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► **To cite this version:**

Mayu Aoki, Yukiko Asami-Johannson, Carl Winslow. Learning to speak mathematically at the Japanese supplementary school in Sweden. Thirteenth Congress of the European Society for Research in Mathematics Education (CERME13), Alfréd Rényi Institute of Mathematics; Eötvös Loránd University of Budapest, Jul 2023, Budapest, Hungary. hal-04392953

HAL Id: hal-04392953

<https://hal.science/hal-04392953>

Submitted on 14 Jan 2024

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Learning to speak mathematically at the Japanese supplementary school in Sweden

Mayu Aoki¹, Yukiko Asami-Johansson², Carl Winsl w¹

¹University of Copenhagen (IND), Denmark; mayu13ak@gmail.com

²University of G vle, Sweden

Natural language is known to play a crucial and specific role for children in school mathematics. It not only carries a special vocabulary but also subtle differences between natural languages that may lead to surprising challenges for learners who are not simply taught mathematics in their mother tongue. In this paper, we shall consider some cases from the teaching of mathematics at Japanese schools abroad, while following at the same time a regular school in the local language (in this case, Swedish). It turns out that Japanese teachers spontaneously emphasise certain subtle particularities, not only in how mathematical symbols are “read out” in natural language, but also in the terminology required to explain crucial techniques – a terminology which may have no equivalent in their pupils’ regular school experience or, indeed, in daily life Japanese.

Keywords: Teaching fractions, Japanese language, the Japanese Supplementary School, anthropological theory of didactic.

Introduction

The problem of how learners’ natural language background affects their learning in school mathematics is a classical one, especially when considering learners whose out-of-school language (or mother tongue) does not coincide with the language of instruction at school; such learners are especially prevalent in countries with several languages, which is quite common in Africa, and also in Western countries with large migrant communities (Barwell et al., 2016; Hodgen et al., 2022). Moreover, globalisation has led to more temporary cases of expatriation, linked to parents working for a limited time in a company or agency of their home country. Some countries, like Japan, have established schools abroad for children of citizens in the latter situation, in order to facilitate a smooth return to scholarisation in the country of origin, once the stage abroad has ended. Following the expansion of Japanese companies overseas, and a corresponding increase in Japanese expatriate families, the Ministry of Education, Culture, Sports, Science and Technology (MEXT) began (from 1956) to support daily or supplementary schools all over the world, with certified teachers being stationed temporarily or hired locally to serve at these schools. Daily schools operate as ordinary schools in Japan, while supplementary schools operate only on Saturday, while the pupils attend national or international schools during weekdays (MEXT, 2022).

The supplementary schools, which form the context of this paper, can naturally not offer a full curriculum, and as one could expect, Japanese language is a central subject (which is particularly demanding in terms of its highly demanding writing system, with several thousand characters in regular use). More surprisingly, the supplementary schools also usually offer mathematics lessons in Japanese, despite the fact that their pupils are also taught mathematics at their regular school. Why is this? One reason is certainly the particular structure of the Japanese Course of Study in mathematics,

the consequent needs for being prepared for resuming ordinary school upon return, and the universally acknowledged importance of success in mathematics for subsequent performance in higher education. The aim of this paper is to show that learning what one could informally call “school mathematical language” in Japanese is also an important function of this teaching. We do so by analysing a couple of episodes from lessons observed in a 5th grade class taught at the Japanese Supplementary School in Sweden (JSS_{SE}). We stress that all pupils in JSS_{SE} are bilingual in the sense that they speak both Swedish and Japanese in their daily lives, but the natural language used in the classroom is Japanese.

The results in this paper suggest that the purpose and functions of mathematics teaching at expatriate schools may not be limited to teaching pupils the curriculum taught in their home country, but also to deal with subtle language phenomena that are specific to that country’s school mathematics. So unlike more well-studied phenomena related to second language learners of mathematics, this paper deals with some striking cases that show how learning to ‘speak mathematically’ depends on the natural language.

Theoretical framework and research questions

Our study adopts the anthropological theory of the didactic (ATD) proposed by Yves Chevallard from the early 1980s (Bosch & Gascòn, 2006), in particular, the notion of *praxeology*. From the perspective of ATD, mathematical knowledge and practice can be described by using the notion of praxeology. Praxeologies consist of two parts: *praxis* and *logos*. The praxis is made up of a type of tasks (T) and techniques (τ), and the logos consists of technologies (θ) and theories (Θ). The set of several tasks (t) becomes a type of tasks (T), typically a family of tasks that can be solved by some specific technique (τ). The technique (τ) is explained by a technology (θ), that is unified and justified by the more abstract discourse of a theory (Θ). Concretely, for instance, converting two fractions to forms with the same denominator constitute a type of tasks (T), and there are two techniques (τ_1 and τ_2). The first technique does this by using the multiple of the original denominators as a common denominator (τ_1). The second one is determining the least common multiple of the denominators (τ_2). The discourse of those techniques constitutes a technology (θ), and the theory (Θ) that justifies this technology is multiplication. The praxis and the logos interact with each other, and as we will describe later, the technology may depend strongly on special natural language terms which are widely established in school mathematics but do not appear in daily language. ATD serves to link concrete discourse (logos) to the mathematical praxis, situated in a specific school institution. We can now formulate the research questions of this paper as follows.

RQ1: What difficulties with the technology related to the praxis of adding and subtracting fractions (with different denominators) appear in lessons at the JSS_{SE}?

RQ2: How are pupils’ difficulties related to specific properties of the mathematical Japanese terminology? What role could be played by differences between this and corresponding Swedish terminology the pupils are familiar with?

Context, data and methods

The episodes studied here took place during the two lessons on 22nd and 29th October 2022 in a 5th grade class with 19 pupils. The teacher we observed was Japanese and had experience teaching mathematics in Japanese ordinary schools. The lessons were given, as usual, in Japanese, but some pupils sometimes stated some sentences in Swedish. The lessons are based on a part of the chapter 10 in a grade five textbook 5B “Let’s Extend Addition and Subtraction of Fractions” (Fujii & Majima, 2021, pp. 2–19), published by TOKYOSHOSSEKI. These lessons we observed are taught using a whiteboard and a projector. The teacher mainly uses the whiteboard, but the projector is sometimes used for showing the textbook and pupils’ written productions. The teacher interacts with the pupils to solve problems from the textbook. During the lesson, the pupils take notes and do not open the textbook. We could roughly say that the phases of the lesson follow the standard flow of Japanese ordinary schools’ lessons, namely: 1. *Reviewing the previous lesson* 2. *Presenting the problem for the day* 3. *Pupils working individually or in groups* 4. *Discussing solution methods* 5. *Highlighting and summarizing the major points* (Stigler & Hiebert, 1999, pp. 79–80). For the lesson design, the teacher usually follows the *Teacher’s Guide* provided by the publisher of the textbook they use. The Teachers’ Guide provide suggestions for the flow of the lessons and for main tasks that are considered to be “appropriate” from the perspective of the Japanese *noosphere* (Bosch & Gascòn, 2006), by which we mean roughly curriculum developers in Japan.

We base this study on the praxeological analysis of the parts of the national Course of Study of Japan that deals with the teaching of fractions, which was presented in Aoki and Winsløw (2022). The model specifies, in particular, different themes and sectors taught throughout primary school. We analyse the technology related to the praxis of addition and subtraction of fractions with different denominators, linking that praxis also to other themes like equivalent fractions.

The first author video-recorded the lessons and took field notes, and the video material is analysed in two steps. In the first step, the episodes where pupils’ erroneous technology appears were chosen and transcribed in Japanese. Here, “erroneous” refers to both mathematical errors and inadequate use of Japanese to refer to specific aspects of the mathematical praxis. In step 2, those transcripts are translated into English, but some parts are remained in Japanese and Swedish to indicate the results of the analysis explicitly. In the transcriptions, the descriptions of gestures and other actions observed are shown in square brackets. *Italics* and **bold font** are used for Japanese and Swedish, respectively, and then these are translated into English in round brackets.

Findings

We present three episodes to answer RQ1 and RQ2 as follows.

Episode 1: Difficulty of relating symbolic language in Japanese

This episode is related to the task (t_1): Let us think how to calculate $\frac{1}{2} + \frac{1}{3}$ (Fujii & Majima, 2021, p. 3). The transcription starts with pupil 1 explaining his ideas to the other pupils in front of the whiteboard.

- Pupil 1: [Pointing at the 2 of $\frac{1}{2}$] First of all, this calls *bunbo* (the denominator, in Japanese) doesn't this? We do the same denominators. If it is the same, it is easy, isn't it. And then, [pointing at the 1 and 2 of $\frac{1}{2}$ the top and the above] well.. **gänger** (multiply, in Swedish)
- Teacher: If you multiplied.
- Pupil 1: If you multiplied, multiplied same, and it is the same number, isn't it. So... [writes $\frac{1}{2} \cdot \times$] *ichi bun* (1 over, in Japanese) right, multiplied something? **Multiplicera** (multiply, in Swedish). Eh, [pointing at the 2 and 3 of $\frac{1}{2} + \frac{1}{3}$] if 2 times 3, both are 6.
- Teacher: Multiply.
- Pupil 1: Because it is multiplied, and 2 times 3 is...eh... *bun...bunsu* (fractions, in Japanese)...eh...
- Teacher: *Bunbo* (the denominator, in Japanese)? *Bunshi* (the numerator, in Japanese)?
- Pupil 1: *Bunbo* (the denominator, in Japanese)! *Bunbo* (the denominator, in Japanese)! Eh... 2 times 3 is 6, right? [Writes $\frac{1 \cdot 3 = 3}{2 \cdot 3 = 6}$] 3 and 6 so... *san*, no, *roku bunno san* (3 over 6, in Japanese) So, *roku bunno san* (3 over 6, in Japanese) is the same with *hanbun* (a half, in Japanese).
- Pupil 1: Then, [pointing at $\frac{1}{3}$] this one also, eh... **nämna** (the denominator, in Swedish)
- Teacher: *Bunshi* (the numerator, in Japanese).
- Pupil 1: *Bunshi* (the numerator, in Japanese).
- Teacher: Ah, *bunbo* (the denominator, in Japanese)!
- Pupil 1: *Bunbo* (the denominator, in Japanese)! If we do *bunbo* (the denominator, in Japanese), we can do it easier. So, this one [points at $\frac{1}{3}$], 3 times something is 6 and it is 2, isn't it? So, [writes $\frac{1}{3 \cdot 2 = 6}$] 3 times 2 is 6. And, because the top and the bottom is the same, [writes $\frac{1 \cdot 2 = 2}{3}$] 1 times 2 is 2. Okay?
- Pupil 2: Yes, yes. It is correct.
- Pupil 1: So, *ni...roku bunno ni* (6 over 2, in Japanese). And [looks at $\frac{3}{6} + \frac{2}{6}$] *san bunno roku tasu ni bunno roku* (6 over 3 plus 6 over 2, in Japanese), ah! *roku bunno ni* (2 over 6). So, this one [pointing at $\frac{3}{6} + \frac{2}{6}$] is the same with this [pointing at $\frac{1}{2} + \frac{1}{3}$].

In the above episode, we can identify two main difficulties. The first one is that pupil 1 has difficulty reading out fractions in the Japanese way (we found this particular problem with other pupils during the lesson). When fractions are read out in Japanese, one reads the denominator first, and then the numerator. This is different from the order used in Swedish and English (as first the numerator, then the denominator is read out). This difficulty has been discussed by Bartolini Bussi et al. (2014), and was confirmed in the Japanese and Swedish case. Secondly, pupil 1 struggles to present his ideas to the others in Japanese, in particular, to use the specific mathematical terminologies, such as a denominator and a numerator. Pupil 1 is very active in presenting his idea to the others (not only in this episode, but also during the whole lesson); however, pupil 1 resorts to Swedish terms quite often, compared to other pupils. We consider that pupil 1 actually has no difficulty in solving t_1 , in other words, pupil 1 knows how to solve t_1 , but faces difficulty with the technology corresponding to the

practice block, as it requires Japanese vocabulary which is specific to mathematics and is rarely, if ever, used in out-of-school language.

Episode 2: The differences in mathematical terminologies between Japanese and Swedish

This episode is related to the task (t_2): There is $\frac{3}{5}L$ of juice and $\frac{1}{4}L$ of juice. How many L is the difference between the two? This task is originally from the textbook (Fujii & Majima, 2021, p. 7), but the teacher changed the content of the problem from milk to juice. The girl, the pupil 1 orally presents the calculation $\frac{3}{5} - \frac{1}{4} = \frac{3 \times 4}{5 \times 4} - \frac{1 \times 5}{4 \times 5} = \frac{12}{20} - \frac{5}{20} = \frac{7}{20}$. After that, they discuss why she did 5 times 4, in other words, why the denominator was aligned to 20.

- Teacher: Pupil 1 did 5 times 4. Why?
Pupil 2: Because, regarding the previous task, pupil 3 was doing like this.
Teacher: Haha, yes. Pupil 3 was doing so. Why?
Teacher: [Writes slash mark from 5 and 4] this one is connected to something we learned before when we studied fractions, isn't it? Pupil 4 said it a bit earlier.
Pupil 5: *Koubaisuu* (the common multiple of the original denominators, in Japanese)
Teacher: How about pupil 6?
Pupil 6: *Saisyokoubaisuu* (the least common multiple, in Japanese).
Teacher: This is *saisyokoubaisuu* (the least common multiple, in Japanese).
Pupil 7: I am, well... if 5 times 4, both 5 and 4, you know all of them go in, right? from 5 and 4. That is how I thought of it. Well, they don't teach such things in Swedish schools, but I thought about it on my own.
Teacher: Yes, that is right. So, what pupil 7 just said has a specific name in Japan, and it is connected to the idea of *koubaisuu* (the common multiple of the original denominators, in Japanese), which we all have learned when we studied fractions. [pointing at 5 and 4] So, you just have to find the common *baisuu* (the multiple, in Japanese) of these 5 and 4. You should find that *koubaisuu* (the common multiple, in Japanese).
Pupil 8: The smallest one.
Teacher: Yes, that is right. The smallest one. A number in multiples of 5 and 4. We learned that before, didn't we? *koubaisuu* (the common multiples, in Japanese).

After the above interactions, the teacher and the pupils confirm the following:

- Teacher: You should find this one (the common multiple). Well... as I said earlier, when *tsuubun* (finding a common denominator, in Japanese), this expression will appear many times from now on among those problems in the exercise book and the textbook. "*Tsuubun suru*" "*Tsuubun shite* and calculating this fraction". These mean, yes, pupil 9.
Pupil 9: Then, below, *Bun*, we align the *bunbo* (the denominator, in Japanese).
Teacher: Yes, that is right. When we *tsuubun* (finding a common denominator, in Japanese), then let us write it in the notebook. This is the important word. Write it in your notebooks.

In the process of solving t_2 , they discuss why the denominators are aligned 20, and the technique (τ_1): finding the least common multiple of the denominators, rewriting them into the same denominator, and then subtracting those fractions is finally developed. During the discussion, the mathematical vocabulary used to describe and explain τ_1 , such as finding *saisyokoubaisuu* (the least common multiply, in Japanese), *koubaisuu* (the common multiply, in Japanese) and *tsuubun* (a noun: finding a common denominator, in Japanese) are developed. Note here that in the Japanese context, the entire procedure of the operation "finding a common multiply of two different denominators, and

make them a common denominator” is rificated into one specific mathematical terminology: *tsubun*. This noun does not exist in Swedish mathematics terms (see, e.g. A Glossary of Swedish school mathematics terms by Kiselman & Mouriwitz, 2008). Based on studies of Swedish textbooks (Asikainen et al., 2018), along with the experience as a teacher educator by the second author, we have ascertained that in Swedish ordinary schools, the operation (the technique) for solving addition/subtraction of fractions with different denominators is taught along with the process of determining some common denominator. However, it is not explained in a special mathematical terminology such as *tsubun* as in the Japanese context. Therefore, we consider that the praxis is similar in both countries, but the part of the logos, in particular, the level of technology is different. When we compare Japanese and Swedish contexts, we can point out that this technology relies on the specific mathematical terminology *tubun* in Japan, which is supposed to be shared by teachers and pupils. In this episode, the teacher probably unconsciously recognises the importance of this technology and emphasises the need for a key vocabulary that does not translate the terms they know in Swedish.

Episode 3: The difference in the logos leads to the level of technique

This episode is in the second lesson, where the teacher starts reviewing what pupils have learned last Saturday. The teacher confirms that fractions are read out in the order of the denominator first, and then the numerator, and how to pronounce numerators and denominators in Japanese. After that, the teacher asks the pupils how to calculate $\frac{2}{3} + \frac{1}{6}$. The pupil 1 answers that “Make it the common denominators, so I do *saisyokoubaisuu* (the least common multiple, in Japanese), and then I do use the same of it”. His explanation in Japanese is ungrammatical, but the teacher interprets it as aligning the denominator and confirms that it is called *tsuubun* (finding a common denominator, in Japanese). After the above interactions, pupil 2 writes $\frac{2}{3} + \frac{1}{6} = \frac{2 \times 6}{3 \times 6} \dots (\tau_1)$ first, but changes to $\frac{2}{3} + \frac{1}{6} = \frac{2 \times 2}{3 \times 2} + \frac{1}{6} = \frac{4}{6} + \frac{1}{6} = \frac{5}{6} (\tau_2)$ as other pupils have told him to do so. They briefly confirm that τ_1 is also a correct technique. However, the teacher emphasises that *tsuubun suru* (finding a common denominator, in Japanese) means aligning the denominator, and tells the class that “as indicated by pupil 1 this (which we interpreted as 6) is *saisyokoubaisuu* (the least common multiple, in Japanese). It is *saisyokoubaisuu* (the least common multiple, in Japanese) of 3 and 6.” However, some of the pupils have further thoughts. Consequently, the interactions continue as follows:

- Pupil 3: But why 2? Why 3 times 2? Why not like 3 times 4?
 Teacher: Why here, 3 multiplied by 2?
 Pupil 2: Because, it is the smallest number.
 Pupil 4: Well... the denominators? It will be the same... well... if you multiply the top and the bottom, it will become the same denominators, so it is easier if you do it with the denominator aligned.
 Teacher: Yes, when you align *bunbo* (the denominator, in Japanese), as pupil 1 said, *saisyokoubaisuu* (the least common multiple, in Japanese) of 3 and 6. We want to align 6. Then, do you remember this? [writes $\frac{2 \times 2}{3 \times 2} = \frac{4}{6}$] We learned this last week, though. If you multiply both the denominator and the numerator by the same number, the magnitude of the fraction does not change. Do you remember that? Then we want to align 6. So, for aligning 6, we multiply it by 2. Okay?

In the above episode, we confirm that the type of task (T_1) is addition and subtraction of fractions with different denominators (the type of task is to find $\frac{a}{b} \pm \frac{c}{d}$), and that two techniques have developed. The first one passes by $\frac{a \times d}{b \times d} \pm \frac{c \times b}{d \times b}$ (τ_1), and the second proceeds through finding the least common multiple of b and adding those fractions (τ_2). We note that pupil 3 is confusing τ_1 with τ_2 (in the first line), not to deny that, in fact, τ_1 and τ_2 are both correct techniques for T_1 . However, the teacher (and probably also pupils) seem to prioritise τ_2 . Based on pupil 3's utterance, pupil 1 spontaneously finds *saisyokoubaisuu* (the least common multiple, in Japanese) as *tsuubun* (finding a common denominator, in Japanese). In fact, according to the Japanese Teacher's Guide, τ_1 is set up first, and then τ_2 is specified as the goal technique to be acquired for T_1 . In addition, the Teacher's Guide provides a special section entitled *tsuubun* (finding a common denominator, in Japanese) and *saisyokoubaisuu* (the least common multiple, in Japanese). These structures observed in the Teachers' Guide indicate that the term *tsuubun* is also related to τ_2 as a technology (which justifies the validity of the techniques) of addition and subtraction of fractions, which in the Japanese context is determined by the Japanese noosphere. As a result, this orientation became implicitly prevalent among the teachers and pupils, and therefore, they unconsciously refer to and use τ_2 as a priority. As we mentioned in episode 2, the term *tsuubun* does not exist as a single noun in the Swedish context, and this may complicate its acknowledgement as part of a technology that justifies τ_2 of T_1 as the case mentioned above. Consequently, the technique using *saisyokoubaisuu* (the least common multiple, in Japanese) appearing with τ_2 is not commonly referred to in the Swedish context. Episode 3 thus shows how the differences of the logos (and the terminology on which technology is based) influence the praxis (use of the techniques) of a mathematical praxeology.

Conclusion

In this study, we have presented three episodes to address RQ1 and RQ2. Based on the analysis of these episodes, the result showed that firstly, reading fractions out in Japanese is difficult for pupils because the order of the terms differs in Japanese and Swedish. Secondly, it is a difficult for pupils to present their ideas to others in Japanese since specific mathematical terminologies are used in the context of T_1 : addition and subtraction of fractions with different denominators. These show the difficulties pupils face at the levels of the logos, particularly in the technology, as it is highly dependent on special natural language terms. In addition, we have found that the technology related to the technique for solving T_1 requires a mathematical terminology which involves slightly different mathematical ideas. As a result, the teacher emphasises such terminology to the pupils. The first author has had experience of observing several lessons in JSS in Denmark, and found that the teachers often ask pupils to take notes related to specific meanings of mathematical terminologies. There is thus a mixture of syntactic and semantic differences between the mathematical logos to be acquired in the two languages, and the teaching focuses on these spontaneously. Indeed, the Japanese teachers may not – at least initially – be aware of them, since they (unlike their pupils) are hardly ever familiar with the school's mathematical logos in other languages than Japanese. Raiker (2002) pointed out the importance of establishing mathematical meanings of mathematical terms for pupils, as it is crucial for the development of mathematical reasoning. The emphasis on learning to “speak mathematically in Japanese” may then be a significant aim of the supplementary schools. While learning the writing (in *kanji* or “Chinese characters”) of mathematical terms is also emphasized in regular Japanese

schools, the subtle linguistic differences of school mathematical logos in different languages, thus take on a separate importance at Japanese schools abroad.

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