

# Mathematical Modeling of Pulsed Electric Field Cardiac Ablation

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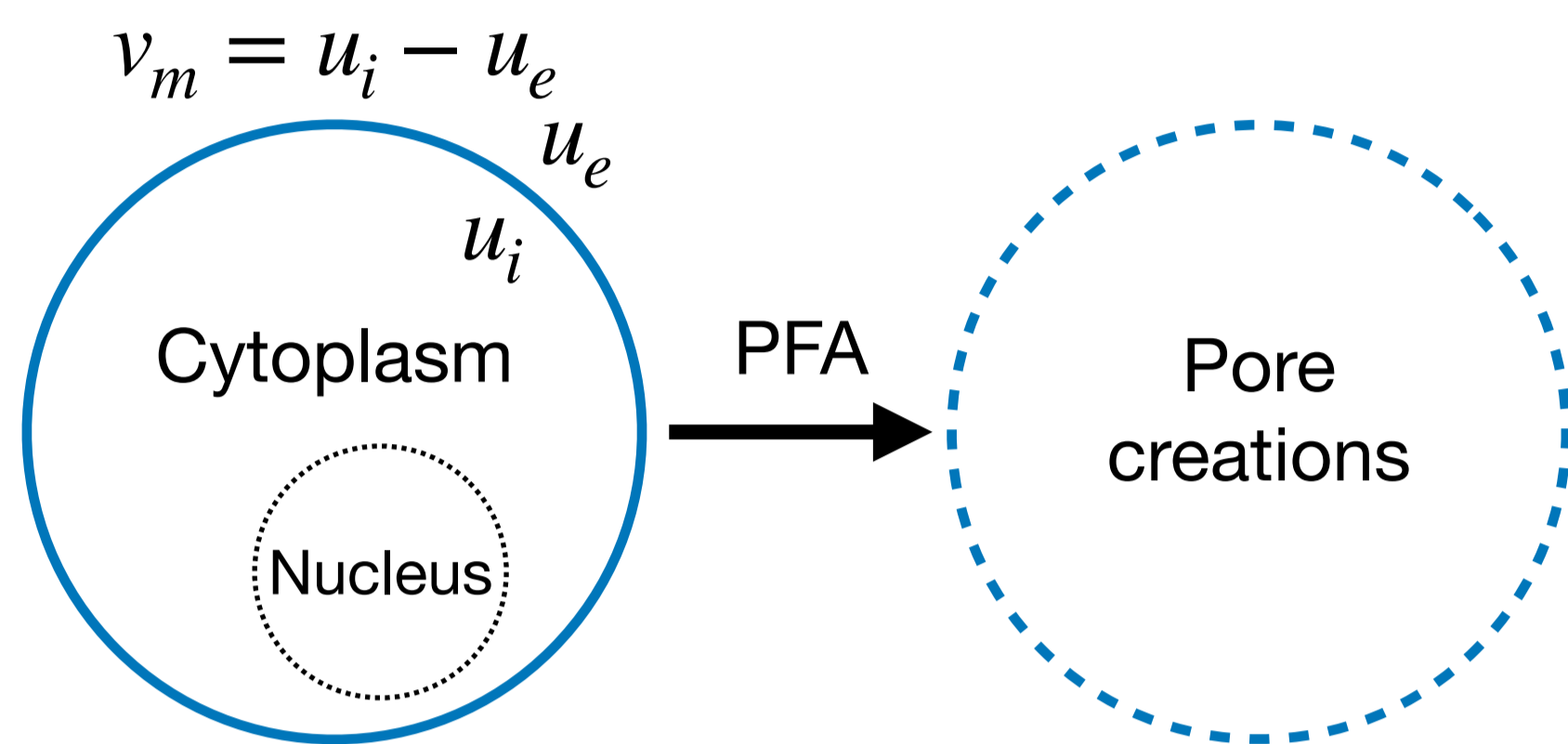
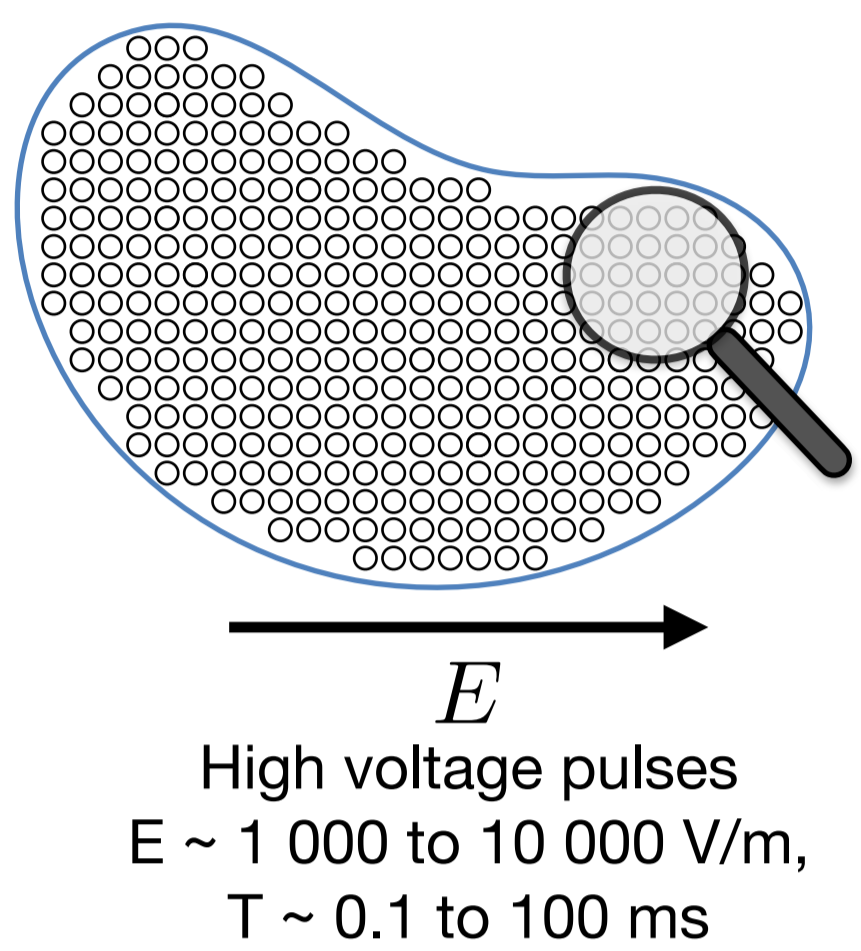
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## Context & Objectives

### Clinical context

- Eliminate sites of **arrhythmias** origin as atrial fibrillation (AF) or ventricular tachycardia (VT) using **cardiac ablation**
- Focus on **Pulsed electric Field Ablation (PFA)** based on **irreversible electroporation** [1]



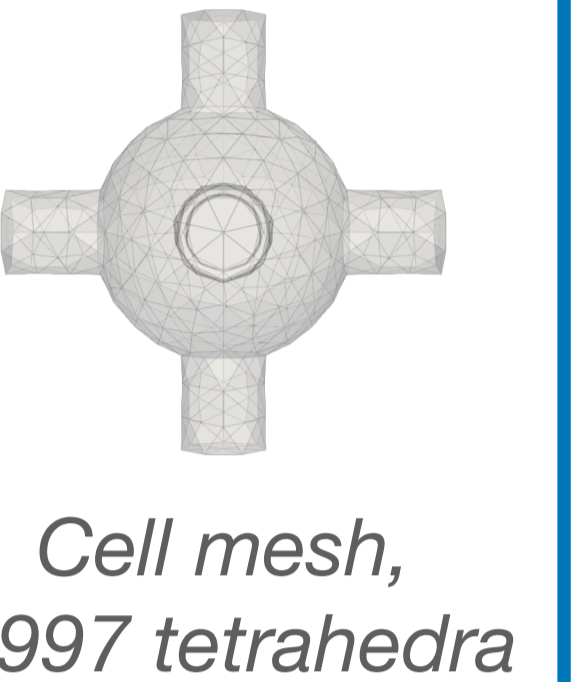
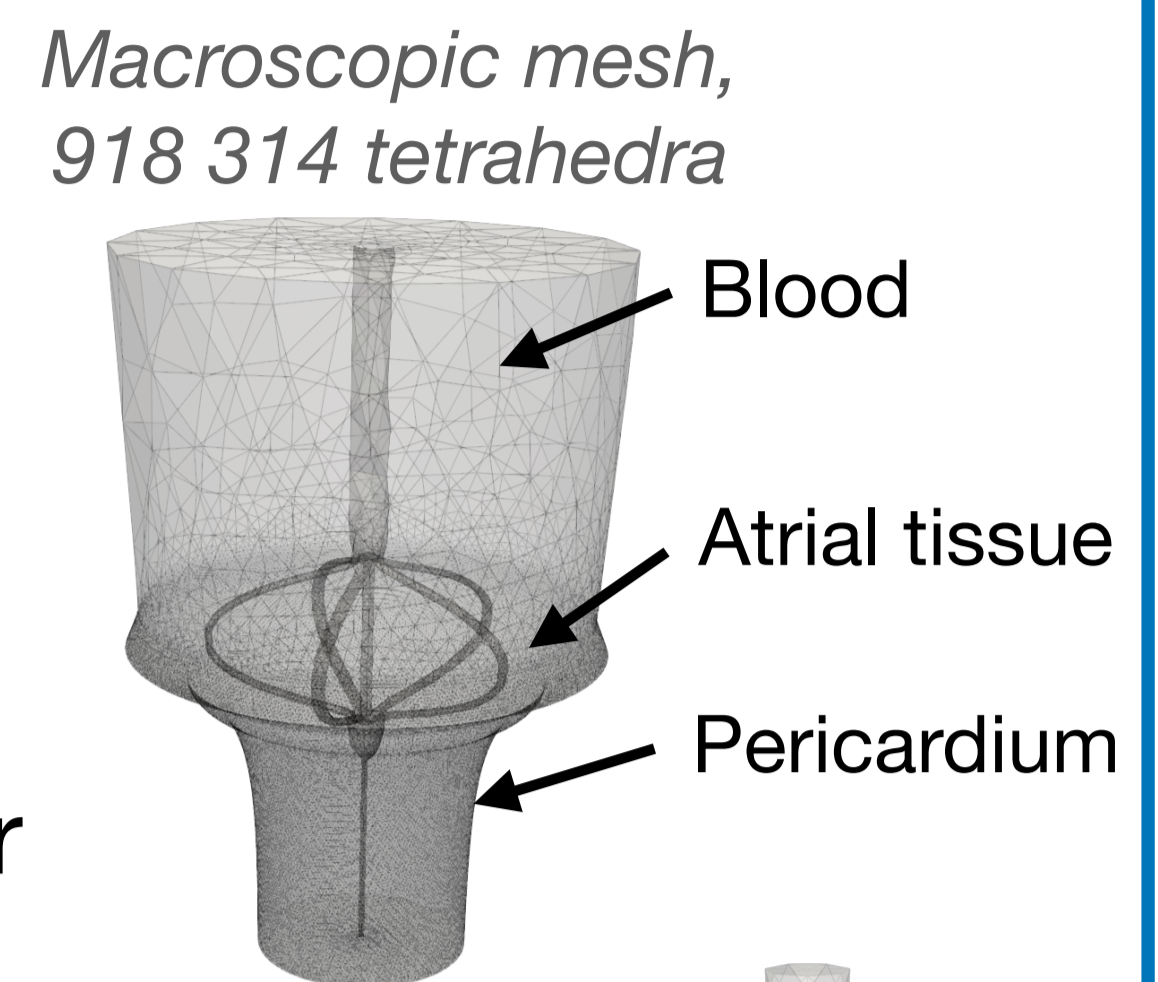
Irreversible electroporation leading to **cell death** occurs when  $|v_m|$  reaches a threshold.

### Objectives

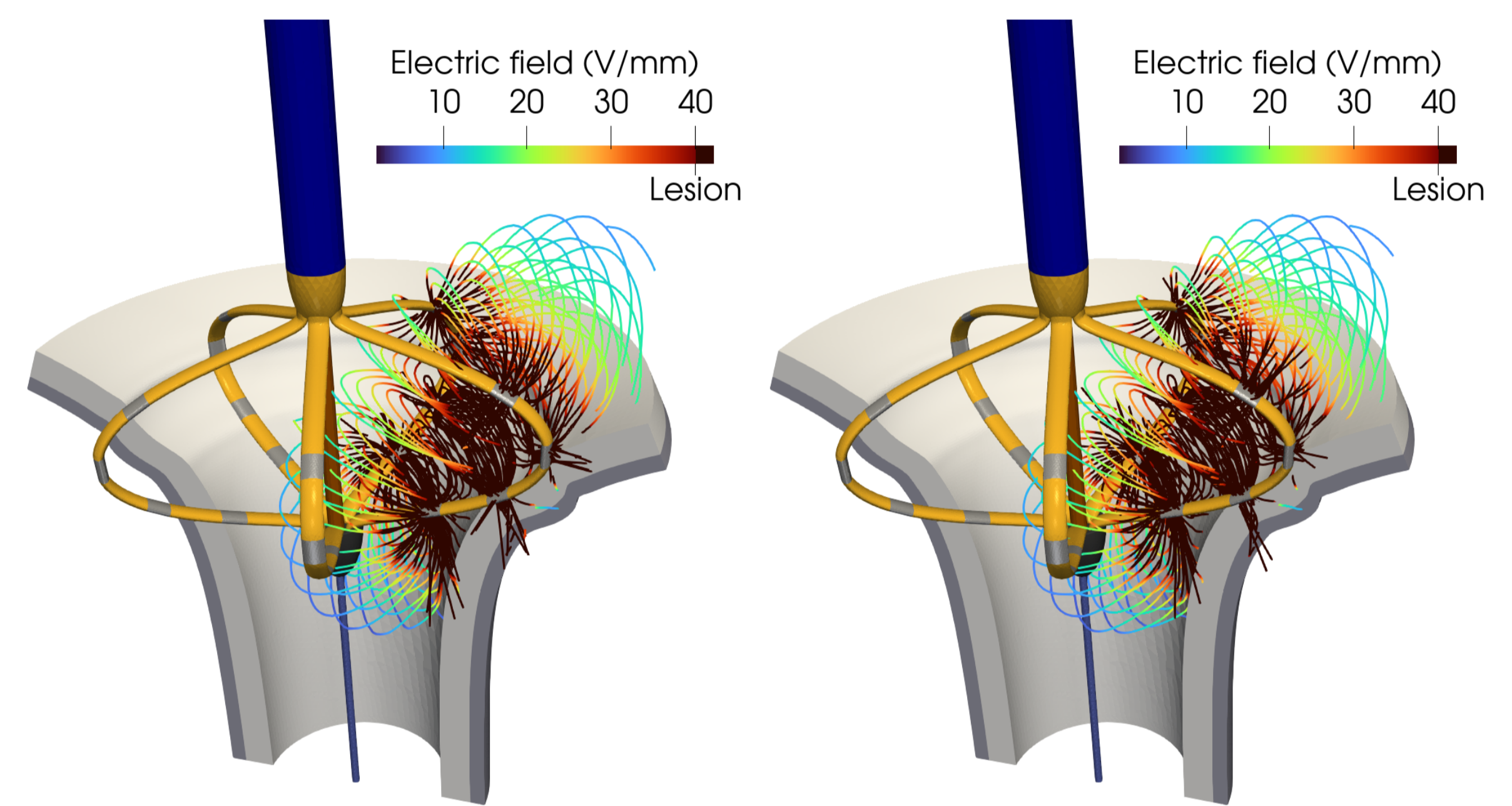
- Write a mathematical PFA model with **physical** interpretation & compatible with **cardiac electrophysiology model**
- Perform **numerical simulations** of the model in a realistic context

## Numerical Illustration

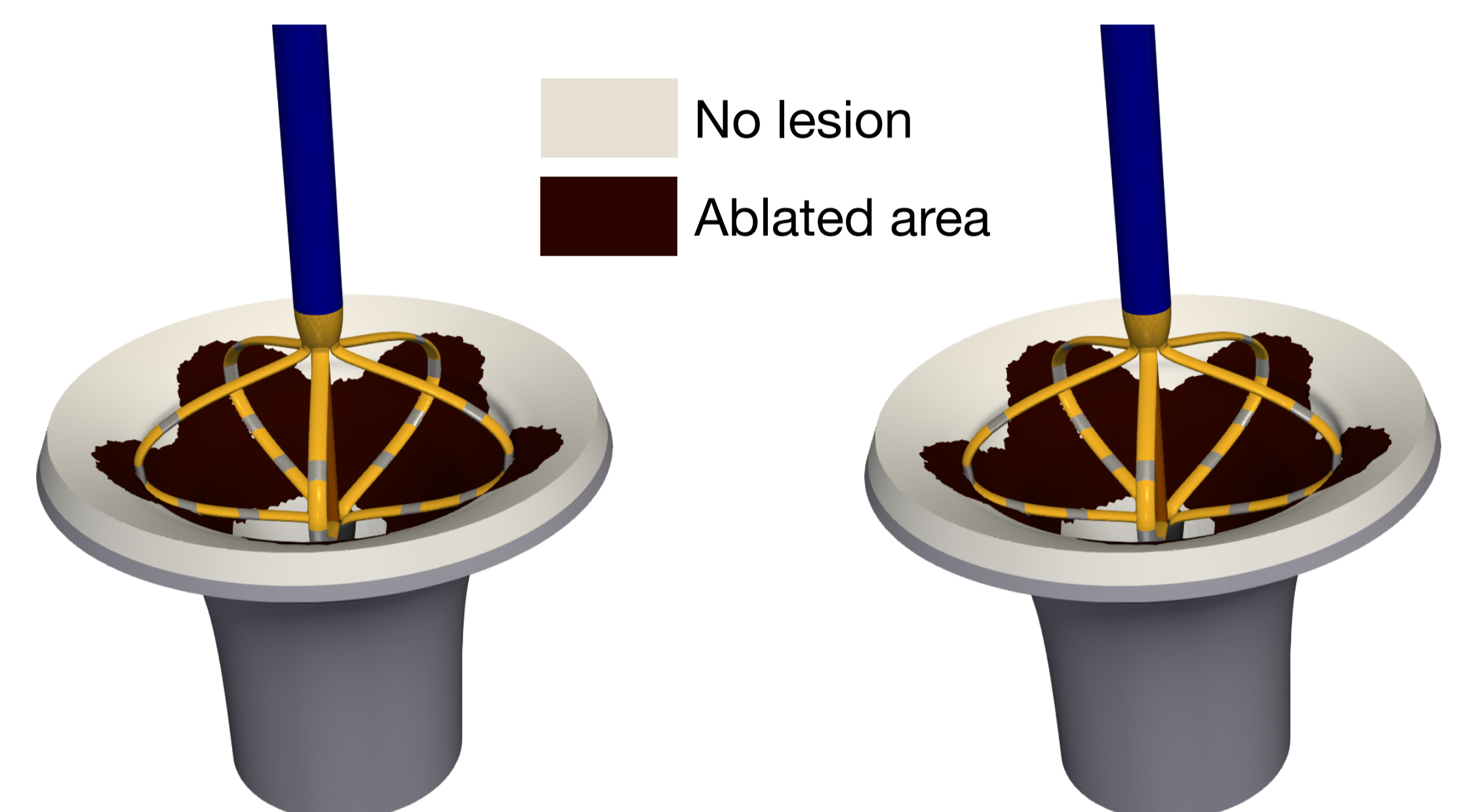
- Static case
- Periodic cell
- Ionic term neglected
- $S_m$  as a sigmoid
- Realistic context: use of the Farapulse [4] catheter inside the **entry of a pulmonary vein (LA)**
- Comparison with the standard model widely used in oncology [3]



$$\nabla \cdot (\sigma(|\nabla u|) \nabla u) = 0, \quad \Omega$$



Comparison of the electric field generated with one pair of branches - standard model (left), new model (right)

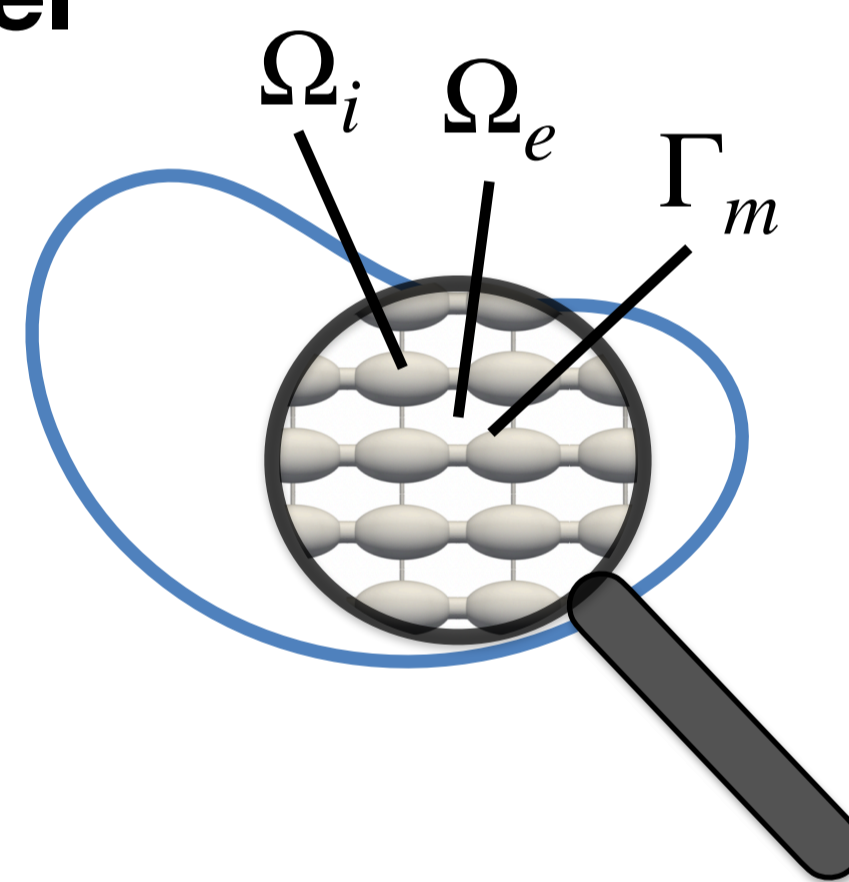


Comparison of the ablated area after the activation of all 5 pairs of branches - standard model (left), new model (right)

## PFA Modeling & Homogenization

### Microscopic bidomain model

$$\begin{aligned} -\nabla \cdot (\sigma_i \nabla u_i) &= 0, & \Omega_i \\ -\nabla \cdot (\sigma_e \nabla u_e) &= 0, & \Omega_e \\ \sigma_i \nabla u_i \cdot n_i &= \sigma_e \nabla u_e \cdot n_i, & \Gamma_m \\ -\sigma_i \nabla u_i \cdot n_i &= C_m \partial_t (u_i - u_e) + I_{ion} (u_i - u_e) \\ &+ S_m (|u_i - u_e|) (u_i - u_e), & \Gamma_m \end{aligned}$$



$S_m$  is the **membrane conductance**

**Not useful** in practice (~ $10^{12}$  cardiomyocytes in the heart)

**Classical microscopic bidomain model (based mostly on physical laws) with a new term modeling PFA**

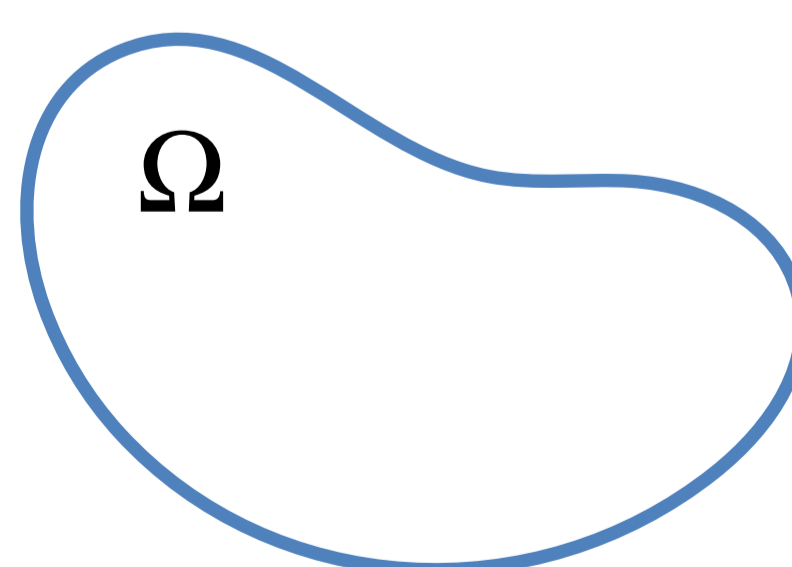
$$\varepsilon = \frac{\text{cell size}}{\text{tissue size}} \sim \frac{10^{-4}}{10^{-2}} = 10^{-2}$$

### Mathematical homogenization

- Two-scale convergence [2]
- Same process as classical bidomain model with different scaling
- Leads to the following **effective model (macroscopic + cell)**

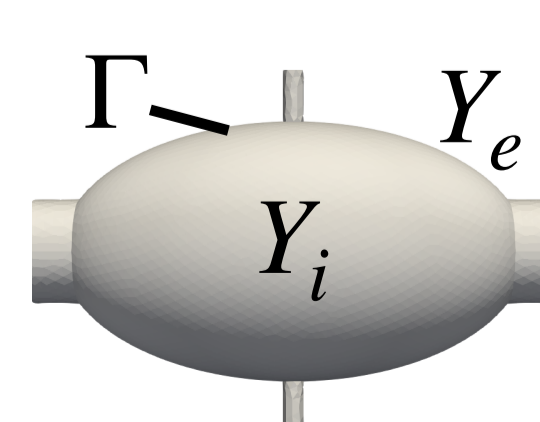
### Effective macroscopic bidomain model...

$$\begin{aligned} -\nabla \cdot (\Sigma_i \nabla u_i) &= \nabla \cdot (\Sigma_e \nabla u_e) \\ \nabla \cdot (\Sigma_i \nabla u_i) &= A_m (C_m \partial_t (u_i - u_e) + \frac{1}{\varepsilon} I_{ion} (u_i - u_e) \\ &+ \frac{1}{|\Gamma|} C_m \partial_t \int_{\Gamma} I[\chi] \cdot \nabla u_e \\ &+ \frac{1}{|\Gamma|} \int_{\Gamma} S_m (|I[\chi] \cdot \nabla u_e + v_m|) (I[\chi] \cdot \nabla u_e + v_m)) \end{aligned}$$



### ...coupled with a cell problem

$$\begin{aligned} -\Delta \chi_{i,e}^j &= 0, Y_{i,e} & [\chi] &= (\chi_i^j - \chi_e^j)_j \\ \nabla \chi_{i,e}^j \cdot n_i &= -e^j \cdot n_i, \Gamma & \Sigma_{i,e} &= \sigma_{i,e} \left( \frac{|Y_{i,e}|}{|Y|} I + \frac{1}{|Y|} \int_{Y_{i,e}} \nabla \chi_{i,e} \right) \\ \int_{Y_{i,e}} \chi_{i,e}^j &= 0. & & \end{aligned}$$



Volume fraction of the microscopic conductivity      Microscopic term (periodic case)

## Conclusions & Perspectives

### Conclusions

- Effective PFA bidomain model with physical interpretation & compatible with cardiac electrophysiology model
- Appearance of a transport term
- Results comparable with the standard model in a realistic case

### Perspectives

- Understand the differences with the standard model by introducing an effective conductivity
- Investigate the effect of cell geometry

## Bibliography

- [1] Reddy VY et al. Pulsed Field Ablation of Paroxysmal Atrial Fibrillation. JACC Clin Electrophysiol 2021.  
 [2] Collin A, Imperiale S. Mathematical analysis and 2-scale convergence of a heterogeneous microscopic bidomain model. Math Models Methods Appl Sci 2018.  
 [3] A. Ivorra, J. Villemejeane, and L. M Mir. Electrical modeling of the influence of medium conductivity on electroporation. Physical chemistry chemical physics, 2010.  
 [4] Reddy, V, Neuzil, P, Koruth, J. et al. Pulsed Field Ablation for Pulmonary Vein Isolation in Atrial Fibrillation. J Am Coll Cardiol. 2019 Jul, 74 (3) 315–326.