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# Dynamic similarity during human running: About Froude and Strouhal dimensionless numbers

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## A B S T R A C T

Dynamic similarity is a widely used concept in the fluid mechanics field, and consists in placing two different-sized systems in equivalent experimental conditions. This enables removal of the effects of size and prediction of the behavior of a full size system from a scale model. The aim of this study was to test whether the Froude number (Nfr) or the Strouhal number (Str) could be used as a criterion for dynamic similarity during running. Fifteen male subjects ran barefoot on a runway in three experimental conditions (i) all subjects ran at the same speed  $V = 3.5 \text{ m s}^{-1}$ ; (ii) the speed was determined from Nfr; (iii) the stride frequency was determined from Str. Antero-posterior (Fy) and vertical (Fz) ground reaction force components were assessed. The similarity between the subjects was analysed from scale factor sets computed from anthropometric and kinetic data. The use of Str implied strong inter-subject similarity for temporal parameters (mean  $r = 0.96$ , time to Fz peak, time to Fy braking peak, Fy zero fore-aft shear, time to Fy propulsive peak) while Nfr induced fewer and lower similarities (mean  $r = 0.75$ , Fy zero fore-aft shear, time to Fy propulsive peak, Fy braking impulse) that only concerned antero-posterior parameters. This study brought experimental evidence that neither Nfr nor Str were sufficient for dynamic similarity during running, but that each of them made its own contribution. These findings suggested that the concomitant use of Nfr and Str should be assessed to induce inter-subject dynamic similarity during running.

### Keywords:

Running  
Dynamic similarity  
Dimensionless number  
Froude  
Strouhal

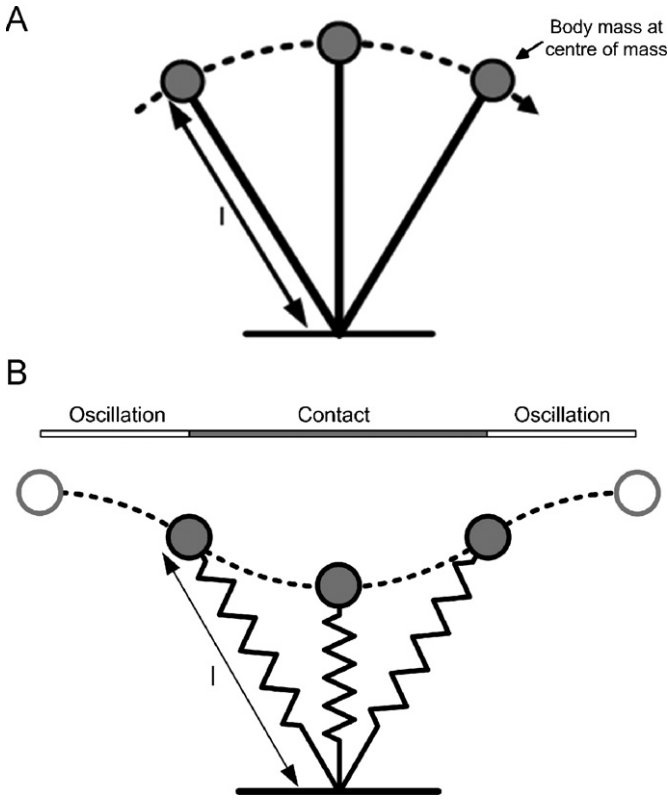
## 1. Introduction

The concept of dynamic similarity has the same role in the study of movement as geometric similarity has in the study of shape. When two shapes are geometrically similar, one could be made identical to the other by multiplying all the lengths by the same scale factor  $C_L$ . In the same way, if two movements are dynamically similar, one could be made identical to the other by multiplying all lengths ( $[L]$  dimension) by one scale factor  $C_L$ , all masses ( $[M]$  dimension) by another scale factor  $C_M$ , and all times ( $[T]$  dimension) by a third scale factor  $C_T$ . The scale factors for these three reference dimensions ( $L$ ,  $M$ , and  $T$ ) allow the scale factor for all other mechanical parameters to be determined. This scaling principle could thus be used both for inter-individual comparison in humans, animals and robots, and to estimate locomotion patterns for extinct species and those on other planets (Minetti, 2001; Vaughan and Blaszczyk, 2008; Vaughan and O'Malley, 2005).

While walking, all individuals can be considered as acting like an inverted pendulum. This model consists of a point mass (representing the body mass at the centre of mass) oscillating at the end of a massless rigid segment length ( $l$ ) (Fig. 1A; Cavagna et al., 1977; Dickinson et al., 2000). In view of this model, it appears that gravity is an important factor in determining the speed where the walk-to-run transition occurs (Alexander, 1992; Usherwood, 2005). Typically during a walking cycle, the centre of mass moves in a circular arc, which implies a centripetal force equal to  $mV^2/l$  (with  $m$ : body mass;  $V$ : forward speed;  $l$ : rigid segment length) acting on the centre of mass. The ratio between the centripetal force and the gravitational force ( $mg$ , with  $g$ : gravitational acceleration) is called the Froude number (Nfr):  $\text{Nfr} = (mV^2/l)/(mg) = V^2/(gl)$ . Nfr corresponds to the dimensionless expression of the speed, humans and other animals are able to use a walking gait only when the centripetal force is lower than or equal to the gravitational force, i.e., only at speeds where Nfr is less than 1. Thus, it appears that the theoretical walk-to-run transition should occur at  $\text{Nfr} = 1$  for animals of a wide range of sizes (Alexander, 1989, 1992). Experimental evidence has shown that the walk-to-run transition appears when Nfr equals about 0.5 in humans (Alexander, 1989; Kram et al., 1997; Thorstensson and

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**Fig. 1.** (A) The inverted pendulum model for the mechanics of walking. (B) The spring-mass model for the mechanics of running. This model consists of a point mass (representing the body mass at the centre of mass) bouncing on a massless linear spring.  $l$ : initial spring length.

Robertson, 1987), Equal values of Nfr were shown to ensure dynamic similarity between walking humans (Bisiaux et al., 2003; Moretto et al., 2007), and between different animals species during running (Farley et al., 1993). Moreover, dynamic similarity allows a decrease in dimensionless data variability (Bisiaux et al., 2003; Moretto et al., 2007), which is very useful for an analysis to distinguish between groups. Such results have not yet been obtained during human running. Although dynamic similarity has been observed in animals running with equal Nfr (Bullimore and Burn, 2006; Farley et al., 1993), Nfr has never been used for the study of dynamic similarity during human running.

Since Nfr is based on the pendulum-like mechanics of walking, it does not consider any elastic phenomenon that has an important role during bouncing gaits like running (Cavagna et al., 1977). One way to consider the elastic phenomenon is to take the non-linear stress-strain relationship of tendons, which allows the reduction in deviations from dynamic similarity occurring when using Nfr during bouncing gaits (Bullimore and Burn, 2006; Bullimore and Donelan, 2008). Another way is to consider the runner as a spring-mass system, a model commonly used in the literature (e.g., Dalleau et al., 1998; Farley and Gonzalez, 1996; Lee and Farley, 1998; Morin et al., 2005) since it was introduced by Blickhan (1989). The model consists of a point mass bouncing on a massless spring (Fig. 1B). The virtual “leg spring” undergoes length variation during the ground contact phase, which illustrates runners’ bounces and enables them to store and return elastic energy (Cavagna et al., 1977). In order to take account of this elastic phenomenon for the dynamic similarity hypothesis, Alexander (1989) suggested referring to the Strouhal dimensionless number  $Str = ((\text{Frequency} \times \text{Length}) / \text{Speed})$ .  $Str$  represents a dimensionless frequency, and in its

expression, while the leg length appears to be the only length that can be used, any frequency and any speed component of the centre of mass may be used. According to Alexander (1989), two geometrically similar individuals running with the same values of  $Str$  should move in a dynamically similar fashion. In a sub-gravity field, the use of  $Str$  including the natural frequency of the spring-mass system and the vertical landing speed did not enable dynamic similarity to be observed (Donelan and Kram, 2000). Stride frequency and forward speed are two important parameters of the running pattern, which are also considered by the spring-mass system. Moreover, to the best of our knowledge,  $Str$  has never been used for dynamic similarity at Earth gravity.

Our study aimed to evaluate the respective effect of Nfr and  $Str$ , and to compare them to a control condition in order to identify the most relevant experimental conditions for dynamic similarity between runners. We hypothesized that, as Nfr made it possible during walking (Bisiaux et al., 2003; Moretto et al., 2007), Nfr and  $Str$  could make it possible to induce dynamic similarity between subjects when running.

## 2. Methods

### 2.1. Population

Fifteen healthy male subjects, all regular runners, volunteered to take part in the study. All participants signed an informed consent document. Their characteristics were (mean  $\pm$  sd): age  $23 \pm 9$  years, height  $1.79 \pm 0.06$  m, leg length  $0.94 \pm 0.03$  m, and body mass  $69.7 \pm 7.4$  kg. Leg length was measured as the distance from the ground to the greater trochanter when standing.

### 2.2. Experimental conditions

The subjects performed running tests under three experimental conditions ( $EC_{3.5}$ ,  $EC_{Nfr}$ , and  $EC_{Str}$ ).  $C_L$  and  $C_M$  were imposed by the anthropometry of our subjects, and were computed from the leg length and the mass, respectively. For instance, the scale factor for lengths between two subjects  $S_1$  and  $S_2$  was computed from the ratio of their leg length  $l_1$  and  $l_2$ , respectively:  $C_L = l_1/l_2$ . Each experimental condition enabled us to impose an inter-subject scale factor for times ( $C_T$ ) through the speed (in  $EC_{3.5}$  and  $EC_{Nfr}$ ) or the stride frequency (in  $EC_{Str}$ ).

#### 2.2.1. Experimental control condition ( $EC_{3.5}$ )

The same speed  $V = 3.5 \text{ m s}^{-1}$  was imposed on all subjects. The  $V = 3.5 \text{ m s}^{-1}$  value has often been used as a recreational jogging speed in studies on running gait (e.g. McNair and Marshall, 1994). All subjects running at the same absolute speed, the inter-subject scale factor for speeds was  $C_{\text{speed}} = C_L C_T^{-1} = 1$ , thus the scale factor for times ( $C_T$ ) equalled the scale factor for lengths ( $C_L$ ). The same speed being imposed on different-sized subjects, no dynamic similarity was expected in this condition, and  $EC_{3.5}$  was used as a control condition. In this condition, stride frequency was freely chosen by the subjects.

Leg lengths ( $l_i$ ), speed ( $V$ ), and stride frequencies ( $f_i$ ) recorded in  $EC_{3.5}$  enabled us to determine the two other experimental conditions  $EC_{Nfr}$  and  $EC_{Str}$ .

#### 2.2.2. Experimental condition Nfr ( $EC_{Nfr}$ )

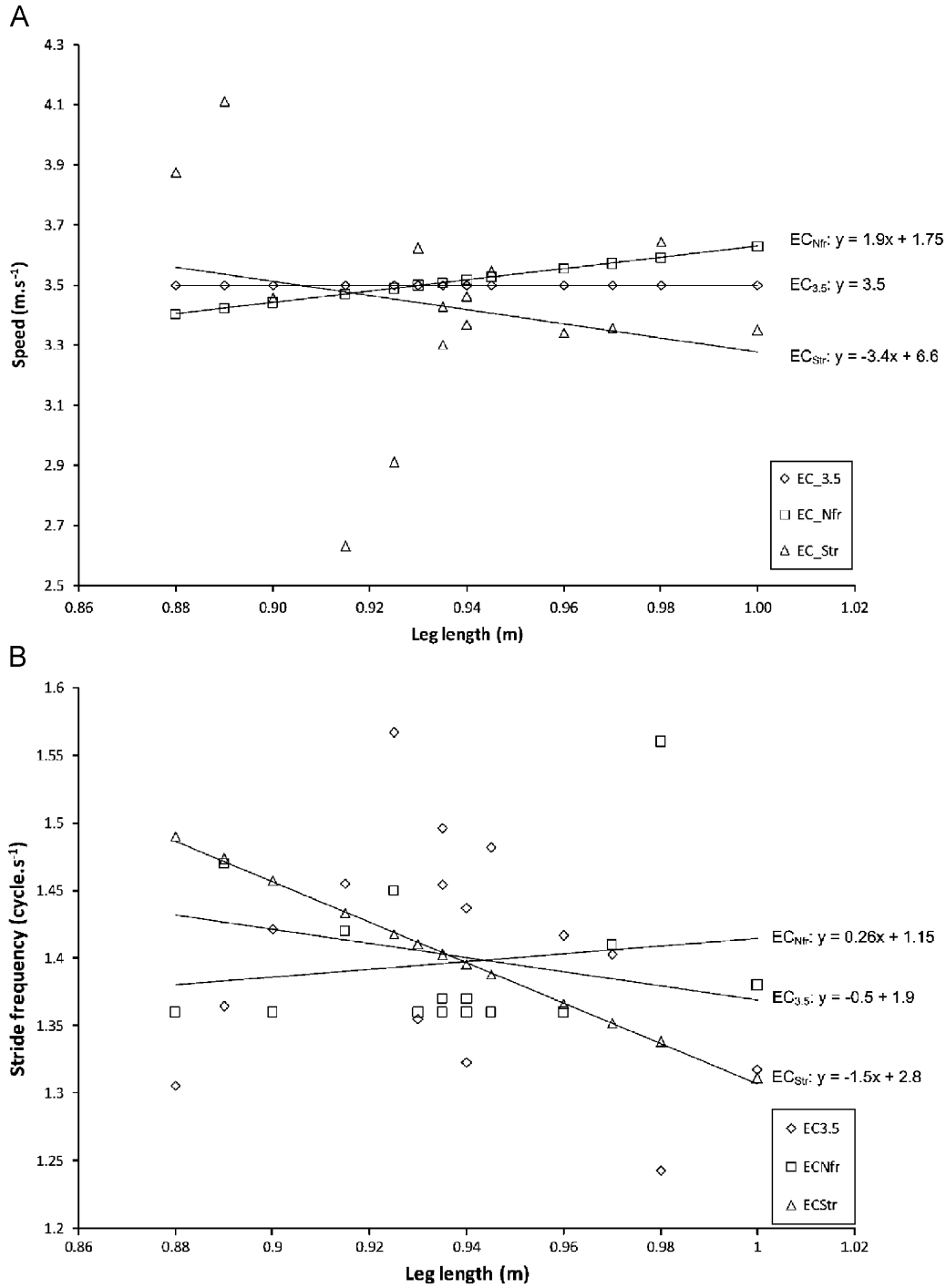
The  $V = 3.5 \text{ m s}^{-1}$  running speed imposed in  $EC_{3.5}$  and subjects’ leg lengths  $l_i$  enabled us to compute an overall mean Nfr ( $\overline{Nfr} = (1/n) \sum_{i=1}^n Nfr_i$ ) for our population, where  $n$  is the number of subjects: 15. We then used  $\overline{Nfr}$  to determine a speed ( $V_{sim_i}$ ) proportional to leg length ( $l_i$ ) for each subject:

$$V_{sim_i} = \sqrt{\overline{Nfr} g l_i} \quad (1)$$

This method made it possible to keep the same value of  $\overline{Nfr}$  in  $EC_{3.5}$  and  $EC_{Nfr}$  while enabling us to reorder running speed as a function of leg length in  $EC_{Nfr}$  (Fig. 2A). The speeds determined from Nfr were called “similar speeds” ( $V_{sim}$ ).

In  $EC_{Nfr}$ , stride frequency was freely chosen by the subjects, and similar speeds enabled us to establish a scale factor for times ( $C_T$ ) between the motions of two subjects. Indeed, as shown by Eq. (2), the scale factor of similar velocities ( $C_L C_T^{-1}$ ) is equal to  $C_L^{0.5}$ , consequently, the scale factor for times is  $C_T = C_L^{0.5}$ .

$$\frac{V_{sim_2}}{V_{sim_1}} = \frac{\sqrt{\overline{Nfr} g l_2}}{\sqrt{\overline{Nfr} g l_1}} = \sqrt{\frac{l_2}{l_1}} = C_L^{0.5}$$



**Fig. 2.** (A) Running speed as a function of subjects' leg length. In EC<sub>3.5</sub>, all subjects ran at the same speed indifferently of their leg length. In EC<sub>Nfr</sub>, the speed imposed on subjects depended on their leg length. In EC<sub>Str</sub>, the running speed was freely chosen by subjects. (B) Stride frequency as a function of subjects' leg length. In EC<sub>3.5</sub>, stride frequency was self-selected by the subjects. In EC<sub>Str</sub>, the stride frequency imposed on subjects depended on their leg length. In EC<sub>Nfr</sub>, the stride frequency was freely chosen by subjects.

### 2.2.3. Experimental condition Str (EC<sub>Str</sub>)

Leg lengths ( $l_i$ ), speed ( $V$ ), and self-selected stride frequencies ( $f_i$ ) recorded in EC<sub>3.5</sub> enabled us to compute an overall mean Str ( $\overline{\text{Str}} = (1/n) \sum_{i=1}^n \text{Str}_i$ ) for our population, where  $n$  is the number of subjects: 15. We then used  $\overline{\text{Str}}$  to determine a stride frequency ( $F_{\text{sim}_i}$ ) proportional to the leg length ( $l_i$ ) for each subject:

$$F_{\text{sim}_i} = \frac{\overline{\text{Str}} V}{l_i} \quad (3)$$

The computation of similar stride frequencies was made from values recorded in EC<sub>3.5</sub>, thus  $V$  was equal to  $3.5 \text{ m s}^{-1}$  in Eq. (3). This method made it possible to keep the same value of Str in EC<sub>3.5</sub> and EC<sub>Str</sub> while enabling us to reorder stride

frequencies as a function of leg length in EC<sub>Str</sub> (Fig. 2B). The stride frequencies determined from Str were called "similar stride frequencies" (Fsim).

In EC<sub>Str</sub>, similar stride frequencies enabled us to establish a scale factor for times ( $C_T$ ) between the motions of two subjects. The running speed was freely chosen, and since  $C_L$  and  $C_T$  were imposed by the anthropometry and the similar stride frequencies, respectively, the scale factor for speeds was expected to be  $C_L C_T^{-1}$ , i.e., speeds were expected to be similar between subjects.

As shown by Eq. (4), the scale factor for frequencies is  $C_T^{-1} = C_L^{-0.5}$ , thus the scale factor for times is  $C_T = C_L^{0.5}$ .

$$\frac{F_{\text{sim}_2}}{F_{\text{sim}_1}} = \frac{(\overline{\text{Str}} V_{\text{sim}_2} / l_2)}{(\overline{\text{Str}} V_{\text{sim}_1} / l_1)} = \frac{V_{\text{sim}_2}}{V_{\text{sim}_1}} \frac{l_1}{l_2} = C_L^{0.5} C_T^{-1} = C_L^{-0.5}$$

This three-step procedure enabled us to compare experimental conditions while the subjects ran at the same mean dimensionless speed ( $\overline{Nfr}$ ) in EC<sub>3.5</sub> and EC<sub>Nfr</sub>, and at the same mean dimensionless stride frequency ( $\overline{Str}$ ) in EC<sub>3.5</sub> and EC<sub>Str</sub>.

### 2.3. Running test

Subjects ran on a 15-m-long runway equipped with a flush-to-the-floor mounted force plate (Kistler, 9281 B21 model) sampled at 1 kHz and covered by ethylene vinyl acetate (EVA) foam. The foam allowed us to mask the force platform location in order to avoid a targeting artefact and also allowed subjects to run barefoot to avoid a shoe-type effect on measurements.

During EC<sub>3.5</sub> and EC<sub>Nfr</sub> conditions, the speed was imposed by a metronome tone (bleeps) and marking cones placed every 3 m on the runway. Subjects were asked to pass a cone at every bleep while the interval of the bleeps was calculated so that each subject ran at  $V = 3.5 \text{ m s}^{-1}$  (EC<sub>3.5</sub>) or  $V_{sim_i}$  (EC<sub>Nfr</sub>). We verified that the subjects matched the imposed speed by computing their real running speed  $V = d/t$  (with  $d$ : the distance between two photo-electric cells two meters apart and arranged on both sides of the force plate,  $t$ : the time elapsed to travel distance  $d$ , measured by the photo-electric cells).

During EC<sub>Str</sub> condition,  $F_{sim}$  was imposed by a metronome tone. The subjects were asked to hit the ground at every bleep. We verified that the subjects matched the imposed stride frequency by using an electronic stride frequency counter.

The subjects carried out a familiarisation session (Karamanidis et al., 2004) before each condition, and then performed five recorded trials per condition in order to ensure consistency of ground reaction force (GRF) data. The EC<sub>Str</sub> and EC<sub>Nfr</sub> sessions were randomised to avoid a learning artefact.

### 2.4. Data processing

A threshold of 5 N in the vertical component of GRF ( $F_z$ ) was chosen to identify the onset and the offset of the contact phase. Ground reaction data were low-pass filtered at 50 Hz. The  $F_z$  curve was analysed to check the time of contact (TC); active peak force (PF); time to active peak (TP); loading rate (LR) between 10% and 90% of PF; and vertical impulse (Iz) (Fig. 3A). The antero-posterior GRF component ( $F_y$ ) allowed us to obtain the braking peak (BP), propulsion peak (PP), time to braking peak (TBP), zero fore-aft shear (ZFAS, time at which the antero-posterior component of the ground reaction force ( $F_y$ ) changed direction from backward (braking) to forward (propulsion)), time to propulsion peak (TPP), braking impulse (BI), and propulsion impulse (PI) (Fig. 3B).

### 2.5. Similarity analysis process

Scale factors were computed for each subject pair combination with reciprocal ratios avoided. Hence, for each parameter, a set of 120 inter-subject combinations ( $C_{15}^2 = 120$ : pair combinations among 15 subjects) was obtained. We then referred to these sets as "scale factor sets".

$C_L$ ,  $C_M$ , and theoretical  $C_T$  being known, the scale factor for each kinetic parameter was predicted from them. For example, according to Newton's second law, the dimension of a peak force (accelerated mass) is  $MLT^{-2}$ . Consequently, the scale factor between two peak forces ( $C_{PF}$ , ratio of the peak force  $PF_1$  and  $PF_2$  of two subjects  $S_1$  and  $S_2$ , respectively) can be predicted by the product of the scale factors involved:  $C_{PF} = C_M C_L C_T^{-2}$ . Units, dimensions, and predicted scale factors for each parameter studied are summarized in Table 1.

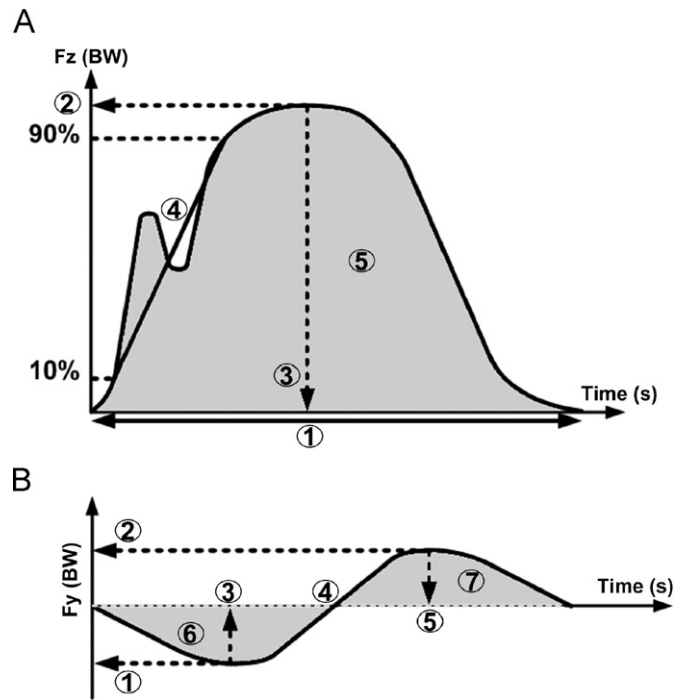
On the one hand, the predicted scale factor sets for kinetic parameters were computed following the same method (Table 1). On the other hand, scale factor sets were also computed from measured kinetic parameters. When, for a given parameter, the measured scale factor set equalled the predicted scale factor set, it could then be concluded that this parameter was similar (proportional) from one subject to another.

### 2.6. Statistical analysis

For each experimental condition, data recorded over the five trials were averaged. Standard statistical methods were used to calculate means and standard deviation. The normality of data distribution was then verified with the Shapiro-Wilks test.

A two-step analysis was conducted to verify the equality between predicted and measured scale factor sets. Firstly, we carried out a paired  $t$ -test ( $p < 0.05$ ) in order to compare means and variance of predicted scale factor sets and measured scale factor sets. Then, we performed a Fisher's "z" coefficient of correlation ( $p < 0.05$ ) in order to verify that the evolution of predicted scale factor sets was the same as that of measured scale factor sets. Only  $r$  values above 0.7 for correlations have an acceptable level of confidence (Vincent, 1999), and were considered in our study.

Thereafter, two scale factor sets which did not significantly differ in means and variance, and which were significantly correlated with a common variance ( $r^2$ )



**Fig. 3.** (A) Vertical ground reaction parameters. Vertical ground reaction force ( $F_z$ ) as a function of time. 1: Time of contact (TC); 2: active peak force (PF); 3: time to peak (TP); 4: loading rate (LR); 5: vertical impulse (Iz). The force is expressed relative to body weight (BW). (B) Antero-posterior ground reaction parameters. Antero-posterior ground reaction force ( $F_y$ ) as a function of time. 1: Braking peak (BP); 2: propulsion peak (PP); 3: time to braking peak (TBP); 4: zero fore-aft shear (ZFAS); 5: time to propulsion peak (TPP); 6: braking impulse (BI); 7: propulsive impulse (PI). The force is expressed relative to body weight (BW).

**Table 1**

Units, dimensions and predicted scale factors of studied parameters.

| Parameters                    | Units                 | Dimensions | Predicted scale factors |
|-------------------------------|-----------------------|------------|-------------------------|
| Leg length                    | m                     | L          | $C_L$                   |
| Body mass                     | kg                    | M          | $C_M$                   |
| Time (TC, TP, TBP, ZFAS, TPP) | s                     | T          | $C_T$                   |
| Speed                         | $\text{m s}^{-1}$     | $LT^{-1}$  | $C_L C_T^{-1}$          |
| Stride frequency              | $\text{cycle s}^{-1}$ | $T^{-1}$   | $C_T^{-1}$              |
| Peak force (PF, BP, PP)       | N                     | $MLT^{-2}$ | $C_M C_L C_T^{-2}$      |
| Loading rate (LR)             | $\text{N s}^{-1}$     | $MLT^{-3}$ | $C_M C_L C_T^{-3}$      |
| Impulse (Iz, BI, PI)          | Ns                    | $MLT^{-1}$ | $C_M C_L C_T^{-1}$      |

Vertical ground reaction force parameters: TC: time of contact; TP: time to peak; PF: active vertical peak force; LR: loading rate; Iz: vertical impulse. Antero-posterior ground reaction force parameters: BP: braking peak; PP: propulsive peak; TBP: time to braking peak; ZFAS: zero fore-aft shear; TPP: time to propulsive peak; BI: braking impulse; PI: propulsive impulse.  $C_L$  and  $C_M$  were imposed by the anthropometry of our subjects, and were computed from the leg length and the mass, respectively.  $C_T$  was dependent on the experimental condition (see Methods).  $C_L$ ,  $C_M$ , and  $C_T$  were used as reference scale factors for basic physical dimensions (L, M, and T, respectively) to compute the predicted kinetic scale factors.

value exceeding 0.5, were considered to be identical. In that case, dynamic similarity between runners was met.

## 3. Results

$Nfr$  and  $Str$  enabled us to reorder the speed (Fig. 2A) and the stride frequency (Fig. 2B) as a function of leg length, respectively.

The average values of speed, stride frequency,  $Nfr$ , and  $Str$  were identical throughout the three experimental conditions (Table 2).

This allowed us to make a comparison between the three conditions.

Two criteria were taken into account to consider an experimental condition as being the most appropriate for dynamic similarity. The quantitative criterion was based on the number of parameters for which the predicted and the measured scale factor sets were identical, while the qualitative criterion was based on the average value of the correlations between the predicted and the measured scale factor sets.

Correlations between the predicted and the measured scale factor sets for spatiotemporal parameters in each experimental condition (Table 3) showed that subjects matched the imposed speed ( $EC_{3.5}$  and  $EC_{Nfr}$ ) and the imposed stride frequency ( $EC_{Str}$ ). As expected,  $EC_{3.5}$  only enabled the inter-subject scale factor for times to be imposed ( $C_T = C_L$ ), and no inter-subject similarity was obtained for stride length, speed, and stride frequency. The absence of correlation between measured and predicted scale factors for speeds confirmed that no running speed similarity existed between the subjects.

Both similar speeds ( $LT^{-1}$  dimension) imposed in  $EC_{Nfr}$  and similar stride frequencies ( $T^{-1}$  dimension) imposed in  $EC_{Str}$  enabled us to obtain inter-subject similarity for the times of contact ( $T$  dimension). Since, for the three experimental conditions, the measured scale factor for times of contact matched the predicted scale factor for times, the scale factor set for times of contact was hence used as  $C_T$  (reference scale factor for times) for computation of the predicted kinetic scale factor sets.

Correlations between predicted and measured scale factor sets for ground reaction force parameters are shown in Table 4. Inter-

**Table 2**  
Mean values of speed, stride frequency, Nfr, and Str in each experimental condition.

|                  | $EC_{3.5}$     | $EC_{Nfr}$     | $EC_{Str}$    |
|------------------|----------------|----------------|---------------|
| Speed            | 3.5<br>(0)     | 3.5<br>(0.06)  | 3.4<br>(0.35) |
| Stride frequency | 1.4<br>(0.09)  | 1.4<br>(0.06)  | 1.4<br>(0.05) |
| Nfr              | 1.3<br>(0.05)  | 1.3<br>(0)     | 1.3<br>(0.28) |
| Str              | 0.38<br>(0.02) | 0.37<br>(0.02) | 0.38<br>(0)   |

Mean (sd).

**Table 3**  
Correlations between the predicted and the measured scale factor sets for spatiotemporal parameters and for time of contact.

| Parameters              | $EC_{3.5}$ | $EC_{Nfr}$ | $EC_{Str}$ |
|-------------------------|------------|------------|------------|
| $C_{speed}$             | –          | 0.853*     | –0.223†    |
| $C_{stride\ length}$    | –0.003†    | –0.410*†   | –0.341†    |
| $C_{stride\ frequency}$ | 0.037†     | –0.267†    | 0.987*     |
| $C_{TC}$                | 0.486*     | 0.455*     | 0.435*     |

\*: Significant correlation ( $p < 0.05$ ) between predicted and measured scale factor sets; †: significant difference ( $p < 0.05$ ) in means and variance between predicted and measured scale factor sets. We considered that measured scale factor set matched the predicted scale factor set when they were significantly correlated, and were not significantly different in mean and variance. When these two last criteria were met, the value was *italicised*. –: No correlation between predicted and measured scale factors for speed since the predicted scale factor for speed is always 1 in  $EC_{3.5}$ .

Scale factor for speeds ( $C_{speed}$ ), for stride lengths ( $C_{stride\ length}$ ), for stride frequencies ( $C_{stride\ frequency}$ ), for times of contact ( $C_{TC}$ ). Speed, stride frequency, and times of contact were measured (see Methods). Stride length was computed from the ratio between speed and stride frequency. We chose the time of contact as the most representative parameter in order to verify the equality between the measured scale factor for times and the predicted scale factor for times which were to be imposed.

**Table 4**  
Correlation between the predicted and the measured scale factor sets for ground reaction force parameters.

| Scale factor set | $EC_{3.5}$ | $EC_{Nfr}$ | $EC_{Str}$ |
|------------------|------------|------------|------------|
| $C_{TP}$         | 0.75 *     | 0.53 *     | 0.96 *     |
| $C_{PF}$         | 0.27       | 0.57 *     | 0.93 *†    |
| $C_{LR}$         | 0.82 *†    | 0.46 *     | 0.97 *†    |
| $C_{Lz}$         | 0.48 *†    | 0.08       | –0.24 †    |
| $C_{BP}$         | –0.14 †    | –0.13      | 0.88 *†    |
| $C_{PP}$         | 0.31       | 0.18       | 0.8 *†     |
| $C_{TBP}$        | 0.48 *†    | 0.86 *†    | 0.94 *     |
| $C_{ZFAS}$       | 0.72 *     | 0.7 *      | 0.98 *     |
| $C_{TPP}$        | 0.79 *     | 0.78 *     | 0.99 *     |
| $C_{BI}$         | 0.54 *     | 0.77 *     | 0.66 *†    |
| $C_{PI}$         | 0.27       | 0.2        | –0.12 †    |

\*: Significant correlation ( $p < 0.05$ ) between predicted and measured scale factor sets; †: significant difference ( $p < 0.05$ ) in means and variance between predicted and measured scale factor sets. We considered that measured scale factor set matched the predicted scale factor set when they were significantly correlated with an  $r$  value higher than  $r = 0.7$  (Vincent, 1999), and were not significantly different in mean and variance. When these two last criteria were met, the value was *italicised*.

Scale factor for time to peak ( $C_{TP}$ ), for active peak force ( $C_{PF}$ ), for loading rate ( $C_{LR}$ ), for vertical impulse ( $C_{Lz}$ ), for braking peak ( $C_{BP}$ ), for propulsive peak ( $C_{PP}$ ), for time to braking peak ( $C_{TBP}$ ), for zero fore-aft shear ( $C_{ZFAS}$ ), for time to propulsive peak ( $C_{TPP}$ ), for braking impulse ( $C_{BI}$ ), scale factor for propulsive impulse ( $C_{PI}$ ).

subject similarity (with  $r$  values greater than  $r = 0.7$ ) was obtained for three parameters (TP, ZFAS, and TPP, mean correlation  $r = 0.75$ ) in  $EC_{3.5}$ , for three parameters (ZFAS, TPP, and BI, mean correlation  $r = 0.75$ ) in  $EC_{Nfr}$ , and for four parameters (TP, TBP, ZFAS, and TPP, mean correlation  $r = 0.96$ ) in  $EC_{Str}$ .

#### 4. Discussion

Our study aimed to compare Nfr and Str to a control condition in order to identify the most relevant experimental conditions for dynamic similarity during running.

As expected, the use of the same absolute running speed for all subjects ( $EC_{3.5}$ ) allowed us to induce only an inter-subject scale factor for some temporal parameters (Table 4), and did not make it possible to induce dynamic similarity between subjects. Our method based on inter-subject scale factor sets confirms that, while running at the same speed, different-sized subjects move in a very different manner (Alexander and Jayes, 1983).

In  $EC_{Nfr}$ , similar speeds ( $LT^{-1}$  dimension) enabled us to observe a scale factor for some temporal parameters. However, Nfr did not appear to be appropriate to induce complete dynamic similarity, because similar parameters were as numerous and similarities as strong in  $EC_{Nfr}$  as in our control condition  $EC_{3.5}$  (Table 4). This was in disagreement with the results of Farley et al. (1993), but in accordance with other studies (Bullimore and Donelan, 2008; Donelan and Kram, 2000) reporting that, although equal values of Nfr are not sufficient for dynamic similarity during running, it is a necessary condition. Our results also showed that similar speeds enabled similarities to be obtained exclusively for parameters from Fy. Similar speeds being normalized forward speeds in our study, this last result shows that the use of Nfr enables inter-subject similarity to be obtained exclusively for parameters along the same axis. As an example, Nfr could be useful to remove the effects of size by normalizing the vertical speed in studies focused on impact, or by normalizing the horizontal speed in studies focused on propulsion and braking phases.

Imposing similar stride frequencies enabled us to obtain similarities that were more numerous, and stronger (mean correlation  $r = 0.96$ ) than in other experimental conditions

(Table 4). Two comments arose. On the one hand, inter-subject similarity was not obtained for all the parameters studied, which proved that Str did not make it possible to induce complete dynamic similarity, as shown in sub-gravity (Donelan and Kram, 2000). On the other hand, similar stride frequencies enabled us to impose an inter-subject scale factor for all temporal parameters (Table 4). This finding suggests that similar stride frequencies ( $T^{-1}$  dimension) enabled us to impose the inter-subject scale factor for times more directly than similar speeds ( $LT^{-1}$  dimension). Str thus appeared to be necessary and could play a key role for dynamic similarity during running.

Our findings have been highlighted through the analysis of inter-subject scale factor sets for all parameters, which made it possible to accurately study the scaling principle inherent in the dynamic similarity. In studies dedicated to dynamic similarity (Bullimore and Burn, 2006; Bullimore and Donelan, 2008; e.g., Donelan and Kram, 1997, 2000; Farley et al., 1993), the inter-subject equality of some dimensionless parameters is usually used to attest to dynamic similarity. The dimensionless parameters are: relative stride length (stride length divided by leg length), duty factor (time of contact divided by stride time), and relative peak force (peak in vertical ground reaction force divided by body weight) (Hof, 1996). These parameters computed from our data are reported in Table 5. The inter-subject variability of relative stride length and relative peak force significantly increased under  $EC_{Str}$  and confirmed the poor inter-subject similarity for lengths and for forces that was also revealed using scale factor sets. Conversely, the variability of the duty factor was not significantly affected in  $EC_{Str}$  compared to  $EC_{3.5}$  and thus did not reveal similarity for times, whereas the scale factor sets revealed strong similarity for times. Moreover, there was no significant difference in dimensionless parameter variability between  $EC_{Nfr}$  and  $EC_{3.5}$ , which showed that the study of dimensionless parameters ignored some inter-subject similarities highlighted using scale factor sets.

Our method enabled us to study inter-subject similarity through all the subject pair combinations, which increased the statistical power and the ability to discriminate between conditions. As mentioned previously (Moretto et al., 2007), dynamic similarity allows a decrease in dimensionless data variability. The opposite approach, which consists in studying dynamic similarity from the decrease of dimensionless parameters, is limited to cases where inter-subject comparison is impossible (extinct species).

In conclusion, our experimental evidence showed that neither Nfr nor Str were sufficient for dynamic similarity during running, but that each of them made its own contribution. As suggested by Alexander (1989), the concomitant use of Nfr and Str could enable their cumulative effect. It has already been shown that the Moretto-Delattre dimensionless number ( $N_{Mo-Dela}$ ), highlighted from mechanical energy exchange on a spring-mass system, can be expressed as a function of Nfr and Str (Delattre and Moretto, 2008). Future experimental investigation should assess the

**Table 5**  
Comparison of variability between conditions for dimensionless parameters generally used to study dynamic similarity.

|     | $EC_{3.5}$    | $EC_{Nfr}$    | $EC_{Str}$     |
|-----|---------------|---------------|----------------|
| RSL | 2.733 (0.183) | 2.733 (0.176) | 2.66 (0.463)*† |
| DF  | 0.337 (0.036) | 0.336 (0.034) | 0.341 (0.059)  |
| RPF | 2.484 (0.188) | 2.504 (0.288) | 2.566 (0.478)* |

Mean (sd). RSL: relative stride length; DF: duty factor; RPF: relative peak force. \*: Variability significantly ( $p < 0.05$ ) different from  $EC_{3.5}$ . †: Variability significantly different from  $EC_{Nfr}$ . Statistical test was  $F$ -Snedecor variance ratio.

concomitant use of Nfr and Str by using  $N_{Mo-Dela}$  in order to improve the determination of dynamically similar conditions. With respect to model building, the similarity for lengths, masses, and times basic dimensions should lead to a lower inter-individual variability of dimensionless data which could be of interest for the comparison of human and animal running patterns.

## Conflict of interest

None.

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