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► To cite this version:

Mine Işiksal Bostan, Reyhan Tekin Sitrava, Zeynep Özel, Seçil Yemen Karpuzcu. Teachers' in-the-moment response to student thinking: The case of pattern generalization. Thirteenth Congress of the European Society for Research in Mathematics Education (CERME13), Alfréd Rényi Institute of Mathematics; Eötvös Loránd University of Budapest, Jul 2023, Budapest, Hungary. hal-04398420

HAL Id: hal-04398420

<https://hal.science/hal-04398420v1>

Submitted on 16 Jan 2024

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Teachers' in-the-moment response to student thinking: The case of pattern generalization

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This study aimed to examine the nature of a middle school mathematics teacher's teaching moves after discussions held in an online teacher training platform in the context of pattern generalization. As a part of the two-year research, a design experiment was used as a research design. Thirty-one middle school mathematics teachers with less than 15 years of experience participated in the research study. In this paper, we focused on one teacher, Sasha's responsive teaching in the scope of instructional practices during her classroom teaching. Findings revealed that Sasha was responsive to students' mathematical thinking and pushed them to make sense of the problem, consider other strategies, connect their thinking to symbolic notation, and solve more comprehensive problems. However, the nature of the response varied according to the teaching moves.

Keywords: In-the-moment response, teaching moves, pattern generalization.

Introduction

In the last decades, many researchers have emphasized that one of the most important teacher competencies is to establish the connection between teachers' cognitive abilities (e.g., teacher knowledge) and their practices in the real learning environment (Kaiser et al., 2017). To establish this connection, Jacobs et al. (2010) claimed that teachers should be able to attend to and interpret students' understanding, and to decide how to make instant decisions. Stated differently, Jacobs et al. (2010) argued that teachers need to have noticing skills that enable them to reflect their knowledge into practice. Although teachers' noticing skills cannot be observed directly, it reflects teachers' teaching moves to support and extend children's mathematical thinking within the context of responsive teaching (Jacobs & Empson, 2016). In this research, we examined middle school mathematics teachers' teaching moves after the discussions held in an online teacher training platform in the context of pattern generalization.

Theoretical Framework

Responsive teaching is considered a type of instruction in which teachers make in-the-moment instructional decisions based on students' understanding rather than applying predetermined judgments. To construct such instruction, Jacobs and Empson (2016) presented five teaching moves in which teachers' knowledge and their noticing skills of children's mathematical thinking play an influential and significant role. The initial category of teaching moves is ensuring the student makes sense of the problem. To accomplish this, it is expected that teachers could ask students what they understand from the problem and discuss the context of the problem or a part of the problem with students without changing the mathematical structure of the problem (Jacobs & Empson, 2016). As a second teaching move, the teacher tries to explore details of the student's existing strategy by

“posing general starter questions to the student, pressing the student for an explanation of specific parts of his/her problem-solving process, linking the student’s representation and the problem context, and expanding the students’ understanding of quantities used during problem-solving” (Jacobs & Empson, 2016, p.190). The third teaching move is encouraging the child to consider other strategies. The teacher employs this form of teaching move to aid students in arriving at accurate solutions and foster the ability to contrast and evaluate varying strategies (Jacobs & Empson, 2016). In the following teaching move, connecting the student’s thinking to symbolic notation, the teacher asks the students to articulate their mathematical concepts or solutions through written mathematical notation, such as expressions or equations (Jacobs & Empson, 2016). The last teaching move is posing a related problem linked to what the students understand to force them to think from different perspectives. The teacher applies this teaching move by changing the existing problem or asking for new problems.

With these teaching moves, Jacobs and Empson (2016) provided teachers with a roadmap to plan effective mathematics teaching based on students’ mathematical thinking. According to this, teachers must first notice students’ mathematical thinking. Then, they reflect this knowledge and skills into the classroom, determine and apply their teaching moves. However, Jacobs and Empson stated that it is challenging for teachers to apply the teaching moves without imposing their understanding. Thus, the teachers need to develop their noticing skills and be supported to apply the teaching moves effectively. Considering this, we aimed to support the teachers in implementing the teaching moves with a teacher training platform enriched with collaborative discussion. In the current study, we presented one of the teacher’s (Sasha) applications of the teaching moves in the real classroom environment in the context of pattern generalization.

Pattern generalization

Algebra is a fundamental mathematical concept encompassing relationships between variables, generalizing them, and representing them using algebraic expressions (Kaput, 1999). In addition, generalizations are the backbone of mathematics, as they are the means by which we derive general principles from specific examples (Mason et al., 2010). The process of generalization, as outlined by Radford (2008), consists of three key steps: (1) identifying a recurring pattern, (2) extending this pattern to all terms in a sequence, and (3) formulating a rule that allows for the direct determination of any term in the sequence. Through this process, students develop their algebraic thinking skills by engaging in activities that require recognition, justification, and reasoning (Radford, 2008). Due to the importance of the pattern generalization process, this topic has an important place in both national and international elementary school mathematics curricula (Ministry of National Education [MoNE], 2018; National Council of Teachers of Mathematics [NCTM], 2014). Despite this recognition, research has shown that students often struggle to express patterns and inverse functional relationships using algebraic expressions (Jurdak & Mouhayar, 2014). Therefore, it is important that teachers make sense of students’ generalization process and effectively respond to them to support their understanding of pattern generalization. Thus, in this research, we aim to investigate the following research question:

What is the nature of a middle school mathematics teacher's instructional moves during classroom teaching after the discussions held in the online teacher training platform in the context of pattern generalization?

Methods

This research study is a part of the two-year research project designed to nurture teachers' noticing skills and responsive teaching moves through their involvement in a teacher training platform enriched with collaborative discussion. Considering this aim, a professional development design experiment (Zawojewski et al., 2008) with three phases (preliminary phase, prototyping phase, and retrospective analysis) was utilized as a research design.

Thirty-one middle school mathematics teachers working in public schools in different provinces of Türkiye and whose teaching experience is less than 15 years participated in the research study. In this paper, due to page limitation, we specifically focused on the teacher's (Sasha) responsive teaching in the scope of instructional practices in her classroom teaching in the case of pattern generalization. Since classroom observation was the main data collection tool, it was important to volunteer to teach pattern generalization in class for being a participant. Among 31 teachers, Sasha volunteered and her school was accessible to the researchers. Sasha graduated from a middle school mathematics teacher education program and has been working as a math teacher in a public school for ten years. She also had a master's degree in the division of Curriculum and Instruction.

During the preliminary stage of the study, a literature review was conducted to develop a teacher training platform where all the teachers could share and discuss their thinking using Microsoft Teams. At that stage, the data collection instruments were also developed. During the prototyping stage, whole group discussions were held through the online teaching platform on particular concepts related to the content areas (e.g., integers, pattern generalizations) asynchronously, and then volunteer teachers performed teaching in their classes. Sasha, one of the volunteer teachers, performed her teaching at school after her involvement in the discussions held online. In the retrospective analysis stage, teachers' noticing skill performance and responsive teaching moves were analyzed and interpreted. In this paper, we mainly focus on Sasha's instructional moves during her classroom teaching after collaborative discussions with the whole group via the online teacher training platform.

Mathematics education experts, who also constitute the design team, developed the content of the online teacher training platform in which 17 written and online scenarios focusing on student solutions were posted during two years on alternative content areas. In the platform, under the researchers' guidance, asynchronously, the groups of teachers carried out the discussion by writing their ideas related to the given written and online scenarios. They collaboratively discussed students' work given in scenarios by referring to other teachers' ideas stated in the platform. Since this research mainly focused on analyzing Sasha's instructional moves performed in her real learning environment regarding pattern generalization after her involvement in online discussions, those discussions were important for the study. Classroom observation was used as a data collection tool to reach the aim.

Within the project's scope, the data were coded based on the teaching moves presented by Jacobs and Empson (2016). Each teaching move was coded separately, and the sub-codes were generated using the teachers' explanations and the related literature (Ellis et al., 2019) using open coding. Due to the

page limitation, we only referred to the names and sub-codes related to the teaching moves in this paper. The categories of teaching moves and the sub-codes are displayed in Table 1. Two of the authors coded the data until they reached a complete agreement. Also, multiple data collection tools (interview data, classroom observation) were used for triangulation.

Table 1: An overview of the framework that Sasha uses for her teaching moves

Categories of teaching moves	Sub-codes
Ensuring the child is making sense of the story problem: explaining students' solutions through questions	Repeat Re-voicing Encouraging Student Re-voicing Encouraging students' understanding through questioning Cue
Exploring details of the child's existing strategy: drawing the student's attention to the details of their responses through the use of questions	Clarification Orientation (or/with Invitation) Connection (or/with Invitation)
Encouraging the child to consider other strategies: asking students questions to encourage them to find alternate/correct solutions and to motivate them to do so	Suggesting alternative solution Suggesting alternative solutions with mathematical ideas
Connecting the child's thinking to symbolic notation: asking students to find alternative/correct solutions and connecting the solutions they find.	Bridging Scaffolding and Bridging
Posing a more comprehensive problem: enhancing the student's level of thinking	Just posing an idea/ supporting another's idea/problem. Posing problem with number/context modification Posing problem/idea for an extension

Findings

Based on the data analysis of classroom observation of Sasha's teaching, it is identified Sasha grounded her teaching on applying teaching moves in the context of responsive teaching presented by Jacobs and Empson (2016). This section describes Sasha's responsive teaching according to the instructional moves.

The first teaching move: Ensuring the child is making sense of the story problem

To comprehend whether the student made sense of the problem or not, Sasha asked the following questions: Ali made a reading plan for himself and read five more pages every day than he had read the previous day. Starting with 12 pages on the first day, on what day did Ali read 132 pages? Sasha tried to ensure that the student comprehended the problem by utilizing questions such as "Can you tell me what you understand from this problem?" and "What is the question asking for? Through these questions, Sasha helped the student understand what was given and what was required. This approach, coded as *encouraging students' understanding through questioning*, was a fundamental component of the first teaching move presented in Table 1.

The second teaching move: Exploring details of the child's existing strategy

The teacher endeavoured to analyze and understand students' strategies through a second responsive teaching move. An example of this teaching movement is given below.

Sasha: What did you find as the answer to the problem?
Student: $5n+7$
Sasha: How did you find $5n+7$? Can you tell me what you did in this question?
Student: On day 1, the number of pages of the book read was 12, and it was increasing by five by 5. So we do $5n+7$.

As seen from the dialogue, Sasha explored the student's existing strategy by posing questions specific to the student's answer. Since she encouraged the student to explain his approach in broad terms rather than asking him to reconsider the solution or explain specific mathematical details in the solution, Sasha's teaching move was classified as clarification (see Table 1).

The third teaching move: Encouraging the child to consider other strategies

To encourage the students to develop alternative solution strategies, Sasha asked her students during her teaching whether there was another way to solve the question, as shown in the following dialog.

Sasha: Well, what other way can we do it?
Student 1: We can do it by experimenting.
Student 2: We can do it with the table method.
Sasha: Which variables do you think we should write in the table?
Student 2: Number of days, number of pages read.
Sasha: What will we look for (in the table)?
Student 2: To the relationship.

In this interaction, Sasha first engaged in a teaching move that encouraged the student to consider alternative methods for solving a problem by posing questions. Furthermore, she also assisted the students in utilizing this alternative strategy by guiding them through questions. This teaching move was classified as suggesting alternative solutions with mathematical ideas (see Table 1) due to her proposing alternative solutions and posing questions that facilitated the student's understanding of the mathematical concepts involved in this strategy.

The fourth teaching move: Connecting the child's thinking to symbolic notation

Sasha made a series of teaching moves during her teaching to connect students' thinking to symbolic notation. An excerpt of them is given below as an example.

Sasha: Then, how can I find the general term for it? What do they all have?
Student: 12
Sasha: And then there was a plus?
Student: Yes, we are putting a plus.
Sasha: What do they all have?
Student: 5
Sasha: Multiplying five by what?
Student: He multiplied 0. He multiplied 1. He multiplied 2. He multiplied 3.
Sasha: Are they constantly changing?
Student: Yes
...
Sasha: I mean, how does it relate to the number of days?
Student: 1 less.
Sasha: Then what will I multiply 5 by?

Student: with $(n-1)$
Student: General rule: $12+5(n-1)$

Sasha's teaching move involved posing questions to facilitate the student's ability to think abstractly and connect the student's existing understanding to the algebraic representation of the relation. This approach represents a bridging, one of the critical components of the fourth teaching move presented in Table 1. Sasha seeks to link the student's existing understanding with symbolic notation.

The fifth teaching move: Posing a related problem linked to what the child understands

In this teaching move, teachers can change the existing problem or ask for new problems to force students to think from different perspectives. After the problem given above, Sasha asked the students two problems. One is given in the following.

Berk, who had 250 liras in his piggy bank, gave up saving money and took out 6 liras from the piggy bank every day for 40 days and spent it. According to this, how many liras were left in Berk's piggy bank after 40 days?

The above-given problem involves a pattern in which the number of pages read increases daily. To expand the student's understanding, Sasha presented additional problems (Problem 1) that involved a decreasing pattern, precisely one in which the amount of money decreases daily. The aim of presenting these problems was to broaden the student's understanding through thought-provoking challenges rather than just providing an idea or a problem involving number/context modification. Thus, this teaching move can be classified as posing problems/ideas for extension (see Table 1) as the best key of the fifth teaching move.

Discussion and Conclusion

Based on the data analysis, we could deduce that Sasha could apply the five teaching moves of responsive teaching in the context of pattern generalization. This result lets us conclude that Sasha was responsive to students' mathematical thinking and pushed them to make sense of the problem, consider other strategies, connect their thinking to symbolic notation, and solve more comprehensive problems. More specifically, Sasha first established that the students understood the existing problem. Then, Sasha encouraged the students to construct a table and determine the pattern rule by posing follow-up questions based on the data presented in the table. This was followed by an extension pattern problem involving alternative types of relationships. Considering the questions asked by Sasha, it might be said that Sasha's responsive instructional practices align with the nature of the pattern generalization process stated in the literature (Jurdak & Mouhayar, 2014; Radford, 2008).

In Table 1, the sub-codes that focus on student understanding were characterized as higher quality in each teaching move. For example, cue (1st teaching move), connection (or/with Invitation) (2nd teaching move), suggesting alternative solutions with mathematical ideas (3rd teaching move), scaffolding and bridging (4th teaching move), and posing problem/idea for an extension (5th teaching move) were more qualified than other sub-codes. The analysis showed that the sub-codes assigned to Sasha's first, third, and fifth teaching moves were more qualified than her second and fourth moves. Thus, it can be concluded that her first, third, and fifth teaching moves were deeper since their responses could be linked directly to what the child understands. On the other hand, she applied the

second teaching move through clarification and the fourth through bridging. Therefore, her second and fourth teaching moves are not deep enough since her responses in these moves require more responsiveness to the student's thinking.

The differences in the quality of each teaching move might be the effect of the discussions held in the online teacher training platform. When these discussions were analyzed, it was found that 31 mathematics teachers, including Sasha, probed more effective questions while discussing how to implement first, third, and fifth teaching moves. Sasha might have been affected by these discussions while applying responsive teaching in the classroom, which was in line with the results of the previous studies (McFadden et al., 2014) that teachers learn from each other and develop themselves professionally on teacher training platforms where alternative perspectives were discussed. More specifically, Sasha's responses to students' solutions in the pre-test and online discussion supported this significant finding. In the pre-test, Sasha was given the correct student solution about pattern generalization. However, Sasha asked the student to find *the rule of a pattern whose first three steps were given in the problem*. Therefore, while demonstrating responding skills in the pre-test, she asked for a drill without considering the student's understanding. On the other hand, during the online discussion, when Sasha was given a student solution about the pattern, including increasing relationships, she mentioned that she first let the student make a table and discover the general term for the pattern. Afterward, Sasha posed the extension problem about the pattern, including decreasing relationship (Ali has 100 TL; he spends 6 TL every day; how much TL is left on the 11th day). Those reflections could be accepted as evidence that Sasha aimed to make the student understand the mistake and extend his understanding. Thus, it can be concluded that the discussion environment supported Sasha's responsive teaching skills.

Although Sasha's teaching moves were determined concerning the teaching moves that Jacobs and Empson (2016) presented, their nature was specified based on the data gathered from the research project. From this point of view, we expanded the emerging framework of teaching moves in the context of responsive teaching offered by Jacobs and Empson (2016) by identifying the sub-codes for each teaching move (see Table 1). Furthermore, Sasha's application of teaching moves in her classroom enabled us to extend the framework of Jacobs and Empson from whole numbers (Jacobs & Ambrose, 2008) and fractions (Jacobs & Empson, 2016) to pattern generalization. However, since this study is limited to one teacher and one content-pattern generalization, it can hardly be generalized. Further research could be performed to investigate the details of moves in other content areas. Based on the findings, we could conclude that effective online discussion environments in which teachers can discuss their ideas and classroom experiences with each other would be valuable in enhancing teachers' in-the-moment responses during classroom teaching. Thus, such environments should be developed to support teachers' opportunities to discuss their ideas with their colleagues and improve their teaching practices.

Acknowledgement

This study has been supported by the Scientific and Technological Research Council (TUBİTAK) (Project No. 218K508) and by the H2020 project MaTeK, no. 951822.

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