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Socio-mathematical norms related to explanations: A case of a gifted and talented mathematics classroom

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This study explores socio-mathematical norms (SMNs) related to explanations in a gifted and talented students' classroom as a response to calls for investigating their classroom micro-culture. Data consist of forty-three mathematics lessons. We defined two dimensions (teacher and student) of SMNs which were reported in the literature and analysed the data based on these dimensions. Findings revealed one SMN regarding acceptable mathematical explanations and justifications that has the potential to support creative mathematical activity through elaboration. However, the classroom community lacked explicit evaluations of ideas based on other creativity components such as fluency, flexibility, and originality. This study recommends some practical and theoretical implications for creating a classroom environment for gifted students.

Keywords: Socio-mathematical norms, gifted students, mathematical explanations, classroom micro-culture, mathematical creativity.

Introduction

There has been an agreement in the mathematics education community that knowing and doing mathematics is a social and cultural activity (Cobb et al., 1997; Yackel & Cobb, 1996). In line with this perspective, this study sees teaching as establishing and maintaining a classroom culture for doing mathematics, and the role of the teacher is “to support students’ enculturation into the ways of speaking, working and thinking that are specific for mathematics” (Nowińska, 2022, p. 3450). Each classroom is considered a community in which different learning practices are adopted by teachers and students (Goos, 2004). In this context, socio-mathematical norms (SMNs) describe “normative aspects of mathematical discussions that are specific to students’ mathematical activity” (Yackel & Cobb, 1996, p. 458). SMNs are concerned with what counts as mathematically different, efficient, or elegant, and what counts as an acceptable mathematical solution or justification (Yackel, 2000). Since norms are tied to the community in which they emerge because of their nature and can change according to the classroom (Yackel, 2000), gifted students’ classroom norms may differ from others. To deepen the mathematical understanding of gifted students, it is recommended to justify the reasoning behind the thoughts or solutions (Singer et al., 2016). Therefore, teachers should initiate the negotiation process about mathematical explanations and justifications in the classrooms of gifted students and should provide the development of normative understandings in terms of conceptual explanations and mathematical creativity.

Mathematical creativity, which is associated with giftedness has been examined by researchers through the evaluation of products based on how novel (originality) or how flexible they are (flexibility), whether they contain in-depth explanations (elaboration), or the number of solutions made (fluency) (Mann et al., 2017). For thoughts or solutions to be evaluated in terms of originality

and flexibility in the classroom community, it is necessary to understand how they differ from each other. Then, the classroom community can reach a mutual understanding of these issues.

This study examines SMNs related to explanations and justifications in a classroom of gifted students. The research aims to contribute to our understanding of norms that could be established in gifted students' classroom micro-culture, and norms that could support creativity (Goldin, 2017).

Socio-mathematical norms related to explanations and justifications

According to the interpretive framework, the social dimensions of the classroom micro-culture and the psychological dimensions of students' individual activities in the classroom are in a reflexive relationship (Cobb et al., 2011). As the teacher and students interpret and respond to each other's actions (psychological perspective), normative activities emerge (sociological perspective) in the classroom. Conversely, negotiated norms of the classroom (sociological perspective) represent how the teacher and students interpret and respond to each other's actions (psychological perspective) (Cobb et al., 2011).

SMNs related to explanations (e.g., Cobb et al., 2011; McClain & Cobb, 2001; Yackel & Cobb, 1996; Yackel, 2001, 2004) focused on a normative understanding of "acceptable mathematical explanations and justifications". Such SMNs in mathematics classrooms of gifted students should be supportive of students' mathematical abilities and creativity. However, in a classroom environment where all teacher answers are accepted as correct or a solution method in the textbook is strictly adhered to, students' creative abilities would likely be limited (Mann et al., 2017). In such a classroom environment, we probably cannot see a negotiation process among classroom participants about what constitutes acceptable mathematical explanations and justifications. Thus, we should consider the teacher's role in establishing norms, and other additional factors (such as textbooks used). If the normative understandings regarding mathematical explanations and justifications are mutually agreed on by the classroom community as conceptual explanations based on mathematical reasoning processes, it would be supportive of gifted students' creative mathematical activity. Therefore, examining the normative understandings of mathematical explanations and justifications would likely give an idea about SMNs that can be supportive of creative mathematical activity.

Methods

Since the investigation of classroom norms requires a long-term and in-depth analysis (Yackel & Cobb, 1996), we conducted a descriptive case study (Yin, 2009) in a single fifth-grade mathematics classroom with twelve (three girls and nine boys) gifted students to be able to reflect the characteristics of the phenomenon (Bleijenbergh, 2010). The school is a private secondary school supported by a foundation for gifted and talented students which selects its students using the WISC-IV intelligence test. The mathematics teacher of the class had seven years of teaching experience. The first author observed the lessons as a non-participant observer aiming to investigate the SMNs of the classroom without any intervention. The primary source of data consisted of forty-three mathematics lessons that were video recorded during the 2018 autumn term.

We analysed the data (transcripts of 43 lesson videos) using the content analysis method. We developed a coding scheme for SMNs related to explanations reported in the literature (McClain &

Cobb, 2001; Levenson et al., 2006; Yackel & Cobb, 1996; Yackel, 2001, 2004) utilising the notions of teacher and student dimensions of a norm (Çakır & Akkoç, 2020). We defined an initial version of the descriptors based on the literature on SMNs related to explanations. Since norms require a two-way negotiation between the teacher and students, we defined descriptors for both dimensions (teacher and student) as evidence of an SMN.

After preparing the verbatim transcripts of lessons, the first step of the data analysis was the identification of episodes in videos where classroom discussions took place related to explanations and justifications. Then, by following Cobb et al. (2011), we focused on regularities in these discussions. Content analysis pointed out an SMN directly related to explanations. Based on the literature, we defined the descriptors of teacher and student dimensions of the SMN. We revised the descriptors as we analysed the data as shown below:

Acceptable mathematical explanations and justifications should be offered (McClain & Cobb, 2001; Yackel & Cobb, 1996; Yackel, 2001, 2004)

Teacher dimension: The teacher's explanations are based on mathematical concepts previously experienced by students. The teacher expects students to do the same. The teacher expects explanations to be based on mathematical reasoning rather than repeating the procedures. Mathematical concepts under investigation and procedures or performed actions about them are conceptually meaningful for students.

Student dimension: Students are aware of the necessity to base their explanations on previously learned mathematical concepts and to justify their solutions. They know that explanations merely based on procedures are not enough. Students attempt to make conceptual explanations. Their explanations are not only for themselves but also meaningful for their peers.

Findings

The data analysis revealed only one SMN related to explanations and justifications that was also reported in the literature. The student dimension of the SMN was coded thirty-eight, and the teacher dimension was coded thirty-four times. We will give illustrative examples of the teacher and student dimensions of the SMN that we observed. These examples will point out the teacher's and students' expectations, awareness of these expectations, and their actions which we will consider as evidence of the SMN.

The data showed that the teacher frequently asked the question "why" in response to students' answers during class discussions, which was considered a clear indication that the answer was not considered sufficient without explanation and that the thoughts were expected to be explained and justified. For example, the teacher expected an explanation regarding the concept of prime numbers, and one of the students was aware that explanations should be based on mathematical foundations. For this reason, this student made an explanation that can be conceptually described as "numbers that cannot be divided by anything other than itself and 1" (psychological perspective). Afterward, the student who answered "even" to the question of whether the sum of the largest and the smallest prime number is odd or even, said, "if we add an even and an odd, it is odd, that is, if we add even numbers or odd numbers, it is even." Upon the teacher's request for an explanation here, it can be said that the

student gave a conceptual explanation by using the concepts he had previously experienced in natural numbers, instead of making a procedural explanation based on a concrete confirmatory example. This explanation about prime numbers, which is a subject of the sixth-grade curriculum, points to “elaboration”, one of the creativity components, as it shows that the student reached a generalization. The students’ mathematical explanations and justifications are evidence of action regarding the related SMN (psychological perspective). The existence of both student and teacher dimensions indicates a two-way negotiation.

The excerpt below illustrates the SMN in the context of a lesson on the order of operations:

Teacher: What is the order of operation?
Kayra: We have to do the operations inside the parentheses first.
Teacher: Operations in the parentheses first. Next?
Mete: Then factorials and exponentials.
Teacher: There is such a thing as factorial. What was the factorial? Kayra, tell us about this.
Kayra: Three factorial is, 3 times 2, and multiply by 1.
Teacher: Okay, you know the concept of factorial. So, what does the factorial concept do for us, and where do we encounter it in daily life?
Mete: In the cinema.
Teacher: For example, in the cinema, Mete said. Okay, let Serkan also tell us about it
Serkan: For example, we will order a, b, and c. Since there are three, we use three factorials. There are as many different orders as 3 factorials.
Teacher: Burak said that he doesn’t know factorial at all. Can anyone explain it to him? (One of the students explained it using the cinema example and drawing it on the board)

As can be seen above, a student’s answer included the concept of “factorial” which is not included in the curriculum at this grade level. This finding sets an example for the SMN about explanations and justifications. With the teacher’s explicit demand for an explanation, the students gave conceptual explanations in response (psychological perspective). We see that one of the students explained the $3!$ as the multiplication of the numbers by going backward from 3 to 1 which was accepted by the teacher. In addition, we can say that the teacher's question about how the factorial concept is used in daily life reflects the expectation of a different explanation from the calculational explanation made by a student. Based on this, it was seen that a student also explained how this concept was used to find how many different ways the letters a, b, and c could be arranged side by side. Here, the student preferred a more conceptual explanation for the use of the concept in daily life (cinema context), in line with the teacher’s expectation, instead of making an explanation about the calculation of the factorial through an example (psychological perspective). After this verbal explanation, the student presented his explanation to the classroom community with the help of a model on the board. It can be said that the teacher’s multiple requests for explanation and justification enabled the students to deepen and elaborate their explanations for their ideas.

Another dialogue that points out to the SMN started when the teacher and students were questioning why a number should be considered a perfect square:

Teacher: What is the smallest three-digit perfect square with different digits?
Students: 121
Teacher: Mete said 121, but I said no, the digits should be different, next?
Students: 144. (Then, students tried to find the next perfect square with different digits)
Serkan: Can I tell you how I did it? ...I knew that 12×12 is 144. I changed the multiplier to 13 and added 12. Then I made the other multiplier 13 and added 13. 169

Serkan's desire to explain his method and then present his mathematical explanation and justification reflects the student dimension of the SMN. While explaining the solution method, he stated that he started from the fact that $12 \times 12 = 144$ and turned the operation into an addition by increasing each factor by one ($12 \times (1+12)$ and $(1+12) \times 13$). In addition, since the student stated that he increased each factor by one while explaining his method, it was understood that he used the distributive property of multiplication over addition twice in a row, from the left and the right. This detail in the student's explanation of the solution (which could be considered flexible) was seen as related to elaboration, one of the creativity components. It is understood from the teacher's reaction that he accepted the solution without giving any feedback about the solution.

Another classroom dialogue exemplifying the SMN is presented below:

Teacher: The question is just a difficult one... How many numbers are there with two natural number divisors greater than 20 and less than 50? Serkan, how did you answer this?
 Serkan: Since it says two natural number divisors, I understood that he meant prime numbers. I looked for prime numbers between 20 and 50.

We can say that the teacher's asking the student "how" he answered the question is an indication of his expectation for a more conceptual explanation when making a conclusion and that results will not be sufficient without an acceptable explanation. The student's explanation in response to the teacher's question indicates that he perceived the teacher's expectation and offered a conceptual explanation instead of a procedural one (such as listing the numbers one by one and checking whether they are prime) considering the grade level (psychological perspective). It can be said that the class community has reached a common agreement on this SMN which was observed with both the teacher and student dimensions (sociological perspective).

Another dialogue reflecting the SMN is given below. Discussions took place after the class watched clips from the Lego movie (Lord & Miller, 2014):

Teacher: What lessons can be drawn from this about mathematics?
 Doruk: The lego pieces in constructions, x, y, and z coordinates are arranged in a row.
 Teacher: Good, they have instructions.
 Kayra: Geometric shapes
 Teacher: You're close, but it's not. Yes, good answers, but I want you to see the bigger picture. Think more broadly, not a part of the video, but think of the whole video...
 Deniz: Teacher, everything should go in a certain order...
 Teacher: Yes, there is a certain order... The word "order" is correct, but it does not exactly have a mathematical meaning... Is there a subject called "order" in mathematics?
 Deniz: Pattern
 Teacher: Isn't that in the video a pattern? Now let's get out of the video, let's go back to our lives. Are there things in our lives that we repeat and call patterns? Is life a pattern? Our life is a pattern... Let's examine the patterns here together.... (turning to the video) He woke up, we are waking up too. (The clock rings in the video, attracting the students' attention as a pattern) I guessed what you would say. Then a question for you: If I slept at 22 o'clock, how many hours did I sleep if I woke up at 7 o'clock?
 Mete: 9
 Teacher: How many minutes did I sleep?
 Doruk: We multiply 9 by 60.
 Teacher: You don't have to answer, we just have to think about how we will do it. Then I ask this question, if a person who starts the day at 7 o'clock stays awake until 4:45 the next day, how many hours did he stay awake?

The teacher did not accept all of the answers regarding mathematical inferences drawn from the video. In addition, the teacher requested a mathematical explanation in response to one of the answers (psychological perspective). The teacher did not accept the explanation made by the student as “everything is in a certain order” and asked for a mathematical explanation. The demand for an acceptable mathematical explanation for the concept can be seen as a clear indication of the teacher dimension of this SMN. Afterward, while students were responding to the clock problem, a student’s mathematical explanation of the process instead of telling the result shows the student dimension of the SMN (psychological perspective). The explanation presented is a procedural one. The teacher’s statement “You don’t have to answer, we just have to think about how we will do it.” indicates that the teacher evaluated the explanation given by the student as “acceptable”. The teacher accepted the procedural explanation instead of demanding a more conceptual one because the problem is not so complex that it needs justification.

Discussion and implications

This study investigated SMNs as a response to calls for investigating both “the nature of classroom culture and the role of the teacher in fostering mathematical expertise” (Singer et al., 2016, p. 35) concerning mathematical explanations in the context of a classroom of gifted students. We observed an SMN related to explanations and justifications. Regarding this SMN, the data indicated that both the teacher and students offered mathematical explanations which were conceptual rather than procedural. From a cognitive perspective, it is not surprising that explanations of gifted students are more conceptual than the students at the same grade level. From a sociocultural perspective, offering acceptable mathematical explanations and justifications had become part of the classroom culture. As norms are closely related to the roles of teachers and students (Cobb & Yackel, 1996; Cobb et. al, 2011), the observed SMN points out the role of the teacher in the classroom which was also compatible with recommended teacher roles in gifted classrooms such as asking students to “clarify, justify, connect, and extend their ideas” (Johnsen & Sheffield, 2021, p. 20). The SMN as being associated with elaboration, which is a component of mathematical creativity, allowed students to “describe, illuminate, and generalize ideas” (Leikin, 2010, p. 163) by explaining and justifying their thinking. So, the SMN has the potential to support creative mathematical activity by elaborating ideas or solutions which could also enable students to deepen their understanding of mathematics by reasoning and justifying (Singer et al., 2016). On the other hand, findings point out just one component of creativity (elaboration). However, ideas and solutions should also be evaluated based on other components such as originality, fluency, and flexibility. This evaluation should be an essential part of the classroom micro-culture. The lack of an SMN regarding the evaluation of creativity components might be one of the reasons why explanations in the classroom dialogues were not at a high level.

Based on our findings we will discuss the implications. The first one is practical and concerned with the SMN: “Acceptable mathematical explanations and justifications should be offered”. For this SMN, explicit evaluations of explanations and justifications were not made in the classroom. Although lacking explicit evaluations, there was a taken as shared understanding of conceptual explanations rather than procedural ones. The second implication is a theoretical one. Our study was conducted within a classroom of a teacher who had been teaching the same classroom for five years.

We focused on the established norms rather than the negotiation process during norm construction. Therefore, we could not observe explicit talk about what counts as an acceptable mathematical explanation and justification. For example, in the discussion of prime numbers, the teacher accepted the student's explanation about the sum of an even and an odd number without any feedback about why it is acceptable.

The reason for not observing explicit evaluations might also be related to our participant teacher's beliefs and pedagogical content knowledge. Researchers suggest more challenging problems (Leikin, 2018) for gifted students, which also could require more original or flexible explanations. Establishing the SMN regarding acceptable mathematical explanations and justifications, with appropriate task selection and explicit evaluations, could enhance gifted students' mathematical creativity and proving abilities. We also suggest establishing norms regarding explanations in regular classrooms according to students' levels and supporting their problem-solving practice.

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