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The complexity of task design for utilising the epistemic potential of multiple languages in developing pattern understandings

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Operationalising the concept of language as a resource in multilingual classrooms has recently involved the development of tasks that focus on the epistemic potential of multiple languages. However, task design is a complex process. Using an example of the development of an algebra task, which aimed to support students' mathematical meaning-making through using multiple languages, the changes made across several iterations are described. Results show how changes to one aspect of the task affected other aspects, including the mathematics and the language which were the foci, reducing the possibilities for deepening students' mathematical understandings.

Keywords: Language as a resource, algebra, patterns, epistemic potential, task design.

Introduction

Recently, there has been a move towards operationalising the concept of a language as a resource in which students' multiple languages can be used to deepen their understandings of mathematics (see for example, Planas, 2021). This has included supporting teachers to consider multiple languages as a resource, by developing classroom tasks which utilise the epistemic potential of multiple languages to deepen students' understandings of mathematical concepts (see for example, Planas, 2021; Prediger & Uribe, 2021; Schüler-Meyer et al., 2023). However, in focusing on the epistemic potential of multiple languages, other aspects of task design, highlighted as important in earlier research (see for example, Margolinas, 2013), may risk being ignored. In this paper, we investigate how different aspects of task design may interact to limit the possibilities for designing topic-specific learning opportunities that utilise multiple languages to deepen mathematical understandings (Schüler-Meyer et al., 2023). By considering how a task was modified over several iterations, it is possible to illustrate how changing one aspect of the task may have ramifications for other aspects and for the likelihood of the task to achieve its intended goal.

Sullivan et al. (2015) stated, "tasks prompt activity which offers students opportunities to encounter mathematical concepts, ideas, and strategies" (p. 82). They distinguished between intended and enacted tasks. Our focus is on the intended task, specifically designed to engage students in activities to support their mathematics learning through utilising the epistemic potential of multiple languages. Although we use data collected from classroom trials of the task, we do not present results about what the students learnt from the task, rather we are interested in how the results of the enactment contributed to changes in the next iteration of the task. Our research questions are:

How does changing one aspect of a task impact on other aspects of the task design?

What is the likely impact of such changes on utilising the epistemic potential of multiple languages to develop mathematical understandings?

In the next section, we discuss Sullivan et al.'s (2015) five dilemmas of task design to identify how changes to aspects of the task can affect the kinds of activities that tasks can contribute to.

Dilemmas in decisions in task design

The dilemmas described by Sullivan et al. (2015) originated in the work of Barbosa and de Oliveira (2013) who identified five arenas of conflicts that appeared when developing tasks in collaborative groups. Sullivan et al. (2015) renamed them dilemmas, as they considered that each one had two alternatives, which could lie on opposite ends of a continuum. The dilemmas were: context; language; structure; distribution; and levels of interaction.

Following Skovsmose's (2001) milieus of learning, context as a dilemma is about whether the task includes a real-life context or only a mathematical context. Sullivan et al. (2015) recognised that tasks could have a semi-reality context, which would lie between the two ends of the continuum. The context could be supportive of the learning process, if known to the students and seen as authentic, or an inhibitor to learning if it reduced students' opportunities for engaging with mathematics.

Language as a dilemma focuses on how the language of the task and its intended solution are interpreted and used. Barbosa and de Oliveira (2013) discussed the degree of rigour of the mathematical language. Tasks which require students to be able to interpret and use algebraic notation are unlikely to be solvable if students have not learnt this. On the other hand, some tasks may require students to bring their own interpretations, such as the one that Sullivan et al. (2015) described as being about fairness, to determine what is the appropriate mathematics for the activity.

Structure as a dilemma is about whether the tasks are open, with multiple solutions that can be found in multiple ways, or closed with only one solution that can be found using only one approach. Sullivan et al. (2015) considered that there would also be tasks between the two extremes, where the students were guided towards particular answers and solution methods, but retained some degree of autonomy over the choices that they made.

Distribution as a dilemma is about the type of mathematics that the task requires and is related to the cognitive demand placed on students. Sullivan et al. (2015) discussed this in terms of whether the task requires the use of a memorised procedure or requires synthesising and evaluating approaches for solving the task. In between those two extremes, there are a range of alternatives.

Levels of interaction as a dilemma focused on what the task requires or expects of participants. Barbosa and de Oliveira (2013) suggested that closed tasks are expected to be done by students working independently, whereas open tasks required more engagement between students with each other and the teacher.

In this study, we consider retrospectively the task design of three iterations of one algebra task, focused on recognising and describing patterns, that was designed using the design heuristics of Schüler-Meyer et al. (2023). As aspects of different dilemmas were changed across the iterations, other dilemmas were also changed, sometimes unexpectedly.


Methods

During 2022, we developed three versions of the table and chairs task in English. It had previously been used for developing algebraic concepts, such as functional thinking (Hunter & Miller, 2022) and generalising patterns (Lenz, 2018). All versions were to be done in groups. The first version (V1),

presented in Schüler-Meyer et al. (2023) (see Figure 1), focused on how the pattern from determining the number of chairs could be described as an explicit rule in different languages. The intention was to deepen insights about multiplication, by focusing on how multiplication was described in the rule.

Task

In the country of Guest-hospitality, if someone turns up at your door, then you must feed them. Every family, therefore, keeps lots of tables and chairs so they can accommodate people for a meal on any specific day. Tables must be placed end to end because it is considered rude to put people on one table away from others. Generally, each rectangular table can accommodate four chairs along the long sides and one chair at each of the short sides, as shown in the diagram.



(1) Describe a rule in your own words for the relationship between the number of tables and the number of chairs, so that every family will know how many chairs they need.

(2) This problem was given to another class and Sandra described the rule as “to work out the number of chairs needed, double the number of tables and add four”. However, her partner Jan didn’t think this was correct. In Mandarin, he said the rule would be “椅子的数量是桌子数量的4倍再加2”, (literally: “The chair number is the number of tables 4 times plus 2”). Which student do you think is correct and why?

(3) In how many different ways and languages could you write the rule? Which one is the easiest to understand? Why do you think that?

Figure 1: Task for comparing language expressions for multiplicative structures of equal grouping

The two other iterations of the task (V2 and V3) used the same introduction, presenting the same context and the diagram. In V2, developed in May 2022, the context and functional relationship between the number of tables and chairs was the same. However, the question was to find the number of tables, rather than the number of chairs. This direction of the function was the same as in Lenz (2018).

(1) If a family of 8 turns up at your house, how many tables and chairs would you need to set up, so you could be a good host?

- a. Draw a picture of how you would arrange the table and chairs.
- b. Is it the same for everyone in your group.

(2) Use words or pictures to describe a rule so that every family will know how many tables they need for the number of people they have to feed.

(3) This problem was given to another class. Sandra described the rule as “to work out the number of tables needed, if it is an even number half the number of people and take away two. If it is an odd number, add one first and then half the number and take away two”.

However, her partner Jan didn’t think this was correct. In Mandarin, he said the rule would be “要算出所需桌子的数量，我们可以分成两种情况。如果人数是偶数，桌子的数量是人数减半再减1；如果人数是奇数，桌子的数量是人数加一 减半再减1.”, (literally: “To calculate needed tables quantity, we can divide into two cases. If people’s quantity is even, tables’ quantity is people’s quantity minus half (of it) then minus one; if people’s quantity is odd, the tables’ quantity is people’s quantity plus one minus half (of it) then minus 1.”).

Which student do you think is correct and why?

(4) In how many different ways and languages could you write the rule? How do different ways of expressing the rule help you to understand the rule? Which one is the easiest to understand? Why do you think that?

Figure 2: Version 2 of the tables and chairs task

Version 3, developed in September 2022, went over several pages, and so is summarised here. The context remained the same, but the initial problem was to determine the number of chairs as in V1. The task began with closed questions about the number of chairs for a specific amount of tables, with

a suggestion to draw the tables. The task then asked students to calculate the number of chairs for 42 tables, without drawing. Table 1 was included as a suggestion for recording the results to support identifying the pattern. This was in alignment with Korntreff and Prediger’s (2022) task.

Table 1: Table provided in version 3 of the task

Number of tables	1	2	3	4	5	6	7	8	...	42
Number of chairs										

The next question was about what changed and what stayed the same as the pattern developed, before asking the students to write their own rule. The students were then asked to determine the validity of an invented student rule, written in the language of instruction, about doubling the number of tables to determine the number of chairs. The students were asked to describe alternative versions of the rule, using other languages. The next part of the task had students use their rule to determine the number of tables, needed for 253 guests, before describing an explicit rule to work out the number of tables for any number of guests. The following question asked if two alternative rules were the same, but just worded differently. The final question requested a rule which was as concise as possible.

V2 and V3, provided in Norwegian, were trialled with one group in June and two groups in September. The students, who were fluent in Norwegian, were at the same multilingual school. Extracts from the transcripts illustrate how aspects of the tasks were interpreted by the students.

Results and discussion

In this section, we present the results, based around the five dilemmas. From identifying the changes across the three versions of the task, it is possible to discuss the impact on other aspects of the task and the complexity involved in designing tasks, for utilising the epistemic potential of multiple languages to develop mathematical understandings.

Context as a dilemma

The context for all three versions was a fictional country and traditional customs. This can be described as a semi-reality context (Skovsmose, 2001) in that students might be familiar with similar customs, as Hunter and Miller (2022) suggested for their version of the task, but were unlikely to have exactly this way of arranging tables and chairs.

In V2, although still a semi-reality context, the focus shifted from determining the number of guests in V1, to being about the number of tables needed. In making this change, the context was supposed to become more realistic, in that usually the number of guests is set and tables need to be found for all of them. However, the students were uncertain about using the number of people in their own family and so bringing the task closer to their own reality.

Student1: Yes, because they don’t say how much your family is. So how many places your family needs.

Teacher: Yes, that is absolutely true. That’s very good. You’re actually going to get to write a bit about that, so it was good that you already saw it there. But then you can on task one, then you can think that it is your family that is about.

Sullivan et al. (2015) raised how the context of a task could derail the students from the mathematical focus. In V3, the beginning of the task went back to determining the number of guests who could be seated at a certain amount of tables via multiplication. However, in this case, the students seemed to ignore the context and the diagram of the tables, which hindered them answering the question.

Teacher: And then they ask, “How many chairs do you need for three tables? And how many are needed for four tables?” Then you are allowed to see here, and then the question is, how many...
Student10: For three tables, aren't there 18 chairs?

Unlike in V2, the context did not support the students to think about the question they were asked to answer. They did not recognise that the cognitive demand of the task required more than just a multiplication.

Language as a dilemma

By providing explicit rules in different languages as well as encouraging students to use their different languages to create them, the tasks were designed to support students to identify differences in how multiplication could be expressed in V1 and to state their own rule for a pattern in V2 and V3. However, the use of technical language and how to structure mathematical argumentation seemed to affect how the students interpreted and responded to the requirements of the task.

One member of the group, who trialled V2, did interpret the fictitious students' rules to determine if they produced the correct amount of tables. However, when giving a rule in another language, this student translated literally from Norwegian into German (their home language) what happened in their family, rather than describe an explicit rule. The other two students seemed to also describe the situation for their own families in Arabic and Somali, but seemed unsure of the correctness of what they said. As the students could not make sense of each other's languages, the epistemic value of multiple languages seemed limited. In V3, a new question was added about producing a concise version of the rule, as suggested by Prediger and Uribe (2021). Without having produced a clear rule, the students could not complete this part of the task. The language of the task was understandable but did not contribute to the students' developing a deeper understanding of the mathematics.

Although the fictional students' rules in V2 seemed to support the students to use other languages, they did not lead to them describing explicit rules. In V3, the fictitious students' rules were given in Norwegian. However, the group which trialled V3 still struggled to interpret the rules, because they were unfamiliar with some mathematical terms, such as “doubling”.

Teacher: Because we are now going to ask if Sandra's rule applies to everything. Is Sandra's rule right? And then we have to discuss with the group.
Student12: So, if we're going to follow ... What can we do to find 42?
Teacher: How many tables were there, 42? 42. Then you can try Sandra's rule. If I do the same calculation down here. 42, and then we will double.
Student12: Times, you mean?
Teacher: Yes.

In this case, the language of the task was difficult for the students to interpret and required the intervention of the teacher, which had not been anticipated in the task design. This led to a recognition of how changing the language to support the students to produce an explicit rule also affected the structure of the tasks and the level of interaction.

Structure as a dilemma

Structure is about how open or closed the task is (Sullivan et al., 2015). None of the tasks were completely open, but the possibilities for the students to make their own choices about how to solve the task became more limited with each version. V1 was the most open, what Skovsmose (2001) called a landscape of investigation, with an expectation that students would want to solve it. When the task changed from being about the number of chairs to being about the number of tables in V2, suggestions were included about how to solve the problem, such as drawing the tables and chairs. Question 4 was also included to focus the students on the ways that different languages expressed explicit rules. However, as noted previously, this was not as helpful as it was intended.

Thus, in V3, the task included more suggestions for how to answer the problem, such as Table 1, reducing further students' possibilities to decide how to respond to the questions. Nevertheless, the intervention of the teacher was still needed to support the students to recognise the pattern.

- Teacher: What do you think then? If you want to draw it, feel free to draw it. Then we have one table. We will have two tables. Three tables.
- Student12: Okay, so it will be like this, like this, like this, like this, like this. [pointing at their drawing]
- Teacher: Yes, it was 14. What did you think then?
- Student12: I don't know. Now I just counted. ... At first I thought it was 18, then I only took 4 minus. In one way or another. For some reason I took minus 4. ...
- Teacher: Okay. Why do you think you took 4 minus?
- Student12: I don't know, no. But it's 14.
- Teacher: It's 14. That's right. You can write 14 there. [pointing at the table on the answer sheet] But then they ask for four tables. Do you have something on your mind before we draw?
- Student11: 18.
- Teacher: Why is it 18?
- Student11: I just added 4.

The teacher encouraged the students to draw the tables and chairs for each increase in the pattern and to fill in the table. However, this led to a focus on working out the result for the next turn in the pattern. The students were led to recognise the recursive pattern of adding 4 each time, but not the explicit rule. This shifted the mathematical focus made available to them through the task.

Distribution as a dilemma

Distribution as a dilemma is about the kind of mathematics made available and how students were expected to engage with it. In V1, the students were expected to develop an understanding about how multiplication can be expressed in different ways, as part of developing a rule about the number of people who can sit at a certain amount of tables. With the shift to a more realistic setting in V2, the pattern required an understanding of division and this made identifying a rule more difficult. The task had been set up to be open. However, the same student, from context as a dilemma, in trying to support the other students to gain some answers to the closed questions, led them to the correct answer by only providing questions which were impossible to answer incorrectly. In V3, more structural supports, such as Table 1, were provided and along with the teacher's intervention, the students did identify the recursive pattern of increasing by four, but not the functional relationship (Lenz, 2018). It may be that expectations that students in grade 7 in Norway would have previous experiences with

forming rules about functional relationships were unrealistic and all three versions of the task placed too much cognitive demand on the students.

Levels of interaction as a dilemma

Although Barbosa and de Oliveira (2013) connected levels of engagement to the degree of openness in the task, all three versions of the tables and chair task were designed as group tasks. However, the trialling showed that when students struggled with meeting the cognitive demand of the task, then the teacher or one of the students took on the role of supporting students to identify an appropriate answer. The extra scaffolding provided in V3 seemed to reinforce the need for an interaction style in which students were funnelled into answering a series of closed questions. This suggests that in designing tasks that are cognitively demanding, such as those which want to utilise the epistemic potential of multiple languages to identify concise ways to formulate a rule for a functional pattern, teachers need to be aware of the most appropriate kind of questions to achieve this.

Conclusion

Task design is an important component in supporting students' meaning making in multilingual classrooms (Schüler-Meyer et al., 2023). The aim of this paper was to extend our understanding of the complexity of task designs which support the realisation of the epistemic potential of multiple languages to deepen students' mathematical understandings. To do this, we chose to answer two research questions: how does changing one aspect of a task impact on task design? What is the likely impact of such changes on utilising the epistemic potential of multiple languages to develop mathematical understandings?

In the analysis, a change in a task in relationship to one dilemma, led to aspects of other dilemmas also changing. The inter-connectedness of the dilemmas illustrates the complexity of task design. Finding and utilising differences between languages to deepen mathematical understandings, when students and/or the teacher do not understand the languages being used, places a lot of responsibility on participants to recognise when differences between the languages make a difference and then to convey to other participants the importance of this recognition. As part of the dilemma of language, the task and its solution are interpreted and produced by a speaker of another language, through the language of instruction in which they may not be fluent. When extra supports are built into the tasks, other aspects, such as the mathematical focus of the task, can change, resulting in the possibilities for deepening the mathematical understandings being reduced. This does not mean that task design for the realisation of the epistemic potential of multiple languages should not be attempted. Rather, our results indicate the necessity for care and time to work in developing these tasks. It also shows that task design needs to be done in alignment with professional development for teachers on how to implement such tasks in classrooms and the kind of classroom norms needed for such tasks to have the best chance for successful implementation.

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References

- Barbosa, J. C., & de Oliveira, A. M. (2013). Collaborative groups and their conflicts in designing tasks. In C. Margolinas (Ed.), *Task design in mathematics education* (Proceedings of the International Commission on Mathematical Instruction Study 22, pp. 541–548), ICMI. <http://hal.archives-ouvertes.fr/hal-00834054>
- Hunter, J., & Miller, J. (2022). The use of cultural contexts for patterning tasks: supporting young diverse students to identify structures and generalise. *ZDM*, 54(6), 1349–1362. <https://doi.org/10.1007/s11858-022-01386-y>
- Korntruff, S. & Prediger, S. (2022). *Zusammenhänge allgemein beschreiben mit Variablen und Termen: Sprachbildendes Unterrichtsmaterial für Klasse 7–10*. [Describe relationships in general with variables and terms: Language-building teaching material for grades 7–10]. Open Educational Resources. <https://sima.dzlm.de/um/8-002>
- Lentz, U. (2018). *Algebraic thinking of sixth graders through the lens of multimodality* [Ph.D. thesis, The University of North Carolina at Charlotte].
- Margolinas, C. (Ed.). (2013). *Task design in mathematics education* (Proceedings of the International Commission on Mathematical Instruction Study 22). ICMI. <http://hal.archives-ouvertes.fr/hal-00834054>
- Planas, N. (2021). How specific can language as resource become for the teaching of algebraic concepts? *ZDM*, 53(2), 277–288. <https://doi.org/10.1007/s11858-020-01190-6>
- Prediger, S., & Uribe, Á. (2021). Exploiting the epistemic role of multilingual resources in superdiverse mathematics classrooms: Design principles and insights into students' learning processes. In A. Fritz, E. Gürsoy & M. Herzog (Eds.), *Diversity dimensions in mathematics and language learning* (pp. 80–97). De Gruyter/Mouton. <https://doi.org/10.1515/9783110661941>
- Schüler-Meyer, A., Meaney, T., Uribe, A., & Prediger, S. (2023). Design heuristic for generating conceptual learning opportunities through multiple languages – Exemplified for algebra. In A. Schüler-Meyer, J. Ingram, & K. Erath, (Eds.), *Proceedings of the ERME Topic Conference: Language in the mathematics classroom* (pp. 102-109). <https://hal.science/hal-03992500>.
- Skovsmose, O. (2001). Landscapes of investigation. *ZDM*, 33(4), 123–132. <https://doi.org/10.1007/BF02652747>
- Sullivan, P., Knott, L., & Yang, Y. (2015). The relationships between task design, anticipated pedagogies, and student learning. In A. Watson & M. Ohtani (Eds.), *Task design in mathematics education: ICMI study 22* (pp. 83–114). Springer.
- Temple, C., & Doerr, H. M. (2012). Developing fluency in the mathematical register through conversation in a tenth-grade classroom. *Educational Studies in Mathematics*, 81(3), 287–306. <https://doi.org/10.1007/s10649-012-9398-6>