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Hintikka, Free Logician Singular Terms in World Lines Semantics

Matthieu Fontaine

Abstract. The combination of quantifiers with a semantics for epistemic operators in a modal framework is one of the major contributions of Hintikka in intensional logic. Hintikka's starting point is his diagnosis of the failure of existential generalization and the substitution of identicals in terms of referential multiplicity. In this paper, I introduce Hintikka as a free logician. Indeed, Hintikka's first-order epistemic logic is grounded on a logic free of ontological presuppositions with respect to singular terms. It is also a logic free of presuppositions of uniqueness of reference. After having focused on the use of quantifiers and singular terms in Hintikka's epistemic logic, I discuss some consequences from a semanticological perspective, but also from a philosophical one. By arguing against the so-called contingent *a priori* truths defended by Kripke, I conclude with a proposal in favour of Hintikka's non-rigid interpretation of proper names.

Keywords. Modality, World Lines, Identity, Existential Generalization, Singular Terms.

1. Introduction

Hintikka was a free logician. Whereas the interpretation of proper names in intensional contexts has often been discussed in relation to the substitution of identicals, Hintikka has never ceased to insist on the failure of existential generalization. In their virulent attack against Kripke, Hintikka & Sandu [18] show that the thesis of the necessity of identity (between proper names) is based upon a fallacy - it is indeed a case of begging the question - and that it is not entailed by the rigidity of proper names. In this paper, we go further by challenging the Kripkean rigidity of proper names with respect to the thesis of the contingent *a priori*. In the context of the world lines semantics, the point is to show that rigidity commits to the validity of existential generalization, which in turn commits to the acceptance of the contingent *a priori*. Before to set the argument, we return to Hintikka's original preoccupations and we

explain the genesis of the world lines semantics as well as its implications. We finally stick the consequences with respect to the substitution of identicals and existential generalization in first-order intensional logic.

More precisely, Hintikka's first-order epistemic logic¹ is defined with respect to a modal framework, that is, a structure built of a plurality of states of affairs, or possible worlds. The fundamental difficulty consists in providing rules for the use of quantifiers and to clarify the meaning of non-logical terms such as predicates and individual constants. On the one hand, we must explain how quantification is possible across a plurality of possible worlds and, above all, how is it possible to quantify over the same individual in different possible worlds. On the other hand, the meaning of non-logical terms cannot be reduced to their extension in one state of affairs. This yields what Hintikka has called a phenomenon of referential multiplicity. That is, the use of proper names in intensional contexts is not associated with the apparition of a unique individual in the various possible worlds. An immediate consequence is the failure of standard laws of first-order logic. In particular, existential generalization, universal instantiation and the substitution of identicals are not valid. This is characteristic of free logic, in which specific presuppositions with respect to singular terms are dropped. This is also the main topics of this paper, namely to explain how Hintikka has grounded his first-order epistemic logic on free logic.

In order to complete the picture, we will also introduce Hintikka's semantics, sometimes referred to as "world lines semantics", in which quantifier range over individuals that are not reducible to their apparitions in the various possible worlds of the modal framework. We will first define the semantics, then we will discuss its philosophical presuppositions and implications. World lines semantics sheds a new light on the use of quantifiers and our conception of modal individuals. Moreover, it enables us to highlight the fundamental difference between rigid and non-rigid interpretations of proper names in intensional contexts. A rigid designator, as Kripke defines it [19], is a singular term which reference is the same in every possible world. Although rigidity has often been discussed in relation to the substitution of identicals, this rule is not valid in the world lines semantics, no matter whether proper are interpreted rigidly or non-rigidly. Actually, the crucial difference is that a rigid interpretation entails the validity of existential generalization, whereas a non-rigid one does not. Therefore, if we want to take part in the debate on the interpretation of proper names, we must focus on existential generalization: do we accept its validity? In this paper, we will put forward an argument against the validity of existential generalization, hence the rejection of rigidity, based on a criticism of the so-called contingent *a priori* truths, originally advocated by Kripke.

¹In this paper, "epistemic logic" is used in a wide sense, i.e. to refer to logics that deal with epistemic operators K and B , which are the formal translation of intentional verbs "to know" and "to believe", respectively. We also call "epistemic contexts" those contexts opened by the occurrence of such verbs in a sentence.

In the second section, we briefly present Hintikka’s initial argument for allowing quantification in “opaque contexts”. In the third section, we identify the causes of the failure of existential generalization and the substitution of identicals in intensional contexts on the basis of Hintikka’s own diagnosis put forward in terms of referential multiplicity. These two sections deal with well-known problems of first-order intensional logics, but they will allow the readers familiar with Kripke’s approach to avoid confusion with Hintikka’s. Indeed, while the former shrugs off the problem of identity across the worlds, the latter takes it as a fundamental starting point. In the fourth section, we see how he remedies to these difficulties by grounding first-order epistemic logics on free logics. In the fifth section, we define the world lines semantics. We will discuss the presuppositions and implications of this semantics in the sixth section. In the seventh section, we show that no matter how proper names are interpreted, substitution of identicals is doomed to fail in the world lines semantics. In the last section, we conclude with an argument against rigidity, by focusing on existential generalization and the contingent *a priori*.

2. Quantifying in Opaque Contexts

In first-order logic, existential generalization and the substitution of identicals are valid inferences. Existential generalization consists in inferring an existentially quantified statement from a singular statement. For example, if someone accepts (1), then he must also accept (2):

- (1) El Chapo has been arrested for international drug traffic.
- (2) There exists someone who has been arrested for international drug traffic.

Now, given that “El Chapo” refers to the same man that “Joaquín Guzmán”, if someone accepts (1), he must also accept (3), by substitution of identicals:

- (3) Joaquín Guzmán has been arrested for international drug traffic.

These valid inferences are formally represented by (4) and (5), respectively:

- (4) $A(k_1) \models (\exists x)A(x)$
- (5) $A(k_1), (k_1 = k_2) \models A(k_2)$

Problems begin when quantifiers and identity are combined with epistemic operators:

- (6) John believes that El Chapo has been arrested for international drug traffic.

Even if “El Chapo” actually refers to the same man that “Joaquín Guzmán”, (7) does not follow any more:

- (7) John believes that Joaquín Guzmán has been arrested for international drug traffic.

It might even be the case that (6) and (8) are simultaneously true:

(8) John believes that Joaquín Guzmán has not been arrested for international drug traffic.

It is not possible to infer (9) from (6) either:

(9) $(\exists x)$ John Believes that $(x$ has been arrested for international drug traffic)

In order for (9) to be true, there must be a suitable value of substitution for the bind variable x , i.e. a determinate individual of which John believes that he has been arrested for international drug traffic. However, who is this person whom John believes he has been arrested for international drug traffic? El Chapo, that is, Joaquín Guzmán? But John does not believe that Joaquín Guzmán has been arrested for international drug traffic. According to Quine [21, 22, 23], these difficulties arise because of the referential opacity generated by the intentional operator “believes that”. Sentences like (6) and (8) can simultaneously be true because they do not express a genuine relation between John and a determinate individual. That is why existential generalization is not allowed. Eventually, quantifying in opaque contexts does not make sense. Nevertheless, Quine acknowledges that quantifying in epistemic contexts corresponds to widespread uses and he recommends, in these cases, to read transparently the intentional operator “believes that”. That is, in general, intentional verbs generate opaque contexts. But if formulations like (9) are used, they must be read transparently, and the substitution of identicals must be allowed.²

Hintikka also endorses referential opacity of epistemic contexts, but he rejects the transparent reading of intensional quantified statements. According to Hintikka, if quantification in epistemic contexts assumes a transparent reading of intentional verbs, then we derive paradoxical situations. For the sake of clarity, let us introduce a terminological distinction used by Hintikka, which can be illustrated by the following two statements:

(10) John knows *who* the head of the Sinaloa Cartel is.

(11) John knows *that* there is a head of the Sinaloa Cartel.

According to Hintikka’s terminology [10] (p. 131), the first statement is a *knowing-who* construction in which the knowledge of a determinate individual is assumed. Such determination is not assumed by (11), which is true if John knows that there is a head of the Sinaloa Cartel whoever he is. Let K (B) be an epistemic operator such that the intended meaning of K (respectively B) is “the epistemic agent knows (respectively believes) that”. These statements can be translated by the following *de re* and *de dicto* quantified formulas, respectively:

(12) $(\exists x)K_{John}(\text{the head of the Sinaloa Cartel} = x)$

(13) $K_{John}(\exists x)(\text{the head of the Sinaloa Cartel} = x)$

Given that (13) is not concerned with a determinate individual, it does not imply (12). Now, similar distinctions should be applied to statements about

²See Quine [23], p. 133.

individuals in epistemic contexts; i.e. an agent might *know that* El Chapo is El Chapo, without *knowing who* El Chapo is. Yet, if we give a transparent reading to the epistemic operator in a formula like (12), then the following argument runs³:

(14) John knows that the head of the Sinaloa Cartel is El Chapo.

If the epistemic operator is read transparently, then (12) follows by existential generalization. Moreover, (14) can be inferred from (15) and (16) by substitution of identicals:

(15) John knows that the head of the Sinaloa Cartel is the head of the Sinaloa Cartel.

(16) The head of the Sinaloa Cartel = El Chapo.

Eventually, if we assume that quantification is allowed only in transparent contexts, then (12) can be derived from (15) (whether it be through (14) or directly). However, whereas (15) is almost trivial and could hardly be false, (12) is not trivially true. In order to block this paradoxical derivation, Hintikka recommends allowing quantification in opaque contexts.

3. Referential Multiplicity

Hintikka acknowledges the opacity of epistemic contexts in the sense that existential generalization and the substitution of identicals do not hold in the scope of intentional verbs like “to know” or “to believe”. However, unlike Quine, he does not consider that *de re* quantification is senseless or that opacity is due to a failure of reference. According to Hintikka, proper names that occur in the scope of epistemic operators are referential. Notwithstanding, we have to accommodate with their referential multiplicity, which can be grasped within a modal framework with respect to a plurality of possible states of affairs, or possible worlds. A possible world, a scenario or an alternative to the actual world is a (partial) description of a possible state of affairs. A sentence like (6) expresses a genuine relation, but it is a relation between John and (possibly) a multitude of individuals that appear in these possible worlds.⁴ This diagnosis, that constitutes the ground of Hintikka’s first-order epistemic logic in Knowledge and Belief, has its roots in a paper of 1957 entitled “Modality as Referential Multiplicity” [9].

More precisely, the content of a propositional attitude such as “ α knows that p”, where α is an epistemic agent and p a proposition, can be grasped only in relation to a plurality of possible worlds. Now, “ α knows that p” is true if and only if p is true in every world compatible with α ’s knowledge. This makes sense in a modal framework (W, R) that consists of a set of

³For the original argument, see Hintikka [10] (p. 142). For a discussion of this argument, see Sleight [24], and Hintikka [11] for the answer.

⁴Hintikka usually expresses himself in terms of alternatives (to actual world) or scenarios. These notions are interchangeable with the more widespread notion of possible world. Notice that Hintikka’s study in [10] takes place in the context of “model systems” (similarly to modal frameworks) and “model sets” (similarly to possible world).

possible worlds W and an accessibility relation R between these worlds. The semantics of K might require the reflexivity of R , e.g. if factivity of knowledge is assumed. Since it is possible to believe something that is not actually true, this is not the case for the semantics of B . The semantics of epistemic operators is defined as follows:

1. $M, w \models K_\alpha p$ ($B_\alpha p$) iff for every $w' \in W$ such that $wR_K w'$ ($wR_B w'$): $M, w' \models p$.
2. $M, w \models \hat{K}_\alpha p$ ($\hat{B}_\alpha p$) iff for at least one $w' \in W$ such that $wR_K w'$ ($wR_B w'$): $M, w' \models p$.

Notice that $\hat{K}p$ ($\hat{B}_\alpha p$) is the dual of Kp (Bp), i.e. $\hat{K}p$ ($\hat{B}_\alpha p$) intuitively means “it is compatible with ’s knowledge (belief) that p ”. The propositional level is not puzzling at all. Problems begin with the introduction of quantifiers and singular terms. First of all, the meaning of an individual constant (or proper name) cannot be reduced to its reference in the actual world. In a modal framework, to know the meaning of an individual constant is to know its reference in every (relevant) possible world. And its references in every world may not be manifestations of the same individual. Therefore, when we interpret a statement in which a proper name occurs in the scope of an intentional verb, we must take into account its referential multiplicity. For example, let us formally translate (6) by (17):

$$(17) B_{John}S(c)$$

Such a formulation does not assume the uniqueness of reference of the individual constant “ c ”. That is, (17) is true if in every world compatible with John’s beliefs someone called “El Chapo” has been arrested for international drug traffic, no matter the reference of “El Chapo” in each of these worlds. Although John might be able to express his own belief in these terms (i.e. by means of the expression “ $S(x)$ ” and “ c ”), his use of “El Chapo” need not be related to a determinate well-identified individual. Perhaps we wish to express something about a determinate individual, e.g. about the one who is referred to by “El Chapo” in the actual world, as in (12) above. Such a belief can be attributed by means of a quantified formula like (18):

$$(18) (\exists x)((x = c) \wedge B_{John}S(x))$$

In (18), we express something about a determinate individual, the same in every world compatible with John’s belief. That is, (18) is true if the one who is actually called “ c ” is such that in every world compatible with John’s belief he has been arrested for international drug traffic. This can be the case even if “ c ” does not refer to the same person in every such world, or if John himself does not call this person by this name. Uniqueness of reference of “ c ” in every world compatible with John’s belief requires a more complex formulation:

$$(19) (\exists x)((x = c) \wedge B_{John}(S(x) \wedge (x = c)))$$

Now, the application of existential generalization to (17) would yield the following result:

$$(20) (\exists x)B_{John}S(x)$$

However, (20) expresses the attribution of a belief directed towards a determinate individual, the same in every world compatible with John's belief. But this uniqueness is not assumed in (17). More generally, a formula like (17) expresses a genuine relation, but it is a relation between John and possibly a multiplicity of individuals. A quantified formula like (20) expresses a relation between John and a unique individual. That is why existential generalization is not valid in epistemic contexts.

The failure of the substitution of identicals is also due to referential multiplicity. The fact that two proper names have the same reference in the actual world does not imply that they have the same reference in every possible world. Since John may not know or believe that "Joaquín Guzmán" and "El Chapo" actually refer to the same man, their references can be different in the worlds compatible with John's belief. Therefore, we cannot substitute "Joaquín Guzmán" to "El Chapo" in (6) *salva veritate*.

More generally, referential multiplicity has also the two following consequences:

$$(21) K_{\alpha}A(k) \not\models (\exists x)K_{\alpha}A(x)$$

$$(22) K_{\alpha}A(k_1), (k_1 = k_2) \not\models K_{\alpha}A(k_2)$$

The same holds for the belief operator B . Although Quine and Hintikka both recognize the failure of these inferences in epistemic contexts, their respective diagnosis are different. According to Quine, intensional statements do not express genuine relations, while Hintikka considers that they express relations to a multiplicity of individuals.

4. Free Logics

The failure of existential generalization is characteristic of free logics. Actually, existential generalization may fail for two reasons:

1. the individual constant k does not refer to an existent individual, or
2. the individual constant k does not refer to a unique individual in every relevant alternative to the actual world.

The first one is not specific to epistemic logic and relates to logics *free of ontological presuppositions* with respect to singular terms. The second one is linked to referential multiplicity and relates to logics *free of uniqueness presuppositions*. These presuppositions can be dropped by asking which conditions should be fulfilled by an individual constant to count as a suitable value of substitution for bind variables. In order to answer this question, Hintikka's strategy consists in looking for a condition $Q(x)$ such that if an individual k satisfied $Q(x)$, then it would be possible to existentially generalize with respect to k . This means that k would satisfy conditions of normal use of quantifiers. So, we are now looking for a condition $Q(x)$ that would make the following inference valid:

$$(23) A(k), Q(k) \models (\exists x)A(x)$$

where $A(x)$ and $Q(x)$ may contain epistemic operators. Henceforth, the use of quantifiers will be defined as follows:

(QC1) $M, w \models (\exists x)\varphi$ iff $M, w \models Q(k)$ and $M, w \models \varphi[k/x]$ for at least one individual constant k .

(QC2) $M, w \models (\forall x)\varphi$ iff if $M, w \models Q(k)$ then $M, w \models \varphi[k/x]$.

If we focus on the ontological presuppositions, we reject existential generalization because quantifiers range over existent individuals while individual constants may not do. For example, from (24), we cannot infer (25):

(24) $B_{Benedikt}(\text{the Slynx is screaming})$

(25) $(\exists x)B_{Benedikt}(x \text{ is screaming})$

Indeed, Benedikt might believe that the Slynx is screaming even if the Slynx does not exist. Therefore, the inference from (24) to (25) would be allowed only if the reference of “the Slynx” existed. More generally, this kind of presupposition can be made explicit for any k by means of the following premise:

(26) $(\exists x)(x = k)$

Actually, (26) expresses the existence of k .⁵ Insofar we are only concerned with ontological presuppositions, the following inference is valid:

(27) $A(k), (\exists x)(x = k) \models (\exists x)A(x)$

In logics free of ontological presuppositions, the clauses for the quantifiers (QC1) and (QC2) can be expressed by substituting $(\exists x)(x = k)$ to $Q(k)$. From a semantic viewpoint, free logics can be defined in different manners. Positive free logicians usually admit that atomic formulas containing individual constants deprived of existent reference can be true (especially, they usually admit identity statements such as $k = k$). By contrast, negative free logicians do not agree with this possibility (they even usually reject the truth of $k = k$ if k does not exist). Neutral free logicians think that such formulas have no truth-value, and sometimes complement the semantics with a supervaluational one. Nonetheless, from a logical viewpoint, they all invalidate existential generalization. As far as I know Hintikka’s position, he does not really defend any of these free logics in particular, even though he thinks that there are non-existent individuals. However, Hintikka is not a Meinongian since he does not accept simply non-existent objects in his ontology; he considers them as inhabitants of other possible worlds.⁶ His main thesis is that individuals can exist in some worlds, but fail to exist in the other possible worlds, including the actual world.

Although existential generalization may fail for ontological matters, it is worth noting that it does not constitute a criterion of ontological commitment. Hintikka [15] criticizes Parsons [20] in this respect. Existential generalization cannot be a criterion of ontological commitment because it can fail

⁵See Hintikka [10](pp. 123-30) and Hintikka [12] (p. 31).

⁶See Hintikka [15].

for reasons that have nothing to do with the failure of existence. For example, the inference from (28) to (29) is not valid:

(28) $K_{John}(\text{El Chapo has been arrested for international drug traffic})$

(29) $(\exists x)K_{John}(x \text{ has been arrested for international drug traffic})$

This is obviously due the rejection of the presupposition of uniqueness of reference of the proper name “El Chapo” that may refer to different individuals in the worlds compatible with John’s knowledge. Therefore, the condition (26) is not sufficient to state the clauses for the use of quantifiers in epistemic logic. In order to validate the inference from (28) to (29), the following additional premise must be added:

(30) $(\exists x)K_{John}(x = \text{El Chapo})$

This premise makes explicit the presupposition of uniqueness of reference of the proper name “El Chapo”, i.e. it makes explicit the fact that it is associated with a unique individual in every world compatible with John’s knowledge. Actually, (30) expresses nothing less than John knows who El Chapo is. Existential generalization can be applied with respect to an individual constant k only if the epistemic agent knows who k is. More generally, the condition $Q(x)$ can now be formulated like (31), by means of which we formulate the valid inference (32):

(31) $(\exists x)K(x = k)$

(32) $K_\alpha A(k), (\exists x)K_\alpha(x = k) \models (\exists x)K_\alpha A(x)$

If α does not know who k is, then the uniqueness of reference of k is not warranted. We now define (QC1) and (QC2) by substituting $(\exists x)K_\alpha(x = k)$ to $Q(k)$ and by adding the restriction that there is no intensional operator different from K in φ . These clauses that have originally been defined by Hintikka in Knowledge and Belief should be defined for every intensional operator and cannot be applied if several operators are embedded.⁷

Hintikka’s free logic is somehow a generalization of logic free of ontological presuppositions. Indeed, the failure of existential generalization for ontological matters is only a particular case in which an individual constant fails to specify a well-defined individual in the actual world. We will come back to that point later, after the introduction of the world lines semantics. Let us simply mention one of the most striking consequences of Hintikka’s free logic:

(33) $(\forall x)K_\alpha A(x)$

What is expressed by this formula is concerned with all individuals known by α , and no all individuals simpliciter. Indeed, given that bind variables only admit as values of substitution individual constants that satisfy the additional condition expressed in (31), every value of substitution used to interpret the universal quantifier in (33) must be such that it also satisfies

⁷In “Existential Presuppositions and Uniqueness Presuppositions” (reprinted in Hintikka [12] (pp. 112-50) Hintikka generalizes these definitions. Such details are not needed here, especially since the world lines semantics we will introduce thereafter solves this difficulty.

(31). As a consequence, the quantifier ranges over individuals known by α . In Hintikka’s epistemic logic of *Knowledge and Belief*, if we wish to quantify over all individuals, and not only those known by α , we must quantify outside the epistemic operator too, as in the following formula⁸:

$$(34) (\forall x)(\exists y)((y = x) \wedge K_\alpha A(y))$$

If we assume the factivity of K , and therefore the reflexivity of R_K , then $(\forall x)A(x)$ will be implied by (34), but not by (33). Some authors like Castañeda [3] or Sleigh [25] have expressed their scepticism with respect to what they saw as an unjustified restriction of the range of quantifiers (“restricted range feature”). However, according to Hintikka [12] (p. 125), such an understanding of the conditions of use of quantifiers in terms of restricted range is erroneous. That quantifiers range over individuals known by the agent is a consequence of a normal use of quantifiers in epistemic contexts; that is, they assume that bind variables are substituted by individual constants for which the uniqueness of reference is presupposed. And this uniqueness presupposition, made explicit by (31), expresses that the agent knows *who* is referred to by such individual constants. It is not a restriction of the range of quantifiers to individuals actually known or existent. This fact will not surprise those who are familiar with logic free of ontological presuppositions, in which (35) is valid:

$$(35) (\forall x)(x = x)$$

Indeed, if quantifiers range over existent individuals, then every value of substitution satisfy $(x = x)$. This is only a particular case of the following formula that is also generally valid in epistemic logic:

$$(36) (\forall x)K_\alpha(x = x)$$

What are the truth conditions of formulas such as (30) or (31)? When can we assume that “El Chapo”, for example, refers to the apparitions of a unique individual in every alternative compatible with John’s knowledge? If we ask to John who is El Chapo, he might answer “El Chapo is Joaquín Guzmán”. Is this sufficient? Before to attempt to answer this question, we have to understand first how identity works in epistemic contexts. Indeed, we have already emphasized the failure of the substitution of identicals in epistemic contexts. The substitution of identicals can be restored by adding another extra premise of the following form, which states that two terms have the same reference in every accessible alternative:

$$(37) K_\alpha(k_1 = k_2)$$

For example, on the basis of (28) and (38), it is possible to infer (39):

$$(38) K_{John}(\text{El Chapo} = \text{Joaquín Guzmán})$$

$$(39) K_{John}(\text{El Chapo has been arrested for international drug traffic})$$

More generally, the following inference is valid in epistemic logic:

$$(40) K_\alpha(A(k_1), K_\alpha(k_1 = k_2)) \vDash K_\alpha A(k_2)$$

⁸See Hintikka [10] (pp. 155 ff.).

Finally, is it sufficient to know an identity as in (38) to know who El Chapo is? In general, the answer is negative. It might be the case that John knows that El Chapo is Joaquín Guzmán without knowing who El Chapo is or who Joaquín Guzmán is. Although John knows an identity of names, knowing (-who) the referent of one of the names is necessary to infer that John knows who is the other. Clearly, (41) is not valid, but (42) is:

$$(41) K_\alpha(k_1 = k_2) \not\models (\exists x)K_\alpha(x = k_2)$$

$$(42) K_\alpha(k_1 = k_2), \exists x)K_\alpha(x = k_1) \models (\exists x)K_\alpha(x = k_2)$$

What is required for the attribution of knowing-who may vary. Sometimes, to know a name or a description will suffice. Other times, an acquaintance might be necessary. In any case, the agent of the epistemic state must be able to identify the apparitions of a unique individual under different circumstances. Whatever the answer is, quantifying in epistemic contexts assumes that criteria of identification have been given. However, it is not necessary to provide a definitive and exhaustive answer. It is not the purpose of a logic free of uniqueness presuppositions to answer this question, no more than we would expect from a logic free of ontological presuppositions to prescribe what we must admit in our ontology.

5. World Lines Semantics

In *Knowledge and Belief*, quantifiers are given a substitutional interpretation. An individual constant is a suitable value of substitution for a bind variable if it is associated with the apparitions of a unique individual across the relevant possible worlds. In epistemic contexts, this relates the use of quantifiers to the notion of knowing-who. The agent of a propositional attitude must be able to identify the apparitions of the same individual under various circumstances. In other words, quantifying in epistemic contexts makes sense only if criteria of cross-identifications have been given, even though the semantics is not concerned with their precise definition. The incorporation of this presupposition is achieved in Hintikka's world lines semantics in which objects of different worlds are connected along a line in order to form a modal individual:

[I]n contexts involving modal notions, individuals have to be considered as members of several different possible worlds. An individual virtually becomes, for logical purposes, tantamount to the 'world line' [...] connecting its manifestations in these possible worlds.

Hintikka [13] (p. 871)

As stressed by Tulenheimo [27] (pp. vi, 27), world lines have been introduced by Hintikka in two incompatible ways. On the one hand, they have been motivated for epistemological reasons, i.e. as a codification of our means of recognizing an individual in various worlds. On the other hand, they have been given a transcendental interpretation, as a precondition of the use of quantifiers in intensional contexts. The transcendental interpretation assumes that world lines are themselves the modal individuals we talk about in modal

contexts. Even if we recognize that criteria of identification must have been given, we now define the world line semantics by following Tulenheimo's transcendental interpretation. We will come back to the epistemological problem of identification in the next section. Since Hintikka has not been very clear in his own terminology, we also follow Tulenheimo by distinguishing between the notion of "(modal) individual", a world line, and the notion of "(local) object", a world-bounded entity. In the modal framework, an individual is now mathematically represented by a function, which argument is a possible world and which value is an object of that world:

[E]ach individual in the full sense of the word is now essentially a function which picks out from several possible worlds a member of their domains as the 'embodiment' of that individual in this possible world or perhaps rather as the role which that individual plays under a given course of events. Hintikka [14] (p. 412)

A modal framework is now like $F = (W, R, D, Q)$, where W is a set of possible worlds and R is an accessibility relation (or a set of relations) between these worlds. D is a domain of individual functions (or world lines). Q is a function that assigns to each possible world w its own domain Q_w of objects. For every $d \in D$, $d(w) \in Q_w$ if $d(w)$ is defined in w . Individual functions are partial functions, that is, although there must be a w such that $d(w) \in Q_w$ for every d , there may be some w' such that $d(w')$ is not defined. We call q a "manifestation of d in w " if q is such that $d(w) = q$. In order to evaluate formulas, we also need an assignment g defined as a function mapping variables onto D and an interpretation that gives the value of individual constants and predicates:

[VALUE OF A SINGULAR TERM]

- $[t]_{M,w,g} = I_w(t) \in Q_w$ where t is an individual constant,
- $[t]_{M,w,g} = g(w)(t) \in Q_w$ where t is a variable.

[INTERPRETATION OF A PREDICATE] If P is an n -ary predicate, then $P_w^n \subseteq Q_w^n$.

It is worth noting that the interpretation of individual constants is not rigid; that is, it may give different values for different worlds. Moreover, the interpretations of predicates and individual constants are defined for every w with respect to its domain Q_w .⁹ The semantics is now defined with respect to a model M (that consists of a framework F plus an interpretation) and assignment function:

[SEMANTICS]

- (i) $M, w, g \models P(t_1, \dots, P t_n)$ iff $\langle [t_1]_{M,w,g}, \dots, [t_n]_{M,w,g} \rangle \in P_w^n$,
- (ii) $M, w, g \models t_i = t_j$ iff $[t_i]_{M,w,g}, [t_j]_{M,w,g} \in Q_w$ and $[t_i]_{M,w,g} = [t_j]_{M,w,g}$,
- (iii) $M, w, g \models \neg\varphi$ iff $M, w, g \not\models \varphi$,

⁹We might define the semantics otherwise. Whereas Hintikka [15] explicitly admits non-existent objects as inhabitants of other possible worlds, he does not give precisions with respect to the interpretation of proper names and predicates.

- (iv) $M, w, g \models \varphi \vee \psi$ iff $M, w, g \models \varphi$ or $M, w, g \models \psi$,
- (v) $M, w, g \models \varphi \wedge \psi$ iff $M, w, g \models \varphi$ and $M, w, g \models \psi$,
- (vi) $M, w, g \models \varphi \rightarrow \psi$ iff $M, w, g \not\models \varphi$ or $M, w, g \models \psi$,
- (vii) $M, w, g \models (\exists x)\varphi$ iff there is at least one $d \in D$ such that $d(w) \in Q_w$ and $M, w, g[x/d] \models \varphi$,
- (viii) $M, w, g \models (\forall x)\varphi$ iff for every $d \in D$ such that $d(w) \in Q_w$: $M, w, g[x/d] \models \varphi$,
- (ix) $M, w, g \models \hat{K}_\alpha\varphi$ iff for at least one $w' \in W$ such that $wR_K w'$: $M, w', g \models \varphi$.
- (x) $M, w, g \models K_\alpha\varphi$ iff for every $w' \in W$ such that $wR_K w'$: $M, w', g \models \varphi$.

The operators \hat{K} and K in the clauses (ix) and (x) can be substituted by other operators, e.g. \hat{B} and B for belief, or \diamond and \square for possibility and necessity.¹⁰

The quantifiers are now given an objectual interpretation. The result is a more general semantics, without the need for special individual constants that would satisfy additional conditions like (31). Notice to conclude this section that although individuals conceived in terms of world lines assume that criteria of identification have been given, there is ultimately no mark of world lines in the language. There are no explicit criteria of cross-identification in the semantics or the modal framework either. Criteria of cross-identification are only a precondition of the use of quantifiers in intensional languages. That is why, once the world lines have been drawn on the basis of criteria of cross-identification, they do not provide any means to recognize the manifestation of an individual in different possible worlds.

6. Identification

In a modal framework, quantifiers range over world lines mathematically represented by means of individual functions. The transcendental interpretation proposed by Tulenheimo has guided their introduction in the previous section. That is, the use of quantifiers in modal languages presupposes that world lines have been drawn. However, drawing world lines presupposes that criteria of cross-identification have been given. Indeed, individuals are not pre-existing entities upon which we would stipulate possible worlds. They are constructions based on cross-world comparisons from which criteria of identification arise.¹¹ In this respect, although the semantic perspective does not require, and perhaps it is independent of, a study of the criteria of identification, this

¹⁰Since it is not clear how world lines should be drawn for metaphysic modality, Hintikka [13] (p. 418) and [16] is rather sceptical regarding metaphysic modality. Nevertheless, he sometimes makes use of these operators.

¹¹Hintikka [12] (p. 109), defends a semantic neo-Kantianism. Individuals depend on modes of identification and have an objective reality. They are human constructions that make possible our transactions with reality.

does not mean that a complementary epistemological perspective is useless. How to understand what could be such criteria is the issue of this section. Let us insist on the fact that such a task relates to epistemological and cognitive considerations and, even if Hintikka is sometimes confusing (e.g. by talking of modal individuals in terms of “individuating functions” or “world lines of cross-identification”), we do not think of world lines as criteria of identification.

From a purely formal viewpoint, our individual functions are not very different from individual concepts. For example, Aloni [1, 2] defines a similar semantics, by making use of the notion of conceptual cover, which purpose is to account more accurately for the contextuality of the knowing-who constructions. However, our notion of individual should not be confused with the notion of individual concept. Indeed, the world lines are a presupposition of our modal language, they are not language relative. The introduction of new individual concepts does not yield the creation of new individuals concomitant with their intension in the domain D . Moreover, the value of an individual concept in different alternatives need not be considered as manifestations of the same individual. It is worth noting that the world line semantics is not descriptivism either. The system of reference of proper names, and other non-logical symbols, is independent of the system of world lines:

On the possible-worlds model, the referential system has to include two partly independent components. On the one hand, the references of our primitive non-logical constants such as singular terms, predicates, function symbols, etc. in each possible world have to be specified. On the other hand, the imaginary ‘world lines’ (which connect the roles of the same particulars in different worlds) have to be drawn. The relative independence of these two tasks, the interpretation of non-logical constants world by world and the drawing of the world lines (which span several worlds), implies that the corresponding two ingredients of the referential system can to some extent be varied independently. Hintikka & Hintikka [17] (pp. 156-60).

Proper names are not synonymous with definite descriptions. They are not used in association with a world line or a well-identified individual either. Moreover, criteria of identification arise from transworld comparisons and cannot be expressed in the object language. Indeed, in order to express these criteria, we should quantify in intensional contexts, across possible worlds. But such quantification presupposes that the criteria of identification have already been given. Therefore, this is doomed to circularity. There is nothing strange here. According to Hintikka & Sandu [18] (p. 249), this is due to the transcendental nature of the question. We can only presuppose that criteria of identification have been given, we cannot express them. We cannot expect from modal languages and semantics that they provide means of identifying an individual in various possible worlds.

All of this does not prevent criteria of identification from containing a descriptive content, even if identification of individuals relies on much more complex epistemological and cognitive processes. When it comes to identify an individual through time and space, criteria of identification can be grounded on the laws of nature and spatiotemporal continuity. When it comes to identify an individual across various possible worlds, there is a rupture and such continuity cannot be invoked.¹² Several parameters, including belief, knowledge, perception, etc., must be taken into account. These criteria also depend on what counts as a possibility and this can be relative to each agent. Eventually, criteria of identification and then modal individuals are relative to agents and modalities: the individuals that appear in the worlds compatible with α 's perceptions (e.g. seeing) need not coincide with the individuals that appear in the worlds compatible with his belief, or with the individuals that appear in the worlds compatible with the perceptions or beliefs of another agent β .¹³

Hintikka [13] (p. 873) has characterized two methods of cross-identification: public identification and perspectival identification. Public identification is identification by description, centred on the object. Perspectival identification is centred on the subject and his direct cognitive relations to persons and objects he perceives. These two methods of identification yield two distinct sets of individuals, although world lines that arise from two different methods of identification may connect the same objects. That is, no matter the method of identification, the manifestations of two differently identified individuals can coincide in some possible worlds. By contrast, on some occasions, an agent will be able to identify someone according to a method of identification, but not according to the other. For example, John might have read something about El Chapo in a newspaper. He would know a lot of things about him, e.g. that he has been the head of the Sinaloa Cartel, he has been one of the richest people of Mexico, he has been arrested in 2014, he escaped in 2015, was arrested again in 2016, etc. But John might not be able to recognize him if he met him in the street. John would be able to publically, but not perspectivally, identify El Chapo. Inversely, John might have met El Chapo close to his property in Mazatlán. Hi might be able to perspectivally identify him without knowing that he is in fact the man called "El Chapo" or that he is in fact the man corresponding to the public description he has in mind for El Chapo. In that case, there are two different world lines. They cross the same possible worlds and connect the same persons, but they are grounded on two different methods of identification.

The two methods of identification give raise to different sets of world lines. That is why Hintikka introduces two pairs of quantifiers. The quantifiers $(\exists x)$ and $(\forall x)$ range over publically identified individuals whereas (Ex) and (Ax) range over perspectivally identified individuals. Thus, we can distinguish between the two following formulations:

¹²See Hintikka [14] (p. 411) .

¹³See Hintikka [11] (p. 416).

(43) $(\exists x)K_{Jean}(x = \text{El Chapo})$

(44) $(Ex)K_{Jean}(x = \text{El Chapo})$

These two different formulations grasp the distinction between an indirect knowledge construction and a direct knowledge attribution, respectively (45) and (46)¹⁴:

(45) John knows who is El Chapo.

(46) John knows El Chapo.

The modal framework now contains two different domains of individual functions, namely the domain of perspectively identified individuals (D_I) and the domain of publically identified individuals (D_P). Then, the clauses (vii) and (viii) of the semantics are directly adapted by stipulating that (Ex) and (Ax) range over D_I and $(\exists x)$ and $(\forall x)$ range over D_P . Moreover, given that the individuals of D_I are subject centred individuals, we must assign to each relevant agent a domain of perspectively identified individuals.¹⁵ Although the clauses for quantifiers do not contain additional conditions such as (31), ontology is still relative to the agents, their belief, knowledge, perception, etc.¹⁶ That is why, the quantifier of a formula such as $(Ax)K_\alpha A(x)$ should range only on individuals perspectively identified by α , even if this is not entailed by the semantics itself but rather our understanding of the content of the domain of individuals.

Finally, the fallibility of the criteria of identification, i.e. that on some occasions an agent does not succeed in drawing a world line through a possible world, must be emphasized. Whereas it might be argued that an object is always identical to itself, two world lines may split or merge. That is, two individuals may manifest themselves in an identical way in one world, but in a different way in another. If we agree with that, then the necessity of identity that has been defended by several authors is not valid¹⁷:

(47) $\not\models (\forall x)(\forall y)((x = y) \rightarrow \Box(x = y))$

Hintikka has sometimes rejected splitting and merging, especially in epistemic contexts, but he has always acknowledged it was not necessary.¹⁸ For belief, it is clear it should be accepted. For example, John might have correct beliefs about El Chapo Guzmán and identify him in a certain way. He might also have correct beliefs about Joaquín Guzmán and identify him in another way. Although they are one and the same person, there may be two different world lines that coincide in the actual world but that split in the worlds compatible with John's belief. Overall, the moral consists here in recognizing the problem of identification across the worlds and to stick the consequences with respect

¹⁴See Hintikka [13] (p. 875).

¹⁵See Tulenheimo [27] for more details (in a slightly different semantics).

¹⁶Hintikka [1970b, 420] compares his proposal with Quine's indeterminacy of ontology.

¹⁷See e.g. Fitting and Mendelsohn [8] (pp. 146 ff.): in a standard framework (without world lines), $(\forall x)(\forall y)((x = y) \rightarrow \Box(x = y))$ only expresses an identity between objects (which cross-world identity is conceived as being unproblematic).

¹⁸See [12] (p. 140).

to the behaviour of individuals in a modal framework. We will come back to identity between proper names in the next section.

To sum up, when we quantify in intensional contexts, and epistemic contexts in particular, we quantify over individuals that appear in different possible worlds. Modal individuals are represented in a modal framework by world lines. In order to draw world lines across the worlds, criteria of cross-identification must have been given. The study of these criteria does not belong to semantics or logic, but to epistemological and cognitive considerations. We must postulate the existence of such criteria by means of which objects of different worlds are connected. In order to define the semantics, we do not need to determine these criteria; we can be content with only presupposing them. Nonetheless, the study of these criteria is the basis of a deeper understanding of individuals and their behaviour in modal contexts. Although the two tasks (semantic and epistemology) are different, they are complementary with respect to our understanding of intensionality.

7. Naming and Identity

Hintikka has probably been one of the most virulent opponents to rigid interpretation of proper names. Indeed, rigidity consists in connecting objects of different possible worlds through the system of references, whereas connecting objects of different possible worlds is the job of the system of identification, i.e. the system assumed by the drawing of world lines. “Rigidity”, if the word made sense, would be relative to a certain class of possible worlds, a modality and a given system of identification. A name can be associated with a known reference, but rigidity can never be a primitive notion. According to Hintikka, Kripkean rigidity is actually senseless, as well as Kripkean metaphysical modality, and “[t]here is no such concept to be accounted for”.¹⁹ Nevertheless, there is room to ask what we would be committed to if we interpreted proper names rigidly, as referring directly to world lines rather than world-bounded objects.²⁰ First, what about identity between proper names?

To begin, if proper names rigidly referred to world lines, one proper name could not refer to the apparitions of individuals identified differently. Indeed, an interpretation (rigid or not) provides only one value. For example, “Joaquín Guzmán” could not simultaneously refer to a publically identified individual and a perspectively identified individual. By contrast, according to Hintikka’s non-rigid interpretation, the value of a proper name can be an object that is the manifestation of two different individuals. Next, non-rigid interpretation of proper names is sometimes invoked against the so-called *a*

¹⁹Hintikka [16] (p. 458).

²⁰That is, if the value of proper names is defined like $[t]_{M,w,g} = I(k) \in D$, where $I(k)$ is an element of D , the same for every possible world. Notice that formulas such that (31) only express a local uniqueness presupposition, i.e. $(\exists x)K_\alpha(x = k)$ tells us that k refers to the apparitions of the same entity in the accessible worlds, according to a given modality, a particular agent, and a system of identification, but it does not say anything about the other possible worlds.

posteriori necessities, and a fortiori the necessity of identity between proper names, which have been defended by Kripke in another context.²¹ Actually, if individuals are conceived as world lines, identity is contingent no matter how proper names are interpreted.

Very briefly, in a Kripkean framework (i.e. where objects travel across the worlds), if “Hesperus” and “Phosphorus” are rigid designators for Venus, then they refer to Venus in every possible world. Identity between Hesperus and Phosphorus is therefore necessary, although it has been discovered *a posteriori*. By contrast, for the same reasons as the substitution of identicals, necessity of identity is invalidated by the non-rigid interpretation of proper names:

$$(48) \not\models (k_1 = k_2) \rightarrow \Box(k_1 = k_2)$$

$$(49) \not\models (k_1 = k_2) \rightarrow K_\alpha(k_1 = k_2)$$

Now, within the world line semantics, rigid interpretation or explicit uniqueness presuppositions do not warrant the validity of (48), (49) or the substitution of identicals. Indeed, the following formulas are perfectly compatible:

$$(50) \text{El Chapo Guzmán} = \text{Joaquín Guzmán}$$

$$(51) (\exists x)((\text{El Chapo Guzmán} = x) \wedge B_{\text{John}}(\text{El Chapo Guzmán} = x))$$

$$(52) (\exists x)((\text{Joaquín Guzmán} = x) \wedge B_{\text{John}}(\text{Joaquín Guzmán} = x))$$

$$(53) \neg B_{\text{John}}(\text{El Chapo Guzmán} = \text{Joaquín Guzmán})$$

The individuals that satisfy (51) and (52) can be different, even if their manifestations coincide in the actual world. Their respective world lines might split in the worlds compatible with John’s beliefs. John might have met Joaquín Guzmán, having called him “Joaquín Guzmán”, and knowing who he is. He might also have met El Chapo under other circumstances, having called him “El Chapo”, and knowing who he is. But this would not entail that John realizes that they are one and the same person. More generally, even if the names are rigidly interpreted, this does not preclude that the two individuals d_1 and d_2 referred to by “Joaquín Guzmán” and “El Chapo” be such that $d_1(w) = d_2(w)$, but that $d_1(w') \neq d_2(w')$. Inversely, (50) and the following formulas are compatible:

$$(54) \neg(\text{El Chapo Guzmán} = \text{Arturo Guzmán})$$

$$(55) (\exists x)((\text{Arturo Guzmán} = x) \wedge B_{\text{John}}(\text{Arturo Guzmán} = x))$$

$$(56) B_{\text{John}}(\text{El Chapo Guzmán} = \text{Arturo Guzmán})$$

Again, John might have met El Chapo Guzmán and Arturo Guzmán. He might also know who they are. He might even have called them “Señor Guzmán”. A long time after, while he would read a paper about the arrest of El Chapo, being ignorant of the death of Arturo Guzmán, he would

²¹It is worth noting that here, and in the next section, we do not directly criticize Kripke. Indeed, Kripke justifies his theses in a framework in which proper names rigidly refer to objects that travel across the worlds. Moreover, Kripke limits his proposal to metaphysic modality. Here, we only propose a comparison between the two interpretations of proper names within the world line semantics.

believe that it is about Arturo Guzmán. Given the truth of (7) and (56), (57) would also be true, despite the truth of (54):

(57) B_{John} (Arturo Guzmán has been arrested for international drug traffic)

Even if proper names are interpreted rigidly, the fact that $d_1(w) = d_2(w)$ does not preclude that $d_1(w') \neq d_2(w')$. Two proper names can be associated with different individuals which manifestations coincide in the worlds compatible with John's beliefs.

In general, in order to validate the substitution of identicals or the necessity of identity, we should presuppose that world lines never split or merge. However, this would amount to presuppose the necessity of identity, and would end in a circular account.²² Without further hypothesis with respect to the behaviour of modal individuals, the substitution of identicals and necessary identities are not valid, no matter the interpretation of proper names of the presuppositions held in their respect. There is nothing strange here, since the system of reference cannot handle semantic relations between possible worlds. Cross-world identity is to be understood in terms of world lines, and if it is agreed that world lines can merge or split, e.g. by acknowledging that the criteria of cross-identification they assume are not infallible, then identity is contingent.

8. Existential Generalization and the Contingent A Priori

The consequences of rigidity are better understood in relation to existential generalization. Leaving aside ontological considerations, rigidity commits to the validity of existential generalization.²³ Non-rigidity does not. When applied to cases of conceptual knowledge, existential generalization allows the derivation of a knowledge concerned with a determinate individual, i.e. a knowing-who. In this section, we argue that existential generalization eventually yields the illusion of contingent *a priori*. Therefore, if we reject the possibility of knowing contingent *a priori* truths, we must reject the validity of existential generalization, and finally the rigidity of proper names. As in the precedent section, it is worth noting that although the contingent *a priori* have been defended by Kripke, we do not directly target him. The aim is rather to show that, within the world line semantics, particular examples of contingent a priori follow from rigidity, and that if we reject them, we must also reject rigidity.

Kripke's [19] (p. 56) example is the following: the length referred to by "one metre" is fixed by stipulating an identity between the length of one

²²See Hintikka & Sandu [18] (pp. 269 ff.).

²³Indeed, rigid designators satisfy by definition the condition $Q(x)$ of section 4. Therefore, if we assume that $I(k) \in D$ and that $I(k) \in P_w$, then there is a d such that $M, w, g \models P(x)$. So, if $M, w, g \models (\exists x)P(x)$, whether $P(x)$ contains intensional operators or not. Actually, we should also leave aside the complications involved by negation, but this does not impact our argument in this section.

metre and the length of a particular stick, the standard metre rod (say the length of S at time t), at a determinate instant. Since we are fixing that length by stipulation, we know that “the standard metre rod is one metre long” is true *a priori*. We know that without measuring S at time t. Once the length of “one metre” has been fixed, “one metre” rigidly refers to that length. The length of S might change, not the length of one metre. Under other circumstances, the length of S at time t might have been different. Therefore, that S is one metre long at time t is a contingent fact. Even though we know *a priori* that “the standard metre rod is one metre” is true, this statement is contingent. According to Dummett [6] (p. 124), this is the sign that something goes wrong with rigidity. Indeed, *a priori* knowledge does not depend on the facts of a particular world. So, what is known *a priori* can be known without experience, no matter the world in which we are. At the same time, a contingent statement is not true in every possible world. Therefore, there would possible worlds in which it is possible to know *a priori* something that is false. This is unacceptable. Dummett argues that when we know *a priori* the truth of “the standard metre rod is one metre”, we do not know the same that when we know the contingent fact expressed by “the standard metre rod is one metre”. The question can now be asked in these terms:

What, then, is the fact, whose contingency we express by saying that the standard metre rod might have been shorter than 1 metre, but which is not expressed when we say a priori that it is 1 metre long or that it is the length which it has? Dummett [6] (p. 124)

The notion of ingredient sense, introduced by Dummett [7] (p. 48) might help us to understand that question.²⁴ The ingredient sense of a sentence is defined as the contribution of a sentence to the truth-value of more complex sentences of which it is part. Therefore, what must be explained is how it can be the case that a sentence constructed by the operator “necessarily” and “the length of S at t is one metre” in its scope is false, whereas a sentence constructed by the epistemic operator “it is known a priori” and the same sentence “the length of S at t is one metre” in its scope can be true. In our semantics, the question can be asked with respect to the object of knowledge. That is, how knowing the truth of the same sentence, namely “the length of S at t is one metre”, could involve different kind of knowledge: the knowledge of a contingent fact or an *a priori* knowledge. In our semantics, the object of knowledge can be clearly disambiguated. In the first case, knowledge is concerned with a determinate individual, namely the stick S, and its length. The length of S at time t is contingent. But this length cannot be known *a priori* since we should know which (as in the knowing-who constructions) this length is. This knowledge can be expressed as follows:

$$(58) (\exists x)(x = \text{the length of S at t}) \wedge K_{\alpha}(x = \text{one metre})$$

Direct knowledge (perspectival) can be expressed as follows:

²⁴For a more detailed study of the application of the notion of ingredient sense to the rigidity thesis, see Stanley [26], in particular pp. 574 ff.

(59) $(Ex)(x = \text{the length of S at } t) \wedge K_\alpha(x = \text{one metre})$

This knowledge cannot be *a priori* either, since it assumes that is acquainted with an object of a particular world. In the second case, knowledge is concerned with an identity that holds between two lengths, whatever they are. No knowledge of the world, of what is actually the metre or the standard metre rod, their respective lengths, is required. By stipulation, their respective lengths are identical. In order to know that, it is sufficient to know that the length of S at time t is identical with the length of one metre. This can be expressed as follows:

(60) $K_\alpha(\text{the length of S at } t = \text{one metre})$

This knowledge can be *a priori*, but it does not suffice to know which the particular length at stake is.²⁵ In order to know the contingent fact expressed by “the length of S at t = one metre”, we should know which the length of S at t is or which the length of one metre is. We are now in a situation to answer precisely to Dummett’s question. When we know the contingent fact expressed by “the standard metre rod is one metre long”, we know which the length of the standard metre rod (or one metre) is. When we know *a priori* that “the standard metre rod is one metre”, we only know that two concepts hold for the same length. The first case is concerned with a determinate individual (knowing-who), the second is not. And there is no relation of implication between these different ways of knowing that “the standard metre rod is one metre” is true.

The difference between (58) and (60) can be understood in terms of Donnellan’s [5] terminology, which distinguishes between the referential use and the attributive use of an expression such as “the length of S at time t”. The referential use is concerned with a determinate individual, like in (58) and (59), the attributive use is not, like in (60).²⁶ Indeed, the attributive use consists in introducing “one metre” as a name for the length of S at time t, whatever this length is. By stipulation, we thus know *a priori* that “the length of S at t = one metre” is true. However, we do not know which this length is. By contrast, the referential use consists in making use of the description with the intention to refer to a determinate length.²⁷ However, in that case, an empirical verification might be required. A mistake might also occur, i.e. the standard metre rod might not have been the intended length. Such a mistake is not possible in the case of the attributive use.²⁸

For the same reasons as (41) is not valid, the conceptual knowledge expressed in (60) does not imply knowing which the length of S at time t is. This knowledge is not concerned with a determinate individual. However,

²⁵See (41) section 3.

²⁶Hintikka [11] (p. 49) proposes this modal formulation of Donnellan’s uses.

²⁷For example, we would have used “the length of S at t” with the intention to refer to one ten-millionth of the length of a half-meridian.

²⁸A similar objection can be found in Casullo [4]. However, Casullo does not propose a formal analysis in the context of an epistemic logic.

if “one metre” is rigidly interpreted, then (61) can be inferred by existential generalization:

$$(61) (\exists x)K_\alpha(\text{the length of } S \text{ at } t = x)$$

Because of existential generalization, we would infer which the length of S at time t is on the basis of a conceptual knowledge of the identity between the length of S at t and one metre. That is, by stipulation, we would acquire knowledge about the world, the knowledge of a contingent fact. Like Hintikka who objected to Quine the inference of a knowing-who from a trivial knowledge in transparent contexts, we reject rigidity because of this inference of the knowledge of a contingent fact from the *a priori* knowledge of a conceptual relation. Indeed, if individual constants are not interpreted rigidly, then the inference from (60) to (61) is blocked. By answering Dummett’s question, we reject the contingent *a priori* nature of the example “the standard metre rod is one metre”. This does not suffice to conclude to the general impossibility of contingent *a priori*, but it is sufficient to reject the validity of existential generalization, and rigidity of proper names.

9. Conclusion

Ontological presuppositions are not the only concern of free logicians. In Hintikka’s work, this is also reflected by his rejection of uniqueness presuppositions. This was the ground of deep insights in epistemic logic and formal approach to epistemology, e.g. with the definition of the knowing-who and identification processes presupposed by quantification in epistemic contexts. This has led to the world lines semantics, in which Hintikka’s free logic takes another turn with the relative independence of the system of reference and the system of individuals. Eventually, being a free logician is overall a matter of rejection of existential generalization. And Hintikka has never ceased to motivate the rejection of this classical inference rule.

By focusing on existential generalization, we have shed the light on another questionable thesis of the New Theory of Reference, namely the thesis of the contingent *a priori*. To be honest, the New Theory of Reference was not concerned with world lines, but the move is the same: the myth of rigidity commits to the validity of existential generalization, and further to the illusion of contingent *a priori*. Here is one of the key questions with respect to the interpretation of proper names in the world lines semantics. Are we prepared to endorse contingent *a priori* truths? If we do not, existential generalization must be rejected. If existential generalization must be rejected, then rigidity too.

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