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On Generalized Homogeneous Leader-following Consensus Control for Multi-agent Systems

Min Li, *Student Member, IEEE*, Andrey Polyakov, and Gang Zheng, *Senior Member, IEEE*

Abstract—In this paper, the leader-following consensus problem for multi-agent systems is considered. Each agent is assumed to be modeled by a linear multi-input system. A novel (generalized homogeneous) consensus control protocol is designed under the assumption that there are some uncertainties in the dynamic of the leader. Some conditions (LMIs) are derived to select the control parameters in order to ensure the input-to-state stability and global finite-time stability of the consensus errors with desired homogeneity degrees. Some simulations are performed to illustrate the obtained results.

Index Terms—Generalized homogeneity, consensus control, finite-time stability, input-to-state stability, multi-agent system.

I. INTRODUCTION

THE multi-agent system (MAS) has been a hot research object because multiple agents perform better working efficiency, reduced sensitivity to possible failures and increased flexibility during task execution. The research on MAS with coordinated control protocols has wide potential applications in many industrial scenarios, such as the control and optimization of unmanned aerial systems [1], network security [2] and smart grids [3].

Consensus is one of the fundamental problems in MAS applications, and it requires the state of each individual agent to reach an agreement with the regulation of coordinated control protocols. The research on consensus issues can be traced back to the 1980s [4]–[7], and it becomes systematized until the concept of the solvability of consensus problems and consensus protocols are introduced by Olfati-Saber and Murray [8], [9]. After that, recent two decades have witnessed the launch of many results (e.g., [10]–[13]).

Homogeneity is a dilation symmetry. All linear and a number of nonlinear models for mathematical dynamics are homogeneous in a certain sense. Homogeneous systems have some useful properties which drive their wide applications in system analysis, regulation, and estimation (e.g., [14]–[16]). To be specific, the local properties of homogeneous systems, such as local stability, can always be expanded into a global sense [14], [17], [18]. In addition, the homogeneous system is

robust with respect to a large class of perturbations [19] and time delays [20]. Moreover, the convergence rate of the homogeneous system is defined by its homogeneity degree, i.e., by adjusting the homogeneity degree, an asymptotic stable homogeneous system could achieve finite-time/asymptotic/fixed-time convergence [15], [21], [22]. For learning more details about the mentioned properties, we recommend a recent survey [23] and the book [20] to the reader.

Given the merits of the homogeneous system, homogeneous control strategies are introduced to consensus issues. The pioneering work is carried out by Wang and Hong [24], where a homogeneous consensus protocol, adapting the well-known controller of Bhat and Bernstein [15], is designed for consensus under an undirected graph. For directed topologies, homogeneous consensus protocols are studied for MAS with and without velocity measurements [25], [26]. The case of switching topologies is studied in [27], [28] for leader-following consensus. The agent model of MAS in the above-mentioned papers is double integrators, and consensus analysis is carried out with weighted homogeneity. Homogeneous consensus control protocols for MAS with high-order integrator dynamics are proposed, for example, in [29], [30]. The main difficulty of their applicability is the absence of an efficient/simple procedure for controller tuning. The mentioned algorithms (as well as most of the algorithms of homogeneous stabilization, see e.g., [15], [31]) just guarantee the existence of an appropriate control provided that the homogeneity degree is close to zero (i.e., if a homogeneous control is close to a linear control). In this case, it is hard to guarantee any improvement compared with the linear control.

In this paper, we propose to develop a generalized homogeneous control protocol for MAS, where each agent is modeled as a linear multi-input system. This agent dynamic is rather general. The actively researched MAS with linear agent dynamics (e.g., first-order system [32]–[36], second-order system [11], [37]–[40] and high-order integrators [41]–[43]) are special cases of this model. In addition, the multi-input agent dynamic gives a considerable possibility for the experimental implementation of the MAS, since a large number of realistic objects can be modeled as multi-input systems, such as the well-known unicycle robot model [44] and the car-like robot model [45].

The generalized homogeneous consensus protocol is proposed by following the idea of an “upgrade” of the classical consensus protocol by Olfati-Saber and Murray [8], [9] to a homogeneous one [20, Chapter 9], [46] in such a way that

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Min Li, Andrey Polyakov and Gang Zheng are with the Inria, Univ. Lille, CNRS, UMR 9189 - CRISTAL, Centrale Lille, F-59000 Lille, France (e-mail: min.li@inria.fr, andrey.polyakov@inria.fr, gang.zheng@inria.fr).

the control quality such as the convergence rates and the robustness can be improved. We show that such an upgrade is possible under the assumption that MAS has a supervisor, which can observe the whole system and broadcast a small amount of information to all followers simultaneously. The supervisor cannot be utilized as a centralized controller because its communication ability is assumed to be very limited and possibly sampled in time. A nonlinear consensus protocol is designed for MAS in such a way that the consensus error equation is generalized homogeneous of a desired degree. Under the assumption that there exist some bounded uncertainties in the dynamic of the leader, input-to-state stability (ISS) and uniform global finite-time stability are guaranteed by a proper selection of homogeneity degrees and control parameters. The control parameters are specified by Linear Matrix Inequalities (LMIs), which can be easily solved in MATLAB. Finally, simulations are performed to compare the classical linear and the homogeneous consensus protocol, and we can see the convergence performance is considerably improved although the homogeneous control protocol consumes more energy.

The rest of the paper is organized as follows. Section II gives some basic knowledge about graph theory, as well the definition of stabilities and generalized homogeneity. The problem to be studied is formulated in Section III. The motivation and the basic idea for upgrading the classical linear consensus protocol to a homogeneous one are shown in Section IV. The supervisor-based homogeneous control protocol is proposed and convergence results are obtained in Section V. Finally, in Section VI, we use some simulations to illustrate the effectiveness of obtained theories and compare the control energy consumption with the classical linear consensus protocol.

Notations:

- *Numbers and Sets:* \mathbb{R} is the set of real numbers; \mathbb{R}_+ is the set of positive real numbers; $\mathbb{R}_{\geq 0} = \{0\} \cup \mathbb{R}_+$; \mathbb{N}_+ is the set of positive integers; $\exp(x) = e^x$, where e represents the Euler number; a series of positive integers $1, \dots, N$ is denoted as $\overline{1, N}$.
- *Vectors and Matrices:* let $n \in \mathbb{N}_+$, \mathbb{R}^n and $\mathbb{R}^{n \times n}$ denote the $n \times 1$ real vector and the $n \times n$ real matrix, respectively; I_N is the $N \times N$ identity matrix; $\text{diag}\{\sigma_i\}_{i=1}^N$ is the $N \times N$ diagonal matrix with the diagonal entry $\sigma_i \in \mathbb{R}$; $\mathbf{1}_N$ is the N -dimensional vector whose elements are all ones; let $P \in \mathbb{R}^{n \times n}$, $\lambda_{\max}(P)$ and $\lambda_{\min}(P)$ denote the maximum and minimum eigenvalue of P , respectively; $P \succ 0$ ($\prec 0$) means that P is symmetric and positive (negative) definite; P is anti-Hurwitz if $-P$ is a Hurwitz matrix; \otimes represents the Kronecker product.
- *Spaces and Norms:* let $x = [x_1, \dots, x_n]^\top \in \mathbb{R}^n$, $x_i \in \mathbb{R}$, $i = \overline{1, n}$, $\|x\|$ is a norm in \mathbb{R}^n ; $\|x\|_2 = \sqrt{x^\top x}$; $\|x\|_p = \sqrt[p]{x^\top P x}$; $\|x\|_\infty = \max_{i=\overline{1, n}} |x_i|$; let function $\alpha(t) : \mathbb{R}_{\geq 0} \mapsto \mathbb{R}^n$, $\alpha \in L^2$ implies that it is Lebesgue integrable, corresponding norm is defined as $\|\alpha\|_{L^2(t_a, t_b)} := \sqrt{\int_{t_a}^{t_b} \|\alpha(t)\|_2^2 dt}$; $\alpha \in L^\infty$ implies that it is a Lebesgue measurable bounded function with $\|\alpha\|_{L^\infty(t_a, t_b)} := \text{ess sup}_{t \in (t_a, t_b)} \|\alpha(t)\|_\infty$; $\alpha \in W^{1, \infty}$ implies that its (weak) derivative up to order 1 has a finite L^∞ -norm, and $\|\alpha\|_{W^{1, \infty}} := \max_{i=0,1} \|\alpha^{(i)}\|_{L^\infty}$, where $\alpha^{(i)}$ is function $\alpha(t)$'s derivative of order i .

- *Functions:* $C(X, Y)$ is the class of continuous functions: $X \mapsto Y$, where X, Y are the subset of normed vector spaces; $C^p(X, Y)$, $p \in \mathbb{N}_+$ denotes the class of functions which are differentiable at least up to order p ; the function $\gamma(t) : \mathbb{R}_{\geq 0} \mapsto \mathbb{R}_{\geq 0}$ is said to be of class \mathcal{K} if it is continuous strictly increasing with $\gamma(0) = 0$; the function $\beta(\rho, t) : \mathbb{R}_{\geq 0} \times \mathbb{R}_{\geq 0} \mapsto \mathbb{R}_{\geq 0}$ is said to be of class \mathcal{KL} if for each fixed t the function $\rho \mapsto \beta(\rho, t)$ is of class \mathcal{K} and for each fixed ρ the function $t \mapsto \beta(\rho, t)$ is strictly decreasing to zero as $t \rightarrow +\infty$.

II. PRELIMINARIES

A. Graph Theory

Let $\mathcal{G} = \{\mathcal{V}, \mathcal{E}, \mathcal{W}\}$ denote a fixed directed graph, where $\mathcal{V} = \{1, \dots, N\}$ is the set of nodes (agents); $\mathcal{E} = \{(i, j) | i, j \in \mathcal{V}\}$ is the edge set, $(i, j) \in \mathcal{E}$ if i can receive the information from j ; n_i denotes the number of incoming edges of node i ; \mathcal{W} is the weighted adjacency matrix with elements w_{ij} , $i, j \in \mathcal{V}$, $w_{ij} > 0$ if $(i, j) \in \mathcal{E}$, and $w_{ij} = 0$ otherwise. In this paper, the self-loop situation is excluded, i.e., $w_{ii} = 0$. The element l_{ij} , $i, j \in \mathcal{V}$ of the Laplacian Matrix \mathcal{L} associated to the graph \mathcal{G} is defined as: $l_{ij} = -w_{ij}$ if $i \neq j$, and $l_{ij} = \sum_{k=1}^N w_{ik}$ otherwise, which implies $\sum_{k=1}^N l_{ik} = 0$. A directed path from node i to node j in the graph \mathcal{G} is a sequence of nodes $\overline{i_1, i_s}$, where $i_1 = i$, $i_s = j$ and $(i_{\kappa+1}, i_\kappa) \in \mathcal{E}$, $\kappa = \overline{1, s-1}$. The root node (the leader) i_L of \mathcal{G} characterizes itself from the other nodes since it has at least one directed path to each of the rest node while $(i_L, j) \notin \mathcal{E}$ for all $j \in \mathcal{V}$.

B. Stability Notation

Definition 1: The system $\dot{x} = f(x)$, $t \geq 0$, $x \in \mathbb{R}^n$, $x(0) = x_0$ is said to be globally uniformly finite-time stable [47] if it is Lyapunov stable and for all $x_0 \in \mathbb{R}^n$, there exists a locally bounded settling-time function $T(x_0) : \mathbb{R}^n \mapsto \mathbb{R}_{\geq 0}$, such that $\|x(t)\| = 0$ for all $t \geq T(x_0)$.

Definition 2: The system $\dot{x} = f(x, q)$, $t \geq 0$, $x \in \mathbb{R}^n$, $q \in \mathbb{R}^\omega$, $x(0) = x_0$ is said to be input-to-state stable (ISS) [48] if there exist $\beta \in \mathcal{KL}$ and $\gamma \in \mathcal{K}$ such that

$$\|x(t)\| \leq \beta(\|x_0\|, t) + \gamma(\|q\|_{L^\infty(0, t)}),$$

for any $x_0 \in \mathbb{R}^n$ and any $q \in L^\infty(\mathbb{R}, \mathbb{R}^\omega)$.

Definition 3: The system $\dot{x} = f(x)$, $t \geq 0$, $x \in \mathbb{R}^n$, $x(0) = x_0$ is said to be globally uniformly nearly fixed-time stable [23] if it is Lyapunov stable and for all $\vartheta > 0$, there exists $T(\vartheta) > 0$ such that $\|x(t)\| < \vartheta$ for all $t \geq T(\vartheta)$, where $T(\vartheta)$ is independent of x_0 .

C. Dilation in \mathbb{R}^n

Homogeneity is an invariance (symmetry) of an object with respect to a class of transformations called dilations. Choosing a proper dilation group $\mathbf{d}(s)$, $s \in \mathbb{R}$ is an essential part of the homogeneity-based analysis, $\mathbf{d}(s)$ is supposed to satisfy the limit property: $\lim_{s \rightarrow \pm\infty} \|\mathbf{d}(s)x\| = \exp(\pm\infty)$ for $x \in \mathbb{R}^n \setminus \{\mathbf{0}\}$. A dilation \mathbf{d} is monotone if the function $s \mapsto \|\mathbf{d}(s)x\|$ is strictly increasing for $x \in \mathbb{R}^n \setminus \{\mathbf{0}\}$ [20]. The examples of dilations are given as follows:

- Standard dilation (Leonhard Euler, 18th century): $\mathbf{d}(s) = \exp(s)I$, $s \in \mathbb{R}$;
- Weighted dilation [17]: Given $r_i > 0$, $i = \overline{1, n}$, $\mathbf{d}(s) = \text{diag}\{\exp(sr_i)\}$, $s \in \mathbb{R}$.
- Linear dilation [20]: $\mathbf{d}(s) = \exp(sG_{\mathbf{d}})$, $s \in \mathbb{R}$, $G_{\mathbf{d}} \in \mathbb{R}^{n \times n}$ is an anti-Hurwitz matrix known as the generator of dilation.
- Geometric dilation [49], [50]: a flow generated by an unstable C^1 vector field.

D. Canonical Homogeneous Norm

Definition 4: The functional $\|x\|_{\mathbf{d}} : \mathbb{R}^n \mapsto \mathbb{R}_{\geq 0}$ defined as $\|\mathbf{0}\|_{\mathbf{d}} = 0$,

$$\|x\|_{\mathbf{d}} = \exp(s_x), \text{ where } s_x \in \mathbb{R} : \|\mathbf{d}(-s_x)x\| = 1, x \in \mathbb{R}^n \setminus \{\mathbf{0}\}$$

is called the canonical homogeneous norm in \mathbb{R}^n , where \mathbf{d} is a *linear monotone dilation*.

The canonical homogeneous norm has the following properties [20]:

- $\|\mathbf{d}(s)x\|_{\mathbf{d}} = \exp(s)\|x\|_{\mathbf{d}}$, for $x \in \mathbb{R}^n$ and $s \in \mathbb{R}$;
- $\|x\|_{\mathbf{d}} = \|-x\|_{\mathbf{d}}$, $\|x\| = 1 \Leftrightarrow \|x\|_{\mathbf{d}} = 1$;
- $\|x\|_{\mathbf{d}}$ is locally Lipschitz continuous on $\mathbb{R}^n \setminus \{\mathbf{0}\}$;
- if $\|\cdot\| \in C^1(\mathbb{R}^n \setminus \{\mathbf{0}\}, \mathbb{R}_+)$, then $\|\cdot\|_{\mathbf{d}} \in C^1(\mathbb{R}^n \setminus \{\mathbf{0}\}, \mathbb{R}_+)$;
- if $\|x\| = \|x\|_P$, where P satisfies

$$P \succ 0, PG_{\mathbf{d}} + G_{\mathbf{d}}^T P \succ 0,$$

then the linear dilation $\mathbf{d}(s) = \exp(sG_{\mathbf{d}})$ is monotone and

$$\frac{\partial \|x\|_{\mathbf{d}}}{\partial x} = \frac{\|x\|_{\mathbf{d}} x^T \mathbf{d}^T (-\ln \|x\|_{\mathbf{d}}) P \mathbf{d} (-\ln \|x\|_{\mathbf{d}})}{x^T \mathbf{d}^T (-\ln \|x\|_{\mathbf{d}}) P G_{\mathbf{d}} \mathbf{d} (-\ln \|x\|_{\mathbf{d}})}, x \in \mathbb{R}^n \setminus \{\mathbf{0}\}.$$

E. Homogeneous System

Definition 5: A vector field $f : \mathbb{R}^n \mapsto \mathbb{R}^n$ (resp. a function $h : \mathbb{R}^n \mapsto \mathbb{R}$) is said to be \mathbf{d} -homogeneous if there exists a $\mu \in \mathbb{R}$ such that

$$\begin{aligned} f(\mathbf{d}(s)x) &= \exp(\mu s) \mathbf{d}(s) f(x), \quad s \in \mathbb{R}, x \in \mathbb{R}^n \\ (\text{resp. } h(\mathbf{d}(s)x) &= \exp(\mu s) h(x), \quad s \in \mathbb{R}, x \in \mathbb{R}^n), \end{aligned}$$

where \mathbf{d} is a dilation and the scalar μ is known as the homogeneous degree of f (resp. of h).

The following result is an adaption of [19], [31] to the case of linear dilation and can be found in [20].

Lemma 1: Assume a continuous vector field $\tilde{f} : \mathbb{R}^{n+\omega} \mapsto \mathbb{R}^{n+\omega}$ takes the form of

$$\tilde{f}(x, q) = \begin{pmatrix} f(x, q) \\ \mathbf{0} \end{pmatrix}, \quad x \in \mathbb{R}^n, q \in \mathbb{R}^{\omega}$$

and \mathbf{d} -homogeneous of a degree μ with respect to a dilation $\mathbf{d} = \begin{pmatrix} \mathbf{d}_x & \mathbf{0} \\ \mathbf{0} & \mathbf{d}_q \end{pmatrix}$ in $\mathbb{R}^{n+\omega}$. If the system $\dot{x} = f(x, \mathbf{0})$ is asymptotically stable at the origin, then the system $\dot{x} = \tilde{f}(x, q)$ is ISS.

The latter lemma shows that the robustness of a homogeneous perturbed system is equivalent to the asymptotic stability of the unperturbed system. This essentially simplifies the robustness analysis of homogeneous systems.

Lemma 2: Let f be a continuous \mathbf{d} -homogeneous vector field of degree $\mu \in \mathbb{R}$. The system $\dot{x} = f(x)$ is globally asymptotically stable if and only if there exists a positive

definite \mathbf{d} -homogeneous Lyapunov function $V : \mathbb{R}^n \mapsto [0, +\infty)$ of degree 1 such that

$$\dot{V}(x) \leq -\rho V^{1+\mu}(x), \quad \rho > 0.$$

The latter lemma is a straightforward corollary of the Zubov-Rosier Theorem [17], [51]. It immediately implies that any asymptotically stable homogeneous system $\dot{x} = f(x)$ is

- globally uniformly finite-time stable for $\mu < 0$ with time estimate $T(x_0) \leq \frac{-1}{\rho\mu} V^{-\mu}(x_0)$;
- globally uniformly asymptotically stable for $\mu = 0$;
- globally uniformly nearly fixed-time stable for $\mu > 0$, $T(\vartheta) = \frac{1}{\rho\mu} \vartheta^{-\mu}$, $\vartheta > 0$.

III. PROBLEM FORMULATION

Consider a MAS consisting of $N + 1$ agents, whose communication topology is described by a fixed digraph $\mathcal{G} = \{\mathcal{V}, \mathcal{E}, \mathcal{W}\}$, where the leader is represented by agent 0, and the rest agents labeled by $\overline{1, N}$ are followers. The dynamic of the leader is an ordinary differential equation (ODE):

$$\dot{\xi}_0(t) = f_0(t, \xi_0), \quad f_0 \in C(\mathbb{R} \times \mathbb{R}^n, \mathbb{R}^n) \quad (1)$$

and the dynamics of the followers are

$$\dot{\xi}_i(t) = A \xi_i(t) + B u_i(t), \quad i = \overline{1, N} \quad (2)$$

where $A \in \mathbb{R}^{n \times n}$, $B \in \mathbb{R}^{n \times m}$, $\xi_i(t) \in \mathbb{R}^n$ is the state vector, $u_i(t) \in \mathbb{R}^m$ is the control input.

Assumption 1: The pair (A, B) is controllable.

Assumption 2: The state of the leader as well as whose first derivative are uniformly bounded, i.e., $\xi_0 \in W^{1, \infty}(\mathbb{R}, \mathbb{R}^n)$.

In this paper, we assume that MAS has an external *supervisor* which can measure the states of all agents and it can broadcast (send) a *common* scalar signal back to the group. A concrete scenario of such a MAS is a swarm of drones observed by the camera of a supervisor. Positions of all drones can be detected/estimated by the supervisor which however cannot realize a centralized control due to communication constraints. Nevertheless, a broadcast of a common signal to the MAS is allowable (e.g., asking all of them to boost or slow down the motion). Mathematically such an assumption can be formalized as follows.

Assumption 3: All followers can receive a common scalar signal $u_g(t) \in \mathbb{R}$, which might depend on the state of the whole MAS and can be utilized for the control purpose.

Notice that even for $m = 1$ we cannot just select $u_i = u_g$ and design a purely centralized control protocol, since the obtained large-scale system is not controllable. To achieve consensus, it is necessary to use as well the communication graph and local interactions of the agents. Such a combination, i.e., a common centralized scalar signal and the distributed information of local agents, leads to a “quasi-decentralized” consensus protocol.

Our *first goal* is to develop a generalized homogeneous consensus control protocol for the considered MAS. By definition, this means that, *given* $\mu \in \mathbb{R}$, *we need to design a supervisor’s signal* u_g *and agents’ control laws* $u_i = \tilde{u}_i(u_g, \xi_i, \xi_{j_1}, \dots, \xi_{j_{n_i}})$, *where* $j_{\zeta} \in \mathcal{V} : (i, j_{\zeta}) \in \mathcal{E}$, $\zeta = \overline{1, n_i}$, $i = \overline{1, N}$ *such that for* $e = [(\xi_1 - \xi_0)^T, \dots, (\xi_N - \xi_0)^T]^T$ *and for the error equation*

$$\dot{e} = f(e, \xi_0, \dot{\xi}_0), \quad (3)$$

with $f: \mathbb{R}^{Nn} \times \mathbb{R}^n \times \mathbb{R}^n \mapsto \mathbb{R}^{Nn}$, one holds

- $\exists \tilde{\mathbf{d}}$ – a linear dilation in \mathbb{R}^{Nn} such that the vector field $e \mapsto f(e, \mathbf{0}, \mathbf{0})$ is $\tilde{\mathbf{d}}$ -homogeneous of degree μ ;
- the error equation (3) is ISS with respect to $q = (\xi_0, \dot{\xi}_0)$.

Notice that the asymptotic stability of the homogeneous error equation (3) with $\xi_0 = \dot{\xi}_0 = \mathbf{0}$ implies its finite-time stability for $\mu < 0$ and nearly fixed-time stability for $\mu > 0$.

Our *second goal* is to design a generalized homogeneous consensus control protocol such that *the error equation (3) is globally uniformly finite-time stable provided that the upper estimate of $\|\xi_0\|_{W^{1,\infty}}$ is known and the leader has the same model as other agents, i.e., in (1)*,

$$f_0(t, \xi_0) = A\xi_0(t) + Bu_0(t). \quad (4)$$

Usually, such a property can be guaranteed only using a sliding mode control technique (see, e.g., [52]), which may lead to a “chattering” during the transition phase (i.e., before reaching a consensus). Using the input u_g we design a consensus control protocol in such a way that the sliding mode arises only at the consensus point (i.e., when $e = \mathbf{0}$).

Since the supervisor is an external “agent” with respect to our MAS then it is hard to guarantee a stable communication channel for transmission of the signal u_g in a real-life scenario. Our *third goal* is to analyze the stability and robustness of the control system in the case of a sampled (possibly aperiodic) transmission of the control signal u_g .

IV. THE MOTIVATION AND BASIC IDEA TO THE HOMOGENEOUS CONSENSUS PROTOCOL

The conventional linear consensus protocol can only guarantee asymptotic consensus (see, e.g., [11], [33], [53]). This means consensus can be achieved as time goes to infinity. Such a long-term transient process may influence the control performance. However, the design of the consensus protocol with fast convergence rates may involve challenges and complexities. Recall (see Section I and II) that homogeneous systems can be finite-time stable or even fixed-time stable. This motivates us to find the approach to get some novel consensus protocol that can ensure the closed-loop consensus error equation (3) to be homogeneous of a desired degree. Then, the fast convergence could be performed by the MAS.

To demonstrate our basic idea, let us consider first the design of the well-known linear consensus control protocol [32].

Lemma 3: Let Assumption 1, 2 hold. Let $(X, Y) \in \mathbb{R}^{n \times n} \times \mathbb{R}^{m \times n}$ be a solution of the linear matrix inequality

$$X \succ 0, \quad I_N \otimes (AX + XA^\top) + \tilde{\mathcal{L}} \otimes BY + (\tilde{\mathcal{L}} \otimes BY)^\top \prec 0, \quad (5)$$

where

$$\tilde{\mathcal{L}} = \begin{pmatrix} -l_{11} & \dots & -l_{1N} \\ \vdots & \ddots & \vdots \\ -l_{N1} & \dots & -l_{NN} \end{pmatrix}. \quad (6)$$

Let the linear consensus protocol be

$$u_i = K \sum_{j=0}^N w_{ij} (\xi_j - \xi_i), \quad i = \overline{1, N} \quad (7)$$

with $K = YX^{-1}$. Then the error equation (3) is ISS with respect to $q = (\xi_0, \dot{\xi}_0)$.

Although the proof of the above lemma is straightforward, we still give the detailed proof in Appendix A since we will reuse it later for the homogeneous design.

Let us recall the basic idea of upgrading a linear control to a homogeneous one. The simplest scalar control system

$$\dot{x} = u, \quad x \in \mathbb{R}, \quad u \in \mathbb{R}$$

with the linear control

$$u = -kx, \quad k \in \mathbb{R}_+$$

is globally asymptotically stable. If the control u is transformed to the homogeneous controller

$$u = -k|x|^{1+\mu} \text{sign}(x),$$

then for $\mu = 0$ we will have the original linear controller, for $\mu < 0$ the closed-loop system is finite-time stable, and for $\mu > 0$ is nearly fixed-time stable. Similarly, for a multi-input system

$$\dot{x} = Ax + Bu, \quad x \in \mathbb{R}^n, \quad u \in \mathbb{R}^m, \quad A \in \mathbb{R}^{n \times n}, \quad B \in \mathbb{R}^{n \times m}$$

it can be shown [46] that the linear stabilization controller

$$u = Kx, \quad K \in \mathbb{R}^{m \times n}$$

can be transformed (“upgraded”) to the homogeneous form

$$u = K_0x + \|x\|_{\mathbf{d}}^{\mu+1} (K - K_0) \mathbf{d} (-\ln \|x\|_{\mathbf{d}}) x, \quad K_0 \in \mathbb{R}^{m \times n} \quad (8)$$

such that the closed-loop multi-input system is finite-time stable for $\mu < 0$ and nearly fixed-time stable for $\mu > 0$, where \mathbf{d} is a linear dilation in \mathbb{R}^n and $\|\cdot\|_{\mathbf{d}}$ is the canonical homogeneous norm. In this paper we generalize this upgrading approach to the consensus control of MAS.

V. SUPERVISOR-BASED HOMOGENEOUS CONSENSUS PROTOCOL

In this section, a generalized homogeneous consensus protocol is designed by upgrading the classical linear one. The main result shows that such an upgrade is feasible if there is a supervisor who monitors the MAS and can broadcast a scalar signal u_g simultaneously to all followers. With this homogeneous control scheme, the ISS and the global finite-time stability of the consensus errors can be guaranteed by tuning the homogeneity degree.

Let the pair $(G_0, Y_0) \in \mathbb{R}^{n \times n} \times \mathbb{R}^{m \times n}$ be a solution of the following algebraic equations

$$AG_0 - G_0A + BY_0 = A, \quad G_0B = \mathbf{0}, \quad (9)$$

such that the matrix $G_0 - I_n$ is invertible. Then the matrix $G_{\mathbf{d}} = \mu G_0 + I_n$ is anti-Hurwitz for any $\mu \leq 1/n$ with

$$A_0 G_{\mathbf{d}} = (\mu I_n + G_{\mathbf{d}}) A_0, \quad G_{\mathbf{d}} B = B, \quad (10)$$

matrix $A_0 = A + BK_0$ is nilpotent, where

$$K_0 = Y_0(G_0 - I_n)^{-1}. \quad (11)$$

Remark 1: According to [20], [54], if Assumption 1 is fulfilled, i.e., the pair (A, B) is controllable, then (A_0, B) is controllable, and such a solution of (9) always exists. In addition, $K_0 = \mathbf{0}$ if matrix A is already nilpotent.

Theorem 1: Let Assumption 1, 2, 3 hold. Let $(X, Y) \in \mathbb{R}^{n \times n} \times \mathbb{R}^{m \times n}$ be a solution of the linear matrix inequality

$$\begin{aligned} I_N \otimes (A_0 X + X A_0^\top) + \tilde{\mathcal{L}} \otimes B Y + (\tilde{\mathcal{L}} \otimes B Y)^\top < 0, \\ G_d X + X G_d^\top \succ 0, \quad X \succ 0, \end{aligned} \quad (12)$$

where $A_0 = A + B K_0$, K_0 is given in (11), $\tilde{\mathcal{L}}$ is defined in (6), $G_d = \mu G_0 + I_n$ is anti-Hurwitz with G_0 satisfying (9), $\mu \leq 1/n$. Let $\|e\|_{\tilde{\mathbf{d}}}$ be the canonical homogeneous norm induced by the norm $\|e\|_P = \sqrt{e^\top P e}$, $P = I_N \otimes X^{-1}$. Then the error equation (3) with $u_g = \|e\|_{\tilde{\mathbf{d}}}$, $K = Y X^{-1}$ and

$$u_i = K_0 \xi_i + u_g^{\mu+1} K \mathbf{d}(-\ln u_g) \sum_{j=0}^N w_{ij} (\xi_j - \xi_i), \quad i = \overline{1, N} \quad (13)$$

has the following properties:

- $\tilde{\mathbf{d}}$ -homogeneous of degree $\mu \in [-1, 1/n]$ if $\xi_0 = \tilde{\xi}_0 = \mathbf{0}$, $\tilde{\mathbf{d}}(s) = \exp(s G_d) = I_N \otimes \mathbf{d}(s)$ and $\mathbf{d}(s) = \exp(s G_d)$;
- globally asymptotically stable for $\xi_0 = \tilde{\xi}_0 = \mathbf{0}$;
- ISS with respect to $q = (\xi_0, \tilde{\xi}_0)$ for $\mu \in (-1, 1/n]$;
- globally uniformly finite-time stable if $\mu = -1$, f_0 satisfies (4), $\|A_0 \xi_0 - \tilde{\xi}_0\|_{L^\infty} \leq U_{\max}$, and

$$U_{\max} < U_0 := \frac{\lambda_{\min}(\Pi_1)}{2\sqrt{N n \lambda_{\max}(X^{-1})}}, \quad (14)$$

with

$$\Pi_1 = -P^{\frac{1}{2}} (I_N \otimes (A_0 X + X A_0^\top) + \tilde{\mathcal{L}} \otimes B Y + (\tilde{\mathcal{L}} \otimes B Y)^\top) P^{\frac{1}{2}}.$$

The settling time is estimated as

$$T(e(0)) \leq \frac{\|e(0)\|_{\tilde{\mathbf{d}}} \lambda_{\max}(\Pi_2)}{2(U_0 - U_{\max}) \sqrt{N n \lambda_{\max}(X^{-1})}}, \quad (15)$$

where $\Pi_2 = P^{\frac{1}{2}} (I_N \otimes (G_d X + X G_d^\top)) P^{\frac{1}{2}}$.

The detailed proof is provided in Appendix B.

The following corollary follows from the asymptotic stability and $\tilde{\mathbf{d}}$ -homogeneity of the error equation (3) for $\xi_0 = \tilde{\xi}_0 = \mathbf{0}$.

Corollary 1: Let all conditions of Theorem 1 be fulfilled. Let $\xi_0 = \tilde{\xi}_0 = \mathbf{0}$. Then the error equation (3) is

- globally uniformly finite-time stable for $\mu < 0$;
- globally uniformly asymptotically stable for $\mu = 0$;
- globally uniformly nearly fixed-time stable for $\mu > 0$.

It is worth stressing that for $\mu = -1$ the control protocol (13) is a high-order sliding mode algorithm having a discontinuity at $e = \mathbf{0}$ (i.e., when the consensus is reached). The use of the input u_g in the consensus protocol allows us to avoid large (infinite) gains in the transient phase (before reaching consensus). This reduces overshoots [20] and numerical chattering which are typical for sliding mode algorithms [55]. Moreover, for $\mu = -1$ the control inputs of all controlled agents are bounded.

Corollary 2: Let $u = [u_1^\top, \dots, u_N^\top]^\top$, if $\mu = -1$ then we have

$$\|u\|_2 \leq \|K_0\|_2 \|\xi\|_2 + \sqrt{\lambda_{\max}((P^{-1/2})^\top \tilde{K}^\top \tilde{K} P^{-1/2})}, \quad (16)$$

where $\xi = [\xi_1^\top, \dots, \xi_N^\top]^\top$, $\tilde{K} = \tilde{\mathcal{L}} \otimes K$.

Proof: Recall (13), for $\mu = -1$, we have $u = \tilde{K}_0 \xi + \tilde{K} \tilde{\mathbf{d}}(-\ln \|e\|_{\tilde{\mathbf{d}}}) e$, with $\tilde{K}_0 = I_N \otimes K_0$. Then,

$$\begin{aligned} \|u\|_2 &\leq \|\tilde{K}_0 \xi\|_2 + \|\tilde{K} \tilde{\mathbf{d}}(-\ln \|e\|_{\tilde{\mathbf{d}}}) e\|_2 \\ &\leq \|K_0\|_2 \|\xi\|_2 + \sqrt{e^\top \tilde{\mathbf{d}}^\top (-\ln \|e\|_{\tilde{\mathbf{d}}}) \tilde{K}^\top \tilde{K} \tilde{\mathbf{d}} (-\ln \|e\|_{\tilde{\mathbf{d}}}) e} \\ &\leq \|K_0\|_2 \|\xi\|_2 + \sqrt{\lambda_{\max}((P^{-1/2})^\top \tilde{K}^\top \tilde{K} P^{-1/2})} \times \\ &\quad \sqrt{e^\top \tilde{\mathbf{d}}^\top (-\ln \|e\|_{\tilde{\mathbf{d}}}) P \tilde{\mathbf{d}} (-\ln \|e\|_{\tilde{\mathbf{d}}}) e}. \end{aligned}$$

By the definition of the canonical homogeneous norm we have $e^\top \tilde{\mathbf{d}}^\top (-\ln \|e\|_{\tilde{\mathbf{d}}}) P \tilde{\mathbf{d}} (-\ln \|e\|_{\tilde{\mathbf{d}}}) e = 1$. Hence, the inequality (16) can be obtained. ■

Remark 2: If $K_0 = \mathbf{0}$ (i.e., matrix A is nilpotent) and $\mu = -1$ then u is globally uniformly bounded (see the formula (16)).

In some cases, a MAS is already regulated by some linear control protocol. The following corollary presents an algorithm for its upgrading to a locally homogeneous one.

Corollary 3: Let all conditions of Theorem 1 be fulfilled. Then the error equation (3) with $u_g = \|e\|_{\tilde{\mathbf{d}}}$, $K = Y X^{-1}$ and

$$u_i = K_0 \xi_i + \begin{cases} K \sum_{j=0}^N w_{ij} (\xi_j - \xi_i) & u_g > 1, \\ u_g^{\mu+1} K \mathbf{d}(-\ln u_g) \sum_{j=0}^N w_{ij} (\xi_j - \xi_i) & u_g \leq 1, \end{cases} \quad (17)$$

$i = \overline{1, N}$, has the following properties:

- $\tilde{\mathbf{d}}$ -homogeneous of degree $\mu \in [-1, 1/n]$ for $u_g \leq 1$;
- globally asymptotically stable for $\xi_0 = \tilde{\xi}_0 = \mathbf{0}$;
- ISS with respect to $q = (\xi_0, \tilde{\xi}_0)$ for $\mu \in (-1, 1/n]$;
- globally uniformly finite-time stable if $\mu = -1$, f_0 satisfies (4), $\|A_0 \xi_0 - \tilde{\xi}_0\|_{L^\infty} \leq U_{\max}$, where U_{\max} is given in (14).

Proof: With the control (17), the closed-loop error equation $\dot{e} = f(e, \xi_0, \tilde{\xi}_0)$ coincides with a $\tilde{\mathbf{d}}$ -homogeneous system of degree $\mu = 0$ (linear system) for $u_g \geq 1$ and a $\tilde{\mathbf{d}}$ -homogeneous system of degree $\mu \in [-1, 1/n]$ for $u_g \leq 1$.

Since the mapping $u_g \mapsto \mathbf{d}(-\ln u_g)$ is uniformly continuous, and for $u_g = 1$ we have $\|e\|_{\tilde{\mathbf{d}}} = e^\top P e = 1$ and

$$u_g^{\mu+1} K \mathbf{d}(-\ln u_g) \sum_{j=0}^N w_{ij} (\xi_j - \xi_i) = K \sum_{j=0}^N w_{ij} (\xi_j - \xi_i),$$

then the locally homogeneous control (17) is continuous. Let us consider the candidate Lyapunov function $e \mapsto V(e)$ as

$$V(e) = \begin{cases} e^\top P e & e^\top P e > 1, \\ \|e\|_{\tilde{\mathbf{d}}} & e^\top P e \leq 1. \end{cases}$$

Since the function V is locally Lipschitz continuous on $\mathbb{R}^n \setminus \{\mathbf{0}\}$ then combining the constructions of Lemma 3 and Theorem 1 we can complete the proof. ■

The possible fragility of the proposed scheme is related to the use of a supervisor, which is considered as an external object with respect to MAS. The existence of a robust communication channel for broadcasting of the supervisor's signal u_g seems to be a rather restrictive assumption in some cases. That is why it is important to investigate the robustness of the proposed control scheme with respect to losses and/or aperiodic sampling of the supervisor's control signal u_g .

Corollary 4: Let $t_0 = 0 < t_1 < t_2 \dots$ be an arbitrary sequence of time instances such that $t_k \rightarrow +\infty$ as $k \rightarrow +\infty$. Let the control protocol be defined by Theorem 1 and

$$u_g(t) = \|e(t_k)\|_{\tilde{\mathbf{d}}}, \quad t \in [t_k, t_{k+1}).$$

Then the error equation (3) is

- globally asymptotically stable for $\xi_0 = \tilde{\xi}_0 = \mathbf{0}$;
- ISS with respect to $q = (\xi_0, \tilde{\xi}_0)$ for $\mu \in [-1, 1/n]$.

Proof: For $t \in [t_k, t_{k+1})$, let us consider the function $e(t) \mapsto V_k(e(t))$ defined as follows

$$V_k(e(t)) = e^\top(t) \tilde{\mathbf{d}}^\top (-\ln \|e(t_k)\|_{\tilde{\mathbf{d}}}) P \tilde{\mathbf{d}} (-\ln \|e(t_k)\|_{\tilde{\mathbf{d}}}) e(t).$$

Notice that $V_k(e(t)) = 1$ for $t : \|e(t)\|_{\bar{\mathbf{a}}} = \|e(t_k)\|_{\bar{\mathbf{a}}}$. The time derivative of V_k along error equation (3) with $\xi_0 = \dot{\xi}_0 = \mathbf{0}$ satisfies the inequality

$$\dot{V}_k(e(t)) \leq -\lambda_{\min}(\Pi_1) \|e(t_k)\|_{\bar{\mathbf{a}}}^\mu V_k,$$

for $t \in [t_k, t_{k+1})$. The latter means $V_k(e(t_{k+1})) < 1$ and $\|e(t_{k+1})\|_{\bar{\mathbf{a}}} < \|e(t_k)\|_{\bar{\mathbf{a}}}$. Let us show that the monotone sequence $\|e(t_k)\|_{\bar{\mathbf{a}}}$ converges to zero as $k \rightarrow +\infty$. Suppose the contrary, i.e., there exists $q_* > 0$ such that $\|e(t_k)\|_{\bar{\mathbf{a}}} \rightarrow q_*$ as $k \rightarrow +\infty$. Let us consider the function $e \mapsto V_*(e)$ defined as

$$V_*(e) = e^\top \tilde{\mathbf{d}}^\top (-\ln q_*) P \tilde{\mathbf{d}} (-\ln q_*) e.$$

Notice that $V_*(e(t_k)) \rightarrow 1$ as $\|e(t_k)\|_{\bar{\mathbf{a}}} \rightarrow q_*$. Denoting $\delta_i = u_i - K_0 \xi_i - q_*^{\mu+1} K \mathbf{d} (-\ln q_*) \sum_{j=0}^N w_{ij} (\xi_j - \xi_i)$, $i = \overline{1, N}$ we derive

$$\dot{V}_* \leq -\lambda_{\min}(\Pi_1) q_*^\mu V_* + \frac{2\sqrt{NV_* \lambda_{\max}(B^\top X^{-1} B)}}{q_*} \max_{i=\overline{1, N}} \|\delta_i(t)\|_2.$$

Since $\sup_{t \in [t_k, t_{k+1})} \|\delta_i(t)\|_2$ converges to zero as $k \rightarrow +\infty$ and $V_*(e(t_k)) \rightarrow 1$ as $k \rightarrow +\infty$ then there exists $k_* \geq 1$ and $c_* > 0$ such that $\dot{V}_*(e(t)) \leq -c_*$ for all $t \geq t_{k_*}$. The latter means that $V_*(e(t)) \rightarrow 0$ as $t \rightarrow +\infty$. Based on the obtained estimates, the ISS with respect to $q = (\xi_0, \dot{\xi}_0)$ can be proven using the ISS Lyapunov function technique [56]. ■

Remark 3: Corollary 4 has no restriction (like a dwell time) to the sampling period $t_{k+1} - t_k$. So, the consensus error with the presented consensus protocol would remain asymptotically stable and robust in the case of sampled broadcasting of the supervisor's signal. Even in the worst case, when the signal u_g is lost (i.e., $u_g = \text{const}$ for all $t \geq t_*$, t_* is the moment u_g is lost), the system will be simply governed by a linear consensus protocol. In other words, the system will not crash if the supervisor's signal will be damaged, sampled, or completely lost. Therefore, the presence of the supervisor does not increase the fragility of the MAS in comparison with the case of the classical linear consensus protocol.

VI. SIMULATION RESULTS

We consider the MAS (2) with 4 agents regulated by the locally homogeneous consensus protocol (17). The communication topology, as depicted in Fig.1, is fixed. The agents

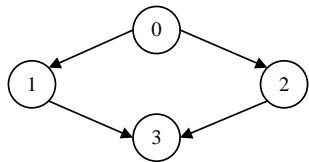


Fig. 1. The topology of the MAS.

are labeled by $i = \overline{0, 3}$, where the agent 0 is the leader and the agent $\overline{1, 3}$ are followers. Here we let the dynamic of the leader be (4) and the followers be (2). The dynamic system matrix A and B are randomly chosen provided the pair (A, B) is controllable:

$$A = \begin{pmatrix} 11 & -11 & 5 \\ 11 & -11 & 4 \\ 1 & 2 & 0 \end{pmatrix}, \quad B = \begin{pmatrix} 1 & 1 \\ 0 & 1 \\ 0 & 2 \end{pmatrix}.$$

Let $u_0 = Y \xi_0 + 0.2 \cdot [\sin(25t) \cos(25t)]^\top$ with

$$Y = \begin{pmatrix} 0 & 16 & -1 \\ -16 & 1 & -4 \end{pmatrix}.$$

According to equation (9) and (11), we obtain

$$K_0 = \begin{pmatrix} 3.3134 & -3.7867 & 0.2622 \\ -3.4293 & 2.3477 & -1.1159 \end{pmatrix}.$$

Let $\mu = -1$, we have

$$G_{\bar{\mathbf{a}}} = \begin{pmatrix} 1.0000 & 1.0366 & -0.5183 \\ 0.0000 & 1.7210 & -0.3605 \\ 0.0000 & -0.5580 & 1.2790 \end{pmatrix}.$$

Let the agent state vector and the consensus error vector be $\xi = [\xi_0^\top, \xi_1^\top, \xi_2^\top, \xi_3^\top]^\top$ and $e = [e_1^\top, e_2^\top, e_3^\top]^\top$, respectively. Let $e_i^{(\zeta)}$ denote the ζ_{th} , $\zeta = \overline{1, 3}$ component of e_i .

Let $w_{ij} = 1$, $(i, j) \in \mathcal{E}$, then we have

$$\mathcal{L} = \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 1 & 1 & -2 \end{pmatrix}.$$

We obtain

$$X = \begin{pmatrix} 2.0588 & 2.1825 & 0.2371 \\ 2.1825 & 4.5417 & 1.8298 \\ 0.2371 & 1.8298 & 8.0728 \end{pmatrix},$$

$$Y = \begin{pmatrix} 18.1713 & -20.8153 & 0.6775 \\ 0.1474 & 5.2376 & 7.4799 \end{pmatrix},$$

by solving (12) with YALMIP. By employing bisection method, $u_g = \|e\|_{\bar{\mathbf{a}}}$ is obtained from

$$\|\tilde{\mathbf{d}}(-\ln(\|e\|_{\bar{\mathbf{a}}}))e\|_P = 1,$$

with $P = I_3 \otimes X^{-1}$, and $\tilde{\mathbf{d}}(\cdot)$ is generated by $G_{\bar{\mathbf{a}}} = I_3 \otimes G_{\bar{\mathbf{a}}}$. The trajectory of $A_0 \xi_0 - \dot{\xi}_0$ is as Fig.2, which is bounded by $U_0 = 0.7$, where U_0 is obtained according to (14).

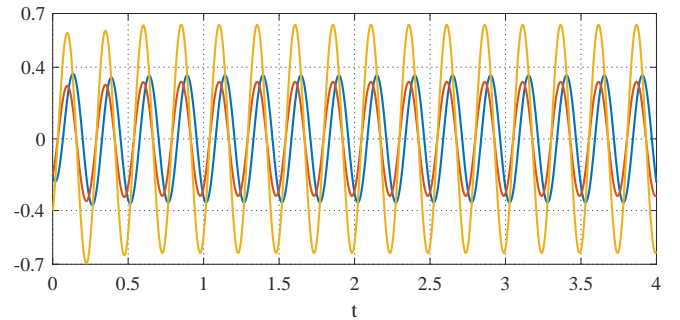


Fig. 2. Evolution of $A_0 \xi_0 - \dot{\xi}_0$.

Let the initial states of the considered MAS be $\xi(0) = [0, 0, 0, 0.95, 1.00, 0.75, 1.25, 0.65, 1.20, 0.70, 0.73, 1.15]^\top$. The implicit Euler method is employed with step $h = 0.001$ s to solve the closed-loop error equation (3), (17) on MATLAB. Meanwhile, we employ the classical linear consensus protocol (7) as the reference. Comparison simulation results on $e_i^{(\zeta)}$, $\zeta = \overline{1, 3}$ are presented in Fig.3-5. In the case of the linear consensus protocol, the consensus error asymptotically converges to a zone located in the neighborhood of the origin, while with the homogeneous control protocol, the error

converges to zero in a finite time. The evolution trajectory of u_g is as Fig. 6, which implies the locally homogeneous control protocol (17) switches from the linear case to the homogeneous case at $t \approx 0.535$ s. The evolution trajectories of the norm of the control signals are depicted in Fig.7.

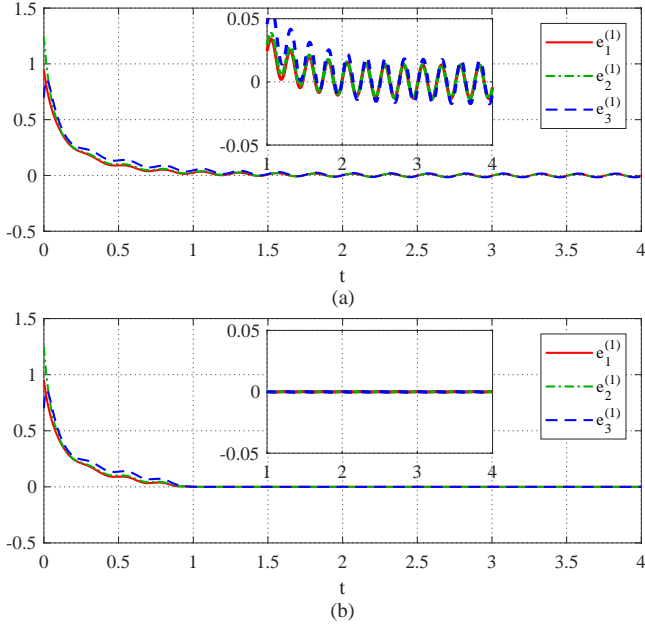


Fig. 3. Evolution of $e_i^{(1)}$ for $i = \overline{1,3}$ with (a) the linear control protocol (7); (b) the locally homogeneous control protocol (17).

Based on the obtained simulation results, we propose to further evaluate the energy consumption and the control performance improvement using the L^2 -norm and the L^∞ -norm of the linear/homogeneous consensus errors and the linear/homogeneous control inputs. They are computed numerically for $x(t) = [x_1(t), \dots, x_\Gamma(t)]^\top \in \mathbb{R}^\Gamma$, $\Gamma \in \mathbb{N}_+$ where $t \in (t_a, t_b)$, let $h = t_i - t_{i-1}$, $i = \overline{1, M}$, $t_0 = t_a$, $t_M = t_b$, by the following formulas

$$\|x(t)\|_{L^2(t_a, t_b)} \approx \sqrt{h \sum_{i=1}^M x^\top(t_i) x(t_i)},$$

$$\|x(t)\|_{L^\infty(t_a, t_b)} \approx \max\{|x_\gamma(t_i)|, \gamma = \overline{1, \Gamma}, i = \overline{1, M}\}.$$

Calculated results are presented in Tables I-III. Firstly, from 0s to 4s, the L^2 -norm of the errors for MAS with homogeneous consensus protocol is about 20.20% lower than for MAS with linear consensus protocol, and this proportion becomes even more pronounced in the time interval of 1s to 4s, reaching 98.62%. Furthermore, the L^∞ -norm of homogeneous errors is about 85 times less than the linear from 1s to 4s, this is mainly attributed to the robustness property of the homogeneous control respect to the bounded perturbation. However, it is clear that to guarantee such an improvement the homogeneous consensus protocol consumes about 20% more energy, and needs about 25% higher magnitude in the steady state.

Therefore, we conclude that even in the presence of bounded uncertainties in leader dynamics, the MAS regulated by the locally homogeneous control protocol (17) with $\mu = -1$ achieves finite-time leader-following consensus. This level of consensus cannot be attained through linear control methods

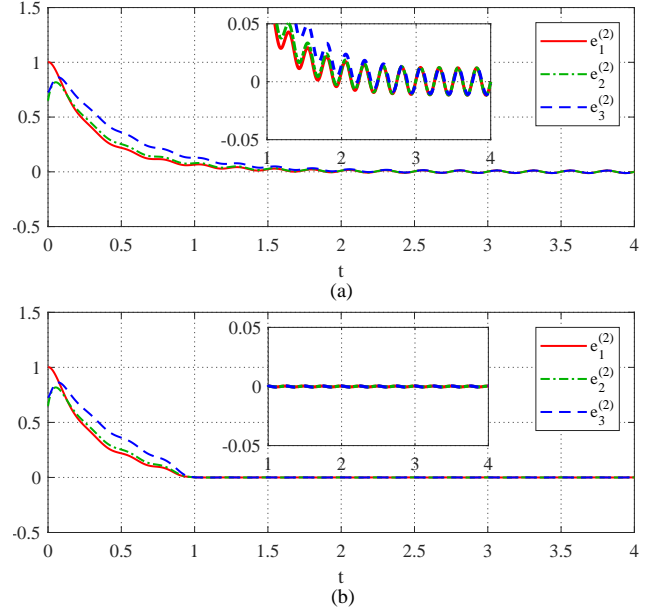


Fig. 4. Evolution of $e_i^{(2)}$ for $i = \overline{1,3}$ with (a) the linear control protocol (7); (b) the locally homogeneous control protocol (17).

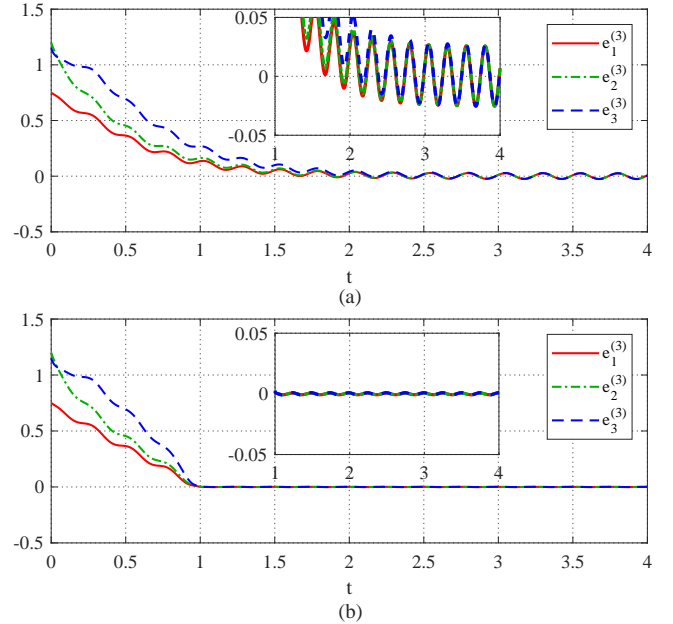


Fig. 5. Evolution of $e_i^{(3)}$ for $i = \overline{1,3}$ with (a) the linear control protocol (7); (b) the locally homogeneous control protocol (17).

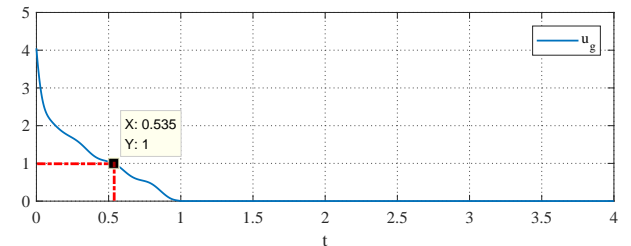


Fig. 6. Evolution of u_g .

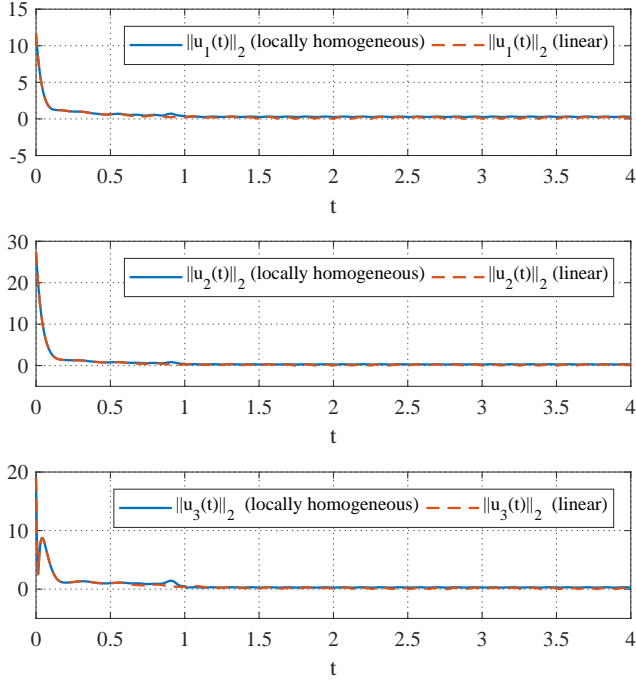


Fig. 7. Evolution of $\|u_i(t)\|_2$ for $i = \overline{1,3}$ with the locally homogeneous control protocol (17) and the classical linear control protocol (7).

TABLE I

THE L^2 -NORM AND L^∞ -NORM OF e_1 AND u_1 .

	$\ e_1\ _{L^2(0,4)}$	$\ e_1\ _{L^2(1,4)}$	$\ e_1\ _{L^\infty(1,4)}$	$\ u_1\ _{L^2(0,4)}$	$\ u_1\ _{L^\infty(1,4)}$
<i>lin</i>	0.6313	0.0987	0.1373	1.5694	0.2753
<i>homo</i>	0.5085	0.0015	0.0019	2.0031	0.4097

TABLE II

THE L^2 -NORM AND L^∞ -NORM OF e_2 AND u_2 .

	$\ e_2\ _{L^2(0,4)}$	$\ e_2\ _{L^2(1,4)}$	$\ e_2\ _{L^\infty(1,4)}$	$\ u_2\ _{L^2(0,4)}$	$\ u_2\ _{L^\infty(1,4)}$
<i>lin</i>	0.7469	0.1108	0.1639	2.3822	0.3070
<i>homo</i>	0.6059	0.0015	0.0019	2.8165	0.4101

TABLE III

THE L^2 -NORM AND L^∞ -NORM OF e_3 AND u_3 .

	$\ e_3\ _{L^2(0,4)}$	$\ e_3\ _{L^2(1,4)}$	$\ e_3\ _{L^\infty(1,4)}$	$\ u_3\ _{L^2(0,4)}$	$\ u_3\ _{L^\infty(1,4)}$
<i>lin</i>	1.0013	0.1670	0.2684	2.1373	0.4495
<i>homo</i>	0.7845	0.0022	0.0029	2.6060	0.4857

alone. However, the implementation of the proposed homogeneous control protocol comes at the cost of increased energy consumption.

VII. CONCLUSION

A leader-following consensus issue for MAS with agents being general linear plants is addressed in this paper. A novel nonlinear control protocol is proposed by upgrading the classical linear consensus control protocol with the concept of generalized homogeneity. With uncertainties in the dynamic of the leader, ISS and finite-time stability of error equations are guaranteed by non-zero homogeneity degrees. By comparing

with the linear consensus protocol, convergence performance is considerably improved although more energy is consumed. This novel control protocol has potential application prospects in various distributed control systems, such as multi-robot systems and distributed power generations. The utilization of this protocol is efficient in increasing the convergence rates and robustness of these systems.

APPENDIX

A. Proof of Lemma 3

Proof: Since $e_i = \xi_i - \xi_0$, $i = \overline{1, N}$, then we have

$$\dot{e}_i = Ae_i + Bu_i + A\xi_0 - \dot{\xi}_0.$$

Combining (7), the error equation becomes

$$\dot{e}_i = Ae_i - BK \sum_{j=1}^N l_{ij} e_j + A\xi_0 - \dot{\xi}_0,$$

which can be compactly written as

$$\dot{e} = f(e, \xi_0, \dot{\xi}_0) = (I_N \otimes A + \mathcal{L} \otimes BK)e + \mathbf{1}_N \otimes (A\xi_0 - \dot{\xi}_0), \quad (18)$$

where \mathcal{L} is defined in (6). At the same time we have

$$\dot{e} = f(e, \mathbf{0}, \mathbf{0}) = (I_N \otimes A + \mathcal{L} \otimes BK)e. \quad (19)$$

Let us consider the Lyapunov function

$$V(e) = e^\top P e, \quad P = I_N \otimes X^{-1} \succ 0.$$

The time derivative of $V(e)$ along the trajectory of the error equation (19) is

$$\dot{V}(e) = e^\top \Pi P e,$$

$$\Pi = I_N \otimes (AX + XA^\top) + \mathcal{L} \otimes BY + (\mathcal{L} \otimes BY)^\top.$$

Since the pair (X, Y) satisfies (5), then error equation (19) is asymptotically stable, and we conclude that the error equation (18) is ISS with respect to $q = (\xi_0, \dot{\xi}_0)$. ■

B. Proof of Theorem 1

Proof: Since

$$\dot{e}_i = A\xi_i + Bu_i - \dot{\xi}_0, \quad i = \overline{1, N}$$

combining (13), we have

$$\dot{e}_i = A_0 e_i - \|e\|_{\bar{\mathbf{d}}}^{\mu+1} BK \mathbf{d}(-\ln \|e\|_{\bar{\mathbf{d}}}) \sum_{j=1}^N l_{ij} e_j + A_0 \xi_0 - \dot{\xi}_0,$$

The latter can be compactly written as

$$\begin{aligned} \dot{e} &= f(e, \xi_0, \dot{\xi}_0) \\ &= I_N \otimes A_0 e + \|e\|_{\bar{\mathbf{d}}}^{\mu+1} \mathcal{L} \otimes BK \mathbf{d}(-\ln \|e\|_{\bar{\mathbf{d}}}) e + \mathbf{1}_N \otimes (A_0 \xi_0 - \dot{\xi}_0), \end{aligned} \quad (20)$$

in this case we can obtain the error equation with $\xi_0 = \dot{\xi}_0 = \mathbf{0}$,

$$\dot{e} = f(e, \mathbf{0}, \mathbf{0}) = I_N \otimes A_0 e + \|e\|_{\bar{\mathbf{d}}}^{\mu+1} \mathcal{L} \otimes BK \mathbf{d}(-\ln \|e\|_{\bar{\mathbf{d}}}) e. \quad (21)$$

The vector field $f(e, \xi_0, \dot{\xi}_0)$ is $\bar{\mathbf{d}}$ -homogeneous of degree $\mu \in [-1, 1/n]$ with respect to $e \rightarrow \bar{\mathbf{d}}(s)e$, $\xi_0 \rightarrow \mathbf{d}(s)\xi_0$, $\dot{\xi}_0 \rightarrow \exp(\mu s)\mathbf{d}(s)\dot{\xi}_0$ for $s \in \mathbb{R}$. The latter implies that $f(e, \mathbf{0}, \mathbf{0})$ is $\bar{\mathbf{d}}$ -homogeneous of degree μ with respect to $e \rightarrow \bar{\mathbf{d}}(s)e$.

Select $\|e\|_{\tilde{\mathbf{a}}}$ as the Lyapunov candidate, whose derivative along equation (21) is

$$\frac{d\|e\|_{\tilde{\mathbf{a}}}}{dt} = \frac{\|e\|_{\tilde{\mathbf{a}}} e^{\top} \tilde{\mathbf{d}}^{\top} (-\ln \|e\|_{\tilde{\mathbf{a}}}) P \tilde{\mathbf{d}} (-\ln \|e\|_{\tilde{\mathbf{a}}}) e}{e^{\top} \tilde{\mathbf{d}}^{\top} (-\ln \|e\|_{\tilde{\mathbf{a}}}) P G_{\tilde{\mathbf{d}}} \tilde{\mathbf{d}} (-\ln \|e\|_{\tilde{\mathbf{a}}}) e}. \quad (22)$$

According to (10), we have $A_0 \mathbf{d}(s) = \exp(\mu s) \mathbf{d}(s) A_0$ and $\mathbf{d}(s) B = \exp(s) B$, equation (22) becomes

$$\frac{d\|e\|_{\tilde{\mathbf{a}}}}{dt} = \frac{\|e\|_{\tilde{\mathbf{a}}}^{\mu+1} e^{\top} \tilde{\mathbf{d}}^{\top} (-\ln \|e\|_{\tilde{\mathbf{a}}}) P \Pi_0 P \tilde{\mathbf{d}} (-\ln \|e\|_{\tilde{\mathbf{a}}}) e}{e^{\top} \tilde{\mathbf{d}}^{\top} (-\ln \|e\|_{\tilde{\mathbf{a}}}) P (I_N \otimes (G_{\tilde{\mathbf{d}}} X + X G_{\tilde{\mathbf{d}}}^{\top})) P \tilde{\mathbf{d}} (-\ln \|e\|_{\tilde{\mathbf{a}}}) e},$$

where $\Pi_0 = I_N \otimes (A_0 X + X A_0^{\top}) + \mathcal{L} \otimes B Y + (\mathcal{L} \otimes B Y)^{\top}$. Since the matrix $G_{\tilde{\mathbf{d}}}$ and the pair (X, Y) satisfy (12), then the error equation (21) is globally asymptotically stable. Moreover, according to Lemma 1, for $\mu \in (-1, 1/n]$, the error equation (20) is ISS with respect to $q = (\xi_0, \dot{\xi}_0)$.

If $\dot{\xi}_0 = f_0 = A \xi_0 + B u_0$, then $A_0 \xi_0 - \dot{\xi}_0 = B(K_0 \xi_0 - u_0)$ and

$$\mathbf{d}(s)(A_0 \xi_0 - \dot{\xi}_0) = \exp(s)(A_0 \xi_0 - \dot{\xi}_0), \quad s \in \mathbb{R}.$$

In this case, the vector field $f(e, \xi_0, \dot{\xi}_0)$ is $\tilde{\mathbf{d}}$ -homogeneous with respect to $e \rightarrow \tilde{\mathbf{d}}(s)e$, $\xi_0 \rightarrow \exp(s(\mu+1))\xi_0$, $\dot{\xi}_0 \rightarrow \exp(s(\mu+1))\dot{\xi}_0$ for $s \in \mathbb{R}$. By letting $\mu = -1$ we derive

$$\frac{d\|e\|_{\tilde{\mathbf{a}}}}{dt} = \frac{-e^{\top} \tilde{\mathbf{d}}^{\top} (-\ln \|e\|_{\tilde{\mathbf{a}}}) P^{\frac{1}{2}} \Pi_1 P^{\frac{1}{2}} \tilde{\mathbf{d}} (-\ln \|e\|_{\tilde{\mathbf{a}}}) e + 2V_1^{\top} V_2}{e^{\top} \tilde{\mathbf{d}}^{\top} (-\ln \|e\|_{\tilde{\mathbf{a}}}) P^{\frac{1}{2}} \Pi_2 P^{\frac{1}{2}} \tilde{\mathbf{d}} (-\ln \|e\|_{\tilde{\mathbf{a}}}) e},$$

where $\Pi_1 = -P^{\frac{1}{2}} \Pi_0 P^{\frac{1}{2}}$, $\Pi_2 = P^{\frac{1}{2}} (I_N \otimes (G_{\tilde{\mathbf{d}}} X + X G_{\tilde{\mathbf{d}}}^{\top})) P^{\frac{1}{2}}$, $V_1 = P^{\frac{1}{2}} \tilde{\mathbf{d}} (-\ln \|e\|_{\tilde{\mathbf{a}}}) e$ and $V_2 = P^{\frac{1}{2}} (\mathbf{1}_N \otimes (A_0 \xi_0 - \dot{\xi}_0))$. Since

$$V_1^{\top} V_1 = e^{\top} \tilde{\mathbf{d}}^{\top} (-\ln \|e\|_{\tilde{\mathbf{a}}}) P \tilde{\mathbf{d}} (-\ln \|e\|_{\tilde{\mathbf{a}}}) e = 1,$$

$$V_2^{\top} V_2 = (\mathbf{1}_N \otimes (A_0 \xi_0 - \dot{\xi}_0))^{\top} P (\mathbf{1}_N \otimes (A_0 \xi_0 - \dot{\xi}_0)),$$

$$V_1^{\top} V_2 \leq \sqrt{V_2^{\top} V_2} \leq U_{\max} \sqrt{N n \lambda_{\max}(X^{-1})},$$

then

$$\frac{d\|e\|_{\tilde{\mathbf{a}}}}{dt} \leq \frac{\lambda_{\min}(\Pi_1) - 2U_{\max} \sqrt{N n \lambda_{\max}(X^{-1})}}{e^{\top} \tilde{\mathbf{d}}^{\top} (-\ln \|e\|_{\tilde{\mathbf{a}}}) P^{\frac{1}{2}} \Pi_2 P^{\frac{1}{2}} \tilde{\mathbf{d}} (-\ln \|e\|_{\tilde{\mathbf{a}}}) e} < 0.$$

Hence, for any $\|A_0 \xi_0 - \dot{\xi}_0\|_{L^{\infty}} \leq U_{\max}$ with (14) satisfied, the canonical homogeneous norm $\|e\|_{\tilde{\mathbf{a}}}$ is a Lyapunov function for the error equation (20) and

$$\frac{d\|e\|_{\tilde{\mathbf{a}}}}{dt} \leq -(U_0 - U_{\max}) \frac{2\sqrt{N n \lambda_{\max}(X^{-1})}}{\lambda_{\max}(\Pi_2)}.$$

Then, the error equation (20) is globally uniformly finite-time stable with the settling-time function estimated by (15). ■

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Min Li received M.E. degree in Control Science and Engineering in 2021 from Huazhong University of Science and Technology, Wuhan, China. He is currently working toward Ph.D. at Inria, Lille, France. His main research interests include distributed control and estimation based on generalized homogeneity.



Andrey Polyakov received Ph.D. in Systems Analysis and Control from the Voronezh State University, Russia in 2005. Till 2010 he was an associate professor with this university. In 2007 and 2008, he was working for CINVESTAV in Mexico City. From 2010 up to 2013 he was a leader researcher of the Institute of the Control Sciences, Russian Academy of Sciences. In 2013 he joined Inria, Lille, France. Andrey Polyakov has co-authored more than 200 papers in peer-reviewed journals and conferences as well as there books: *Generalized Homogeneity in Systems and Control*, *Road Map for Sliding Mode Control Design*, *Attractive Ellipsoids in Robust Control*. His research interests include various aspects of nonlinear control and estimation theory, for example, finite/fixed-time stability, generalized homogeneity, input-to-state stability and Lyapunov methods for both finite dimensional and infinite dimensional systems. He is a member of IFAC TC 2.3. Non-Linear Control Systems.



Gang Zheng received the B.E. and M.E. degrees in Communication and Systems from Wuhan University, China, in 2001 and 2004, respectively, and the Ph.D. degree in automatic control from ENSEA, Cergy-Pontoise, France, in 2006. Since 2007, he has held postdoctoral positions at INRIA Grenoble, at the Laboratoire Jean Kuntzmann, and at ENSEA. He joined INRIA Lille as a permanent researcher from September 2009. His research interests include control and observation of nonlinear systems, and its applications to rigid and soft robotics. Gang Zheng is a senior member of IEEE.