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Dialogues and the Games of Logic: A Philosophical Perspective

Volume 3

Logic of Knowledge. Theory and
Applications

Edited by

Cristina Barés Gómez, Sébastien Magnier
and Francisco J. Salguero

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DIALOGUES AND THE GAMES OF LOGIC: A PHILOSOPHICAL PERSPECTIVE

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For more than two decades now the scientific community has seen a growing number of important results in the emerging field centered on the interactions between game theory and logic. We feel that it is time for the philosopher to assess in which ways those results bear on the fundamental questions of the philosophy of logic. In particular, what insight have we gained on such central notions such as meaning, truth, validity, proof, proposition, form and formality, logicity. We would thus like to open a series of books that we conceive as a gathering place where philosophical ideas concerning those topics can be expressed, scrutinized and confronted. We therefore invite submissions related to all aspects of the philosophical consequences of the interactions between games and logic. More specific topics include (but are not limited to): game-theoretical and game semantics, dialogical logic, ludics, Ehrenfeucht-Fraïssé games, game-theoretical notions of logical form, game-theoretical notion of proposition, logical games between semantics and proof theory, games and modal logic.

Submission Details. Authors are strongly encouraged to use \LaTeX , using the usual ‘article’ document class. Acceptable formats are (in order of preference): \LaTeX (.tex), general text formats (.rtf and .txt), OpenOffice (.odt), Word (.doc). For proposals contact Shahid Rahman with the subject matter “GDL-proposal”.

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Volume 1, 2011. Redmond, J. and Fontaine, M. *How to Play Dialogues. An Introduction to Dialogical Logic.*

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FOREWORD

This volume includes revised versions of presentations at the *International Symposium - Argumentation in Intensional contexts: Knowledge, Belief, Dialogue* organised by the UMR 8163 "Savoirs, Textes, Langage", Université Lille 3, France; and the "Grupo de Lógica, Lenguaje e Información" (HUM-609), Universidad de Sevilla, Spain. We are grateful to these two institutions for their supports. The congress was held in Sevilla, Facultad de Filosofía, May 20th-21th 2010.

Through this foreword, we thank all of the authors included here for taking part in the Workshop, for getting their papers in on time, and for revising them according to referees' recommendations. We owe a large debt of gratitude to all blind reviewers who collaborated to improve the papers. We are particularly grateful for their helpful comments and various suggestions on previous version of each paper. Without those silent contributions such a publication would not be possible. We also address our thanks to Nicolas Clerbout for his hard-work in the final edition of the book.

This book would not have been possible without the constant support and encouragement of the heads of both research groups: Pr. Dr. Shahid Rahman and Pr. Dr. Ángel Nepomuceno. Thanks for all.

The Editors.

Table of Contents

Dialogues and the Games of Logic: A Philosophical Perspective	i
Foreword	iii
<i>Preface</i>	
Shahid RAHMAN	ix
Part I Intensional Contexts: Interaction and Models	1
<i>A Logical Framework for Individual Permissions on Public Announcements</i>	
Hans Van DITMARSCH & Pablo SEBAN	3
Logic of permitted and obligatory private announcements	5
Use of the Formalism in Special Cases	22
Conclusion and perspectives	26
<i>Adaptive Logics and the Solution of Abductive Problems Triggered by Anomaly</i>	
Laura LEONIDES	29
Preliminaries	30
Adaptive Logics	31
LATA ^r : An adaptive logic for anomaly–triggered abductive prob-	
lems	38
Conclusions and further work	43
<i>A Dynamic-Epistemic Approach to Abductive Reasoning</i>	
Fernando SOLER-TOSCANO & Fernando Raymundo VELÁZQUEZ-	
QUESADA	47
Abductive reasoning	47
From a classical to a dynamic epistemic approach	48
Different problems for different kinds of agents	55
Knowledge and belief	64
A semantic model	66
Summary and future work	75

Logic of Time Division on Intervals of Finite Size

Tero TULENHEIMO	79
Logic of time division	83
Characterization of the expressive power of TD	89
Properties of TD	93
Comparing TD with first-order logic	97
Extending TD : and next	101
Conclusion	109

Part II Argumentation in Intensional Contexts 115*The Dialogic of Actually*

Nicolas CLERBOUT	117
Preliminaries	118
Standard Modal Dialogical Logic (for S5)	123
Dialogical semantics for \mathcal{L}_A	130
Concluding remarks and further developments	134
Appendix. Some precisions about the proofs	136

A Dialogical Approach of Iterated Belief Revision

Virginie FIUTEK	141
The logic \mathcal{L}_{PLS^*}	143
Dialogical logic	146
Validity of axioms of \mathcal{L}_{PLS^*}	152
Conclusion and further work	155

*PAC vs. DEMAL**A Dialogical Reconstruction of Public Announcement Logic with Common Knowledge*

Sébastien MAGNIER	159
Public Announcement Logic	161
DEMAL: A Dialogical Reconstruction of PAC	164
Examples	172
Conclusion and further developments	177

Part III Philosophical and Linguistic Perspectives 181*Belief Conditional in the Ugaritic Language*

Cristina BARÉS GÓMEZ	183
The Ugaritic language and our methodology of research	183
The conditional structure in the Ugaritic language	188

Formalizing the conditional structure.	193
Conclusion	199
<i>To Be Is To Be Chosen</i>	
<i>A Dialogical Understanding of Ontological Commitment</i>	
Matthieu FONTAINE & Juan REDMOND	203
Explicit Existence in Free Logics	205
Explicit Choice in Natural Deduction	207
Do it in Dialogic! The Introduction Rule	208
Positive, Negative and Neuter Dialogical Free Logics	212
Supervaluation and Superdialogues	214
To be Chosen and To be Symbolic	217
Conclusion	220
<i>Logical Dialogues from Middle Ages</i>	
Aude POPEK	223
Burley's rules of <i>Positio</i>	225
Burley's division of <i>Positio</i>	232
<i>Positio</i> of disjunction	236
Disjunction of <i>Positio</i>	239
Conclusion	242
Appendix	242

To Be Is To Be Chosen

A Dialogical Understanding of Ontological Commitment

MATTHIEU FONTAINE* & JUAN REDMOND†

ABSTRACT. One of the most important contributions of dialogical logic is its understanding of quantification by means of the notion of choice. Adopting a critical position towards the introduction of an existence predicate in free logics, we will outline a new understanding of existence within the dialogical framework: *To be is to be chosen*.

Indeed, against the standard view in free logic, the aim of this paper is to grasp the meaning of quantification by taking into account the relationship between the choice of a constant and the resulting utterance.¹

1 Introduction

The philosophical reflection on non-existence is an issue that has been tackled at the very start of philosophy and constitutes since the publication of Russell's "On Denoting" (Russell, 1905) one of the most thorny debates in analytical philosophy. Russell's choice was clever: he was keen to show how the new instruments of logic might offer an original approach to some venerable metaphysical and epistemological problems such as the problem of judgment of existence. Actually, the paper gave Russell the opportunity to stress the main contribution of the "modern logic": the notion of "quantifier" that could now bring an unexpected twist to Kant's remark that "existence is not a real predicate".

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¹The following paper is part of an unpublished paper written some years ago in French (Fontaine et al., 2009) and is the original source of some central ideas of recent publications and researches on the dialogical approach to the logic of nonexistents and fictionality. See also (Rahman, 2001) and (Rahman et al., 1997).

Roughly, Russell's definite description theory consists in conceiving grammatical names not as genuine names but rather as disguised definite descriptions. Replacing names by definite descriptions, Russell solves the problem of negative existential statements² by means of quantification and empty predicates. Let "Pegasus" be synonymous for "the two-winged horse", all that we can say is that it is not the case that there exists something satisfying the description. Consequently, though "Holmes does not exist" is true, all of the following statements are false:

1. Holmes is a detective.
2. Holmes is not a detective.
3. Holmes is a character imagined by Conan Doyle.
4. Holmes is Holmes.

The uniform falsity of these statements is linked to the lack of reference of the name "Holmes". However, as stressed by Leonard (Leonard, 1956), although quantifiers enable us to deal with empty predicates, Russell's solution (and more generally classical logic) is grounded on a tacit existential presupposition with respect to singular terms.³ This presupposition is expressed in a formal language by the two following principles:

$\forall x\varphi \rightarrow \varphi[x/k_i]$ (Specification)

$\varphi[x/k_i] \rightarrow \exists x\varphi$ (Particularization)

Intuitively, the former means that if every object in the domain satisfies φ , then k_i satisfies φ . Conversely, the latter means that if k_i satisfies φ , then there exists an object in the domain that satisfies φ . For example, if Holmes satisfies the property of being a detective, then there exists someone who is detective. In other words, these principles establish that every singular term k must refer to something lying in the range of ontologically loaded quantifiers.

²How to deny the existence to something that does not exist? Do we have first to assume a reference of which we say it does not exist? If there is not reference at all, what would be denying the existence about?

³"The modern logic has made explicit the logic of general existence, but it has retained a tacit presupposition of singular existence." (Leonard, 1956).

Different kinds of free logics have challenged these principles. Semantically, the idea is that some singular terms might be without reference or have a reference lying beyond the range of ontologically loaded quantifiers. In this way, the principles of particularization and specification are not valid anymore. Karel Lambert called those logics *free logics*, for logics free of ontological commitment with respect to singular terms (Lambert, 1960). Lambert distinguishes between negative, positive and neuter free logics, the semantics of which will be sketched thereafter.

2 Explicit Existence in Free Logics

Negative free logic is directly inspired by Russell's theory. Indeed, although singular terms are not conceived as disguised definite descriptions, if a name does not refer to something existent it does not refer at all, it is an empty name. And every statement in which such an empty name occurs is false, excepted from negations of atomic formulas. Syntactically, the language is the same as the one for classical logic to which is added an existence predicate ($E!$) and an identity symbol. We now define briefly this semantics:

[D1] A model M for negative free logic is a tuple $\langle D, I \rangle$ where D is the domain of quantification and I a partial interpretation function (that is, a function which is not defined for every singular term: it is not defined for empty names).

[D2] Interpretation for negative free logic:

1. For every singular term k , either $I(k)$ belongs to D or $I(k)$ is not defined.
2. For every n -ary predicate P , $I(P)$ is an n -tuple of members of D .
3. Every member of D has a name.

[D3] Truth in model for negative free logic:⁴

- (i) $V_M(Pk_1, \dots, k_n) = 1$ if and only if $I(k_1), \dots, I(k_n)$ are defined and $\langle I(k_1), \dots, I(k_n) \rangle \in I(P)$.

⁴We give only the relevant truth conditions. The truth conditions for the other connectives $\neg, \vee, \wedge, \rightarrow, \leftrightarrow$ are defined as usual. It will be the same for the other free logics defined below.

- (ii) $V_M(k_i = k_j) = 1$ iff $I(k_i)$ and $I(k_j)$ are defined and $I(k_i)$ is the same as $I(k_j)$.
- (iii) $V_M(E!k_i) = 1$ iff $I(k_i)$ is defined.
- (iv) $V_M(\forall x\varphi) = 1$ iff $V_M(\varphi[x/k_i]) = 1$ for every k_i such that $I(k_i)$ is defined.
- (v) $V_M(\exists x\varphi) = 1$ iff $V_M(\varphi[x/k_i]) = 1$ for at least one k_i such that $I(k_i)$ is defined.

As a direct consequence, if the interpretation of an individual constant k_i is not defined, then every atomic formula containing this k_i will be false (but its negation is true). Thus, we can easily check how specification and particularization are not valid anymore. If instead formulas containing empty names are said to be undetermined, then we get a neuter free logic.⁵

Unlike negative free logic, positive free logic is such that identity statements such as $k_i = k_i$ are always true. Indeed, positive free logicians think that identity formulas such as the mentioned above express an analytic truth, and this is so whether k_i refers or not. They are thus committed to understand this kind of name not as empty but rather as referring to nonexistent entity. To distinguish between existent and nonexistent individuals, the domain of the discourse is divided in an innerdomain D_I (containing the existents) and an outerdomain D_O (containing the nonexistents). We now briefly define the relevant features of their semantics:

[D4] Interpretation for positive free logic:

1. For every singular term k , $I(k)$ belongs to $D_I \cup D_O$.
2. For every n-ary predicate P , $I(P)$ is an n-tuple of members of $D_I \cup D_O$.
3. Every member of $D_I \cup D_O$ has a name.

[D5] Truth in a model for positive free logic:

⁵Notice that indeterminacy is to be conceived either as strong or weak. In the weak version, indeterminacy of atomic sentences does not affect the clauses for the other connectives. In a strong version, it is contagious to the whole formula. How to deal with indeterminacy is in fact another matter, and can be done by means of supervaluation method (see Section 6).

- (i) $V_M(Pk_1, \dots, k_n) = 1$ iff $\langle I(k_1), \dots, I(k_n) \rangle \in I(P)$.
- (ii) $V_M(k_i = k_j) = 1$ iff $I(k_i)$ is the same as $I(k_j)$.
- (iii) $V_M(E!k_i) = 1$ iff $I(k_i) \in D_I$.
- (iv) $V_M(\forall x\varphi) = 1$ iff $V_M(\varphi[x/k_i]) = 1$ for every k_i such that $I(k_i) \in D_I$.
- (v) $V_M(\exists x\varphi) = 1$ iff $V_M(\varphi[x/k_i]) = 1$ for at least one k_i such that $I(k_i) \in D_I$.

These different ways of dealing with existence in logic have the merit of rendering explicit the ontological commitment of (some) singular terms in the object language. However, the criticism against these logics is precisely grounded in this use of the existence predicate. Indeed, we think that such an understanding of existence is linked to a specific omission namely the relation between the act of choosing a singular term and the resulting assertion. In what follows, we will display a way of dealing with ontological commitment that gives a central role to the notion of choice. This can be tracked back to the natural deduction system of Jaśkowski.

3 Explicit Choice in Natural Deduction

The first attempt to implement choices in natural deduction is to be found in Jaśkowski (Jaśkowski, 1934). This natural deduction system was built in order to be applied to inclusive logics — that is logics with possibly empty domain. The problem is the following: if the domain is possibly empty, how to be sure that an individual constant is available to instantiate a quantifier? The solution of Jaśkowski relied on the idea that, while instantiating a quantifier in a proof-process, we have to make explicit the supposition of a singular term. More precisely, Jaśkowski built a system that allows to introduce in the object language two new metalinguistic symbols that express two different kinds of suppositions:

1. Any supposition of a formula is prefixed by the symbol \mathcal{F} .
2. Any supposition of a singular term is prefixed by the symbol \mathcal{C} .

In this paper, we are concerned with the second kind of supposition: the supposition of a singular term. According to Jaśkowski, universal instantiation can be applied only if the supposition of a singular term has been previously introduced in the object language. This idea suggests the following formulation of tableau rules for free logic:

δ -type rules		γ -type rules	
k_i is new		k_i is arbitrary	
$T \exists x\varphi$	$F \forall x\varphi$	$T \forall x\varphi$	$F \exists x\varphi$
$T Ck_i$	$T Ck_i$	$T Ck_i$	$T Ck_i$
	
$T \varphi[x/k_i]$	$F \varphi[x/k_i]$	$T \varphi[x/k_i]$	$F \varphi[x/k_i]$
		(the utterance of $T \varphi[x/k_i]$ assumes the occurrence of $T Ck_i$)	

As a consequence, the validity of specification assumes the occurrence of the supposition of the adequate singular term. Thus, the following formulation of specification does not hold: $\forall x\varphi \rightarrow \varphi[x/k_i]$:

1. $F \forall x\varphi \rightarrow \varphi[x/k_i]$
2. $T \forall x\varphi$
3. $F \varphi[x/k_i]$

The tableau-proof ends without closing at step 3 since $T Ck_i$ does not occur in the branch. In fact, Jaśkowski did not really condition the use of quantifiers in the style of nowadays free-logic, he rather added as a rule that a singular term supposition can always be assumed and thus obtained classical logic. Jaśkowski acknowledges that the act of choice should be rendered explicit as a separated distinguishable step in a proof. Unfortunately, he implemented this idea by the introduction of a new type of formula. The resulting system at the very end is not that different of introducing an existence predicate: replace Ck_i with $E!k_i$ (where $E!$ is the existence predicate) and a standard free logic with an existence predicate obtains. As we will discuss below, we endorse the idea that the meaning of quantifiers involves two steps, however one of these steps is an action, the choice, and the other is a proposition, the result of the act of choice.

4 Do it in Dialogic! The Introduction Rule

From the dialogical point of view free logic is the logic where the choices of the proponent are conditioned by previous choices of the opponent. Existence is thus herewith understood as the result of an interaction. Moreover, we will not assume that the domain of discourse is a static collection of data given by a model, but that this domain result from a dialogue conceived as a process of construction. It is precisely these

dynamic features of dialogical logic that allows avoiding the use of the existence predicate.

For an arbitrary φ , the particle rules that establish the *local semantics*⁶ for quantifiers are the following:

Utterance	Attack	Defense
$X - ! - \exists x\varphi$	$Y - ? \exists$	$X - ! - \varphi[x/k_i]$
$X - ! - \forall x\varphi$	$Y - ? \forall/k_i$	$X - ! - \varphi[x/k_i]$

The local meaning of quantifiers displayed above can be seen as consisting in a double move, involving an action, the choice of a singular term, and the utterance of the formula that results after that choice. Indeed, in the case of the existential quantifier, the defender chooses an individual constant and then utters the resulting (instantiated) formula. In the case of the universal quantifier, the challenger chooses an individual constant and asks the other player to instantiate the formula. Stated in such a way, there is no restriction with respect to the availability of individual constants used by both of the players. Notice that strategically, the opponent will usually introduce new individual constants and the proponent will try to use individual constants previously given by the opponent. But this has nothing to do with the local rules that are stated independently from the player at stake.

If the standard rules for classical and intuitionistic logic are assumed we obtain, as shown below, the validity of the principles of specification and particularization:

	O			P	
				$Ak_1 \rightarrow \exists xAx$	0
1	Ak_1	0		$\exists xAx$	2
3	$? \exists$	2		Ak_1	4

⁶We will not present here the whole rules for dialogical logic, furthermore, we will ignore in our examples the issue on “ranks” since they are not really relevant for our issue. For a presentation of the dialogical rules, see N. Clerbout’s contribution in the present volume. See also Fontaine and Redmond (2008), Keiff (2009) and Rahman and Keiff (2005).

	O			P	
				$\forall xAx \rightarrow Ak_1$	0
1	$\forall xAx$	0		Ak_1	4
3	Ak_1		1	$? \forall / k_1$	2

In order to obtain a dialogical free logic, we must add restrictions on the application of these rules. In other words, the particle rules for quantifiers remain the same - that is the local meaning is not different from the one of classical and intuitionistic logic, however the so-called *global* meaning displayed by the structural rules will be different. More precisely, dialogical free logic requires a structural rule, stated below and called *Introduction Rule*, that establishes that the (proponent's) availability of singular term is a function of the choices made by the opponent during the dialogue.⁷ By contrast with the logics we mentioned before, ontological commitment of quantifiers is captured by means of the application of a particular rule that fixes the meaning of those quantifiers in the context of a game (for short, by means of the application of a particular structural rule). We can therefore delete from our language the existence predicate "*E!*". In the dialogical versions of free logic presented in the present paper we will assume on the one hand that quantifiers are ontologically loaded (i.e. they only range over existents) and on the other hand that individual constants are not necessarily so.⁸

In order to formulate the *Introduction Rule*, we define first the notion of introduction (of singular terms):

[D6] Introduction - A singular term k_i played by X is said to be introduced if and only if:

1. X utters the formula $\varphi[x/k_i]$ while defending a formula of the form $\exists x\varphi$ and k_i has not been used to attack or to defend a quantifier before,
2. X chooses k_i by means of the move $\langle ? \forall / k_i \rangle$ while challenging a formula of the form $\forall x\varphi$ and k_i has not been used to attack or to defend a quantifier before.

⁷See Rahman (2001).

⁸In Rahman's original paper (Rahman, 2001), two different kinds of quantifiers were used, namely ontologically loaded and non-ontologically loaded quantifiers. In the present paper, we will only assume ontologically loaded quantifiers.

[RS-6] Introduction Rule: Only O is allowed to introduce singular terms.

Intuitively, this rule means that availability of a singular term is determined through the choices of the opponent. If we allow us to use a model-theoretic vocabulary, we could describe this process as one in which the opponent was building a counter-model by his concessions and by the same way building the domain of existent entities. Thus, ontological commitment is now understood with respect to the introduction rule: only the individual constants introduced by the opponent are ontologically loaded. As a consequence of [RS-6], the principles of specification and particularization are not valid anymore, as it is shown in the following dialogues:

	O			P	
				$Ak_1 \rightarrow \exists xAx$	0
1	Ak_1	0		$\exists xAx$	2
3	$? \exists$	2			

Explanation: Although Ak_1 has been conceded by O (move 1), P cannot defend himself when O challenges the existential quantifier (move 3) by means of k_1 because O did not introduce it. Therefore, O wins the dialogue and the particularization is not valid.

	O			P	
				$\forall xAx \rightarrow Ak_1$	0
1	$\forall xAx$	0			

Explanation: P cannot attack the universal quantifier stated by O (move 1) since no individual constant has been introduced so far, that is no individual constant is available for P.

These two dialogues show how specification and particularization do not hold in dialogical free logic. In fact, these principles are not valid in positive and negative free logics, but in neuter free logic things are a bit different. Indeed, in a neuter free logic, they are undetermined. The aim of the next section is to develop a more fine-grained system able to introduce further distinctions between different approaches to free logic.

5 Positive, Negative and Neuter Dialogical Free Logics

In the context of positive and negative free logics *without identity*, we just have to apply the standard dialogical rules enlarged by the *Introduction Rule* [RS-6]. However, the main difference between negative and positive free logics rests on the understanding of identity. As we mentioned above, while in negative free logic identity is conceived as synthetic and dependent on the existence of the entity at stake, in positive free logic, identity is thought to be analytic and independent on the existence. Thus, in order to differentiate these logics, we will implement different rules for identity in our dialogical framework.

The following rule involving identity yields *dialogical positive free logic*:⁹

[RS-FL₊] At the start of every dialogue for positive free logic, O concedes $k_i = k_i$, and P can ask to substitute k_i by any k_j played in the dialogue.

With this rule, the opponent is forced to concede that identity holds for every individual constant occurring in a dialogue. Consequently, the proponent is free to state without any justification that $k_j = k_j$ for every individual constant that occurs in a dialogue, including those that have not been introduced.

In *dialogical negative free logic*, identity holds only for the existents, that is for the individual constants that have been previously introduced. In order to implement this notion, we add the following rule:

[RS-FL₋] At the start of every dialogue for negative free logic, O concedes $\forall x(x = x)$.

Notice that because of the Introduction Rule, the proponent will be allowed to challenge the conceded universal only if the opponent has previously introduced an individual constant.

What else do we need to develop a neuter free logic? In dialogical neuter free logic, every formula where an individual constant occurs the ontological commitment of which has not been specified will be

⁹We also assume a rule for substitution of individual constants. Briefly, this rule says that if a player X concedes $k_i = k_j$ and further concedes $\varphi[x/k_i]$, the player Y is allowed to ask him to substitute k_j with k_i in the formula φ and X will have to defend himself by uttering $\varphi[x/k_j]$. It follows from the formal rule for atomic formulae that while applying this rule, the role of X will be played by the opponent and the role of Y will be played by the proponent.

undetermined. Stated in more dialogical words, if certain individual constants that occur in a dialogue have been used but not introduced, then there is no winner. More precisely, a dialogue for dialogical neuter free logic will be played with the [RS-FL₋] rule of dialogical negative free logic¹⁰ but with a different winning rule, namely the following one:¹¹

[RS-FL_N] Winning Rule - X wins iff both of the following conditions are fulfilled:

1. It is Y's turn to play and he cannot or the dialogue contains two occurrences of the same atomic formula respectively labeled X and Y.
2. Every individual constant k_j that occurs in the dialogue has (eventually) been introduced or is identical with a constant k_j that has (eventually) been introduced.

In all other cases, nobody wins (and the thesis is claimed to be undetermined).

Applying this rule, particularization is undetermined (and not valid). The dialogue is the same as the one displayed above (see section 4) though the result is different: there is no win, neither for the proponent nor for the opponent. Indeed, despite the impossibility for P to answer the O's challenge on the existential quantifier (move 3), O does not win because the constant k_1 occurs and this constant has not been introduced in the whole dialogue.

All these dialogues show how to understand ontological commitment by means of ontologically loaded quantifiers. This framework displays how to deal with nonexistents in a dialogue and what are the consequences of taking them in account. This is a logic without the tacit presupposition decried by Leonard (Leonard, 1956). Moreover unlike the traditional free logics, it is not subject to the usual criticism against the use of an existence predicate. As a conceptual result, we achieve a new dynamic understanding of ontological commitment grasped in

¹⁰That is we deal with identity as in dialogical negative free logic.

¹¹We rely here on a strong notion of indeterminacy according to which as soon as an indeterminacy occurs in a formula it contaminates the whole formula. A weaker notion of indeterminacy allows to win a dialogue despite the indeterminacy of a particular individual constant insofar as there is no move of the opponent where an atomic formula has been uttered and this particular constant occurs.

terms of these choices and their interaction essential for the meaning of quantification. In a dialogue, *to be is to be chosen*.

Let us come back once more to Jaśkowski's calculus and let us compare it with the dialogical approach. In the dialogical framework, as stressed above, the local meaning of a quantifier consists in a double move: choosing a singular term and uttering the resulting formula. This double act inherent to the meaning of quantification matches Jaśkowski's double step: the assumption of a singular term and the instantiation of the quantifier. However, contrary to Jaśkowski's approach, the dialogical approach does not convert this action into a formula: quantification is, according to our view, the interaction of actions and propositions.

6 Supervaluation and Superdialogues

Van Fraassen (van Fraassen, 1966) and Bencivenga (Bencivenga, 1986) contested on one hand the metaphysical commitment of positive and negative free logics and on the other hand the truth-values gaps that produce the indeterminacy of neuter free logic. According to Bencivenga, logic should be neutral in relation to metaphysical assumptions on the basic underlying semantics. It is crucial to point that in the dialogical context the above approach to neuter free logic can be seen as jeopardizing the whole project of a dialogical logic. Indeed, originally, dialogical logic was developed with the aim to provide a dialogue-definite notion of propositions (namely to be part of dialogue where necessarily exactly one of both players wins) even for those propositions that might be indeterminate from the truth-value point of view: consider the dialogical approach to intuitionistic logic, where the opponent wins and the dialogue is definite, though from the truth-value point of view third-excluded is neither true nor false. A solution would be to change the notion of non-validity for neuter free logics in such a way that a formula is not valid iff (1) the opponent wins and (2) every individual constant k_i that occurs in the dialogue has (eventually) been introduced or is identical with a constant k_j that has (eventually) been introduced. If (2) is not the case, we say that the formula is indeterminate (however it is *dialogue-defined*). However, it is not that clear if the solution is really satisfying, this leads us to the approach we call below *superdialogues*.

In the following we will describe Bencivenga's own proposal that extends van Fraassen's method of supervaluation to first-order logic (van Fraassen, 1966). Let us start with van Fraassen's theory of supervalua-

tions where models consist in *partial valuations*, *classical extensions* and *supervaluations*:

- *Partial valuations* allow truth-value gaps such as in some three-valued logics. In other words an assignment of truth to some propositions, falsity to the others and no value to the rest.
- Consider all ways of extending a partial valuation to a total valuation by arbitrarily assigning values (consistent with the truth conditions - if a given proposition is arbitrarily made true, then any disjunction containing this proposition will be made true too) to those propositions that the partial function yields a lack of value. Call these the *classical extensions* of the original partial valuation.
- A supervaluation is defined as follows: A proposition is true according to the supervaluation if it is true in all classical extensions; false according to the supervaluation if it is false in all classical extensions and has no (super-)value if it takes different values in different classical extensions.

Logical consequence: A proposition is a logical consequence of another proposition if there is not partial valuation every classical extension of which makes all the premises true and the conclusion false.

Validity: A proposition is valid according to supervaluation if there is no partial valuation the classical extension of which renders that proposition false.

Take for instance the principle of non-contradiction. We assume that the proposition φ might be undetermined — such as a first-order formula in which an empty name occurs — and a partial valuation as described in the matrix below by the lines 2 to 4. The third line is the case with undetermined truth value:

1	φ	$\neg\varphi$	$\neg(\varphi \wedge \neg\varphi)$
2	1	0	1
3	#	#	#
4	0	1	1
5	1	0	1
6	0	1	1

The first of the two possible classical extensions (line 5) assign true to φ and accordingly false to its negation while the second extension (line 6) assigns the dual values. According to both of these extensions $\neg(\varphi \wedge \neg\varphi)$ is true, and thus so is its supervalue. Non-contradiction is thus valid according to the supervaluation.

One way to read supervaluation is to read classical extension as *if-valuation*. In other words, some propositions are neither true nor false, e.g. those involving fictional terms, but we do — by means of classical extensions — as if they were true or false. However, the framework is still incomplete. Supervaluations are only efficient at the propositional level. What about quantifiers? What about $k_i = k_i$? Is it true, false or does it lack a value? If we consider equalities to be atomic propositions then identity will lack a supervalue. Bencivenga accomplished the task to extend the supervaluational framework to first-order logic by combining outerdomains with supervaluations.

Bencivenga (Bencivenga, 1986) does not consider at all classical extensions. Instead he considers all ways of assigning a denotation to the empty names, and the total valuations which will result from that.

Let us assume that k_i in $\varphi[x/k_i]$ is empty, and thus the interpretation function undetermined. The idea is to evaluate this formula with respect to an hypothetical context in which k_i would have a reference, whatever it is.

To be able to that, we consider a partial model U that consists in a domain D and an interpretation I which is partial with respect to singular terms. Then, we add an extension of this model, U' , which ascribes to the partial interpretation I an extension I' . This superinterpretation I' assigns an arbitrary value to initially empty singular terms. Let us take for example $\neg(Pk \wedge \neg Pk)$: If $I(k) = \#$, then $V(Pk) = \#$ and $V(\neg(Pk \wedge \neg Pk)) = \#$. To validate the principle of non-contradiction, we complete I with an I' in order to assign an arbitrary value to k . Now, whatever would k denote, it would render $\neg(Pk \wedge \neg Pk)$ true. The same holds for $k = k$ because whatever k might be, it would be identical with itself.

However, a problem remains: the principles of particularization and specification would now be validated. Bencivenga recommends the following solution: If it is relevant to take into account the extension for the expression lacking interpretation in the initial model, it is not the same for the one which have value in the initial model. Indeed, if for example $\forall x\varphi$ has a value in the initial model, this value has the priority over the extension. Then, we could have that $\forall x\varphi$ is true in the

initial model and $\varphi[x/k]$ as undetermined. Moreover, by applying I' to k , we could have that $\varphi[x/k]$ is false. Hence there is a counter-model to specification.

Another more simple explanation given by Woodruff (1971), and also endorsed by Read (1995), consists in taking into account a *free extension* rather than superinterpretation. In this framework, the extension I' of the partial interpretation I ranges over an outerdomain and applies to individual constants whereas the initial interpretation I ranges over the innerdomain and applies to quantifiers. Therefore, the interpretation of k might lay beyond the range of the quantifiers and that is why specification is no longer valid.

The implementation of the Bencivenga-Woodruff strategy in the dialogical approach is pretty straightforward and amounts to the concatenation of a neuter free logic with a positive free logic. We might call this concatenation a *superdialogue*. Superdialogues result from applying the following rules:

[RS_{SD}-1] Every dialogue is first played by applying the rules that define dialogical neuter free logic.

[RS_{SD}-2] If a dialogue finished with [RS_{SD}-1] has been won neither by O nor by P, the dialogue starts again with the rules for positive free logic.

Hence, specification and particularization are still non-valid whereas a formula like $\exists xAx \rightarrow (\exists xAx \vee Ak_1)$ is valid — notice that the latter is not valid in (dialogical) neuter free logic.

7 To be Chosen and To be Symbolic

Dialogical free logics as presented previously are too restrictive. Because of the introduction rule, every existentially quantified formula turns out to be non-valid. In a model theoretical framework this can be seen to be a consequence of the possibility of having empty domains.¹² Therefore, each time the proponent has to choose first, he will lose. As a result, the equivalence between formulas like $\exists x(Ax \rightarrow \forall xAx)$ and $\exists x\neg Ax \vee \forall xAx$ does not hold anymore. This suggests that in a dialogue, even if the domain is possibly empty, a kind of move, that we will call *symbolic*, should be allowed.¹³ Such moves should indicate that the

¹²Such kind of free logics are *inclusive*.

¹³Although we use “symbolic” in a somewhat different manner, this notion has its roots in the philosophy of Hugh MacColl. For more details on this point, see Rahman and Redmond (2008), Section 1.2.1, pp. 27 sq.

decision about the ontological status of the constants involved have not been taken yet — recall that the ontological status is determined by the opponent's choices. Indeed, sometimes it is due to the interaction that takes place during a dialogue that new information arises about the ontological status of a given individual constant. In order to describe a dialogical system where the status of some individual constants at some stage of the dialogue might have an undetermined ontological status the rule [RS-6] has to be replaced with the rule [RS-FL_S] formulated as follows:

[RS-FL_S] While defending an existential quantifier or challenging an universal quantifier P must use totally new individual constants or individual constants previously introduced by O.

We also need the following definitions:

[D7] An individual constant is said to be totally new if and only if it does not occur in the initial thesis, or if it has not been previously introduced.

[D8] We call symbolic an individual constant totally new or an individual constant occurring in the initial thesis.

[D9] An individual constant is said to be ontologically loaded if and only if it has been introduced by O.

The principles of particularization and specification are still non-valid though the explanation is somewhat different to the one of the dialogues for free logic described in the precedent sections:

	O			P	
				$Ak_1 \rightarrow \exists xAx$	0
1	Ak_1	0		$\exists xAx$	2
3	$?\exists$	2			

Explanation: According to [RS-FL_S], P must defend the challenge on 3 either with a totally new constant or with a constant previously introduced by O. Yet k_1 , the individual constant required to answer the O-challenge occurs in the initial thesis and has not been introduced. Therefore, P cannot defend the existential quantifier and O wins: Particularization is not valid.

As already mentioned, in relation to non-validity of particularization and universal specification the dialogues that make use of symbolic individual constants do not differ from those of free logic described in the previous sections. By contrast, the case of existentially quantified formulas such as $\exists x(Ax \rightarrow \forall xAx)$ makes a difference. As it is shown in the dialogue below, a symbolic constant without determined ontological status occurs at the beginning of the dialogue. However, in the course of the dialogue the opponent introduces another individual constant which is used by the proponent to update the defense of the quantifier with this very same individual constant that is ontologically loaded since it has been chosen by the opponent:

	O			P	
				$\exists x(Ax \rightarrow \forall xAx)$	0
1	? \exists	0		$Ak_1 \rightarrow \forall xAx$	2
3	Ak_1	2		$\forall xAx$	4
5	? \forall/k_2	4		Ak_2	8
				$Ak_2 \rightarrow \forall xAx$	6
7	Ak_2	6			

Explanation: P defends the existential quantifier (move 2) with a totally new individual constant. There begin the symbolic use of the individual constant k_1 and the dialogue continues without having determined its ontological status. Later, O introduces an individual constant, namely k_2 , to challenge the universal quantifier (move 5). So, P updates his defense of the existential quantifier with this ontologically loaded individual constant (move 6). The dialogue continues with the usual rules and P wins (move 8).

The interesting point of the dialogical framework is that we can continue the dialogue despite a while of indeterminacy. One essential feature of this dynamic free logic is indeed the possibility for the proponent to update his defense making profit of new pieces of information conceded by the opponent's action. This symbolic process followed by an update gives the opportunity to the proponent to develop a winning strategy for existentially quantified formulas in a free logic context. Notice that by contrast with supervaluation, the idea is not to give an arbitrary interpretation to singular terms, but to allow for the use of singular term, the ontological status of which is not determined.

This dialogic is now properly called dynamic. Indeed, in addition to a meaning based on actions, the ontological status of individual constants can change during the game with respect to the opponent's choices. In a model-theoretic formulation, we might say that the dialogue begins with a symbolic domain, where the division between existents and nonexistents is not settled — we can think of it also as the union of the innerdomain and outerdomain. After the introduction of k_2 (move 5), the game is played with an (ontologically loaded) domain which contains at least one individual.

In dialogic, the application of the introduction rule can also be seen as producing a change of the context of argumentation. Indeed, after the introduction of k_2 , we play with ontologically loaded quantifiers.

8 Conclusion

To conclude, notice that we never understand quantifiers from a model-theoretic viewpoint, as if it should be objectual or substitutional, nor from a proof-theoretic viewpoint. Indeed, we propose here an alternative semantics proper to game-theoretic approaches in which meaning is given in a relation between an act of choice and a resulting assertion. Moreover, the individual constants are not interpreted with respect to a model. Therefore, it does not matter whether or not the ontological commitment has to be understood with respect to a name or with respect to an object. The ontological commitment here makes sense only in a game; through a sequence of challenges and defenses.

So far so good, but some conceptual problems remain: How to define the conditions for the winning strategies and the notion of validity in this dialogical free logic? Indeed, we face an alternative.

(1) On the one hand, we could understand the meaning of the quantifiers dynamically. That is the ontological commitment of the quantifiers would be understood in such a way that it could change with respect to specific moves in the dialogue. From this perspective, at the start of the dialogue, the quantifiers are not ontologically loaded, that is their ontological commitment can be said to be symbolic. More precisely, the quantifiers are said to be symbolic if and only if the decisive constant to win the dialogue is symbolic. In the dialogue displayed above (see section 7) the last individual constant played, namely k_2 , is ontologically loaded, that is, it has been introduced by the opponent. Thus the existential quantifier is eventually considered to be ontologically loaded, though at the start this quantifier was symbolic.

By contrast, if the decisive individual constants are still symbolic at the end of the dialogue, we say that the ontological commitment of the quantifiers is still symbolic and that the quantifiers are understood as not being ontologically loaded. Let us see this in an example:

	O		P	
			$\exists x(Ax \rightarrow Ax)$	0
1	? \exists	0	$Ak_1 \rightarrow Ak_1$	2
3	Ak_1	2	Ak_1	4

Explanation: The individual constant k_1 played by P in order to defend the existential quantifier (move 2) has not been introduced. However, he wins the dialogue anyway by stating Ak_1 (move 4), an atomic formula previously conceded by O (move 3). According to our definitions the existential quantifier is not ontologically loaded.

(2) On the other hand, we might think that the symbolic status qualifies individual constants and not quantifiers. Furthermore we might think that the quantifiers must always *eventually* be ontologically loaded. That is, the proponent can start using symbolic constants but at the end of the game, the constants occurring in the very last atomic formulas must be ontologically loaded. In this case the winning rule must be formulated as follows: The proponent wins only if he wins by using an ontologically loaded constants. In this case, the proponent would lose $\exists x(Ax \rightarrow Ax)$, but he would have won the game for $\exists x(Ax \rightarrow \forall xAx)$.

Finally, let us point out that the dialogical free logic presented here is not properly speaking a logic of fictionality. Indeed, though this logic is able to express the difference between symbolic and ontologically loaded constants, it does not furnish any tool to determinate if an individual is a fiction or not. It simply shows how to develop a dialogical argument if the thesis involves fictions. In order to extend this free logic to the study of fictionality, some further technical developments are required (as the introduction of a fictionality operator for example) as well as a more precise ontological definition of what a fictional entity is.¹⁴

¹⁴In order to develop the first main ideas for a dialogic of fictionality Fontaine and Rahman (2010) implement A.-L. Thomasson's (1999) approach to fictional entities — conceived as ontologically dependent entities — and combine it with a fictionality operator. Rahman and Tulenheimo (2010) developed a model-theoretical semantics that combines the notion of ontological dependence with fictionality operators.

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