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A new Dimensionless Number for Dynamic Similarity during Running

Nicolas Delattre¹, Pierre Moretto^{1,2}

Topics: Biomechanics.

Abstract: This abstract aims to present the assessment of a new dimensionless number for the study of dynamic similarity (DS) during running by using a computer simulation. Previous studies on walking (Bisiaux *et al.*, 2003, Moretto *et al.*, 2007) induced DS between subjects and reported a decrease of data variability by using the Froude number, a dimensionless number founded on mechanical energy transfers occurring during walking. Delattre *et al.* (2007) reported that the use of the Strouhal number (another dimensionless number based on the spring-mass model (SMM) for the mechanics of running) enabled to induce DS between running subjects. However it seems to us that this approach could be refined. As it has been made to highlight Nfr from the inverted pendulum model, we proposed to consider mechanical energy transfers occurring at the centre of gravity of a simulated SMM. We built two different-sized and proportional SMM, and computed E_p (potential energy), E_K (kinetic energy), E_E (elastic energy). Results showed that the ratio $R=(E_p+E_K)/E_E$ was constant and reached identical value for both SMM (4.1 ± 0.72 and 4.1 ± 0.71) from 30 to 70% of the simulation time. Taking the limits of the simulation into account, we conclude that the dimensionless number R could theoretically be used to induce DS during running. Future investigation has to assess R for DS *in vivo*.

Key words: running, spring-mass model, dimensionless, dynamic similarity, mechanical energy.

1- Background

Dynamic similarity is a widely used concept in physics and engineering to test a design with a scale model prior to building final design. The concept of dynamic similarity has the same role in studies of movement as geometric similarity has in the study of shape.

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When two systems are dynamically similar, one could be made identical to the other by multiplying all lengths by a scale factor C_L , all masses by another scale factor C_M , and all times by a third scale factor C_T . Two subjects moving in a dynamically similar fashion thus can be considered as two scale models. This concept was applied to locomotion (Alexander et al., 1983), and appeared to be very useful because it allowed making quantitative comparison between individuals of a wide range of sizes.

Dynamic similarity between two systems is possible only in particular circumstances, which depend on the nature of forces involved. For example, because gravitational forces are important during walking, dynamic similarity between two individuals requires them to have equal values of the Froude number $Nfr=v^2/g.l$ (with v : forward speed, g : gravitational acceleration, l : leg length). Nfr represents the dimensionless expression of the speed and was highlighted from the pendulum-like mechanics of walking (Figure 1A). This model consists of a point mass (representing the body mass at the centre of gravity) oscillating at the end of a massless rigid segment length (l) representing the leg. According to the inverted pendulum model, the maximum speed of walking is imposed by the gravity and the leg length (Alexander, 1992). Indeed, during a step, Cg follows a curved trajectory and has an acceleration v/l (with v : forward speed, l : leg length) toward the ground contact point. When the leg is vertical, this acceleration is directed vertically downwards and can not be greater than g , the gravitational acceleration: $v^2/l \leq g \Leftrightarrow v^2/g.l \leq 1$. This inequality set a speed limit for walking, and predicts the walk-to-run transition should theoretically appear when Nfr equals to 1. Experimental data showed that Humans naturally switch from a walk to a run when Nfr equalled about 0.5 (Alexander, 1989, Thorstensson *et al.*, 1987).

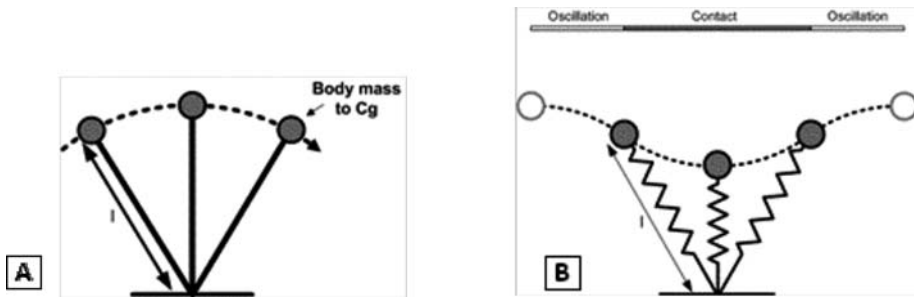


Figure 1 - A: the inverted pendulum model for the mechanics of walking. B: the spring-mass model for the mechanics of running.
 l : initial leg length before touch down.

Fundamental of the pendulum-like mechanics of walking is the exchange of kinetic energy ($E_k=0.5mv^2$, with m : body mass, v : the forward speed) and the gravitational potential energy ($E_p=mgh$, with m : body mass, g : gravitational acceleration, h : height of Cg). Between touchdown and mid-stance, the forward speed of the centre of mass (Cg) decreases as this latter moves upwards. In this phase, kinetic energy is converted to gravitational potential energy. Between mid-stance and toe-off, the forward speed of Cg increases as this latter moves downwards. In this phase, gravitational potential energy is

converted back in kinetic energy. E_K and E_p evolutions have the same magnitude and are out of phase (Cavagna *et al.*, 1966, Farley *et al.*, 1998). The ratio of these two energies is thus constant, and reduces to Nfr. Equal values of Nfr were shown to ensure dynamic similarity between humans during walking (Bisiaux *et al.*, 2003, Moretto *et al.*, 2007), and between different animals species during running (Farley *et al.*, 1993). As a matter of principle, it is not expected that equal values of Nfr make possible dynamic similarity during running, because Nfr is founded on the pendulum-like mechanics of walking, and thus does not take any elastic phenomenon into account. Some studies (Bullimore *et al.*, 2008, Donelan *et al.*, 2000) showed that Nfr was necessary, but not a sufficient condition for dynamic similarity during running.

In addition to E_p and E_K , elastic energy ($E_e=0.5k.\Delta l^2$, with k : leg stiffness; Δl : variation in leg length) was shown to have an important role for mechanical energy conservation during running (Cavagna *et al.*, 1964). Tendons and other springs can be used to store elastic energy and then to return it by elastic recoil. The spring-mass model (Figure 1B) enables to take E_e into account, and has been widely used to describe the mechanics of running.

Considering that two runners behave each like such a vibrating system, Alexander (Alexander, 1989) suggested that dynamic similarity between them requires they have the same values of the Strouhal number $Str=(\text{Frequency}.\text{Length}/\text{Speed})$. Str represents a dimensionless frequency, and in its expression, while the leg length appears to be the only length that can be used, any frequency and any speed component of the center of gravity (C_g) can be used. Donelan and Kram (Donelan and Kram, 2000) showed that dynamic similarity in simulated sub-gravity was not met with the use of Str including the natural frequency of the spring-mass system and the vertical landing speed. Delattre *et al.* (Delattre *et al.*, 2007) used Str including stride frequency and forward speed and reported that Str made possible to meet inter-subject similarity only for some kinetic parameters. Another dimensionless number, the Groucho number $Gr = v_y \cdot f_0 / g$ (with v_y : vertical landing speed, f_0 : natural frequency of the spring-mass system, and g : gravitational acceleration), was proposed (McMahon *et al.*, 1987). Gr represents a dimensionless vertical landing speed and was primarily used to distinguish between hard and soft running modes. However it has been shown that Gr did not predict dynamic similarity during human running (Donelan and Kram, 2000).

It appears that neither Nfr, nor Str , nor Gr can be used for perfect dynamic similarity during running. On the contrary, Nfr is suitable for dynamic similarity during walking, because it is founded on mechanical energy transfer occurring in the pendulum-like mechanics, to which walking has been likened. To the best of our knowledge, there are no available studies dealing with dynamic similarity which have focused on mechanical energy transfers occurring on a spring-mass model.

This study aimed to consider mechanical energy transfer occurring at the centre of gravity of simulated spring-mass models in order to highlight a new dimensionless number that could be applicable for dynamic similarity during running. During the stance phase of running, the E_p and E_K variations are in phase, whereas E_K and E_e are out of phase (Biewener, 2006, Cavagna *et al.*, 1964, Lee *et al.*, 1998). Therefore, the ratio $R=(E_p+E_K)/E_e$ is expected to be constant during the stance phase of running. Like for

Nfr during walking, this new dimensionless number will have (i) to be constant during the running cycle, and (ii) to equal the same value for two dynamically similar spring-mass models.

2- Methods

We created a computer simulation with Working Model 2D 7.0 (Design Simulation Technologies, Inc., USA). We built a first spring-mass model (SMM #1) based on data of Ferris *et al.* (Ferris *et al.*, 1999) (Table 1). We then built a second spring-mass model (SMM #2) dynamically similar to the first. As a starting point, we defined the inter-models scale factors for lengths (L), masses (M), and times (T) dimensions as C_L , C_M and C_T respectively. L, M and T are fundamental dimensions, thus all other mechanical parameters can be defined from them. Therefore, the scale factor for all other mechanical parameters can be determined from C_L , C_M and C_T . For example, according to Newton's second law, a force (F) is expressed in Newtons, and its dimension is $M.L.T^{-2}$. Accordingly, between two subjects moving in a dynamically similar fashion, the scale factor for forces (C_F) equal the product of concerned scale factors: $C_F=C_M.C_L.C_T^{-2}$.

We defined the initial spring length of SMM #2 as to be 0.985m, which implied that C_L equalled 1.13. Assuming that body densities of the two models were identical, the scale factor for masses was $C_M=C_L^3=1.45$, and the mass of SMM #2 was 76.9kg (Table 1). The scale factor for times was determined as follows: E_p and E_k have the same dimension $M.L^2.T^{-2}$, and thus the same inter-subject scale factors. On one hand, the ratio between E_{p1} and E_{p2} of two subjects S_1 and S_2 involves the simplification of constant g, and returns to the ratio of masses that multiplies ratio of lengths, i.e. $C_M.C_L$. On the other hand, E_k equals to $0.5.m.v^2$, has $M.L^2.T^{-2}$ as dimension that involves that scale factor for kinetic energies is $C_M.C_L^2.C_T^{-2}$. Scale factors between E_p and E_k are the same, thus:

$$C_M.C_L=C_M.C_L^2.C_T^{-2} \Leftrightarrow C_L.C_L^{-2}=C_T^{-2} \Leftrightarrow C_T^{-2}=C_L^{-1} \Leftrightarrow C_T=C_L^{0.5}$$

Scale factor for times (C_T) corresponds to the square root of the scale factor for lengths (C_L) and thus equals 1.06. Scale factors for the other parameters of the simulations (spring stiffness, initial Theta, horizontal and vertical speeds) were computed from C_L , C_M , and C_T .

Parameter	Unit	Dimension	SMM #1	Scale factor	SMM #2
Initial spring length	m	L	0.87	1.13	0.985
Mass	kg	M	53	1.45	76.9
Contact time	s	T	0.282	1.06	0.3
Spring stiffness	kN.m ⁻¹	M.T ²	6.9*	1.28	8.84
Initial Theta	deg	Dimensionless	26.6	1	26.6
Initial horizontal speed	m.s ⁻¹	L.T ⁻¹	3	1.06	3.19
Initial vertical speed	m.s ⁻¹	L.T ⁻¹	-1	1.06	-1.06

Table 1 - characteristics of the two simulated spring-mass models (SMM #1 and SMM #2). Initial Theta corresponds to the angle between the vertical and the spring. Parameters of SMM #1 were those of Ferris *et al.* (Ferris *et al.*, 1999) with *: hard surface with adjusted leg stiffness. In dynamically similar conditions, the scale factor for angles equals to 1, meaning that angles between systems are identical in the same phase of the movement (Moretto *et al.*, 2007).

Data from each simulation enabled us to compute the gravitational potential energy $E_p=m.g.h$, the kinetic energy $E_K=0.5m.v^2$, and the elastic energy $E_E=0.5k.\Delta l^2$ (with m: mass, g: gravitational acceleration, h: vertical Cg position, v: speed, k: spring stiffness, Δl : variation in spring length).

3- Results

For each of the two models, Theta and spring length had the same value at the initial (touchdown) and at the final (toe-off) instants of the simulation (Table 2).

Parameter (unit)	SMM #1		SMM #2	
	Initial	Final	Initial	Final
Theta (deg)	26.6	26.6	26.6	26.6
Spring length (m)	0.87	0.87	0.985	0.985

Table 2 - theta and spring length at initial and final instants of the simulation for SMM #1 and SMM #2.

The Table 3 shows that scale factors computed from measured parameters matched the scale factors predicted from C_L , C_M , and C_T . There is a single scale factor for lengths (l: spring length, f_x : horizontal displacement of the centre of gravity during the stance phase, Δl_{max} : maximal spring compression), and a single scale factor for energies (E_p , E_K , E_E) between the two models.

Parameters	SMM #1	SMM #2	Scale factor	
			Measured	Predicted
Δx	0.78	0.886	1.14	1.13
Δl_{max}	0.211	0.239	1.13	1.13
E_p	342.6	562.9	1.64	1.64
E_K	173.1	284.5	1.64	1.64
E_E	153.6	252.6	1.64	1.64

Table 3 - Measured parameters and their scale factor between SMM #1 and SMM #2 compared to the predicted scale factor. Δx : horizontal displacement of the centre of gravity during the stance phase; Δl_{max} : maximal spring compression; E_p : gravitational potential energy; E_K : kinetic energy; E_E : elastic energy. Predicted scale factor for lengths (Δx , Δl_{max}) was $C_L=1.13$. Predicted scale factor for energies (E_p , E_K , and E_E) was $C_M.C_L^2.C_T^{-2}=1.64$.

The Figure 2 shows that, during the first half of stance phase, E_p and E_K decreased, while E_E increased. Conversely, during the second half of stance phase, E_p and E_K increased while E_E decreased. For both SMM #1 and SMM #2, the $R=(E_p+E_K)/E_E$ ratio reached the same mean value and was constant (4.1 ± 0.7) between 30 and 70% of contact time. When out of the 30-70% of time contact interval, the ratio was not constant anymore and reached much high values.

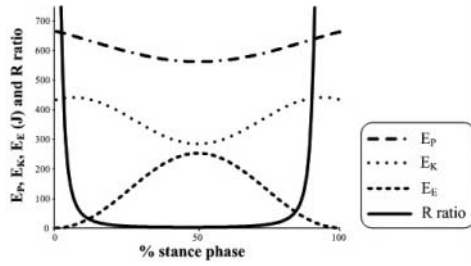


Figure 2 - Evolution to time of E_p , E_k , E_e , and R ratio for SMM #2.

4- Discussion

The two models SMM #1 and SMM #2 bounced symmetrically (Table 2), which proved that the scaling principle used to determine parameters of SMM #2 led to a consistent simulation with a mechanical energy exchange which is typical of running gait (Figure 2) (Biewener, 2006, Cavagna *et al.*, 1964). Moreover, our method enabled us to make SMM #2 dynamically similar to SMM #1, with a single scale factor for lengths (l , Δx , Δl_{\max}), and a single scale factor for energies (E_p , E_k , E_e) between the two models (Table 3) as suggested by Alexander and Jayes (Alexander and Jayes, 1983).

Our results showed that the $R=(E_p+E_k)/E_e$ ratio i) was constant during the stance phase of running, and ii) reached the same value for two different-sized spring-mass models which are in similar dynamic conditions. The high ranked values obtained for R ratio before 30% and after 70% of stance phase were due to the absence of elastic energy (E_e , denominator of our R ratio) at the first and at the last instant of the simulation because the spring was not yet compressed ($\Delta l=0$). This phenomenon occurred because the spring-mass model did not allow any initial elastic energy to be considered prior to leg touch down. This was a limit of simulated spring-mass model, because in human running, a leg muscles pre-activation occurs before heelstrike (Komi, 2000, Wakeling *et al.*, 2001) and thus enables a potential elastic energy storage prior to ground contact.

By taking our approach in the opposite direction, our findings revealed that when two spring-mass models behaved in such a way that they had the same value of the $R=(E_p+E_k)/E_e$ ratio, there was between them a scale factor for lengths, another scale factor for masses, and a third scale factor for times, i.e. they moved in dynamically similar fashion. It thus appeared that our R ratio could be used as dimensionless number to induce dynamic similarity between different-sized runners, because it included mechanical energy transfers occurring during running (Biewener, 2006, Cavagna *et al.*, 1964, Lee and Farley, 1998). More generally, because it was based on the spring-mass model, the R ratio could be used for the study of dynamic similarity during bouncing gaits in both humans and animals. Previous studies on walking (Bisiaux *et al.*, 2003, Moretto *et al.*, 2007) showed that dynamic similarity led to a lower inter-subject variability when data were expressed in their dimensionless form. This lower data variability could be useful to enhance the ability of a statistical analysis to distinguish between groups. It could be very interesting to obtain these results during running gait.

We propose to call the R dimensionless ratio “Mo-De-La” in reference to its authors (Moretto, Delattre, Lafortune), and to evoke the idea of running modelling: Modela number = $\frac{E_P + E_K}{E_E}$. Because this dimensionless number was based on mechanical energy transfers occurring at the Cg of a simulated bouncing mass, future investigation should aimed to validate it during *in vivo* human running.

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