear dynamic state and measurement system, the Kalman filter (KF) provides the best linear unbiased estimator in terms of mean squared error in a recursive form, under certain conditions of non-correlation of the state and measurement noise. The most common solution for nonlinear measurement systems is to resort to a linearization of the system, leading to what is called the extended Kalman filter (EKF). In the presence of strong non-linearities, the EKF loses its optimality, and other more robust solutions (such as sigma-point filters [5] [6]) have been introduced recently. In all cases, the main assumption is a perfect knowledge of the system: (i) known state and measurement models, as well as their parameters, (ii) known inputs, and (iii) known noise statistics (i.e., first and second-order moments).

The novelty of this communication resides in addressing a scenario where the measurement model is unknown. Particularly when the measurement is represented by a complex empirical covariance matrix (Sample Covariance Matrix, SCM) based on a finite number of samples. In such cases, although each individual sample may adhere to a linear parametric model, the finite horizon SCM cannot be analytically expressed in terms of the parameters of this linear model which a priori makes it unsuitable for incorporation into a state/measurement model. In fact, this lack of analytical formulation can be circumvented in the case of instantaneous linear observations from multiple sources in the presence of additive noise (stochastic observation model), when the sources are uncorrelated and the state to be estimated is the power of the sources.

Under the assumption of a deterministic dynamic state model (no state noise) and i.i.d. instantaneous observations, we propose a statistical linear fitting for the measurement model, allowing the SCM to be expressed in terms of the state. This linear fitting introduces an additive noise, equivalent to the measurement noise, and whose covariance matrix depends on the current state, a non-standard case for a measurement model. We first derive a general form for the covariance of the linear fitting noise and explicit it for Gaussian random vectors.

It is shown that an implementable version of this linearly fitted system allows for an accurate estimation of the dynamic state of the system if the initial state estimator is sufficiently accurate. In addition, the robustness to initialization is investigated by considering a degraded initialization. A lower bound on the achievable MSE matrix is given when considering the measurement covariance matrix perfectly known.

This new KF formalization for complex SCMs through statistical linear fitting is applied to dynamic imaging in a radio astronomical context. The series of images to be estimated represents dynamic celestial sources that emit incoherent, monochromatic signals. Observations acquired over time are complex SCMs of the signals received by the antenna array.

II. Underlying Measurement Model

At each instant t_k, we consider the observation of the linear mixture of Q sources, denoted $\mathbf{s}_k \in \mathbb{C}^{Q\times 1}$, in the presence of additive noise $\mathbf{n}_k$:

$$\mathbf{z}_k = \mathbf{A}_k\mathbf{s}_k + \mathbf{n}_k, \tag{1}$$

where $\mathbf{A}_k \in \mathbb{C}^{M\times Q}$ is the system response matrix, $\mathbf{s}_k$ and $\mathbf{n}_k$ are complex circular centered random vectors, and $\mathbf{z}_k$ is the underlying observation vector. The covariance matrix of the observations is denoted $\mathbf{C}_{\mathbf{z}_k}$,

$$\mathbf{C}_{\mathbf{z}_k} = \mathbb{E}\left[\mathbf{z}_k\mathbf{z}_k^H\right] = \mathbf{A}_k\mathbf{C}_{\mathbf{s}_k}\mathbf{A}_k^H + \mathbf{C}_{\mathbf{n}_k},\ \mathbf{C}_{\mathbf{s}_k} = \mathbb{E}[\mathbf{s}_k\mathbf{s}_k^H]. \tag{2}$$

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However, in some applications, estimation is based only on the sample covariance matrix (SCM) of given observations. In that case, it is commonly assumed that the observation vector $\mathbf{z}_k$ is sampled N times within a short time interval around t_k during which the sources are considered static, resulting in N i.i.d samples $\mathbf{z}_k(n)$ from which the SCM is computed as:

$$\hat{\mathbf{C}}_{\mathbf{z}_k} = \frac{1}{N}\sum_{n=1}^{N} \mathbf{z}_k(n)\mathbf{z}_k(n)^H, \tag{3}$$

which converges in probability to $\mathbf{C}_{\mathbf{z}_k}$ as N tends to infinity. Assuming that all $\mathbf{n}_k$ and $\mathbf{s}_l$ for $k, l \geq 1$ are independent and that the sources s_k^q, $q \geq 1$, are mutually independent, the goal is to estimate instantaneous intensities $\mathbb{E}[\mathbf{s}_k\mathbf{s}_k^H] = diag\,(\mathbf{x}_k)$ based on the SCM at the time slots $\{t_1, \cdots, t_k\}$ when the intensities $\mathbf{x}_k$ are governed by a dynamic model. The N observation samples are obtained conditionally on a specific realization of the state $\mathbf{x}_k$, which may be stochastic in the general case. However, in this communication we restrict ourselves to the case where the state $\mathbf{x}_k$ is deterministic (no additive state noise).

III. Linear fitting of the measurement model

In order to benefit from the performance improvement brought by Kalman filtering regarding the estimation of $\mathbf{x}_k$, it is necessary to resort to a dynamic system described by state and measurement equations, ideally linear, that is for deterministic state estimation:

$$\mathbf{x}_k = \mathbf{F}_{k-1}\mathbf{x}_{k-1} \tag{4a}$$
$$\mathbf{y}_k = \mathbf{H}_k\mathbf{x}_k + \mathbf{v}_k. \tag{4b}$$

The evolution matrices $\mathbf{F}_{k-1}$ and the observation matrix $\mathbf{H}_k$ are assumed to be known. One can show that, in order to obtain a KF yielding the lowest minimum MSE matrix, the complex empirical covariance matrix should be considered via its real and imaginary parts. Hence, we then define $\mathbf{y}_k$ as:

$$\mathbf{y}_k = \begin{pmatrix} \mathfrak{Re}\left\{\mathbf{vec}\left(\hat{\mathbf{C}}_{\mathbf{z}_k}\right)\right\} \\ \mathfrak{Im}\left\{\mathbf{vec}\left(\hat{\mathbf{C}}_{\mathbf{z}_k}\right)\right\} \end{pmatrix}. \tag{5}$$

In a non-asymptotic regime, when working with the finite horizon SCM $\hat{\mathbf{C}}_{\mathbf{z}_k}$, the validity of the measurement model (4b) is compromised. Consequently, our approach involves seeking a linear model that can effectively capture the properties of the SCM. For the measurement matrices, the asymptotic relationship (2) linking $\mathbf{C}_{\mathbf{z}_k} = \mathbb{E}[\hat{\mathbf{C}}_{\mathbf{z}_k}]$ and $\mathbf{x}_k$ suggests taking $\mathbf{H}_k$ of the form:

$$\mathbf{H}_k = \left[\begin{pmatrix} \mathfrak{Re}(\mathbf{a}_{k,1}^* \otimes \mathbf{a}_{k,1}) \\ \mathfrak{Im}(\mathbf{a}_{k,1}^* \otimes \mathbf{a}_{k,1}) \end{pmatrix} \cdots \begin{pmatrix} \mathfrak{Re}(\mathbf{a}_{k,Q}^* \otimes \mathbf{a}_{k,Q}) \\ \mathfrak{Im}(\mathbf{a}_{k,Q}^* \otimes \mathbf{a}_{k,Q}) \end{pmatrix}\right], \tag{6}$$

where $\mathbf{a}_{k,q}$ are the column vectors of the matrix $\mathbf{A}_k$ (1). The statistical linear fitting noise is then defined as

$$\mathbf{v}_k \equiv \mathbf{y}_k - \mathbf{H}_k\mathbf{x}_k. \tag{7}$$

This defines a linear discrete state space (LDSS) model for which a Kalman Filter is derived in the following section.

IV. Identification of first and second order statistics of the linear fitting noise

The next step consists of evaluating the first and second-order statistics of the linearization noise $\mathbf{v}_k$ involved in KF recursion. Since $\mathbf{x}_k$ is deterministic, the linear model (4b) is characterized by the identities

$$\mathbb{E}[\mathbf{v}_k] = \mathbb{E}[\mathbf{y}_k - \mathbf{H}_k\mathbf{x}_k] = \mathbb{E}[\mathbf{y}_k] - \mathbf{H}_k\mathbf{x}_k \tag{8a}$$
$$\mathbf{C}_{\mathbf{v}_k} = \mathbf{C}_{\mathbf{y}_k - \mathbf{H}_k\mathbf{x}k} = \mathbf{C}_{\mathbf{y}_k} \tag{8b}$$

A. Mean of the linear fitting noise

Since $\hat{\mathbf{C}}_{\mathbf{z}_k}$ is an unbiased estimate of $\mathbf{C}_{\mathbf{z}_k}$, one has

$$\begin{aligned}\mathbb{E}[\mathbf{y}_k] &= \left[\mathfrak{Re}\left\{\mathbf{vec}\,(\mathbf{C}_{\mathbf{z}_k})^T\right\}, \mathfrak{Im}\left\{\mathbf{vec}\,(\mathbf{C}_{\mathbf{z}_k})^T\right\}\right]^T \\ &= \mathbf{H}_k\mathbf{x}_k + \mathbf{v}_k^a,\end{aligned} \tag{9}$$

where

$$\mathbf{v}_k^a = \left[\mathfrak{Re}\left\{\mathbf{vec}\,(\mathbf{C}_{\mathbf{n}_k})^T\right\}, \mathfrak{Im}\left\{\mathbf{vec}\,(\mathbf{C}_{\mathbf{n}_k})^T\right\}\right]^T. \tag{10}$$

Which from (7), implies that $\mathbb{E}[\mathbf{v}_k] = \mathbb{E}[\mathbf{y}_k] - \mathbf{H}_k\mathbf{x}_k$ and therefore

$$\mathbb{E}[\mathbf{v}_k] = \mathbf{v}_k^a. \tag{11}$$

B. Covariance of the linear fitting noise

As the state $\mathbf{x}_k$ is deterministic, one gets $\mathbf{C}_{\mathbf{v}_k} = \mathbf{C}_{\mathbf{y}_k}$ with

$$\mathbf{C}_{\mathbf{y}_k} = \frac{1}{4N}\mathbf{C}\begin{pmatrix} \mathfrak{Re}\left\{\mathbf{z}_k^* \otimes \mathbf{z}_k\right\} \\ \mathfrak{Im}\left\{\mathbf{z}_k^* \otimes \mathbf{z}_k\right\} \end{pmatrix}, \tag{12}$$

where $\otimes$ denotes the Kronecker product, such that $\mathbf{z}_k^* \otimes \mathbf{z}_k = \mathbf{vec}(\mathbf{z}_k\mathbf{z}_k^H)$. Equivalently, one can write

$$\mathbf{C}_{\mathbf{y}_k} = \frac{1}{4N}\mathbf{V}\mathbf{C}_{\psi_k}\mathbf{V}^H,\ \psi_k = \begin{pmatrix} \mathbf{z}_k^* \otimes \mathbf{z}_k \\ (\mathbf{z}_k^* \otimes \mathbf{z}_k)^* \end{pmatrix}, \tag{13a}$$

where

$$\mathbf{C}_{\psi_k} = \begin{bmatrix} \mathbf{C}_{\mathbf{z}_k^* \otimes \mathbf{z}_k} & \mathbf{C}_{\mathbf{z}_k^* \otimes \mathbf{z}_k, \mathbf{z}_k \otimes \mathbf{z}_k^*} \\ \mathbf{C}^*_{\mathbf{z}_k^* \otimes \mathbf{z}_k, \mathbf{z}_k \otimes \mathbf{z}_k^*} & \mathbf{C}^*_{\mathbf{z}_k^* \otimes \mathbf{z}_k} \end{bmatrix} \tag{13b}$$

and

$$\mathbf{V} = \begin{bmatrix} \mathbf{I}_{M^2} & \mathbf{I}_{M^2} \\ -j\mathbf{I}_{M^2} & j\mathbf{I}_{M^2} \end{bmatrix}. \tag{13c}$$

C. Case of Gaussian random vectors

For complex circular Gaussian sources $\mathbf{s}_k \sim \mathcal{CN}(\mathbf{0}, diag(\mathbf{x}_k))$ and Gaussian noise $\mathbf{n}_k$, the underlying observations are also centered complex circular Gaussian, i.e. $\mathbf{z}_k \sim \mathcal{CN}(\mathbf{0}, \mathbf{C}_{\mathbf{z}_k})$. Hence (13b) simplifies from identities:

$$\mathbf{C}_{\mathbf{z}_k^* \otimes \mathbf{z}_k} = \mathbf{C}_{\mathbf{z}_k}^T \otimes \mathbf{C}_{\mathbf{z}_k}, \tag{14a}$$
$$\mathbf{C}_{\mathbf{z}_k^* \otimes \mathbf{z}_k, \mathbf{z}_k \otimes \mathbf{z}_k^*} = \left(\mathbf{C}_{\mathbf{z}_k}^T \otimes \mathbf{C}_{\mathbf{z}_k}\right)\mathbf{P} \tag{14b}$$

where

$$\mathbf{P} = \sum_{m=1}^{M}\sum_{m'=1}^{M} (\mathbf{e}_m \otimes \mathbf{e}_{m'})\,(\mathbf{e}_{m'} \otimes \mathbf{e}_m)^T, \tag{14c}$$

and $\mathbf{e}_m$, $1 \leq m \leq M$, are vectors of the canonical basis of $\mathbb{R}^M$. Finally, $\mathbf{C}_{\mathbf{y}_k}$ takes the form

$$\mathbf{C}_{\mathbf{y}_k} = \frac{1}{4N}\mathbf{V}\begin{bmatrix} \mathbf{C}_{\mathbf{z}_k}^T \otimes \mathbf{C}_{\mathbf{z}_k} & \left(\mathbf{C}_{\mathbf{z}_k}^T \otimes \mathbf{C}_{\mathbf{z}_k}\right)\mathbf{P} \\ \left(\left(\mathbf{C}_{\mathbf{z}_k}^T \otimes \mathbf{C}_{\mathbf{z}_k}\right)\mathbf{P}\right)^* & \left(\mathbf{C}_{\mathbf{z}_k}^T \otimes \mathbf{C}_{\mathbf{z}_k}\right)^* \end{bmatrix}\mathbf{V}^H. \tag{15}$$

Thus, the KF taken into account mean and covariance matrix of the linear fitting noise calculated in (11) and (15) reads

Algorithm 1 Kalman Filter recursion

Inputs : $\hat{\mathbf{x}}_{k-1|k-1}$, $\mathbf{P}_{k-1|k-1}$, $\mathbf{y}_k$
Outputs: $\hat{\mathbf{x}}_{k|k}$, $\mathbf{P}_{k|k}$

Prediction :
$\mathbf{P}_{k|k-1} = \mathbf{F}_{k-1}\mathbf{P}_{k-1|k-1}\mathbf{F}_{k-1}^H$
$\hat{\mathbf{x}}_{k|k-1} = \mathbf{F}_{k-1}\hat{\mathbf{x}}_{k-1|k-1}$
Update :
$\hat{\mathbf{y}}_{k|k-1} = \mathbf{H}_k\hat{\mathbf{x}}_{k|k-1} + \mathbf{m}_{\mathbf{v}_k}$, where $\mathbf{m}_{\mathbf{v}_k}$ is given by (11)
$\mathbf{S}_{k|k} = \mathbf{H}_k\mathbf{P}_{k|k-1}\mathbf{H}_k^H + \mathbf{C}_{\mathbf{v}_k}$, where $\mathbf{C}_{\mathbf{v}_k}$ is given by (15)
$\mathbf{K}_k = \mathbf{P}_{k|k-1}\mathbf{H}_k^H\mathbf{S}_{k|k}^{-1}$
$\mathbf{P}_{k|k} = (\mathbf{I} - \mathbf{K}_k\mathbf{H}_k)\mathbf{P}_{k|k-1}$
$\hat{\mathbf{x}}_{k|k} = \hat{\mathbf{x}}_{k|k-1} + \mathbf{K}_k(\mathbf{y}_k - \hat{\mathbf{y}}_{k|k-1})$

V. Application in Radio Astronomy

We consider a radio interferometric array similar to the Very Large Array (VLA), located in New Mexico [7]. The VLA consists of a three-branch interferometric array, as shown in Fig 1a, with a total of $p = 3 \times 9$ antennas. This instrument is designed to capture signals from celestial sources, resulting in a composite signal due to the superposition of multiple astrophysical sources. The image we are interested in has Q pixels, representing a discretization of the sky. Each pixel corresponds to a specific direction of arrival (DOA) from which the signal intensity needs to be estimated.

Let $s_q(t_k)$ denote the signal emitted by the q-th pixel at a time slot t_k. The signals $s_q(t_k)_{1\leq q\leq Q}$ are assumed to be mutually independent and identically distributed (i.i.d.), circular complex Gaussian signals, which are concatenated in $\mathbf{s}(t_k) \in \mathbb{C}^{\mathbb{Q}}$. Here, $s_q(t_k) \sim \mathcal{CN}(0, (\mathbf{x}_k)_q)$, where $(\mathbf{x}_k)_q$ represents the intensity of the q-th pixel at time slot t_k. The objective is to estimate these intensities $\mathbf{x}_k$ from the measured visibilities at each time slot t_k, which will be achieved by the newly introduced KF.

The measurement noise originating from the interferometer is modeled as a centered i.i.d. complex Gaussian signal. Noises are also concatenated in a $M \times 1$ dimensional vector denoted as $\mathbf{M}(t_k) \in \mathbb{C}^p$. Under the narrowband condition, the signal received at the j-th antenna can be represented by (1) [8]

$$\mathbf{z}_k = \mathbf{A}_k\mathbf{s}_k + \mathbf{n}_k = \sum_{q=1}^{Q}\mathbf{a}_q(t_k)s_q(t_k) + \mathbf{n}_k, \tag{16}$$

where $\mathbf{A}_k = \exp\left(-j2\pi/\lambda\boldsymbol{\mathcal{R}}\boldsymbol{\mathcal{L}}^T\right)$ is the steering vectors matrix, $\boldsymbol{\mathcal{R}}$ is the $M \times 2$ matrix of antenna positions, and $\boldsymbol{\mathcal{L}}$ is the $Q \times 2$ matrix of source direction vectors. At each t_k, the received signal is composed of N i.i.d. samples $\{\mathbf{z}(n)_{t_k}\}_{1\leq n\leq N}$ from which the SCM $\hat{\mathbf{C}}_{\mathbf{z}_k}$ (also refered to as astronomical visibilities) is calculated [9].

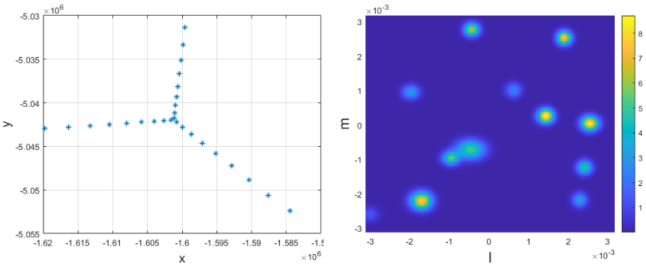

Fig. 1: (a) VLA antennas positions in terrestrial coordinates (each branch is 21km long), and (b) image representing the sky at the initial time $t = 0$

A. Simulations

The considered images have a size of $Q = 128 \times 128$ pixels and consist of ten astrophysical sources modeled by bivariate normal distributions (Figure 1b). At each time slot t_k, a rotation of angle θ_k is applied to the previous image, simulating a dynamic state model. Recall that the main contribution of this study is to statistically characterize the linear fitting noise when the measurements are complex SCMs. While obtaining the mean (11) of the linear fitting noise is trivial, the expression of its covariance matrix is unknown since it depends on the current state to be estimated. The proposed method is therefore compared to a KF in which the measurement noise covariance matrix is fitted in a standard way. In fact, since the matrix is positive definite, it is often parameterized as a diagonal matrix [10]. More precisely, under the assumption of independent and identically distributed (i.i.d.) measurement noise, this matrix takes the form $r \times \mathbf{I}_d$, where the coefficient r is empirically adjusted. In this case, the measurement model is misspecified, and we refer to it as the misspecified Kalman filter (MKF).

B. Initialization and practical implementation

In practice, implementing a KF requires an appropriate initialization of the state $\widehat{\mathbf{x}}_{1|1}$. In our approach, we initially attempt to initialize it using normalized beamforming, as it is commonly used in this context. The estimated image and the resulting KF and MKF performances initialized with the normalized beamforming are presented in Fig 2 and Fig 3a-3b. However, an unbiased estimator $\widehat{\mathbf{x}}_{1|1}$ is of particular interest since this property is conserved by the KF. It can be obtained by a distortionless response filter (DRF). Among the DRFs, the minimum variance distortionless filter (MVDRF)

$$\mathbf{K}_1 = \left(\mathbf{H}_1^H\mathbf{C}_{\mathbf{v}_1}^{-1}\mathbf{H}_1^H\right)^{-1}\mathbf{H}_1^H\mathbf{C}_{\mathbf{v}_1}^{-1} \tag{17}$$

minimizes the covariance and is thus preferred in this work. The MVDRF estimation is constrained by the number of antennas of the interferometric network. In particular, for M antennas the MVDRF is able to efficiently reconstruct images of size $M(M-2)$ pixels. Therefore, we consider reduced images of size $Q = 30 \times 30$ pixels when initializing with the MVDRF. We assess the impact of MVDRF initialization on the performance in Fig 3c-3d.

Since the measurement noise covariance matrix $\mathbf{C}_{\mathbf{v}_k}$ depends on the state $\mathbf{x}_k$, the initialization of the KF requires the estimation of an initial state $\widehat{\mathbf{x}}_1$. In this work, $\widehat{\mathbf{C}}_{\mathbf{v}_1}$ is obtained with a beamforming estimator of $\mathbf{x}_1$ [11] for both the MKF and the KF. At each recursion step, the filter is computed by substituting $\mathbf{C}_{\mathbf{v}_k}$ by an estimate $\widehat{\mathbf{C}}_{\mathbf{v}_k}$ built from the KF prediction $\widehat{\mathbf{x}}^b_{k|k-1}$.

C. Results

The estimates provided by both MKF and KF are shown in Figure 2. It can be observed that the use of the KF visually improves the initial estimation obtained through spatial filtering, while this improvement is not evident in the case of the MKF. These results are further supported by numerical calculations of two reconstruction error measures: the root mean squared error (RMSE) [12] and the normalized cross-correlation (NCC) [13]. The RMSE is sensitive to pixel-wise intensity estimation errors, while the NCC is sensitive to errors in the image structure estimation. The KF demonstrates better performance in both measures. The results are displayed in Figure 3.

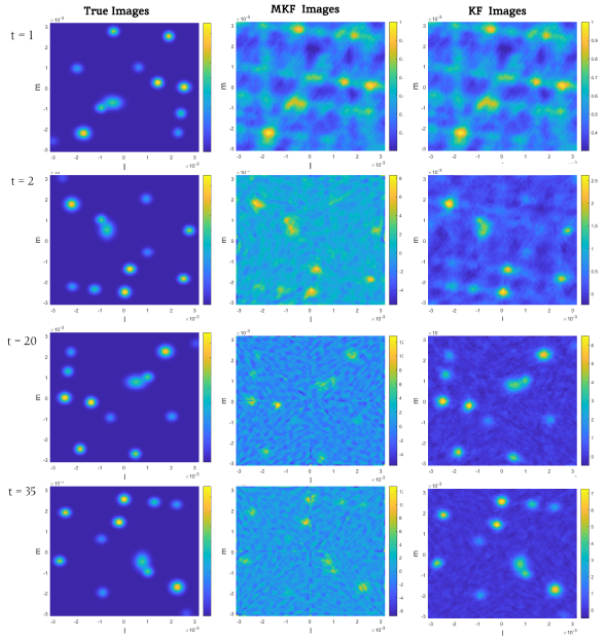

Fig. 2: Estimated images at different time slots. Each row corresponds to a short time interval, with the true image on the left column, the image estimated by the MKF in the middle, and the image estimated by the KF on the right.

It can be observed that the MKF is more sensitive to initialization compared to the KF Fig 3. While the MKF performs poorly when initialized with a degraded estimate obtained through beamforming Fig 3a-3b, it performs reasonably well when initialized with the MVDR Fig 3c-3d. This highlights the importance of choosing an appropriate initialization technique for the MKF to achieve satisfactory results. On the other hand, the KF demonstrates consistently good performance in both cases. This emphasizes the greater robustness and reliability of the KF compared to the MKF.

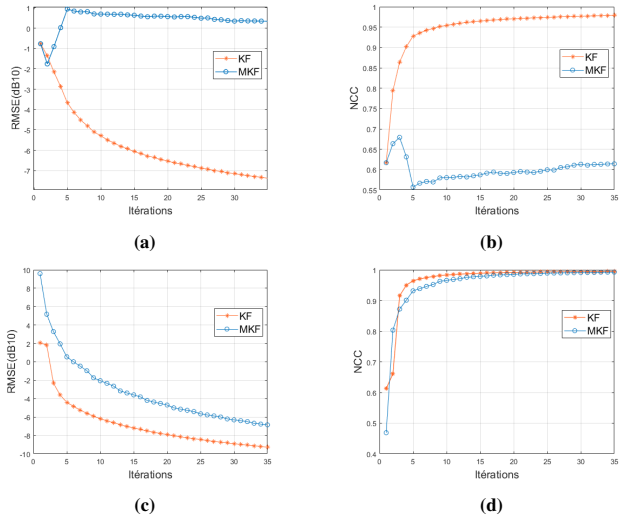

Fig. 3: Evolution of the RMSE and NCC. The first line shows KF and MKF performance with normalized beamforming initialization, while the second line presents performance with MVDR initialization.

The achievable performance of a system can be bounded when assuming perfect knowledge of the state parameters. A lower bound on the RMSE is obtained by substituting the true state vector, $\mathbf{x}_k$, into the covariance matrix $\mathbf{C}_{\mathbf{v}_k}$. The lower bound in Fig 4 provides insights into the minimum level of error that can be expected when the true state is known. Furthermore, it is noteworthy that the KF with the linear fitting noise converges to this lower bound after a few iterations.

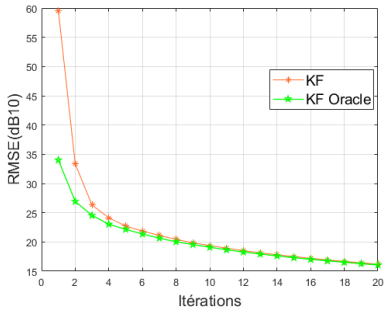

Fig. 4: Performances of the proposed KF with estimated (red) and true (green) $\mathbf{C}_{\mathbf{v}_k}$.

VI. Conclusion

In this paper, we introduced a statistical linear fitting model for the formalization of a Kalman filter when dealing with second-order observations. Our focus was on developing the required calculations to characterize the linear fitting noise in the case of a Gaussian distribution for the sources. The performance of the proposed algorithm is evaluated in a radio astronomy context. Results demonstrate that our approach outperformed a conventional misspecified Kalman filter with a standard model of the measurement noise covariance matrix.

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