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On the need for a methodology of didactical content analysis

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This essay outlines reasons why research in mathematics education requires didactical content analysis, understood as the rational evaluation of the educational suitability of different mathematical approaches towards a given curricular content on the basis of didactical background theories. It continues to argue that such work is currently being done outside of research and still being used in research. A reason for that is investigated via the example of a diminishing German manifestation of didactical content analysis. It becomes obvious that studies in didactical content analysis have become difficult or impossible to publish due to the lack of a methodology for the research field. Eventually, initial thoughts for such a methodology are presented.

Keywords: Mathematics education, course content, pedagogical content knowledge, research methodology.

Introduction

I will argue for the need of the development of a methodology of didactical content analysis in order to install it as a legitimised research approach in mathematics education. Thereby, I understand didactical content analysis as the rational evaluation of the educational suitability of different mathematical approaches towards a given curricular content on the basis of didactical background theories.

After an initial presentation of didactical content analysis and the demonstration of its relevance for research in mathematics education, a closer look at *Stoffdidaktik* (pronounced [ˈʃtɔfdiˌdaktɪk]) as the most prominent manifestation of didactical content analysis will help to understand that didactical content analysis lacks the methodological reflection necessary to be accepted as a research method in mathematics education today. It will be argued that studies which are fundamental for the understanding of central issues in the discipline are excluded from mathematics education research, constitute no prominent field of work for researchers in mathematics education, and do not undergo scientific control by peer-reviewing. This weakens the academic quality of our understanding of the mathematical potential of curricular contents. It is in reaction to this problem that the development of a methodology will prove necessary for legitimising didactical content analysis as a research approach.

What is didactical content analysis?

While mathematical theory building often aims at unifying mathematical theories along values such as precision, rigour, efficiency and aesthetics, educational approaches to mathematics need to diversify mathematical theories in order to negotiate approaches to mathematics that reconcile educational requirements with academic standards in the mathematics classroom. Conceivable activities of didactical content analysis thus include

- the confrontation of different proofs of a theorem on the background of their suitability for gaining specific competences in mathematical argumentation and proof,
- the consideration of different definitions of mathematical concepts on the background of students' foreknowledge,

- the comparison of mathematical problems on the background of their suitability to develop mathematical contents by self-active problem solving, and
- the study of local axiomatisations for the preparation of a concise elementary and yet deductively rigorous arrangement of propositions within a mathematical field.

We can think of didactical content analysis as a two-level approach. On a first level, we juxtapose different *solutions to problems in mathematical theory building* concerning a specific content of school mathematics. We search for different solutions in mathematical publications, in the history of mathematics, and in schoolbooks, and we pursue original ideas. Indeed, as the study of elementary mathematics and the question how specific contents of mathematics can be approached on the basis of school mathematics do not constitute vivid working fields of research mathematicians, such genuinely mathematical work is usually left to the scholar in mathematics education research.

On a second level, we want to formulate recommendations for the selection of a specific solution to a problem of mathematical theory building for its use in the mathematics classroom. In light of the plurality of different perspectives used to understand the complexities of teaching and learning mathematics, there is no objective position from which such a recommendation can be made. Instead, recommendations depend on the *didactical background theories* on the basis of which the different solutions of mathematical problems are evaluated. Typical perspectives include, among others, educational goals in problem solving, mathematical modelling or argumentation and proof, as well as psychological, sociological, epistemological, linguistic, or semiotic conceptualisations of the conditions of learning mathematics. The influence of didactical background theories can be reflected upon explicitly, but often they also unconsciously guide the evaluation of mathematical solutions.

Note that didactical content analysis is a *rational* approach. Research on the question in which form mathematical content should be presented to students can have many forms, empirical and rational. For example, design research in mathematics education has blossomed as an empirical approach for two decades (Gravemeijer & Prediger, 2019). In contrast, didactical content analysis deliberately does not aim at empirical insights. Still, it is not ignorant towards empirical insights, as it builds on various empirically informed background theories on the teaching and learning of mathematics. Didactical content analysis works differently than empirical studies, requiring a different kind of expertise, a smaller amount of resources, and particular research methods. Thereby, didactical content analysis is not an alternative to empirical studies in mathematics education, but complements empirical studies by providing a rational basis for them (Griesel, 1972).

How is didactical content analysis relevant?

Didactical content analysis constitutes an indispensable aspect of research in mathematics education for various reasons. First, it allows for the academic reflection of how both didactical theories and the conventions of mathematics manifest themselves in the form of school mathematics. For example, German grammar school curricula include theorems on angles inscribed in circles such as Thales' theorem, but as the Euclidean axiom system of geometry is no longer accepted in mathematics and as Hilbert's axiom system is too complex, the school topic lacked the requirements for proving these theorems. Here, a study in didactical content analysis by Schupp (1977) succeeded in providing an axiomatisation of school geometry as transformation geometry, which allows for a mathematically

rigorous engagement on the basis of little more than set theory. Thereby, Schupp selected and developed a mathematical approach that aligned with his contemporary insights about teaching and learning proof and with the conventions of academic mathematics.

Second, didactical content analysis provides a theoretical background for empirical studies on the design of learning situations, as designers “will have to problematize the topic under consideration from a disciplinary perspective” as an initial step of planning (Gravemeijer & Prediger, 2019, p. 35). For example, in her design study on exponential functions, Thiel-Schneider (2018) argued that Kirsch’s (1977b) theoretical study of different characterisations of exponential functions proved especially helpful for the analysis of individual processes of concept development (p. 190).

Third, didactical content analysis allows teachers to “didactically reconstruct mathematics” in the sense of gaining familiarity with the benefits and limitations of different mathematical approaches towards a specific content (Jahnke, 1998, p. 72, my translation). This is essential for understanding, evaluating and supporting the divergent mathematical ideas of learners. In this sense, Kirsch’s (1977b) study on different mathematical characterisations of exponential functions can inform teachers in general about the ways in which they or their students might approach this content. This point has recently found strong support from empirical studies when Baumert et al. (2010) showed that student achievement in mathematics depends significantly on the teacher’s pedagogical content knowledge, which Shulman (1987) had defined as “the blending of content and pedagogy into an understanding of how particular topics, problems, or issues are organized, represented, and adapted to the diverse interests and abilities of learners, and presented for instruction” (p. 8). In addition to Shulman’s already telling definition, it is noteworthy that Bergsten (2020), Sträßer (2020) and Wittmann (2014) saw clear analogies between pedagogical content knowledge and Stoffdidaktik, the German manifestation of didactical content analysis, which I will address presently.

It should be added that didactical content analysis is helpful (although not strictly necessary) for some other purposes. To give an example, an elementarisation of mathematics by didactical content analysis may allow prospective teachers to actively engage in authentic activities of mathematical research (such as formulating and debating proofs, sharpening definitions and axiomatising mathematical fields) without extensive studies of university mathematics. (This is not to say that prospective teachers should not study university mathematics. However, the extent to which this is demanded and offered varies strongly between countries and institutions, as does the extent to which such studies lead to a virtuosity in authentic activities of mathematical research and not merely to operational skills.) To give another example, didactical content analysis can contribute to the endeavour of making mathematics intellectually accessible for the general public (Jahnke, 1998).

Stoffdidaktik as a manifestation of didactical content analysis

Bergsten’s (2020) interest in “philosophical, historical, and didactical analyses of mathematical content and of how it is selected, adapted, or transformed in the process of recontextualization by requirements due to educational constraints” (p. 498) documents the affinity of his focus to didactical content analysis. Apart from Stoffdidaktik, Bergsten addressed in greater detail

- the Dutch paradigm of Realistic Mathematics Education, which I will not consider as a manifestation of didactical content analysis as its commitment to applications of mathematics restricts it to only a few didactical background theories;
- the French paradigm of *ingénierie didactique*, which Sträßer (1996) judged to have its strength in linking didactical content analysis with empirical approaches rather than in didactical content analysis itself; and
- the US-American program of Mathematical Knowledge for Teaching, which Bergsten (2020) held to be “less systematic” than Stoffdidaktik and to lack “reference to different possible mathematical background theories” (p. 380).

This leaves Stoffdidaktik as the most elaborated paradigm of didactical content analysis. The existence of this research field and the fact that it is still considered necessary and helpful for today’s research (e.g., by Thiel-Schneider, 2018) underline the relevance of didactical content analysis and present Stoffdidaktik as an interesting corpus of former work in the field. Apart from the large body of publications on didactical content analysis in the tradition of Stoffdidaktik, the field has developed some degree of self-reflection, which includes not only programmatic texts but diverse criticism.

The lack of methodology

One of the objections brought forward against Stoffdidaktik revealed a fundamental problem of didactical content analysis and a sufficient condition for its demise as a legitimate research approach. A few words on the nature of this demise appear necessary. Jahnke (2010) analysed the articles published in the *Journal für Mathematik-Didaktik*, the only predominantly Germanophone journal for mathematics education research in the Web of Science. He counted the number of pages of journal articles “dealing wholly or partly with mathematical contents and their teaching and learning” (p. 441, my translation) for each decade from 1980 to 2009. The proportion of content-related article pages decreased from 25% in the 1980s to 18% in the 1990s and 8% in the 2000s. Sträßer (2020) noted that, “in the last quarter of the twentieth century, Stoffdidaktik has lost its importance as one of the most important and widespread research approaches in the German-speaking community” (p. 808), adding that “nowadays, Stoffdidaktik is mainly published in journals aiming at practicing teachers” (p. 806). Looking at research journals and teacher-oriented journals in English, I get an equal impression: While contributions to what I call didactical content analysis hardly ever appear in research journals, they are rather common in teacher-oriented journals, as, for example, Tomotheus and O’Brien’s (2020) discussion of “their approach to teaching dividing by fractions” (p. 9) in the teacher-oriented journal *Mathematics Teaching*. This exclusion of didactical content analysis from the scientific discourse is problematic, as it fails to secure scientific quality standards, decreases the visibility of studies in didactical content analysis, and discourages scholars from pursuing studies in the field.

We return to that objection, which will explain why Stoffdidaktik lost its status as a research approach. Wittmann (2014) remembered that the acceptability of studies in Stoffdidaktik for a publication in major journals has decreased since the 1980s, as different, presumably more rigorous research approaches became common in journal articles. He stated that, “in the effort to achieve ‘scientificity’, contributions related to mathematics and classroom practice were increasingly excluded” (p. 15, my translation). Already Führer (1999) had observed that

when [...] our community, worrying about its scientificity, began to orient itself more towards the standards of educational science and psychology than towards philosophy, sociology or the ‘pre-scientific’ traditions of German elementary school or grammar school didactics, Stoffdidaktik almost became a scientifically disqualifying attribute towards the end of the eighties and the beginning of the nineties. (p. 80, my translation)

Concerning the declining number of Stoffdidaktik publications in the *Journal für Mathematik-Didaktik*, Sträßer (2015), once one of the journal editors, commented “that the editors can only publish those papers that are submitted and pass the review process with a positive result” (p. 32, my translation), indicating a lack of scientific acceptability of Stoffdidaktik submissions. He also detected “a lack of programmatic texts on Stoffdidaktik” and stated that “I have complained in conversations with various colleagues for a long time that in recent years there has been no reflection anywhere on the characteristics of (certainly not only German) Stoffdidaktik” (p. 30, my translation). I would go further and argue for a causality: The inability of Stoffdidaktik to sufficiently explicate its characteristics as a research approach is a necessary cause of the decline in the acceptance of Stoffdidaktik as a research approach. As mathematics education research matured as a scientific discipline, gained awareness for the necessity to discuss theoretical frameworks and research methods, and reflected on the frameworks and methods of the diversifying approaches adopted from other academic disciplines, the lack of reflection of how Stoffdidaktik works became an obstacle for its acceptance as legitimate research.

While there is no doubt that journals publishing research in mathematics education, such as the *Journal für Mathematik-Didaktik*, demand an explicit presentation and legitimation of the suitability of the methods used, it remained opaque which methods are being used in Stoffdidaktik and what their potentials and limitations for the understanding of the teaching and learning of mathematics might be. Indeed, publications in and on Stoffdidaktik often include the naming of methods without any further discussion of their nature, relevance and legitimacy. For example, Tietze (1994) stated that “elementarizing, simplifying, and visualizing are central issues” in Stoffdidaktik (p. 42). In his encyclopaedia entry on “Mathematical Approaches” in mathematics education, Bergsten (2020) referred to nothing else than this quote from Tietze (1994) for a description of the methodological approach to Stoffdidaktik, which suggests that more elaborated descriptions do not exist. In fact, I know of no presentation of a methodology of Stoffdidaktik in specific or of didactical content analysis in general.

Although there is no methodology of mathematical content analysis, there are a few contributions discussing specific methods of mathematical content analysis. In his encyclopaedia entry on Stoffdidaktik, Sträßer’s (2020) only remark concerning the methods of Stoffdidaktik referred to an ICME plenary given by Kirsch (1977a), in which the latter discussed aspects of the simplification of academic mathematics for teaching purposes. Tietze (1994) dedicated one page to a differentiation between three different ways of elementarising definitions from academic mathematics for their use in mathematics classrooms. In his discussion of axiomatisation, Freudenthal (1963) presented a condensed form of axiomatisation as a method for ordering contents of school mathematics. Bigalke (1991) applied this idea to the development of definitions. However, none of these isolated contributions explicitly aim at conceptualising the methods of didactical content analysis.

Towards a methodology of didactical content analysis

I have established that didactical content analysis is a necessary approach to research in mathematics education but that it currently lacks the methodological awareness that is required to be accepted as proper research. Wolters and Mittelstraß (2013) defined a research method as “a procedure planned by means and purpose, which leads to technical skill in the solution of theoretical and practical tasks and for this reason can be understood as the result of a systematically built up skill, as an operational knowledge, stabilised by the ability to pass it on” (p. 379, my translation). Reflection on methods in such a way can be understood as “methodology”. We lack such reflections on the methods of didactical content analysis. However, Wolters and Mittelstraß observed that “only when doubts arise, methods become necessary for the elimination of doubt, which should result in the (re-)acquisition of relevant security, of reliable orientation in acting and thinking” (p. 380, my translation). Therefore, the current state of affairs can be seen as a moment of a normal methodological evolution, where doubts have been documented and provoke work on the methodology of a research approach.

What would a methodology of didactical content analysis entail? At the moment, the overall “purpose” of the approach is well-defined, but the “means and purpose” of specific methods are not, nor are they “systematically built up” as “an operational knowledge” or discernibly “stabilised”. Every method of didactical content analysis can be understood as the synthesis of the analysis of different solutions to content-specific problems of mathematical theory building on the one hand and didactical background theories on the other hand. The methodological legitimisation of a specific method will then require to explain the procedure of the interaction of mathematical and didactical perspectives, and it will have to justify its relevance for the teaching and learning of mathematics on the basis of the involved background theories.

As an example, let me sketch the following scenario: Consider somebody is comparing different definitions for the concept of continuity and wants to ground a choice on the basis of a linguistic theory of the difficulties of formalised language. (We know that multiple quantification, which is necessary for the Weierstrass definition, is hard to understand for many learners.) We will apply a procedure derived from our linguistic theory to the definitions we found in order to identify a suitable definition. The suitability of that definition will be warranted by our linguistic background theory. This is how our method of “choosing linguistically preferable definitions” can be described concerning “means and purpose”, how it can be understood as being “systematically built up”, how it establishes “an operational knowledge” and gets “stabilised” by its explanation.

A general problem for any methodology of didactical content analysis lies in the fact that we are not facing *one* method but a complex web of interacting activities. Already at the first level, the level of mathematical solutions, we face a variety of problems of mathematical theory building, which lead to insights about different mathematical aspects of a curricular content. Then again, we do not confine ourselves to *one* didactical background theory on the basis of which to judge the suitability of the identified mathematical solutions but allow for the use of many different didactical background theories. Even worse, there is no definable limit to the lists of mathematical activities and didactical background theories, which means that we face the potentiality of an unlimited number of combinations that might eventually count as distinct methods of didactical content analysis. Despite

this unavoidable incompleteness, research can gradually reach an ever more comprehensive understanding of the methodology in question. A pragmatic solution is to start with identifying a selection of methods of didactical content analysis for detailed methodological analysis. This selection should include the methods commonly used and should refer to the most popular didactical background theories in mathematics education.

The resulting analysis could be pursued inductively, trying to reconstruct the methodology of already established manifestations of didactical content analysis such as Stoffdidaktik, and it could be pursued deductively, starting with the questions towards a mathematical content that arise from a chosen set of didactical background theories.

The methodological reflections could culminate in the formulation of a framework, which enumerates recurrent problems of mathematical theory building and, for each entry in that list, presents different selection criteria for solutions derived from didactical background theories together with references to detailed accounts of the involved procedures and theoretical legitimisation. Studies in didactical content analysis can then refer to this framework to organise and methodologically legitimise their work, and they can easily find more detailed discussions of methodological issues in the references provided in the framework.

References

- Baumert, J., Kunter, M., Blum, W., Brunner, M., Voss, T., Jordan, A., Klusmann, U., Krauss, S., Neubrand, M., & Tsai, Y.-M. (2010). Teachers' mathematical knowledge, cognitive activation in the classroom, and student progress. *American Educational Research Journal*, 47(1), 133–180. <https://doi.org/10.3102/0002831209345157>
- Bergsten, C. (2020). Mathematical approaches. In S. Lerman (Ed.), *Encyclopedia of mathematics education* (pp. 498–505). Springer. https://doi.org/10.1007/978-3-030-15789-0_95
- Bigalke, H.-G. (1991). Lokales Ordnen beim Definieren eines bekannten Begriffs [Local ordering when defining a known term]. In H. Postel, A. Kirsch, & W. Blum (Eds.), *Mathematik lehren und lernen* (pp. 61–70). Schroedel.
- Freudenthal, H. (1963). Was ist Axiomatik, und welchen Bildungswert kann sie haben? [What is axiomatics and what educational value can it have?] *Der Mathematikunterricht*, 9(4), 5–29.
- Führer, L. (1999). „Design Science“ als dynamisierte Wissenschaftsmethodik und Sozialform innerhalb der Mathematikdidaktik [“Design science” as a dynamised scientific approach and social form within mathematics didactics]. In C. Selzer & G. Walther (Eds.), *Mathematikdidaktik als design science* (pp. 78–85). Klett.
- Führer, L. (2015). Stellungnahme zu Gert Schubrings erfreulich pointierter Kritik „der“ stoffdidaktischen Tradition [Statement on Gert Schubring's pleasingly pointed critique of “the” tradition of Stoffdidaktik]. *Mitteilungen der Gesellschaft für Didaktik der Mathematik*, (99), 23–25.
- Gravemeijer, K., & Prediger, S. (2019). Topic-specific design research: An introduction. In G. Kaiser & N. Presmeg (Eds.), *Compendium for early career researchers in mathematics education* (pp. 33–57). Springer. https://doi.org/10.1007/978-3-030-15636-7_2

- Griesel, H. (1972). Die mathematische Analyse als Forschungsmittel in der Didaktik der Mathematik [Mathematical analysis as a research tool in the didactics of mathematics]. In *Beiträge zum Mathematikunterricht 1971* (pp. 72–81). Schroedel.
- Jahnke, T. (1998). Zur Kritik und Bedeutung der Stoffdidaktik [On the critique and significance of material didactics]. *mathematica didactica*, 21(2), 61–74.
- Jahnke, T. (2010). Vom mählichen Verschwinden des Fachs aus der Mathematikdidaktik [About the gradual disappearance of the subject from mathematics didactics]. In A. Lindmeier & S. Ufer (Eds.), *Beiträge zum Mathematikunterricht 2010* (pp. 441–444). WTM.
- Kirsch, A. (1977a). Aspects of simplification in mathematics teaching. In H. Athen & H. H. Kunle (Eds.), *Proceedings of the Third International Congress on Mathematical Education* (pp. 98–120).
- Kirsch, A. (1977b). Zur Behandlung von Wachstumsprozessen und Exponentialfunktionen in der Unter- und Oberstufe [On the treatment of growth processes and exponential functions on lower and upper secondary level]. *Didaktik-Reihe der Österreichischen Mathematischen Gesellschaft*, 1, 17–37.
- Shulman, L. S. (1987). Knowledge and teaching: Foundations of the new reform. *Harvard Educational Review*, 57(1), 1–23. <https://doi.org/10.17763/haer.57.1.j463w79r56455411>
- Sträßer, R. (1996). Stoffdidaktik und ingénierie didactique [Stoffdidaktik and educational engineering]. In G. Kadunz, H. Kautschitsch, G. Ossimitz, & E. Schneider (Eds.), *Trends und Perspektiven: Beiträge zum 7. Internationalen Symposium zur „Didaktik der Mathematik“ in Klagenfurt vom 26.-30.9.1994* (pp. 369–376). Hölder-Pichler-Tempsky.
- Sträßer, R. (2015). Mathematikdidaktik – mehr als das Design praktikabler Kurse für den Mathematikunterricht [Mathematics didactics - more than just designing practical courses for maths lessons]. *Mitteilungen der Gesellschaft für Didaktik der Mathematik*, (99), 30–32.
- Sträßer, R. (2020). Stoffdidaktik in mathematics education. In S. Lerman (Ed.), *Encyclopedia of mathematics education* (pp. 806–809). Springer. https://doi.org/10.1007/978-3-030-15789-0_144
- Thiel-Schneider, A. (2018). *Zum Begriff des exponentiellen Wachstums* [On the concept of exponential growth]. Springer. <https://doi.org/10.1007/978-3-658-21895-9>
- Tietze, U.-P. (1994). Mathematical curricula and the underlying goals. In R. Biehler, R. W. Scholz, R. Sträßer, & B. Winkelmann (Eds.), *Didactics of mathematics as a scientific discipline* (pp. 41–53). Kluwer.
- Tomothaus, J., & O'Brien, S. (2020). Division by a fraction. *Mathematics Teaching*, (271), 9–12.
- Wittmann, E. C. (2014). Die Ideologie der Selbstbeschränkung in der Mathematikdidaktik [The ideology of self-restraint in mathematics didactics]. *Mitteilungen der Gesellschaft für Didaktik der Mathematik*, (96), 15–18.
- Wolters, G., & Mittelstraß, J. (2013). Methode [Method]. In J. Mittelstraß (Ed.), *Enzyklopädie Philosophie und Wissenschaftstheorie* (Vol. 5, pp. 379–383). Metzler.