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SOLAR SYSTEM PHENOMENOLOGY OF ENTANGLED RELATIVITY

O. MINAZZOLI^{1,2}

¹Université Côte d'Azur, Observatoire de la Côte d'Azur, CNRS, Artemis, bd. de l'Observatoire, 06304, Nice, France.

²Bureau des Affaires Spatiales, 2 rue du Gabian, 98000 Monaco.
- olivier.minazzoli@oca.eu

ABSTRACT. Entangled Relativity is a non-linear reformulation of Einstein's general theory of relativity (General Relativity) that is more parsimonious in its formulation. It accurately recovers the results of General Relativity in many astrophysical situations, particularly in the Solar System, where the theory currently cannot be distinguished from General Relativity. This paper will explore how this might change in the near future.

1. INTRODUCTION

Entangled Relativity is essentially a non-linear reformulation of General Relativity. Its name does not refer to *quantum entanglement*, but rather to the fact that matter and curvature (gravity) are intertwined at the foundational level of the theory's formulation. In this framework, one cannot define matter and curvature separately; both must be present simultaneously to even define the theory. This differs from General Relativity, where one can envisage a world with only gravity: an entire universe in a vacuum. This notably leads to an infinite set of non-trivial solutions to the vacuum field equations, ranging from black hole solutions to universes entirely filled with black holes—a kind of potential absolutely dark universe.

However, there is a compelling reason to be dissatisfied with a relativistic theory that permits vacuum solutions, even if they are not exactly realized in nature due to the presence of matter fields in our universe. This reason is rooted in the violation of one of the three founding principles that Einstein used to construct his general theory of relativity: the *principle of relativity of inertia*, also known as *Mach's principle*. In particular, General Relativity violates this principle, as Einstein eventually acknowledged several years after de Sitter found a vacuum solution to Einstein's equation with a cosmological constant. To better understand this issue, I recommend reading [Hoefffer, 1995] and [Pais, 1982].

While this might seem anecdotal in comparison to the other merits of the theory, Entangled Relativity, by forbidding the definition of the theory without matter fields, naturally satisfies this principle. Therefore, Entangled Relativity is not only more parsimonious than General Relativity but also more closely aligns with Einstein's vision of a satisfactory relativistic theory. In his own words: "In a consistent theory of relativity there cannot be inertia relatively to "space" but only an inertia of masses relatively to one another" [Einstein, 1917]. This metaphysical demand explains the introduction of the principle of relativity of inertia, which states that "[the metric]-field is completely determined by the masses of the bodies", such that, indeed, "[with this principle], according to the field equations of gravitation, there can be no [metric]-field without matter. Obviously, [this principle] is closely connected to the spacetime structure of the world as a whole, because all masses in the universe will partake in the generation of the [metric]-field". [Einstein, 1918]

1.1 Mathematical formulation

The definition of Entangled Relativity is based on its path integral:

$$Z_{ER} = \int [Dg] \prod_i [Df_i] \exp \left(-\frac{i}{2\epsilon^2} \int d_g^4 x \frac{\mathcal{L}_m^2(f, g)}{R(g)} \right), \quad (1)$$

where $\int [D]$ relates to the sum over all possible (non-redundant) field configurations, R is the usual Ricci scalar that is constructed upon the metric tensor g , $d_g^4 x := \sqrt{-|g|} d^4 x$ is the spacetime volume element, with $|g|$ the metric g determinant, and \mathcal{L}_m is the Lagrangian density of matter fields f —which could be the current *standard model of particle physics* Lagrangian density, but most likely a completion of it. It also depends on the metric tensor, a priori through to the usual *comma-goes-to-semicolon rule* [Misner, Thorne and Wheeler, 1973]. The only parameter of the theory is the quantum of energy squared ϵ^2 . In order to recover standard quantum field theory in a limit that corresponds to our observable universe, ϵ has to be the (reduced) Planck energy [Minazzoli, 2023].

1.2 Comparison with standard physics

Eq. (1) should be compared with the path integral of the Core theory¹, which is expressed as

$$Z_C = \int [Dg] \prod_i [Df_i] \exp \left[\frac{i}{\hbar c} \int d_g^4 x \left(\frac{R(g)}{2\kappa_{GR}} + \mathcal{L}_m^{SM}(f, g) \right) \right], \quad (2)$$

where f_i are the matter fields of the standard model (SM) of particle physics—such as fermions and gauge bosons, and the Higgs. There are three universal constants in this formulation: the quantum constant \hbar (Planck’s), the causal structure constant c and the constant of gravity $G = c^4 \kappa_{GR} / (8\pi)$ (Newton’s). From these constants, one can construct an energy scale, a mass scale, a time scale and a length scale, known as the Planck energy (E_P), mass (m_P), time (t_P) and length (l_P) respectively:

$$E_P = \sqrt{\frac{\hbar c^5}{G}}, m_P = \frac{E_P}{c^2}, t_P = \sqrt{\frac{\hbar G}{c^5}}, l_P = c t_P. \quad (3)$$

The difference between the two theories lies in how curvature (gravity) and matter are coupled within the quantum phase Θ of the path integral, where $Z = \int [Dg] \prod_i [Df_i] \exp(i\Theta)$. Classical physics corresponds to the variational paths for which the quantum phase is stationary $\delta\Theta = 0$. The reason being that for classical, or “*macroscopic*”, phenomena, destructive interferences cancel any contribution from other paths to the path integral, whereas constructive interferences are maximal for paths that lead to a stationary phase. Usually, one talks about the *Principle of Least Action*, because if \hbar is a fundamental constant, then $\Theta = S/\hbar$ and $\delta\Theta = 0 \Leftrightarrow \delta S = 0$, where S has the dimension of an action. In other words, classical physics is quantum physics in some limit, and the *Principle of Least Action* of classical physics simply is a consequence of quantum physics.

1.3 The Parsimonious Nature of Entangled Relativity

In the formulation of Entangled Relativity, as expressed in Eq. (1), there are only two universal fundamental constants: the squared energy constant ϵ^2 and the causal structure constant c . This represents one less constant than is required in standard physics. Specifically, the coupling constant κ_{GR} between matter and curvature in General Relativity disappears due to the non-linear coupling of the two in Entangled Relativity. Consequently, the formulation of Entangled Relativity is more parsimonious in terms of constants than that of standard physics (General Relativity with matter fields).

¹That is, the current standard model of physics, as named by [Wilczek 2016].

However, this leads to two significant implications. Firstly, there is no quantum of action in the formulation of Entangled Relativity, meaning that \hbar cannot be a fundamental constant in this framework. Instead, it must emerge as a limit of the theory—see Sec. 2.3 for details. More crucially, the absence of a quantum of action means one cannot construct a notion of elementary length or time scales using only these two dimensionful constants. This is particularly relevant as many of the challenging issues in quantum general relativity are linked to the concept that a smooth and continuous spacetime may cease to exist at the Planck length and time scale (l_P and t_P in Eq. (3)). This dilemma led to various hypothesis, such as the concept of *spacetime foam* or the existence of a fundamental discrete structure composed of elementary *spacetime atoms* [Loll, Fabiano, Frattulillo, Wagner, 2022].

1.4 Field equations

Classical physics corresponds to the paths in the path integral that lead to a stationary quantum phase $\delta\Theta = 0$. Therefore, extremizing the quantum phase with respect to the various fields lead to the classical field equations that follow [Ludwig, Minazzoli and Capozziello, 2015]:

$$G_{\mu\nu} = \kappa T_{\mu\nu} + f_R^{-1} [\nabla_\mu \nabla_\nu - g_{\mu\nu} \square] f_R, \quad (4)$$

where

$$T_{\mu\nu} := -\frac{2}{\sqrt{-g}} \frac{\delta(\sqrt{-g}\mathcal{L}_m)}{\delta g^{\mu\nu}}. \quad (5)$$

and $G_{\mu\nu} := R_{\mu\nu} - 1/2 R g_{\mu\nu}$ is the usual Einstein tensor, with

$$\kappa = -\frac{R}{\mathcal{L}_m}, \quad \left(f := -\frac{1}{2\epsilon^2} \frac{\mathcal{L}_m^2}{R}, \quad f_R := \frac{\partial f}{\partial R} = \frac{1}{2\epsilon^2} \frac{\mathcal{L}_m^2}{R^2} = \frac{1}{2\epsilon^2 \kappa^2} \right). \quad (6)$$

Let us note that $\kappa_{GR} = -R/T$ in General Relativity instead. The stress-energy tensor is not conserved in general, as one has

$$\nabla_\sigma \left(\frac{\mathcal{L}_m}{R} T^{\alpha\sigma} \right) = \mathcal{L}_m \nabla^\alpha \left(\frac{\mathcal{L}_m}{R} \right). \quad (7)$$

The matter field equation, for any tensorial matter field χ , gets modified due to the non-linear coupling between matter and curvature as follows

$$\frac{\partial \mathcal{L}_m}{\partial \chi} - \frac{1}{\sqrt{-|g|}} \partial_\sigma \left(\frac{\partial \sqrt{-|g|} \mathcal{L}_m}{\partial (\partial_\sigma \chi)} \right) = \frac{\partial \mathcal{L}_m}{\partial (\partial_\sigma \chi)} \frac{R}{\mathcal{L}_m} \partial_\sigma \left(\frac{\mathcal{L}_m}{R} \right). \quad (8)$$

It has been shown already that these equations lead to a classical phenomenology that is very close (or even indistinguishable) to the one of general relativity in many cases, while it also (supprisingly) have standard quantum field theory as a limit. It all boils down to the *intrinsic decoupling* that was originally found for scalar-tensor theories in [Minazzoli and Hees, 2013]. Indeed, as usual in $f(R)$ theories, the trace of the metric field equation produces the differential equation for the extra scalar degree-of-freedom κ , which is

$$3\kappa^2 \square \kappa^{-2} = \kappa (T - \mathcal{L}_m). \quad (9)$$

Therefore, whenever $\mathcal{L}_m = T$ on-shell, the extra degree-of-freedom is not sourced and become constant in many occurrences, and one recovers general relativity minimally coupled to matter, and without a cosmological constant, to a very good accuracy. Let us recall that $\mathcal{L}_m = T$ for a universe that would entirely be made of dust and electromagnetic radiation for instance, which turns out

to be a very good approximation of the current content of our universe. As a side note, whenever $\mathcal{L}_m = T$, one recovers the relation $\kappa = -R/T$ of General Relativity.

1.5 Equivalent classical formulation

The whole set of equations can be recovered by the following phase instead

$$\Theta \propto \int d_g^4 x \frac{1}{\kappa} \left(\frac{R}{2\kappa} + \mathcal{L}_m \right), \quad (10)$$

where κ is a dimensionful scalar-field. Equivalently, if one wants to deal with a dimensionless scalar-field that looks more usual instead, the phase can be written as follows

$$\Theta \propto \int d_g^4 x \left(\frac{\varphi^2 R}{2\bar{\kappa}} + \varphi \mathcal{L}_m \right), \quad (11)$$

where $\bar{\kappa}$ is a normalisation dimensionful constant. Or, alternatively again,² in order to look more like an usual Brans-Dicke theory, or like the more general theory explored in [Minazzoli and Hees, 2013], from the following phase:

$$\Theta \propto \int d_g^4 x \left(\frac{\Phi R}{2\bar{\kappa}} + \sqrt{\Phi} \mathcal{L}_m \right). \quad (12)$$

In particular, one has $\Phi \propto f_R$, and the scalar field differential equation reads

$$3\Phi^{-1}\square\Phi = \frac{\bar{\kappa}}{\sqrt{\Phi}} (T - \mathcal{L}_m), \quad (13)$$

where $\bar{\kappa}$ is a dimensionful normalisation constant, and with

$$\sqrt{\Phi} = -\bar{\kappa} \frac{\mathcal{L}_m}{R}. \quad (14)$$

This alternative formulation of the theory looks much more familiar, and therefore allows one to get a better intuition about how it may work. But ultimately, there are no difference between the two formulations— $f(R, \mathcal{L}_m)$ -like or Einstein-dilaton-like.

2. SOLAR SYSTEM PHENOMENOLOGY

2.1 Post-Newtonian metric

Modeling Solar System bodies by perfect fluids with conserved rest-mass energy densities $\nabla_\sigma(\rho_0 U^\sigma) = 0$, where U^α is the proper four-velocity of the fluid, the on-shell value for the matter Lagrangian is $\mathcal{L}_m = -\rho$, where ρ is the total energy density defined by [Minazzoli and Harko, 2013]

$$\rho = \rho_0 \left(1 + \int \frac{P(\rho_0)}{c^2 \rho_0^2} d\rho_0 \right). \quad (15)$$

From there, one can see that the source term of the scalar-field equation (13) only is pressure, which is a $\mathcal{O}(c^{-2})$ term with respect to ρ_0 in post-Newtonian regimes. Therefore, at the post-Newtonian level, the scalar-field is weakly sourced by matter. In particular, one has

$$\Phi^{-1}\square\Phi = \frac{\bar{\kappa}}{\sqrt{\Phi}} P. \quad (16)$$

²Provided that $\varphi > 0$ everywhere, which should be the case in the entire observable universe because it has been shown that φ does not vary more than a few percent in the densest objects in the universe that are not hidden behind an event horizon [Arruga, Rousselle and Minazzoli, 2021].

Then, from Eq. (4), one can check that one has

$$g \left(R^{ij} - \frac{1}{2} g^{ij} R \right) = \mathcal{O}(c^{-4}), \quad (17)$$

where g is the metric's determinant. Therefore, as in General Relativity, there exist a system of coordinates that satisfies the Strong Spatial Isotropy Condition (SSIC) [Damour, Soffel and Xu, 1991]—that is $-g_{00}g_{ij} = \delta_{ij} + \mathcal{O}(c^{-4})$. This notably means that the post-Newtonian parameters γ and β are both equal to one, such that the post-Newtonian metric can be written as follows

$$g_{00} = -1 + 2\frac{w}{c^2} - 2\frac{w^2}{c^4} + \mathcal{O}(1/c^6) \quad (18a)$$

$$g_{0i} = -4\frac{w^i}{c^3} + \mathcal{O}(1/c^5) \quad (18b)$$

$$g_{ij} = \delta_{ij} \left(1 + 2\frac{w}{c^2} \right) + \mathcal{O}(1/c^4), \quad (18c)$$

Injecting this metric in the metric field equation gives the equation on the potential w and w^i that follow

$$\begin{aligned} w &= w_{GR} - \frac{1}{c^2} G \int \frac{P(\mathbf{x}') d^3x'}{|\mathbf{x} - \mathbf{x}'|} + \mathcal{O}(1/c^4), \\ &:= w_{GR} + \frac{1}{c^2} \delta w + \mathcal{O}(1/c^4), \end{aligned} \quad (19a)$$

$$w^i = w_{GR}^i + \mathcal{O}(1/c^2) \quad (19b)$$

where w_{GR} and w_{GR}^i are the expressions of the potentials predicted by general relativity, and $8\pi G := c^4 \bar{\kappa}$. The scalar-field equation on the other-hand is

$$\frac{\phi}{\Phi_0} = 2\delta w + \mathcal{O}\left(\frac{1}{c^2}\right), \quad (20)$$

where $\phi \equiv c^4(\Phi - \Phi_0)$ and Φ_0 the background “astrophysical” value of Φ . Let us note that the derivation follows the one in [Minazzoli and Hees, 2013], since Eq. (12) turns out to be a special case of the class of scalar-tensor theories considered in [Minazzoli and Hees, 2013].

2.2 Trajectories and Shapiro delay

The non-conservation of the stress-energy tensor in Eq. (7) implies an additional gravitational force acting on free-fall particles. However, this additional force turns out to exactly cancel out the modification of the metric with respect to the metric of General Relativity. The result is that the equation of motion for free-fall objects is the same as in General Relativity at the post-Newtonian level, despite the metric being different at the post-Newtonian level—because of the δw term. Indeed, using the conservation of the rest-mass energy density $\nabla_\sigma(\rho_0 U^\sigma) = 0$, the non-conservation equation reduces to

$$U^\sigma \nabla_\sigma U^\mu = -\frac{1}{2} (g^{\mu\sigma} + U^\mu U^\sigma) \frac{\partial_\sigma \Phi}{\Phi}, \quad (21)$$

where $U^\alpha = dx^\alpha/d\tau$ is the proper four-velocity of the particles, which leads to

$$\frac{d^2 x^i}{dt^2} = a_{GR}^i + c^{-2} \left[\partial_i \delta w - \frac{1}{2} \frac{\partial_i \phi}{\Phi_0} \right] + \mathcal{O}(1/c^4) \quad (22)$$

$$= a_{GR}^i + \mathcal{O}(1/c^4), \quad (23)$$

where a_{GR}^i is the standard acceleration in General Relativity.

The unsourced electromagnetic field equation on the other hand reads

$$\nabla_\sigma \left(\sqrt{\Phi} F^{\mu\sigma} \right) = 0, \quad (24)$$

where $F^{\mu\nu}$ is the standard Faraday tensor. One can show that in the geometric optic limit, it implies that photons still follow null-geodesics of spacetime [Minazzoli and Hees, 2013]. But since the metric only differs from the one of General Relativity at the $\mathcal{O}(c^{-4})$ level, it means that the trajectory of light is the same as in General Relativity at the $\mathcal{O}(c^{-2})$ level, which is the only level that can be probed with the current accuracy of Solar System tests. In particular, the Shapiro delay is the same as in General Relativity at the current level of accuracy of radioscience experiments.

It follows that Entangled Relativity is currently indistinguishable from General Relativity from radioscience and ephemerides in the Solar System. This is a rather remarkable result, given the non-linear formulation that one started with in Eq. (1), and given the fact that it did not require adjusting any sort of free parameter. Indeed, the only parameter of the theory is the quantum of energy squared ϵ^2 , which value affects only the paths that are not stationary in the path integral—that is, ϵ^2 affects purely quantum phenomena only.

2.3 The variation of \hbar

Planck's quantum of action \hbar does not appear in the formulation of Entangled Relativity. It necessarily implies that \hbar is not a constant in this theory. We have just seen that the variation Φ in the solar system is even smaller than the variation of gravitational potentials w and w^i . In terms of the original formulation of the theory, it means that the ratio between R and \mathcal{L}_m varies less than the gravitational potentials. This is not surprising, if one keeps in mind that $\kappa_{GR} = -R/T$ is a constant in General Relativity, while one has $\mathcal{L}_m \approx T$ at leading order for perfect fluid with conserved rest-mass energy densities, such that $\kappa \approx -R/T$ in that situation in Entangled Relativity. This means that when one neglects gravity, the variation of the ratio between R and \mathcal{L}_m can also be neglected. In that situation, Eq. (1) would reduce to [Minazzoli, 2023]

$$Z_{ER-QFT} = \int \prod_i [Df_i] \exp \left(\frac{i}{\kappa \epsilon^2} \int d^4x \mathcal{L}_m(f) \right). \quad (25)$$

Because one wants to recover standard quantum field theory when gravity is neglected, it means that

$$\kappa \epsilon^2 = c \hbar. \quad (26)$$

It implies that \hbar actually varies proportionally to κ in general. In other words, it means that \hbar varies akin to a new gravitational field. But, more importantly, one now has determined the value of the quantum of energy ϵ : it is the reduced Planck energy. Let us note that this is similar to how one determines the value of κ_{GR} in General Relativity: by demanding that General Relativity recovers the theory of Newton at leading order—which imposes that $\kappa_{GR} = 8\pi G/c^4$. Here, it is the requirement to recover standard quantum field theory in the limit where gravity can be neglected that dictates the value of ϵ .

Let's evaluate the variation of \hbar in the Solar System, from Eq. (26) and Eqs. (10-12), one has

$$\frac{\delta \hbar}{\hbar} = \frac{\delta \kappa}{\kappa} = -\frac{1}{2} \frac{\delta \Phi}{\Phi}. \quad (27)$$

Then, from Eq. (20) and (19a), one deduces that for a spherical body A one has

$$\frac{\delta \hbar}{\hbar} = \delta \left(\frac{GM_A^P}{c^2 r} \right), \text{ with } M_A^P := 4\pi \int_A \frac{r^2 P(r)}{c^2} dr, \quad (28)$$

which is a mass term defined from pressure rather than from the density. One can evaluate M_P for the Sun and the Earth, they are respectively $M_{\text{SUN}}^P = 2.3 \times 10^{24}$ kg and $M_{\text{EARTH}}^P = 8.0 \times 10^{14}$ kg—to be compared with their masses that are $M_{\text{SUN}} = 2.0 \times 10^{30}$ kg and $M_{\text{EARTH}} = 6.3 \times 10^{24}$ kg. The maximal fractional observable variation of \hbar in the solar system is between the surface of the Sun and a remote observer, and is therefore given by

$$\frac{\delta \hbar}{\hbar} \approx \frac{GM_{\text{SUN}}^P}{c^2 R_{\text{SUN}}} \approx 2.5 \times 10^{-12}, \quad (29)$$

where R_{SUN} is the radius of the Sun. Whether such a small fractional variation can be probed experimentally remains to be investigated. (The details of the derivations will be checked and then communicated in a peer-reviewed journal).

2.4 The c^{-4} Shapiro delay

Preliminary calculations—which, if confirmed, will be communicated in a peer-reviewed journal—indicate that the whole c^{-4} metric of Entangled Relativity surprisingly simply reads as follows

$$g_{\alpha\beta} = g_{\alpha\beta}^{GR} + \delta_{\alpha\beta}^{00} \frac{2\delta w}{c^4} + \mathcal{O}(c^{-5}), \quad (30)$$

where $g_{\alpha\beta}^{GR}$ is the solution of general relativity, δw is defined in Eq. (19a), and $\delta_{\alpha\beta}^{00}$ is a Kronecker symbol. As a consequence, the coordinate propagation time between an emission (e) and a reception (r) in Entangled Relativity would read as follows

$$\begin{aligned} c(t_r - t_e)_{ER} = & R + \sum_A (1 + \gamma_A) \frac{GM_A}{c^2} \ln \left(\frac{\vec{n} \cdot \vec{r}_{rA} + r_{rA}}{\vec{n} \cdot \vec{r}_{eA} + r_{eA}} \right) \\ & + c(t_r - t_e)_{GR}^{(4)} + \mathcal{O}(c^{-5}), \end{aligned} \quad (31)$$

where $c(t_r - t_e)_{GR}^{(4)}$ are the remaining c^{-4} terms that are the same as in General Relativity. Hence, surprisingly, the c^{-4} correction to the Shapiro delay due to Entangled Relativity looks like an usual post-Newtonian correction. However, it is important to emphasize a few key aspects. First, unlike in usual alternative theories, its value is body-dependent. Moreover, it can be fully calculated for each body A without any free parameter at the theoretical level. Finally, the value of γ_A is expected to be very small, again without any free parameter at the theoretical level. Indeed, while the exact derivation and numerical estimations will be published in a peer-reviewed journal, it is roughly estimated that $1 - \gamma_A \propto M_A^P/M_A$, while M_A^P/M_A is at best of the order of 10^{-6} in the Solar System (for the Sun).

While testing the Shapiro delay at the 10^{-6} level in the Solar System is currently beyond the reach of experimental accuracy, the MORE experiment on BepiColombo is approaching this level of precision [Cappuccio et al., 2020]. Therefore, it is not too much of a stretch to consider that it may be possible to verify this prediction in the not-too-distant future.

3. CONCLUSION

Entangled Relativity is a novel general theory of relativity that is more economical than General Relativity in its formulation. Nevertheless, it predicts very small deviations from General Relativity in weak field situations, such as those in the Solar System. Additionally, it implies several significant conceptual shifts at the theoretical level. One notable example is the proposition that the quantum of action, \hbar , would not be a constant, but rather vary as a new gravitational field. In this communication, I explained that the deviations from General Relativity, which do not depend

on any free theoretical parameters, are quite small in the Solar System, yet they might still be detectable in future experiments.

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