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# Talking mathematically – Negotiating mathematical concepts in bilingual settings

Malte Bürgstein, Marei Fetzer and Elke Söbbeke

University of Wuppertal, Germany; [buergstein@uni-wuppertal.de](mailto:buergstein@uni-wuppertal.de)

*Learning mathematics includes to think and to talk mathematically. Against this background, mathematical negotiation processes are closely interwoven with mathematical learning. We assume, that linguistic diversity can serve as a productive provocation and starting point for processes of negotiating and, as a consequence, be a fruitful resource for learning processes. Mathematical terms in different languages might open up possibilities to generate new perspectives on previous knowledge and to differentiate mathematical concepts. In this paper we present the beginning of a research process by analysing differences between the two languages English and German on a subject-specific, linguistic and etymological level in order to identify resources for mathematical learning in primary school from a didactic perspective.*

*Keywords: Mathematical learning processes, bilingual education, English (second language), mathematical concepts.*

## **Theoretical framework: Mathematical learning and the role of language**

Language plays an important role in mathematical learning processes. This impression is part of our everyday experience. In mathematics classrooms, we feel the influence of limited language competence on mathematics performance very concretely. Research confirms this experience (Ufer et al., 2013; Prediger et al., 2015) and turns it into recognition. Among the various family and language-related factors recorded, the aspect of educational language competence had the greatest impact on mathematical learning and performance. From a process-oriented perspective, it can be said that mathematical learning and the development of language proficiency are closely interwoven. This means that the development of language competence is not only an important learning goal, but also an important prerequisite for the learning of mathematics. In this context, two functions of language can be distinguished: the communicative and the cognitive function (Bruner, 1974; Maier & Schweiger, 1999).

Language is an important means of organising one's own thinking and at the same time opening up new possibilities of thinking and knowing (*cognitive function*). To develop new perspectives on one's own previous knowledge, processes of communication are crucial. Although thinking is not represented purely linguistically, good language competencies come with a differentiated vocabulary that supports the development of differentiated mathematical ideas and conceptual understanding. The more differentiated linguistic means a learner has to describe mathematical concepts, the more differentiated insights can be developed about these concepts. Thus, the cognitive function of language is directly related to its *communicative function*. Language is the central means of exchanging mathematical ideas and insights. But what language is used in mathematics education? What language do teachers and children speak in mathematics classrooms? Three different manifestations of language can be identified for mathematics teaching.

In Germany, we use the term *Alltagssprache* to describe the register of everyday language that almost all children already know when they start school. It is suitable for communication in the face-to-face settings and is characterised by conceptual rather oral forms (Oesterreicher, 1997), incomplete sentences and deictic forms. According to Cummins, this manifestation of language is characterised by basic interpersonal communicative skills (BICS) (Cummins, 2000, see also Aukerman, 2007). Mathematical language is the specific language of mathematics as a field of interest (German: *Fachsprache*). It is composed of mathematical terms but is also characterised by mathematics-specific sentence or text forms. Specialised mathematical language is stipulated as a learning objective in the national educational standards. The third language register is what we call *Bildungssprache*. This translates literally into ‘educational language’. It is characterised by conceptual literacy and precise word uses, and enables decontextualised communication across space and time. Cummins describes the required competencies as cognitive academic language proficiency (CALP) (Cummins 2000, see also Aukerman, 2007). Educational language is the language of teaching and instruction; it is the medium of learning in classrooms. Concerning the communicative and cognitive function of language it shows us that, if we focus on challenges in mathematical and language learning, there is a need to discriminate the level of specific mathematical terms and the level of sentences. Mathematical terms need contextualisation to be more than vocabulary. While they need a vivid context on the semantic level, on the linguistic level they need to be embedded in sentences. Thus, mathematical terms help to express and to understand mathematical ideas, to describe precisely or to explain understandably. In school practice, these educational language competencies often seem to be assumed as taken for granted. At the same time, however, the language of education is a language that few German children master when they enter school. Accordingly, the differences in this area are considerable.

In summary, it can be stated that *learning* mathematics includes *thinking* and *speaking* mathematically. In this context, mathematical negotiation processes are an important foundation for learning mathematics. It requires a mathematics teaching that deals productively with the diversity of children’s language competencies.

### **Processes of negotiation and the importance of diversity**

The above explanations show that language is essential for mathematics learning: on the one hand, for processes of subject-specific exchange (*communicative function of language*) and, on the other hand, for constructing of new mathematical knowledge (*cognitive function of language*). Especially young learners cannot derive new knowledge from existing knowledge in a completely autonomous way (Miller, 1986). Rather, this requires processes of collective negotiation, in which all participants stimulate each other and are confronted with new findings, concepts and linguistic means (Fthenakis, 2009). Various studies have shown that effective mathematical communication plays a significant role in the success of learning processes and has a positive effect on knowledge acquisition (Gersten et al., 2009; Stacey & Gooding, 1998). In this context, it is assumed that by providing students with opportunities to share diverse interpretations, ideas and explanations, learning can begin to take place as previous concepts are rethought and new insights developed. Following this, we assume that learning is especially initiated through *heterogeneous* experiences and concepts on *different levels*.

If we now reflect on the linguistic challenges mentioned above, it becomes clear that it is not so much defensive and language-reduced teaching but rather more language-intensive and language-sensitive teaching that can initiate rich linguistic and mathematical learning processes. From this perspective, the use of linguistic diversity as a productive provocation and starting point for the processes of negotiating seems to be a fruitful resource in learning processes.

### **Multilingual settings as a resource for mathematical learning**

Against this background, in this paper, we follow the paradigm, that (1) mathematical learning needs subject-specific exchange and (2) all kinds of diversity can be used as a resource for learning processes. In our classes in daily school life, diversity is reality. Learners show great heterogeneity in many ways. They differ in their linguistic abilities, language background, mathematical performance, social behaviour and ability to concentrate. In this paper, we take a closer look at one dimension of heterogeneity as a source of variety: the dimension of *linguistic diversity*. We follow the assumption that multilingual settings might bear resources for mathematical learning processes. A fact that mainly has been proven for secondary school under the circumstances of using the students' diverse mother tongues. Thinking about mathematical terms in different languages possibly evokes different mathematical ideas. As a consequence, these different mathematical ideas and terms might become a fruitful subject of negotiation, so that learners may develop more differentiated insights and a deeper understanding.

In Germany, there is a wide range of different heritage languages at primary schools. All these cultural and linguistic backgrounds might be a resource for learning based on communication and negotiating different terms. Various studies from secondary education investigated the relationship between multilingualism and mathematical learning (e.g., Prediger et al., 2019; Uribe & Prediger, 2021). They indicate that knowledge and use of multiple languages in mathematical education can contribute to a deeper understanding of mathematical ideas and concepts, provided that the differences in terms, ideas, and approaches to subject content associated with multilingualism are productively addressed in the classroom and constructively used for delineations and connections through cognitive contrasting. However, scientific findings in the field of learning in elementary school are not yet available. But how can we handle this super-diverse multilingualism on a research level in primary education? To get the research started we restrict ourselves to one single language that is not a mother tongue but a second language. In Germany, all students already learn English in primary school. Against this background, the study aims to investigate whether this foreign language has the potential – already in primary school – to promote mathematical learning by consciously using multilingualism as a starting point for designing of mathematical negotiation processes. For this purpose, in this paper, we focus on English as a second language and take a closer look at mathematical terms in German and English. We explore how basic, fundamental mathematical ideas are expressed in both languages, how these terms differ and in what way these differences emphasise various aspects of the mathematical concepts.

### **Methodological approach**

This paper presents the beginning of our research process. We first analyse the differences between the two languages English and German on a *subject-specific*, *linguistic* and *etymological* level to

identify potentials from a *didactic perspective*. Empirical studies will follow in further steps of the research process.

Our methodological approach is a procedure of increasing systematisation. As a first step, we identified basic mathematical concepts in primary school as they are described in our national standards of mathematical learning and compiled a list of key terms. These basic mathematical terms were translated into English regarding different possible contexts and mathematical concepts. At this point, we consulted native speakers to obtain accurate translations and a wide range of possible interpretations. In the next step, free associations on the German and English terms and various relations to the related mathematical concepts were collected. This step took place in different researchers' teams and involved intensive processes of negotiation on differing interpretations and understandings. Based on this, back translations from English into German were necessary to detect any differences and similarities concerning the aspect of contextualisation. This was followed by an etymological analysis. Finally, all steps of the analysis were bundled in an overview to describe types of multilingual resources.

### **Analysis: Comparing languages**

Analysing basic mathematical terms in both languages, English and German, we identified three types of resources:

1. Contextualisation: Different contexts demand different terms in one of the languages, whereas there is only one term for different contexts in the other language. This lack of explicit attribution elicits cognitive irritation and causes us to think about the reasons for this gap. We start thinking not only about the single expression but about the underlying (mathematical) concepts. By talking about this contextualisation, our mathematical understanding continues to differentiate and develops. Contextualization can be realized in all three types of registers: every-day-language, educational language as well as mathematical language.
2. Derivation: A discrepancy of derivations and etymological meanings evokes a different emphasis in understanding and, as a consequence, different mathematical concepts. This causes us to re-think our understanding and discuss basic mathematical ideas in exchange with others. We adopt and develop our mathematical understanding. This type of resource mainly draws on mathematical and educational language.
3. Word analysis: Components of mathematical terms differ, provoking a variation of interpretations and emphasis. Concepts have to be rethought, developed and adopted. Once again, mathematical and educational language are the basic registers in use.

In the following, we present examples for all three types of resources.

#### **Contextualisation**

Mathematical ideas, relations and concepts are represented and expressed through language by the means of specific terms. However, we do not find a one-to-one correspondence, because these terms refer to different aspects of the mathematical concept. In contrast, in one language there might be one term for a certain mathematical concept, whereas in the other there might be more than one. Thinking about the diversity of expressions leads to negotiating the different contexts those expressions refer

to. The variety of conceptualisations evokes cognitive dissonance and makes us further sharpen our understanding of mathematical terms.

As a first example, we present the variety of contextualisation of expressions referring to the number 0. While in German the only term existing for the number 0 is *Null*, in English, there are many terms that are used differently depending on the context. *Zero*, or sometimes *nought* in British English, is mainly used for 0 as number representing amounts, a number within the counting process or when calculating. For telephone, room or bus numbers, dates etc., the term *oh* is sometimes used instead of zero. For example, ‘She was born in 1807 (eighteen oh seven)’ or ‘Their hotel room is 402 (four oh two)’. While *nil* is used for football scores, for tennis scores you say *love*: ‘The score is forty love (40-0)’. Furthermore, in American English, various words for sports scores are used, such as *nothing*, *zero* or *zip* (MacKenzie, 2012).

Among other things, our discussion led to diverse aspects of the number we had in mind while talking about these specific terms and we realised that we in part associated different aspects with them. Numbers in general, are not exclusively quantities but imply many different aspects. It is an important goal of mathematics teaching to make these different ideas of numbers (number aspects) understandable for children. And it is precisely this process of reflecting and talking about these diverse terms of 0 in English, to sharpen and differentiate the understanding of numbers and to incorporate it into our existing knowledge.

What is striking about this kind of resource is the differentiation between everyday language and mathematical language or Fachsprache in English as opposed to German. When we talk about a *Würfel* in German, we refer either to the mathematical geometrical concept or the object we use in games (English: dice). It just depends on the context in which the word is embedded. In English, the word *cube* is used for the mathematical concept whereas the word *dice* is used in the context of games. As Lipski-Buchholz (2019) points out, this kind of contrast can be used as an opportunity to work out why the dice is not a cube in a narrower, mathematical sense. The students can learn that a dice also has six sides, but that these do not form squares because of the rounded corners. They could thus come to the understanding that a high degree of precision is required when defining mathematical objects.

## **Derivation**

In our second type of resource, derivation, a discrepancy of etymological meanings evokes a different emphasis in understanding and therefore variable mathematical concepts. Researching the origin and historical development of mathematical terms that exist today helps us further comprehend their use in mathematics.

The words *digit* in English and *Ziffer* in German can be taken as an example, both meaning any one of the ten numbers from 0 to 9. Looking at the etymology of the word *digit*, it originally arose from the Indo-European language *deik* (to show) and comes from the Latin *digitus* which means finger or toe. In mathematical and numerical contexts, a digit gives us the number of bundles of the given cardinality (numerical value of the digit) while the position of the digit within the number indicates the cardinality of the corresponding bundle (digit value). Taking the number 498 as an example, the digit 8 tells us that there are 8 ones grouped into a bundle, the digit 9 tells us that there are 9 tens

grouped into a bundle of tens, so 90, and the 4 tells us that there are 4 hundreds grouped to a bundle of hundreds, so 400. The German word is borrowed from the medieval Latin *cifra* or the Arabian word *ṣifr*, both meaning something like zero or empty. To re-establish the reference to the decimal place value system, the zero has a central meaning, because an important function of zero is in the formation of a place value system, which revolutionized mathematics. The German term *Ziffer* refers from its origin to this meaning of zero, and accentuates the possibility to encode *values of bundles of different order* by a certain position. For example, the 4 in 498 means  $4 \cdot 10^2$ , while the 0 in 408 means  $0 \cdot 10^1$ . In contrast to German, the English word focuses more on a geometric-spatial dimension by indicating the position of a number. To fully comprehend our decimal place value system, one needs to understand both types of principles. This reflection can be initiated by thinking about the origin and derivation of these diverse terms.

### Word-analysis

In the second type of resource, the analysis of etymological origins of terms is necessary to provide references to various mathematical ideas of a concept. In comparison to that, the third type, word-analysis, enables interpretations of mathematical concepts, that can be taken as a result of more easily accessible and rather semantical differences of word parts.

The distinction of focus when talking about a flat shape with three straight sides can be taken as an example. In German, this kind of shape is called *Dreieck* which, translated literally into English, means ‘three-corner’. In contrast, in English, we speak of a *triangle*. While in German the focus lies on three corners, in English it lies instead on angles. This is the case for all polygons, as polygons in German are named by combining the number of corners with the root *-eck*, which means corner. A pentagon is therefore a *Fünfeck*, a hexagon a *Sechseck*, an octagon an *Achteck* and so on. The English terms are derived from the Greek adjective *polús* which means many and *gonia* meaning angles. We assume that semantical varieties and differences in word parts have an impact on the understanding of mathematical concepts such as the concept of shapes<sup>1</sup>.

The fact that the resource of word analysis applies not only to abstract mathematical contents, but also to mathematical manipulatives, is illustrated by the example of the *number line*, which is called *Zahlenstrahl* in German. Literally translated it means ‘number ray’ or ‘number beam’. In our opinion, the focus and the idea of the manipulative shifts significantly when starting from the individual word parts. A ray rather focuses the reading direction and, in that way, implicitly arranges the numbers from small to big, whereas a line puts the focus on the infinity of numbers and implies integers instead of only natural numbers. In primary school, where integers do not yet play any role, it may appear sensible to think about the number line the way it is intended with the German word. However, in more advanced school years, integers are introduced and the manipulative is still named *Zahlenstrahl*.

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<sup>1</sup> Lipski-Buchholz (2019) claims that mathematical phenomena such as geometric shapes have a fundamentally universal, cross-cultural character, but that their consideration and description always take place from a cultural context.

## Discussion

In this paper we took a closer look at mathematical terms in the two languages German and English. Analysing key terms of basic mathematical concepts, we revealed both slight and profound differences in possible understandings of these terms, which might possibly lead to the development of different mathematical ideas and concepts. We note, that the use of the three types of resources and registers coming along with the various terms and their interpretations of (mathematical) concepts, enables us to think more deeply and more precisely about one mathematical concept. On a theoretical level, the variety we found in terms of both languages could elicit a subject-specific exchange on mathematical concepts and result in a deeper understanding. For example, by requiring the learners to think about the *different contextualisation*, to reflect on *derivations* or the *etymological meaning* of mathematical terms, multiple relationships have to be taken into the consideration. This can guide learners towards more differentiated insights into mathematical concepts. Our analyses give us reason to assume that a closer look at mathematical concepts at the level of words might have far more potential to initiate learning processes than we initially thought. The analyses we conducted as a team of researchers clearly enable us to experience the potential of provoking substantial thematic exchange. Negotiating different understandings and associations, performing etymological analyses on mathematical terms and varying contexts of the use of terms led us to think about and discuss the topic in a deeply mathematically way.

Our theory has not yet been empirically proven; it is merely a report on theoretical analyses. Nevertheless, in processes of communication and subject-specific negotiating, the diversity of terms seems to be the key to learning and understanding mathematical concepts. This leads us to the thesis that thinking and analysing mathematical terms in different languages can be regarded as a resource for mathematical learning. Furthermore, on the basis of our first analysis we presume that, when it comes to languages, it makes no difference whether we know a language like a first language and are familiar with it as our mother tongue. Instead, a second language that we learn at school can serve the same purpose. This potential of multilingualism could be used for mathematical learning processes. Empirical studies are planned in order to support (or contradict) our hypothesis.

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