A teacher's noticing of students' mathematical thinking in action

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This case study examined one middle school mathematics teacher's noticing students' mathematical thinking while she taught quadrilaterals. The video clips obtained by the teacher through a portable camera and her reflections on them after each lesson comprised the data. The analysis indicated that although the teacher attended to various elements in the classroom, such as student characteristics or discourse, she mainly focused on student thinking. She also presented productive interpretations of student thinking; however, there were plenty of moments where the teacher provided only descriptive or evaluative comments. The findings showed that her interpretation was generally rich when her attention was on students' correct answers. The possible reasons for these results were discussed.

Keywords: In-the-moment noticing, student thinking, in-service teachers, quadrilaterals.

Background of the study

There is a common agreement that teachers' ability to notice comprises an essential component of the teaching profession (Santagata et al., 2021; Sherin et al., 2011), an importance that may come from its consequential nature as noticing shapes teachers' future moves (Schoenfeld, 2011). This nature of noticing implies that students' learning directly depends on what the teachers notice and how they respond (Schifter, 2011). In the literature, the most common theoretical perspective on teacher noticing among the four perspectives is the cognitive-psychological one (König et al., 2022). This perspective conceptualises noticing as the combination of several cognitive processes in the individual teachers' minds (König et al., 2022). Sherin and her colleagues (2011) stated that researchers generally regard two main processes: attending to and making sense of particular situations in the classroom (e.g., van Es, 2011). In other words, these researchers focus on what and how teachers notice. Some researchers add one more process, which is decision-making. For example, by adopting a specific focus on noticing students' mathematical thinking, Jacobs et al. (2010) believed that paying attention to and interpreting students' thinking are the beginning points for effective instructional responses.

The available literature also makes a distinction between reflective and in-the-moment noticing. As the name implies, in-the-moment noticing refers to teachers' noticing during their teaching practices. Besides, if the noticing activity happens outside the classroom by making teachers engage in artefacts like video clips, it is called reflective noticing. Schifter (2011) noted that both are significant; however, it is only possible to understand teachers' noticing completely by examining it in natural classroom settings. Moreover, Santagata et al. (2021) recommended conducting more research to understand how teachers notice during their teaching. In line with this suggestion, this study aimed to analyse an in-service middle school mathematics teacher's in-the-moment noticing as a part of the first author's dissertation. Although the larger study focuses on all three facets of Jacobs et al. (2010) conceptualisation, this study emphasises the first two aspects, which are fundamental for the last skill (van Es, 2011). The research questions guiding the current study are as follows: What does a middle

school mathematics teacher attend to while she teaches quadrilaterals to the seventh graders? How does she interpret student thinking that she attended to in the midst of her instruction? How do the teacher's interpretations of students' thinking relate to what she attended to in terms of student thinking?

Due to its complexity, a few studies analysed the teachers' interpretations during instruction (e.g., Taylan, 2017). Most of the in-the-moment noticing studies were about what the teachers attended to (e.g., Sherin et al., 2008). Furthermore, the specific domain of students' mathematical thinking considered in this paper is quadrilaterals and their relationship with each other. The reason for focusing on this concept serves another aim in the larger study. Several studies showed that it is a difficult concept for students and teachers (e.g., Fujita, 2012), and this content received relatively less attention in the available noticing literature. Hence, answering the above questions gives insight into the nature of teacher noticing by examining the teacher's attention and interpretation of students' thinking about quadrilaterals in the natural classroom setting. It also advances the field's understanding of teacher noticing by examining any observed relationship between the teacher's attention and interpretations in the classroom.

Methods

A qualitative case study was employed to describe the teacher's in-the-moment noticing skills of students' mathematical thinking in depth (Yin, 2014). The embedded units of analysis were the teacher's attention and interpretation during their teaching.

Participants

The participant was Teacher Ayşegül (pseudonym). She was one of the two volunteer participants in the larger study. Ayşegül was selected as a case for this study because her attention and interpretation showed more variety than the other teacher. She has worked in a public school in Ankara, Türkiye, and has seven years of teaching experience. However, she did not have much experience teaching quadrilaterals because of the grade levels she taught. Ayşegül expressed that she taught this topic four years ago and did not attend any professional development program about quadrilaterals, geometry, or other teaching and learning issues.

Data collection and analysis

As part of the first authors' dissertation, the data collection process started by informing the teacher about teacher noticing. Subsequently, Ayşegül prepared a lesson with the other two teachers in the study to teach the properties of quadrilaterals and the relationship among them to the seventh graders. To access teachers' in-the-moment noticing, we drew on the methodology suggested by Sherin et al. (2011). In their study, the researchers used portable cameras that provide video recordings directly after they occur. Sherin et al. (2011) observed that these videos and interviews on them give insights into teachers' in-the-moment noticing. Based on this, we requested the teacher to wear a portable camera and push the button in her hand if anything significant regarding student thinking happens while they teach quadrilaterals. The camera continuously records video with audio, and when the teacher presses the button, it saves the video starting from the previous 15 seconds. The recording ends when the teachers push the button a second time, providing a video recording-teacher generated

videos- that is important to the teachers about student thinking. Since we mainly focused on student thinking in the present study, the prompt intentionally included the phrase student thinking, and there was no restriction on the number of recordings. Ayşegül recorded 32 teacher-generated videos during eight lesson hours. After each lesson, we interviewed the teacher individually to discuss the teacher-generated videos. Although the plan was to conduct these interviews on the same day as the recording, it was not possible due to scheduling issues. Therefore, some interviews were conducted within three days of the lessons. The interview questions in Figure 1 were obtained from studies that examine teachers' in-the-moment noticing (e.g., Liu, 2014; Sherin et al., 2011). We also asked about teachers' experience using portable cameras at the beginning of each interview. Besides, a few questions were asked to reflect on all videos at the end of the interviews. The duration of the interview was 1 hour on average. Teacher-generated videos and the following interviews comprised the primary data set of this study.

1) Do you remember why you pushed the button? What did you notice about student thinking? **(Attention)**
2) Can you talk about what happened prior to and/or beyond this clip? What did you think about this student's understanding at that time? **(Interpretation)**
3) Can you mention what you thought about your next steps for this student? How did your noticing influence your decision-making while teaching? Do you believe your noticing matters in terms of your instruction? How? **(Decision-making)**
4) Did you notice any other student thinking at that moment that is/is not seen in this clip? **(Attention)**

Figure 1: Interview questions

Lastly, all lessons were recorded through a standard video camera stationed in the back of the classroom-researcher-generated videos. The researchers applied these videos in case the teacher could not remember the captured moments from their recordings or recalled a new moment. On the other hand, by observing all lessons as a non-participatory observer, the first researcher obtained detailed field notes about when the teacher pushed the button and forgot to stop the recording. These notes gave more evidence about instances that were important for the teacher.

Grounding on van Es's (2011) learning-to-notice student mathematical thinking framework, Taylan (2017) developed an analytical framework to examine teachers' attention and interpretation in the moment of teaching. We analysed the teacher's reflections on captured moments by modifying this framework using other studies about in-the-moment noticing (Colestock, 2009; Liu, 2014; Sherin et al., 2008) and our data. In particular, different from the previous studies, the teacher attended to her expectations in the classroom. Therefore, a new code-named teacher expectation emerged from the data analysis. Moreover, the teacher's reflections yielded two different interpretations- unable to interpret and making claims about student thinking without giving evidence. Lastly, qualitative data analysis software, MAXQDA (VERBI Software, 2021), was used to code and analyse the data. The following section explains the codes with some examples.

Findings

Ayşegül's attention to students' thinking and other elements during her instruction

At the beginning of the first interview, when we asked Ayşegül whether there was a criterion for her to capture video clips, she stated that she was inclined to push the button when some of her good students would take the floor. She said, "I knew that they would say something important". In other words, according to her, she captured generally based on her students' characteristics. However, when

her reflections on captured moments were examined in detail, it was observed that her attention was mainly on student thinking (70.5%). In particular, she generally attended to student understanding and error. Table 1 displays examples of what Ayşegül attended to regarding student thinking. In other cases, she focused on the discourse between students and teachers or among students, student characteristics, or her teaching moves. Differently, she chose to record some video clips because she expected a particular answer from students, either correct or wrong-teacher expectation code. She explained, "In those moments, I pushed the button immediately after my question to record all of their answers." Ayşegül's reflection below illustrates how she paid attention to her expectations in the classroom.

> I just waited there to see if someone would catch the things you posted us earlier. I waited to see if someone would catch the angle of the rectangle on the board, which is 90°. However, they did not come any closer. They did not emphasise why it was 90°. Since they already knew it was always 90°, they just said it was 90.

Table 1: Examples of Ayşegül's attention to students' thinking

Student Thinking	Example
Student understanding (Taylan, 2017)	I captured this moment because these are the special cases of trapezoids. These students could discover their properties and name them.
Student justification (Colestock, 2009)	Here, by using corresponding and alternate interior angles, the student showed that opposite angles of a parallelogram are equal.
Student connections (Taylan, 2017)	We said the intersection of a rhombus and a rectangle is a square. Although these students could not do this in the next step, they actually did it in this step. While one of the students went from the rectangle, the other defined a rhombus. They intersected them (rectangle and rhombus) without realising it.
Student difficulty (Taylan, 2017)	This part indicates how the students could not draw a parallelogram.
Student error (Liu, 2014)	The student said that two irrelevant angles are alternate interiors. She showed the opposite angles in a trapezoid.
Student thinking through a problem/question (Colestock, 2009)	In this video, the student could see the congruent triangles (Ayşegül mentioned the four triangles when the diagonals of a rhombus are drawn.) This will help her establish the angle relations. Also, I captured this moment since, with this idea, she can see why they (the diagonals of a rhombus) are equal.

Ayşegül's interpretation of students' thinking

When we asked Ayşegül what she understood about those students' thinking during her instruction, she provided a variety of fruitful interpretations. More particularly, she presented diverse evidence while explaining her claims about student thinking. She also made connections between student thinking and previous or future moments of mathematics instruction, generally previous moments.

Although less in number, another productive interpretation of her was to connect the students' thinking that she attended to with her teaching goals. Specific examples for all these interpretations are given below.

Table 2: Descriptions and examples of Ayşegül's interpretation of student thinking

Description	Example
No interpretation The teacher states that s/he could not understand what the student knows/understands	What happened here was that when the students were drawing the diagonals of a rectangle, they first drew the angles of the rectangle. Then, they drew the diagonals of the rectangle starting from the corner of the angles. Such a different misconception emerged in this class. … *I do not know why they understand like this. I really have no comment this time.*
Making claims about student thinking without giving evidence The teacher makes a claim about student thinking but does not refer to specific events. The teacher either explains what occurred generally or provides descriptive and evaluative comments	The student was completely gone there. I even said that we were doing it with two parallel and one intersecting line. I asked her if it would work here. I remember the student gave up and left. She said something like, "No, it won't work." *I think that the student has a big misconception. Well, it wasn't a misconception, but she was confused a lot*.
Use of evidence in making claims about student thinking (Taylan, 2017) The teacher refers to specific events or interactions as evidence when she makes a claim about student thinking	The intersection of a rhombus and a rectangle is a square. Although these students could not do this in the next step, they actually did it in this step. *While one of the students went from the rectangle, the other defined a rhombus. They intersected them (rectangle and rhombus) without realising it. They emphasised that the square is a particular case of both.*
Making connections between student thinking and another moment of math. instruction in her class (Taylan, 2017) The teacher talks about student thinking by mentioning another moment of instruction (previous or future).	In this way, *I said that in the future, when we create that diagram about all quadrilaterals, they will see that the two (the rhombus and rectangle) have something in common*. They gave the keywords for that place, but what was expected did not happen in the rest of the lesson.
Making connections between student thinking and teaching goals (Taylan, 2017) The teacher makes connections between student thinking and teaching goals, whether student thinking meets or fails to meet these goals, including goals of the curriculum.	Here, the student can do what we want to do. *The lesson fulfils our purpose. You know, we aimed to provide meaningful learning by making connections among quadrilaterals. It happened here. The student said that since a parallelogram is a trapezoid, it is like this*.

However, there were also cases where Ayşegül could not provide an interpretation or made just some claims about student thinking but did not give any piece of evidence. As exemplified in Table 2, in

those moments, the teacher either stated that she had no comments or presented some descriptive or evaluative comments about student thinking. For instance, she noted that the student had difficulty with angles formed between transversal and parallel lines but did not explain why she thought so. Indeed, these cases comprised 46.5% of all the moments that Ayşegül interpreted.

Furthermore, we observed that some codes occurred together more than others during coding. Therefore, we wondered whether there could be some relationship between the teacher's interpretations and attention. Specifically, it seemed to us that Ayşegül might provide a piece of evidence when she attended to student understanding or students' correct answers. The code relations feature of MAXQDA was used to check these insights. The following figure indicates the number of intersections of codes about student thinking with the interpretation codes. The analysis also included the discourse category because Ayşegül generally attended to various students' thinking when she attended to the discourse in the classroom.

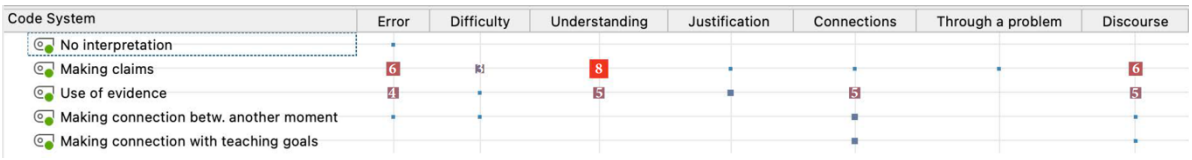

Figure 2: The intersection of attention and interpretation codes

This figure indicates that, as opposed to our insight, Ayşegül did not provide any evidence but just made a claim more when she attended to student understanding (8 cases). Also, there was no clear pattern between what the teacher attended to in terms of students' thinking and their interpretations. The number of cases where the teacher made a claim without presenting evidence and used some evidence to explain her claim was close to each other when she attended to student error, understanding, and discourse. Differently, she made fruitful interpretations when her attention was on students' connections between two concepts (9 vs. 1 case). To check our second insight, whether Ayşegül's interpretation depends on the students' correct answers or not, we gathered codes under the student thinking category into two categories: (possible) correct answers and difficulty and (possible) error. The first category involved student understanding, justification, and making connections codes. The moments where a student stated an idea were evaluated with respect to their correctness. If they were correct for Ayşegül, they were included in the first category. Figure 3 displays the intersection of these categories with the teachers' interpretations.

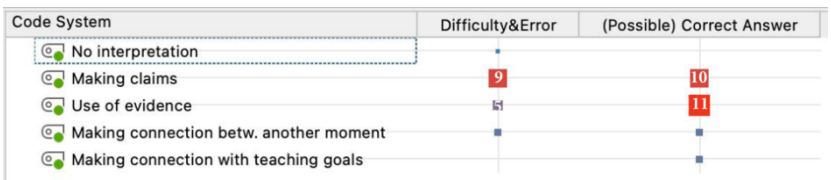

Figure 3: The intersection of two categories with interpretation codes

Indeed, this figure shows some differences from Figure 2. The number of cases where Ayşegül did or did not provide evidence was close to each other (10 vs. 9 cases). However, it may be stated that not always, but in general, the teacher made productive interpretations when she attended to the students' correct answers (11 vs. 5 cases). Ayşegül could not present much evidence when the students had difficulty or an error.

Discussion

This study aimed to investigate what a middle school mathematics teacher attended while teaching quadrilaterals to her seventh graders. It further examined how she interpreted students' thinking during her instruction and the relation between the two aspects of noticing. The findings in terms of the teacher's attention indicated similarities between the other studies in the literature. Specifically, Ayşegül commonly attended to student thinking in the moment of her instruction, consistent with Colestock's (2009) and Taylan's (2017) studies. However, the ratio was less when compared to them. Like Sherin et al.'s (2008) study, she emphasised the discourse category reasonably enough. Nevertheless, not only the discourse between the students but also the several students' thinking within the discourse was important for her. Different from the previous studies, she also attended to a moment based on what she expected to happen. This might be because Ayşegül prepared a plan with the other two teachers in the larger study, and they thought about what the students could do or say during their instruction. With Mason's (2011) statement, she might sensitise herself to notice particular students' thinking in the moment of teaching. Findings regarding the interpretation of students' thinking again show consistency with Taylan's (2017) study. However, in plenty of cases, in almost half of them, Ayşegül either could not provide an interpretation or presented descriptive or evaluative comments. Such comments are observed in the beginning levels of van Es's framework (2011). One of the reasons for this situation could be the teacher's experience in teaching quadrilaterals. Ayşegül stated that she had not taught this content much before. Hence, she might be unfamiliar with the students' thinking that she encountered. Another reason could be the teacher's knowledge of the quadrilaterals since what and how teachers notice directly depends on the teachers' knowledge, resources, goals, and orientations (Schoenfeld, 2011; van Es, 2011).

A related contribution of the study is that it adds to the teacher noticing literature by indicating a relationship between the teacher's attention and interpretation of student thinking. In particular, the findings provided evidence that the teacher made more fruitful interpretations when she attended to the students' correct answers. This might be because the teacher gave more importance to correct answers. However, there was little difference between the number of moments involving students' correct answers and difficulties or errors. It could also be that she regarded teaching as directing/guiding students to the correct answers. Another reason for observing such a relation could be the teacher's knowledge of common students' difficulties or errors in quadrilaterals. Hence, future studies may examine the possible reasons for this situation by analysing the relationship between teacher noticing and knowledge of content and students. Moreover, as mentioned above, the teachers may be guided to think more about students' difficulties and errors during the planning phase since it might enable them to focus more on difficulties and errors during their instruction. However, it should be noted that this study was limited to only one teacher and specific content. Therefore, we suggest further studies to examine whether the same results are valid for the other teachers and contents. Lastly, because of the consequential nature of noticing (Colestock, 2009; Schoenfeld, 2011), as mentioned at the beginning of the paper, it is essential and necessary to examine the third facet of noticing (König et al., 2022). Hence, our next step will be to investigate the teacher's in-the-moment decisions and how they relate to the teacher's attention and interpretation.

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