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An introduction to TWG9: Mathematics and language

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This paper is a summary of the work and discussions of the Thematic Working Group on Mathematics and Language (TWG9). In this paper we provide an overview of the main themes arising from in-depth discussion of presentations and through small group dialogues facilitated during the conference. We highlight the diversity and richness of theoretical approaches and research foci that lead to the constructive discussions we have had at CERME13, while identifying new directions in the future work of TWG9.

Keywords: Language, interaction, norms, multilingualism, professional development.

Introduction

The significance of language in the various fields of mathematics education research has been previously documented (Planas et al., 2019). Language is more than the medium through which mathematics is taught and learnt, as demonstrated by the work of TWG9. Research within this group is contingent on the analysis of how language, interactions and communication influence the teaching and learning of mathematics across the continuum of education and within teacher education. TWG9 included the presentation of 20 papers and 10 posters. The contributions are embodied by a strong diversity in research focus and questions considered, the research contexts examined, and the theoretical and methodological approaches utilised. The 7 sessions were divided into 5 paper sessions and 1 poster session and contributions were discussed individually and in depth. The remaining session provided an opportunity for small group discussion in relation to cooperation and collaboration around the identified common themes. We endeavoured to enable conversations, connections, and to nurture possible future collaborations. The following sections provide an overview of the key themes emerging and contributions from TWG9.

Researching social and sociomathematical norms in mathematics classrooms

Two papers (Ingram; Henschen, Vogler & Teschner) and two posters (Beforth, Lipowsky & Rathgeb-Schnierer; Shaka, Fischer & Rathgeb-Schnierer) presented in TWG9 at CERME13 had a particular focus on social interaction in the learning and teaching of mathematics. Classroom interaction and classroom mathematical practices have been a recurring topic at TWG9 for quite some years, perhaps it is the theme with the longest research tradition reaching back to the interactive paradigm in the 1980s in Germany and the USA. Accordingly, many posters are framing their research within such long research traditions. The particular strength of this research is exemplified by Ingram who shows

the many decisions teachers made during the interaction with their students, cautioning us about what it means that a specific instance of interaction “being supportive of mathematics learning”.

The paper by Henschen, Vogler, and Teschner formed the basis for an in-depth discussion of the concept of sociomathematical norms, that is on normative aspects specific to learners’ mathematical activity in a mathematics (classroom) culture. During the discussion, several important concepts relating to learning in interaction were revisited: mainly the concepts, practices, expectations and (social as well as sociomathematical) norms, and culture. Culture can refer to various sizes of groups of people establishing it, however in our situation always bound to contexts in which mathematics plays a crucial role (for example, culture of university mathematics, culture of marketplace mathematics, (micro) culture of a particular mathematics classroom, (micro) culture of a particular kindergarten children and their teacher, etc.). From an interactionist perspective, a central concept for understanding cultures of mathematics classrooms is reflexivity in interactive interplay (Cobb & Bauersfeld, 1995). On the one hand, individual learners actively contribute to establishing a mathematics classroom microculture and its associated mathematical practices and sociomathematical norms. At the same time, however, these cultures enable and constrain the mathematical activities of individuals. This also implies that the meaning making and understanding of mathematics is tied to practices of negotiating meaning in classrooms.

During the last 30 years, the concepts of sociomathematical norms (Yackel & Cobb, 1996) and mathematical practices (Cobb et al., 2001) have been used to describe various aspects of learning and teaching mathematics in social interaction. Thus, these concepts, norms and practices are widely found in CERME proceedings referring to the contributions of TWG9 and many others. During the small group discussion, the participating researchers extensively discussed the practical question of how (social or sociomathematical) norms can be re-constructed or identified in video, audio, or text data. The challenge is that norms are not directly observable as they are normative aspects of doing mathematics that are often established implicitly in interaction and only rarely made explicit. Norms can be re-constructed, especially when there is a violation of the norm in that a range of discursive structures are used to make the violation visible to all as a source of trouble (Ingram, 2021). Accordingly, an action is rejected in some form. This can occur not only through a direct negative marking but also through (follow-up) questions (e.g., Buttlar, 2017). Another challenge is the distinction between an expectation a classroom culture may have in a specific situation and a norm that is established in a culture and holds over a longer period of time. Thus, for re-constructing a (social or sociomathematical) norm, researchers would need to find several instances in their data in which the norm is made explicit or in which the norm is violated, and an utterance or action is accordingly evaluated. However, these instances are rare and thus the question is to what extent observed behaviours and practices can be used to re-construct norms by following the assumption that norms can be derived by repeatedly observed characteristics of practices. Also of consideration is that practices are interactively and mainly implicitly established in interaction (Erath, 2017). The discussion served as a strong reminder that we need to make our data base and the scope of our empirical findings very explicit in communicating our research on sociomathematical norms. It would be desirable to have long-term data that would allow the re-construction of norms over a longer period and to have time and resources for extensive transcription of video data and thorough analysis.

(Multi-)language-responsive mathematics learning

At CERME13, the topic of language-responsive teaching is represented in many of the papers. Increasingly, more and more research is concerned with the learning and teaching of mathematics to language diverse students, here referred to as multilanguage-responsive teaching (Aoki, Asami-Johannsson & Winsløw; Bürgstein, Fetzer & Söbbeke; Ferrari & Leka; Ferrari, Leka & Meaney; Revina & Schüler-Meyer; Skelton). Other research builds on a long tradition of investigating the intertwined nature of language(ing) and meaning making in mathematics, with a focus on topic-specific language demands (Antonopoulos; Kimber & Smith; Mähner & Erath; Okumus & Yildiz). Similarly in a long tradition, other papers investigate the role of interaction and discourse practices such as explaining or problem-solving in the learning and teaching of mathematics (Ceballos, van den Bogaart, Spandaw, van Ginkel & Drijvers; Ingram; Teledahl).

The theme of multilanguage-responsive learning and teaching of mathematics was well represented, illustrating that many of today's mathematics classrooms are language diverse. Luomaniemi, Gegenfurtner, Kankaanpää, McMullen and Hannula-Sormunen's comprehensive overview of early mathematical interventions for multilingual children illustrates the need to cater for language diverse children in our classrooms. There is wide agreement in the group that language diversity has become the standard instead of the exception in schools in many European countries. Accordingly, investigating how to teach mathematics in a multilanguage responsive way, such that students' languages become a transparent resource (Setati et al., 2008) for learning mathematics becomes increasingly relevant. However, the demands of multilanguage responsive teaching in Europe is distinct from other contexts such as North America and Africa, in that (1) students often do not share their languages, resulting in classrooms with many languages being present, and (2) teachers do not speak the students' languages. Despite these demands of "regular" language diverse classrooms, there are attempts to harness the potential of students' multiple languages as a resource for supporting students' conceptual understanding (e.g., Ferrari & Leka; Ferrari, Leka & Meaney; Revina & Schüler-Meyer).

The interconnection between language and meaning making in mathematics is investigated from different perspectives. In her poster, Jernsletten aims to contribute to our understanding of how teachers' one-to-one communication with students affects students' learning. Other research work aims to identify topic specific language demands (Baschek), building on previous research in this area. However, this research now recognizes that topic-specific demands can also stem from a particular discourse practice such as explaining (Mähner & Erath). The role of mathematical registers is also investigated further and builds on a plethora of previous work presented at CERME. For instance, the role of everyday language for learning sets and infinity is being investigated (e.g., Antonopoulos). In contrast, the role of formal language registers is investigated in terms of inappropriate use of technical vocabulary in Turkey which can potentially hinder meaning making (Okumus & Yildiz). The continued examination of this area of research provides rich insights for our classroom practices and teacher professional development.

Teacher development for language-responsive teaching

The professional development of mathematics teachers has been a growing area of interest in TWG9 in recent years and the number of papers, posters and projects focusing on supporting teachers in developing language-responsive classrooms has been rapidly increasing. In CERME13 there were several papers that consider the learning of pre-service teachers and experienced teachers. While most of these papers focus on classrooms including multiple languages, there is also a growing interest in linguistic diversity in relation to social and economic factors. The growing use of design research and topic-specific considerations of language-responsive teaching is also visible in the papers and posters.

The paper by Kis-Fedi and Büscher and the poster by Gíslason both consider the learning and professional development of pre-service teachers. Kis-Fedi and Büscher focused specifically on functional relationships using a design-based approach to examine the language-responsive elements of these functional relationships. In contrast, Gíslason reported on some work using chatbots in initial teacher education to support pre-service teachers in developing their explanations around some common mistakes made by the AI. The use of design research approaches when working with teachers was also highlighted in the poster presentation by Kristinsdóttir who used silent geometric videos with teachers. They made use of the developing technologies to extend the range of silent video tasks beyond the more traditional films, while also creating a need to communicate some complex mathematics.

The presentations by Planas and Alfonso and Rave-Agudelo and Planas use teacher noticing to focus on the language-responsive teaching of two key topics, linear equations and angles. These papers are drawn from a larger design research project and illustrate how the expertise of researchers, teacher educators and teachers combine in productive ways that can lead to a change in practice. This also connects to the presentation by Mwadzaangati and Adler who reported on a lesson study that focused on similar triangles as discussed above, but also highlighted the need to understand teacher learning and developing practice as a long-term goal. The poster presented by Rønning also drew on the expertise of teachers themselves in understanding the language aspects of teaching the topic of angles.

Zimmer, Ninove and Hanin focused more on classroom interaction, in particular, teacher interventions during scientific debates and discussions. This builds on a long history of classroom interaction research within TWG09 as discussed above and in Erath et al. (2021), exemplifying the specific types of teacher moves that can support or constrain learning in particular interactional contexts. This paper and much of the related research by the members of TWG9 highlight the importance of considering teacher moves in professional development that focuses on language-responsive teaching, to enable teachers to effectively use the opportunities for discussions and explanations that many of the tasks illustrated in the research at the conference enable.

While not focusing specifically on teacher professional development Dafnopoulou offered some insights into mathematics teachers' professional identity when working in a multilingual setting. This highlights the perspectives of teachers that many of the professional development projects within the group are now actively incorporating into their research design.

Language-responsive teaching is a relatively new focus in mathematics education, and we saw a range of definitions which reflect the different research designs and theoretical perspectives being used.

One central principle within the research represented in this group is the idea of language(s) as a resource, and the need to embrace the opportunities to enhance and amplify the learning and teaching of mathematics through a focus on language. Another key principle is that this focus on language is intertwined with the mathematical content. Much of the research within the group focuses on how word choices, opportunities to explain, argue and justify, and to work with multiple languages interact with our understanding of key mathematical concepts. This approach stresses the role of language in meaning-making and as something that needs to be embedded in the learning and teaching of mathematics. Yet this also poses several challenges. Many policies and practices in school still reflect the idea of teaching language in isolation, with keywords and definitions being shared but not used, as well as persistent deficit views and attitudes towards learners who do not necessarily share the language of instruction.

As both researchers and teacher educators we recognise the tension between what is needed for effective professional development and the limited resources available for this development. The need for professional development programmes to take place over time and in ways that respect the voice and agency of the teachers involved and the contexts within which they are working, was highlighted in many of the projects discussed in the group. Yet working with limited resources means that work is often restricted to a short series of workshops. The presentations at the conference demonstrate the importance of working collaboratively with teachers in ways that draw upon their existing expertise, but also in using their existing networks to disseminate research more widely.

The space between informal and formal languages

A classic theme in research on mathematics and language is the relations and differences between informal and formal language, often conceptualised as belonging to either the register of everyday language or the mathematics register, referring to Halliday's (1978) concept. A line of research consists of mapping difficulties and problems to differences between these registers. From this point of view, mathematics learning involves a progression from informal everyday language, riddled with imprecision and multiple connotations of words, to a more precise formal scientific language. At CERME13, most authors complemented and augmented this view with recent insights about seeing mathematics classrooms as sites of multiple languages and registers (with fuzzy borders) in interaction. The mathematics classroom conversational space lies between informal and formal language and provides opportunities for meaning making and learning. Thus, there are papers arguing and illustrating that colloquial language can enrich conceptual understanding when students discuss and analyse words that operate both in informal language (with various connotations) and disciplinary language and when students work with multiple natural languages that each have their webs of everyday meanings.

Three empirical papers explicitly explored task designs intended to exploit the epistemic potential of expressing mathematical concepts and relationships in multiple ways and across different languages. Ferrari and Lekaas present a task about even numbers intended to enhance conceptual understanding by drawing upon different perspectives embedded in the words for even numbers in English and Norwegian. Revina and Schöler-Meyer focus on developing a task on (linear) functional relationship designed to exploit the didactical potentials of connecting "the verbal everyday- and technical register

with representations and by prompting language production in the home language and the language of instruction.” Ferrari, Lekaas and Meaney describe the development of an algebra task and investigate how “different aspects of task design may interact to limit the possibilities for designing learning opportunities that utilizes multiple languages to deepen mathematical understandings.” The authors of these three papers all find that designers must take great care when developing these tasks and that their potential is not likely to be realised without a teacher who recognises students’ repertoires for contrasting and connecting languages and registers, as great linguistic demand is on participants in identifying language-specific nuances of a concept. Bürgstein, Fetzer and Söbbeke wrote a theoretical paper, taking German and English as their contrasting example, arguing that learners can gain insight into mathematical concepts by reflecting on the uses of different terms from different contexts and languages and the etymological meanings of mathematical terms. Farrugia contributes further to this discussion; her poster describes a project to create a Maltese-English glossary of mathematical terms. The theoretical underpinnings include the concept of register, and the explicit point is made that formality and informality of expression need to be considered.

Two papers explicitly address the space between informal and formal language. In analysing geometry teaching episodes, Mwadzaangati and Adler refer to the *school academic register* (Prediger & Zindel, 2017) and investigate movements between the informal and formal registers it lies between. In their episodes, students predominantly responded in formal or school academic registers, mostly naming objects and stating definitions without elaborating on meanings. Only the teachers elaborated on the answers in the informal register, showing that it was not a straightforward task for these teachers to demand or support the elaboration of meaning from students. Mähnert and Erath look in detail at the language means students use to explain the multiplication of two fractions in the sense of “taking part of a part.” These language means belong to *meaning-related language means*, which consist of grammatical constructs and vocabulary required to understand and explain topic-specific school mathematical concepts (Pöhler & Prediger, 2015) and are part of academic school language, which lie somewhere between the informal and formal registers. In Knobbe’s poster, the focus is on how students classified as having special educational needs verbalize their calculation strategies, in the space between the informal and formal language.

Several papers and posters directly compare and contrast the vocabularies or grammatical constructions of informal and formal language. Antonopoulos, Vlachos and Zachariades focus on how terms for sets and infinity are used differently in everyday Greek than in formal mathematics language and how this influences student understandings of formal concepts. They suggest (as Bürgstein, Fetzer & Söbbeke) that students be exposed to etymologies to help bridge the gap between informal and formal language. In his poster, Rønning discusses how words related to angles in Norwegian have different meanings in informal language than in formal language. Okumus and Yildiz depict a prospective Turkish mathematics teacher’s struggle with grammatical features idiosyncratic to formal Turkish mathematics language, illustrating that navigating the informal-formal space can differ for different languages. Planas and Alfonso describe a workshop for secondary-school mathematics teachers aimed at supporting teachers to notice and be responsive to what students need to know and what they struggle to understand around specific contents (linear equations in this case). The focus was on the practices of *naming* (giving word names or phrases from

the mathematical register) and *explaining* (mathematical meanings and relationships within a distinguished mathematical register) in teacher talk. Rave-Agudelo and Planas describe the design of planned workshops for secondary-school mathematics teachers where participants are (among other goals) to be sensitised to the specialised vocabulary of the mathematical register, including the differences in the meaning of words, such as “between”, in everyday language and the mathematical register (of school plane geometry in this case).

The research discussed above primarily focuses on vocabulary and phrases and their meanings. On a more abstract level, Kimber and Smith use systemic functional linguistics to analyse students’ short written descriptions of graphs and how the descriptions relate to the language their teachers used in their lessons. They found a tension in the graph descriptions between formal mathematics discourse, which tends to concern timeless relations between objects rather than material processes, and everyday discourse in which material processes of doing and happening are common. They argue that informal language may have had significant potential (lacking in more formal mathematical language) to communicate the richness of emergent-shape thinking.

The small group discussion on the space between informal and formal language focused mainly on how teachers could benefit from an awareness of this space. For example, should a teacher explain in colloquial language and avoid the specialised vocabulary of the mathematical register, or should they explicitly teach proper technical terms? It seemed essential to the participants that teachers introduce students to technical terms because otherwise, they risk restricting meanings to specific simple contexts. For example, if subtraction is replaced with “take away”, other contexts of subtraction models, such as comparison, will fall by the wayside. Another question is to what extent it could be helpful for students to know or ponder the etymologies of words for formal concepts. The terms may have everyday connotations that confuse or restrict their meaning for students. But these connotations may also reveal important aspects of the formal concept, create interest, and give opportunities for discussions and thinking, not least when students can compare and contrast informal connotations in multiple languages.

Concluding remarks

The themes addressed by TWG9 show the variety of research questions, theoretical perspectives, and methodologies that the papers and posters examined. In respect to the development of the research of TWG9 over the years, we have seen changes and developments in the foci of the research included. These include more studies focused on post-primary mathematics education, the embedding of multilingualism and language varieties as resources for the learning and teaching of mathematics, an increased focus on the role of language and social interaction in teacher education and professional development, and language responsive teaching relating to specific mathematical topics. We would like to thank all the participants of TWG9 who contributed through presentations, planned reactions, and engaging discussions.

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