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# The role of language awareness in an algebraic-thinking task

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*In this article, we present a task about the concept of even numbers that intends to enhance conceptual understanding by drawing upon perspectives found in multiple languages. After justifying its coherence with previously established design principles, we give some examples of 7<sup>th</sup>-grade students' engagement with the task. We analyse how the intended language-related exploration of the task and students' language awareness impacted on their discourse about the concept.*

*Keywords: Algebra, design research, language as resource, multilingualism.*

## Introduction

There is agreement among many researchers today that language plays a crucial role in mathematics learning. Language has multiple roles, among them a communicative role and an epistemic role, as a tool for thinking and knowledge construction. Planas (2018) advocated to look at multiple languages as a resource in classrooms, instead of focusing on language deficits as problems. In a similar line of thought, Barwell (2018) coined the term sources of meaning, emphasising in particular that it is important to study the processes of how meaning arises from the relations between various languages and discourses. Studies have revealed the important role of meaning-related language which bridges everyday language and the technical language of mathematics when learners develop understanding for mathematical concepts (Prediger, 2022). Meaning-related language allows to describe mathematical relationships and structures; it includes topic-specific phrases that explain the meaning of concepts, also in relation to multiple representations and exemplifications outside the mathematical world. Meaning-related language is an important learning medium in the language of instruction, but it is not available to all learners to the same extent. It is therefore necessary to focus on it as a learning goal as well (Erath et al., 2021).

Empirical research has revealed language-benefits in multilingual settings when different languages are compared. One such example in the case of fractions was described by Schüler-Meyer et al. (2019), who investigated how a group of bilingual students profited from using both their home language (Turkish) and the language of instruction (German) when making sense of the concepts of numerator and denominator. Another case study by Prediger and Uribe (2021) in the context of functional thinking found that students' conceptual understanding was enhanced by drawing on their multilingual resources in classroom discussions. However, epistemic potential could go unnoticed in students' unprecise translations to the language of instruction when the teacher did not speak the students' home languages.

This article reports on ongoing research about the design of learning opportunities that support multilingual learners' algebraic thinking by exploiting the epistemic potential of expressing mathematical concepts and relationships in multiple ways and across different languages. In view of the challenges for fully exploiting linguistic potential in classrooms with many non-shared languages, revealed by Prediger and Uribe (2021), we want to investigate which possibilities there are to bring

epistemic potential of several languages into the classroom via the designed material. Our research is guided by the following research question:

*How can language awareness be supported in a task about even numbers drawing on language nuances and how does it influence students' discourses about the meaning of the concept?*

### **Metalinguistic (or language) awareness**

Malakoff and Hakuta (1991) defined *metalinguistic awareness* to be an awareness of the underlying nature of language use that allows the individual to “step back from the comprehension or production of an utterance in order to consider the linguistic form and structure underlying the meaning of the utterance” (p. 147). Metalinguistic awareness (or *language awareness*) encompasses a variety of aspects, including word consciousness, syntactic awareness and awareness about discourse practices (Erath et al., 2021). This resonates with the different perspectives on language such as the lexical dimension (vocabulary and word meanings), the syntactic dimension (grammar and word flexions) and the discursive dimension (Erath et al., 2021). Various components of metalinguistic awareness can enhance mathematics learning. Word consciousness is about sensitivity to words and word parts and how they contribute to the overall meaning of a word or sentence (Scott & Nagy, 2004). With respect to mathematics learning this can involve to study families of related words such as “equation, equate, equal” (Erath et al., 2021). Likewise, translations of words from mathematical register into the language of instruction or home languages can be compared to give a deeper understanding for example of concepts like fractions (Schüler-Meyer et al., 2019). Various studies, e.g. (Abedi & Lord, 2001), have identified syntactical features that can impact on students' understanding of word problems, such as passive voice and conditional clauses, and suggested ways to simplify sentence constructions. An alternative approach was suggested by Dröse and Prediger (2020), who studied how to raise students' syntactical awareness by systematically varying syntactic features in word problems. Finally, students need to be aware of different types of academic discourses like *reporting procedures*, *explaining meanings*, *describing mathematical relationships and structures*, and *arguing*, typically used at school in order to participate in classroom discussion (Prediger, 2022).

### **Task design and implementation**

Based on a review of literature about how to use language in a classroom to bolster mathematics learning, Erath et al. (2021) extracted some common design principles for instructional material and learning situations that proved to be important in empirical studies, among them (DP-A) *engaging students in rich discourse practices*, (DP-B) *connecting language varieties and multimodal representations*, (DP-C) *including students' multilingual resources* and (DP-D) *comparing language pieces (form, function, etc.) to raise students' language awareness*. Design principle (DP-C) refers for example to communicative practices like codeswitching and translanguaging, but recent research, such as the aforementioned article by Schüler-Meyer et al. (2019), presented also examples of using multiple languages as source of meaning-making. To address this epistemic perspective in task design, the three design heuristics shown in Table 1 were presented by Schüler-Meyer et al. (2023), based on the work by Prediger and Uribe (2021). In the following section we explain the design of a task about even numbers and justify its correspondence with these design principles and the design heuristics.

**Table 1: Design heuristics for learning activities (Schüler-Meyer et al., 2023)**

Design heuristics for learning activities to harness the epistemic potential of using multiple languages
<b>Step 1.</b> Identify key facets of a mathematical concept, by investigating informal and formal representations of the concept, standard interpretations and typical student difficulties.
<b>Step 2.</b> Find language-related nuances of this concept by investigating the concept facets' linguistic realisations in multiple languages.
<b>Step 3.</b> Devise learning activities where these language nuances are intended to lead to conceptual insights, for instance in activities of comparing or connecting language-related nuances.

### Task about even numbers and its connection with design principles

As in Step 1 of the design heuristics, an analysis of the key facets of the concept of even numbers was conducted. Even numbers can be conceptualised as 2 groups of  $n$  objects, corresponding to  $2 \cdot n$ , or as  $n$  groups of 2 objects, corresponding to  $n \cdot 2$ , where  $n$  is a natural number. Many print and online sources of mathematical definitions available to students conceptualise even numbers only in a procedural way, for example defining them as those natural numbers that “we can divide by 2 and get a whole number as answer” (Matematikksenteret, n.d.). The linguistic realisations of the concept in English and Norwegian provide the opportunity to support a conceptual understanding of even numbers (Step 2). The English term *even numbers*, similarly to the German *gerade Zahlen*, conveys an idea of “being on the same level”, which fits the idea of two equal groups. The Norwegian term *partall* is composed of two morphemes (*par, tall*) meaning pair/pairs (same form singular and plural) and number. Thus it literally translates as “pair number” which fits the idea of composing groups of 2. The task shown in Figure 1, created in Step 3, takes two situations outside the mathematical world as a starting point, illustrated with two pictures (DP-B). The first situation of building a bridge with building blocks (Part 1) can be related to both conceptualisations of even numbers: to make the bridge perfectly horizontal, both pillars must consist of the same number of building blocks. Alternatively, one and one block of a perfectly horizontal bridge can be paired, starting with the two lowest blocks in the pillars and drawing parallel lines between corresponding blocks in the left and right columns. The most natural conceptualisation of even numbers in the situation that was described about the Ferris wheel (Part 2) is that of making pairs. The nuances of even numbers can be explored by describing the two conceptualisations with meaning-related phrases (such as those in Table 2), thus engaging students in rich discourse practices, as by principle DP-A.

**Table 2: Meaning-related phrases connected to multimodal representations**

	Bridge	Ferris wheel
Conceptualisation $2 \cdot n$	Both pillars consist of the same number of blocks. (1)	As many students sit to the right as to the left. (3)
Conceptualisation $n \cdot 2$	Each block on the left corresponds to exactly one block to the right. (2)	When we make pairs of all the students, no one is left out. (4)

Part 3 of the task was designed to initiate reflections about the two facets of the concept, starting from the concept name in Norwegian and English (Step 2, DP-D) that are multilingual resources available

to all students at the upper-elementary school level in Norway. Presenting a situation where the way of expressing a well-known concept in different languages conveys different shades of meaning (Step 3), can motivate for the students why multiple languages can be relevant in a mathematics class. Here, the language-related nuances are realised already in the name of the concept in the various languages, something likely easier to grasp than a syntactical construction, strengthening language awareness on the level of word consciousness. Students’ home languages can also be exploited in a teacher-led plenary discussion concluding the task, triggering awareness about the literal meaning of the concept word in home languages (DP-C). The potential of the task for relating mathematical representations and linguistic realisations of the concept is summarized in Figure 2, indicating along the arrows where the design principles and meaning-related phrases are at play.

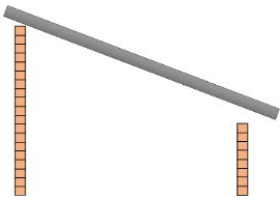

<p><b>Part 1:</b> Pia and Lea build a bridge with building blocks. They use small orange blocks for each of the pillars. In addition, they have got a long grey block that they are using as road over the bridge. Since Pia was faster, her pillar is bigger than Lea’s. Pia and Lea can move the orange blocks from one side to the other. Can they make a bridge that is perfectly horizontal with the blocks they have used so far? Imagine that they construct more bridges of different sizes. Which condition does the number of orange blocks have to satisfy to make a horizontal bridge?</p> 	<p><b>Part 2:</b> Pia and Lea have gone to an amusement park with their class. All students want to go on a ride with the Ferris wheel. Due to security rules no more than two children can be in each cabin. How many children can there be in the class if it is possible that nobody is alone in their cabin?</p>  <p><b>Part 3:</b> Natural numbers that are divisible by 2 are called <i>partall</i> in Norwegian and <i>even numbers</i> in English. Which of the words “partall” and “even number” fits best in the context of Task 1? Explain to the others how you are thinking. Do you agree with each other? Which of the words “partall” and “even number” fits better in the context of Task 2? Explain to the others how you are thinking. Do you agree with each other?</p>
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Figure 1: Task encouraging language awareness (translated from Norwegian)

### Implementation of the task

The episodes presented below are part of the data material of a larger project, aiming at the design of tasks that exploit multilingual resources to support conceptual understanding in mathematics. The task about even numbers was implemented in two 7<sup>th</sup>-grade classrooms in Norway; seven groups of students (Groups A–F) participated, a total of 21 students. Both group work and the plenary discussions introducing and summing up the parts of the task were videotaped. All video recordings were transcribed and translated to English. The transcripts were analysed with respect to students’ language awareness, which became visible in their discourse. We categorised students’ discourse practices according to the categories shown in Table 3.

**Table 3: Categories used for coding student discourse**

Discourse practice	Description
Voicing current action	Students say aloud which steps or calculations they are performing.
Reporting procedures	Students describe retrospectively steps or calculations they performed.
Explaining meanings of terms	Students elucidate the meaning of a concept, possibly related to representations.
Describing mathematical relationships and structures	Students express relationships between different facets of a concept or between different concepts.
Meta-linguistic discourse	Students talk about meanings of words and word parts, such as adjectives, prefixes, subjunctions, modal verbs, etymological origin.

### **Analysis of some episodes from the classroom implementation**

In this section, we give some examples of students' engagement with the task. We analyse the epistemic potential that could be realised in peer group discussions and the obstacles that emerged.

#### **Language awareness and concept as answer**

In accordance with the intentions behind the task design, some groups of students immediately understood that Part 1 and 2 were about the general concept and named the concept in their answers. Other groups, though, blended computational and conceptual approaches. Part 1 of the task started from a specific number of blocks but moved quickly to a call for generalisation of the situation asking to find a condition that the number of blocks must satisfy. Even though this was noticed by Group C ("It doesn't matter as long as...") and a conceptual description ("equally many on each side") was provided (discourse practice *explaining meaning*), the students turned the goal of the task into a computation, trying to find how many even bridges one could make with 26 bricks at hand:

- Student C1: Imagine that you are supposed to make different bridges, which condition must the bridge satisfy to be horizontal?  
Student C2: It doesn't matter as long as there are equally many on each side.  
Student C1: Yes, but how many can you make?

In Part 2, Student C3 showed word consciousness in that he picked up the generalising potential in the word *can* in the wording of the task ("how many children can there be..."):

- Student C3: There can be 22 people, it just needs to be an even number. [...] Because it says how many can [*stresses can with his voice*] there be. Just say an even number.

In the discussion that ensued, the students agreed that any even number would be a legitimate answer and that they could choose one of them as answer, thereby not fully accepting that the answer could be the name of the concept.

A different approach was that of Group A, who interpreted Part 2 about the Ferris wheel as a problem-solving task and applied a computational approach counting the number of cabins of the Ferris wheel, starting from the number of cabins on its upper half visible in the picture, doubling this number to get the total number of cabins and doubling again to get the number of students riding on the wheel. The group seemed to interpret the task as finding how many children there *are* in the class, not (as in the

wording of the task) how many there *can* be. This was also reflected in their language. Computational and contextual words (“take times 2”, “centre” (of the wheel)) dominated. Their discourse practice in Part 2 was restricted to *voicing current action* and *reporting procedures*. Neither the concept name nor meaning-related language about making pairs was mentioned in their discussion about the task. Thus, the discourse of Group A often stayed outside the intended discourse paths drawn in Figure 2.

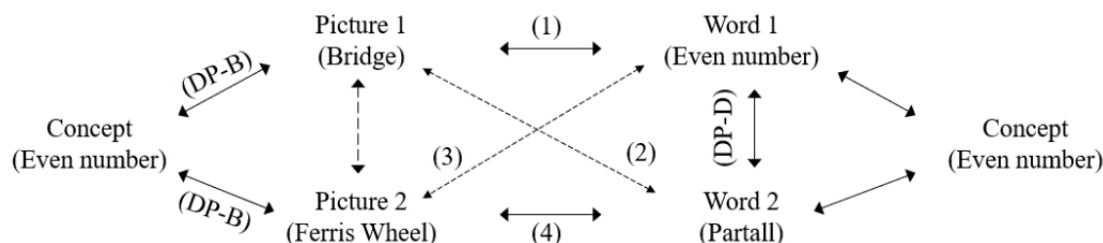


Figure 2: Multimodal representation diagram

### Language awareness and facets of the concept

In the following excerpt, we present an example of how the epistemic potential of the task was realised in the group discussion of Group D when they were working with Part 3:

- Student D1: What the hell do they mean that this should be even number or partall [*Norwegian term for even number*]?  
 Student D4: Even numbers fits for this one [*points to the bridge*], because we are supposed to try to balance it.  
 Student D3: It should have something to do with the context.  
 Student D4: Partall must be in pairs, so that fits with the Ferris wheel. [...]  
 Student D4: They try to even it out here such that it is the same on both sides. Here you must find pairs. [...]  
 Student D3: But isn't even numbers and partall the same? [...]  
 Student D1: Even numbers is just partall in English.  
 Student D4: Ok, but are even and par [*Norwegian term for pair*] the same? [...]  
 Student D3: It means the same. Kanin [*Norwegian term for rabbit*] and rabbit mean the same.

In this excerpt, Student D4 associated the English word *even numbers* with “try to balance it” and “even it out”, conveying clearly the meaning of the adjective *even*. Likewise, he explained that the Norwegian word *partall* fits with a situation where students are grouped in pairs. Student D4 did not look at the words *even number* and *partall* as entities, but decomposed them and compared the meanings of the smaller word parts *even* and *par*, having different meanings, by this displaying word consciousness. With this the basis was laid for looking at the concept of even numbers from two perspectives. But the group reflections did not enter this level of depth, since the other group members looked at the terms *partall* and *even number* as entities, denoting the same mathematical concepts, just as *kanin* and *rabbit* denote the same class of animals. A similar reflection immediately followed in the students’ conversation around the examples *novel* and *short story*. Student D3 argued that they are two different objects, while Student D4 considered them as two facets of the concept *text*. Student D3 seemed to take words as either having the same or a different meaning, lacking awareness about nuances. This limited his understanding of Part 3 of the task. Initiated by Student D3, the discussion of Group D opened for the discourse practice *explaining meaning* (see Table 3) along arrows (1) and (4) in Figure 2, but with only few meaning-related expressions, while the less obvious connections along (2) and (3) were not touched upon at all.

## Discussion and outlook

A task whose goal was to enhance students' understanding of the concept of even numbers through language comparison and reference to multimodal representations, was put to the test with seven groups of students. Its epistemic potential was realised in some groups, but not all. Even though comparing language pieces was only required at the level of words, not syntax, it proved not to be immediate for all students. Some students looked at words only as entities that were translated as units, in this way losing epistemic potential. At the word level also the meaning of the modal word *can* was overlooked by some students. As a design feature it might be beneficial to highlight important words, like modal verbs, to draw students' attention to subtle differences (such as *is/are* versus *can be*) that show whether the expected answer is on a conceptual or computational level. We also suggest that the teacher plays an important role in providing examples, also from everyday language, that highlight the difference between different words meaning the same (synonyms) and words that express different instances of a common superordinate concept (hypernyms and hyponyms).

Besides the wording also the imagery of the task influenced students' interpretation of the task. The image of the Ferris wheel proved to be a distractor from reflections about the concept. Though not the whole wheel was shown, with the intention to discourage approaches based on counting, students applied creative methods to get around this obstacle, moving their discourses from the intended *explaining meaning* to *voicing current action* and *reporting procedures*. In a new implementation of the task, it might be advisable to replace the circular form of the Ferris wheel with a linear structure like a cable car which does not have a natural upper bound on the number of passengers. Also, in other instances of the peer group discussions, students tended to apply computational approaches and find numbers as answers, when conceptual descriptions were the goal. This issue could be tackled if the teacher pays attention to creating language awareness about different discourse practices.

Based on the experiences of the first implementation we outlined above, we conclude that a task based on language nuances at the word level can provide a useful foundation for language and conceptual reflections, but this potential is not likely to be realised in peer group discussions alone. In accordance with findings by Dröse and Prediger (2020) at the syntactic level, we suggest that the teacher plays an important role in developing students' word consciousness and discourse practices, especially in assisting students to describe complex relations and to bring reflections of single students to the attention of the whole class. Further research is needed to provide teachers with examples of mathematical concepts that display multilinguistic variations in a broader spectre of languages and that are accessible to young students.

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