



**HAL**  
open science

# Weak form-based objective functions in hydrology Application to a 1D transport/degradation model

Carole Delenne, Vincent Guinot, Bernard Cappelaere

► **To cite this version:**

Carole Delenne, Vincent Guinot, Bernard Cappelaere. Weak form-based objective functions in hydrology Application to a 1D transport/degradation model. EGU General Assembly 2011, Apr 2011, Vienna, Austria. hal-04385241

**HAL Id: hal-04385241**

**<https://hal.science/hal-04385241>**

Submitted on 10 Jan 2024

**HAL** is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

### Context and Objective

Weak form-based objective functions (WOF) can be a good alternative to the classical distance-based functions (DOF) for model calibration [1]. WOF are monotone with respect to a given parameter when the sensitivity of the model output keeps a constant sign; then:

- The calibration problem is changed from an optimization problem into a root finding problem
- WOF are less prone to local extrema (when one parameter is to be calibrated, they have a single optimum)
- When several parameters are calibrated, their redundancy can be assessed by superimposing the contour lines of different WOF.

The objective is to apply WOF to the calibration of the degradation coefficient ( $k(t)$  depending on time) of a 1D pollutant transport/degradation model.

### Model

Continuity and transport/degradation equations:

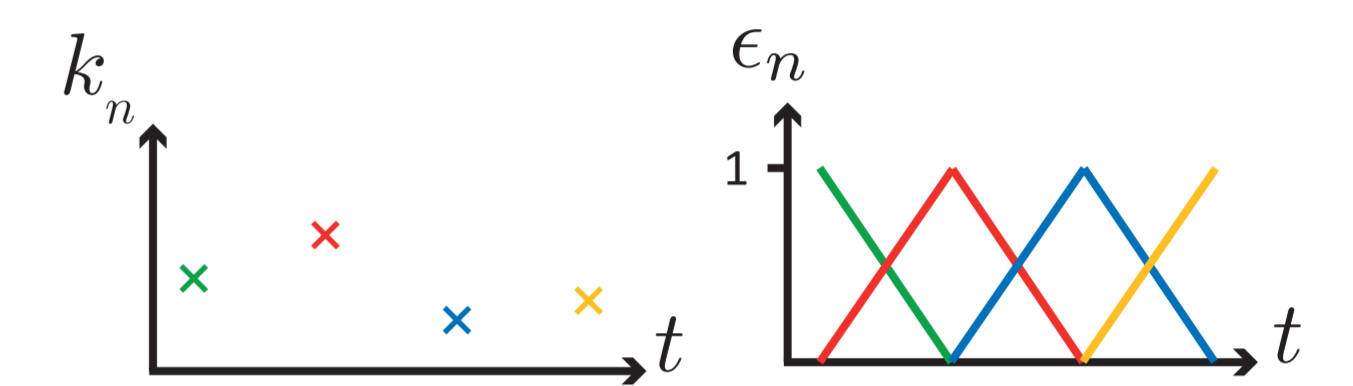
$$\begin{aligned} \text{section: } A & \quad \frac{\partial A(x,t)}{\partial t} + \frac{\partial Q(x,t)}{\partial x} = 0 \\ \text{discharge: } Q & \\ \text{contaminant concentration: } C & \quad \frac{\partial AC(x,t)}{\partial t} + \frac{\partial QC(x,t)}{\partial x} = -k(t)AC(x,t) \\ \text{degradation coefficient: } k & \\ \text{flow velocity: } u=Q/A & \quad \frac{\partial C(x,t)}{\partial t} + u(x,t)\frac{\partial C(x,t)}{\partial x} = -k(t)C(x,t) \end{aligned}$$

Simplified system

#### Hypotheses:

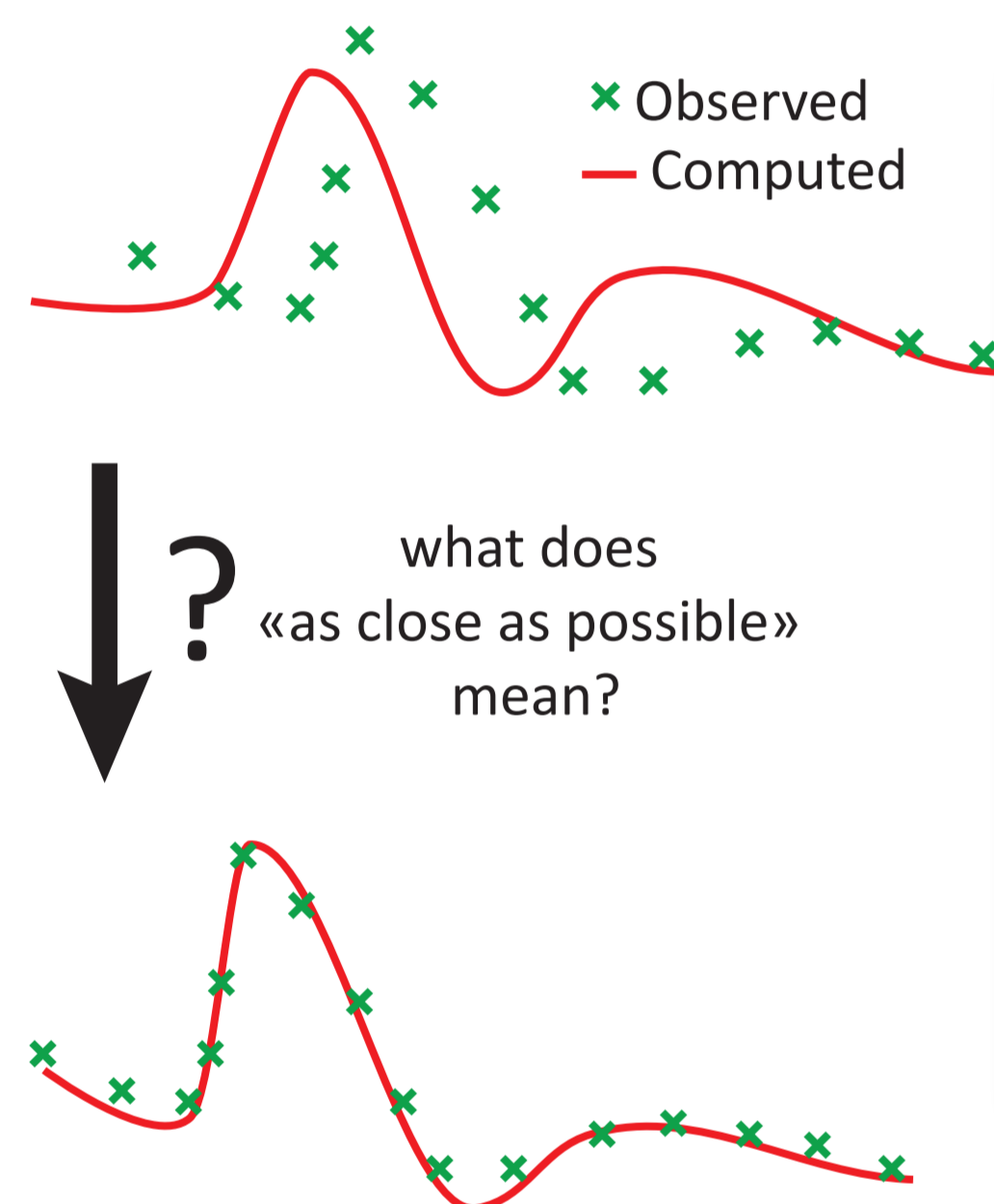
- Water elevation controlled so that  $A$  independent of time  $\Rightarrow Q$  independent of space
- $k(t)$  piecewise linear

$$k(t) = \sum_n k_n \epsilon_n$$



### Calibration process

- Requires observations / measurements
  - The primary variable  $C(x,t)$  cannot be measured easily
  - Use of a secondary variable:  $C(L,t)$  at the outlet
- Model output «as close as possible» to the data
- Classical measures of closeness (DOF):  $D_p = \sum_N |e|^p$
- Weak form-based (monotone wrt  $k$ ):  $W_p = \sum_N e|e|^p$



#### Calibration of 1 parameter $k_n$ with WOF:

- Choose a value for  $p$  and solve  $W_p(k_n)=0$  using e.g. Newton's algorithm:

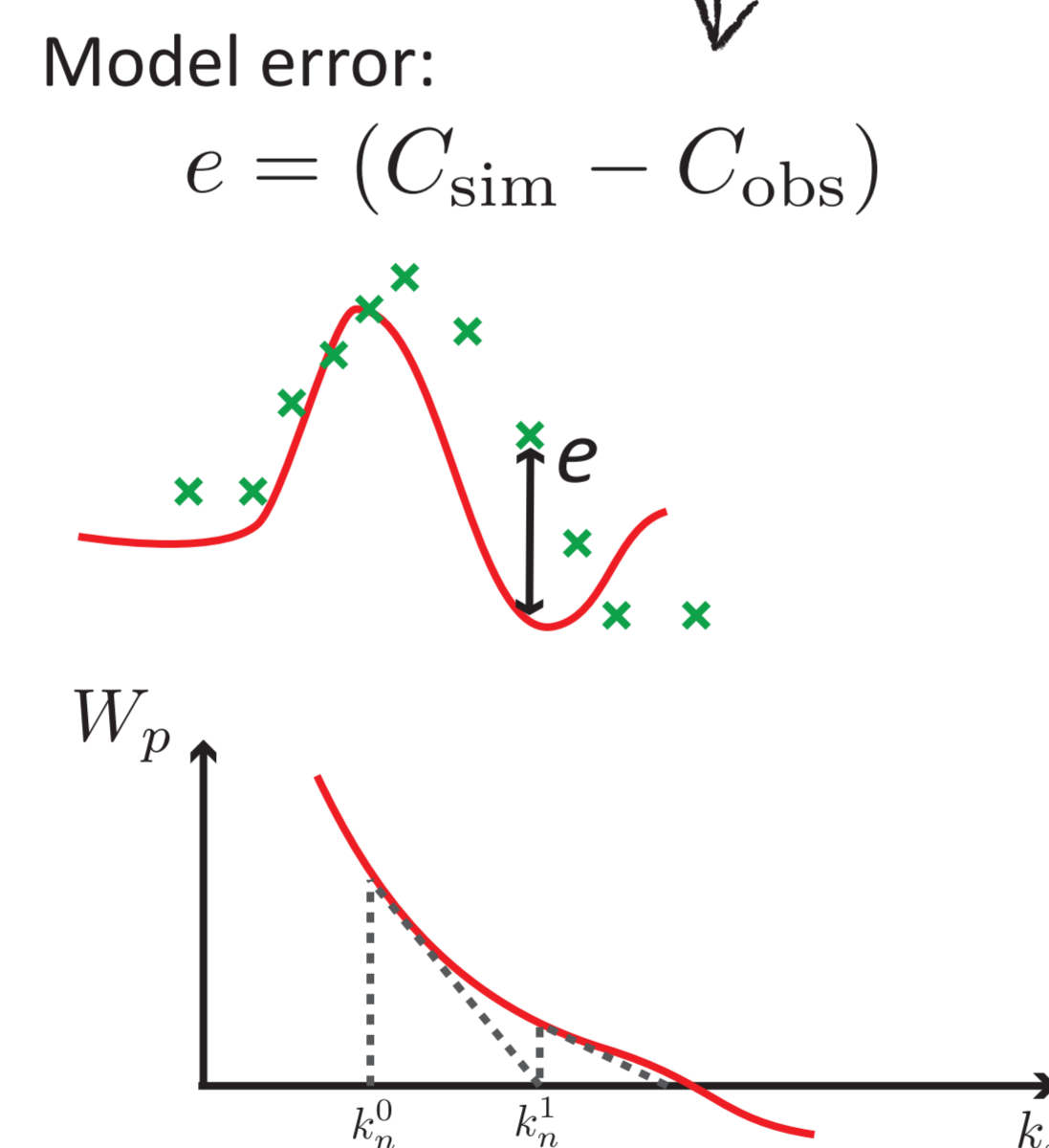
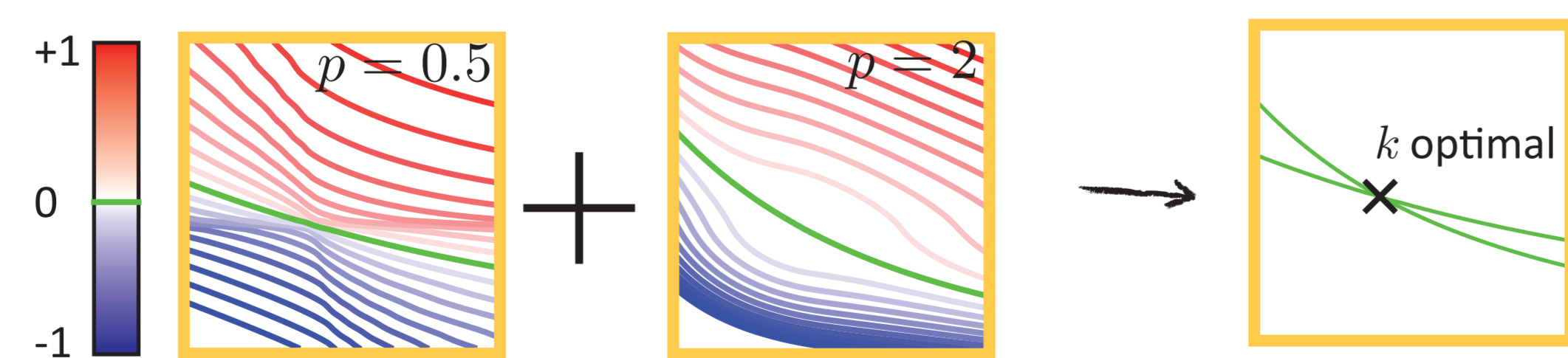
$$k_{i+1} = k_i - \frac{W_p}{W'_p} \quad \text{with } W'_p = p|e|^{p-1}\chi_n \quad \text{and } \chi_n = \frac{\partial C}{\partial k_n}$$

With  $\chi_n$  obtained by solving the sensitivity equation:

$$\frac{\partial \chi_n(x,t)}{\partial t} + u(x,t)\frac{\partial \chi_n(x,t)}{\partial x} = -k(t)\chi_n(x,t) - \epsilon_n C(x,t)$$

#### Calibration of $n$ parameters $k_n$ with WOF

- Find the intersection of  $n$  WOF with  $n$  values for  $p$ .



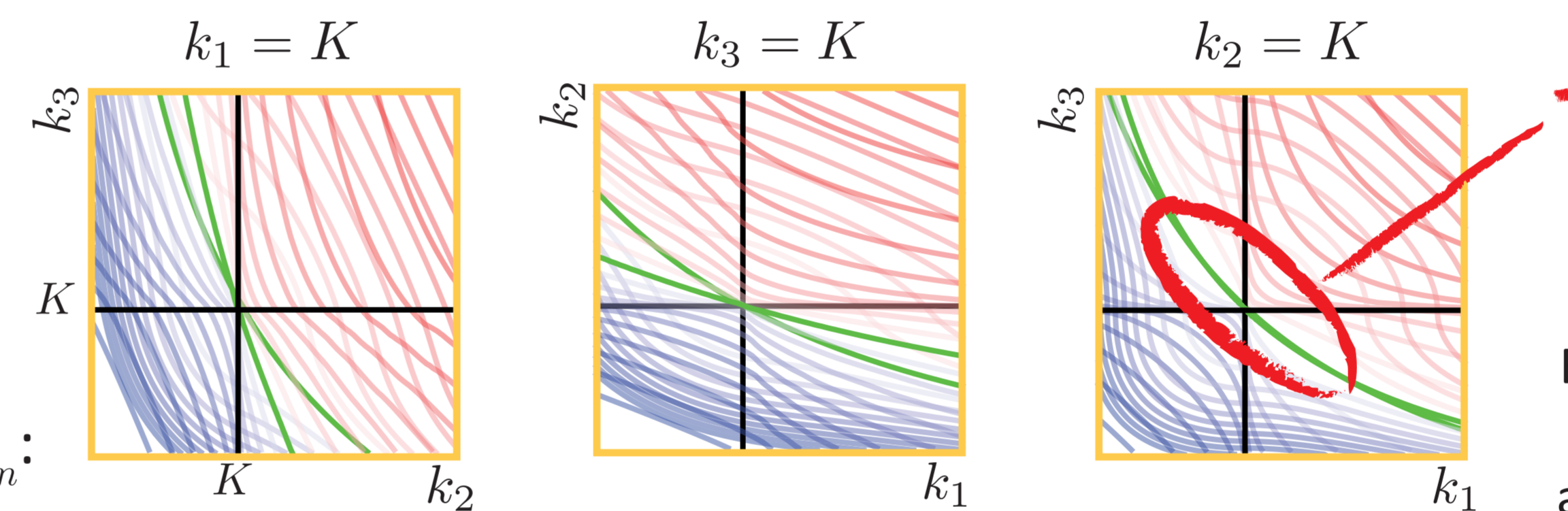
### Application

Test 1 Generation of «observed» values with all parameters constant.

$$\begin{aligned} C(x,0) &= C_0 \\ u(x,t) &= u_{in} \\ C(0,t) &= C_{in} \\ k(t) &= K \end{aligned}$$

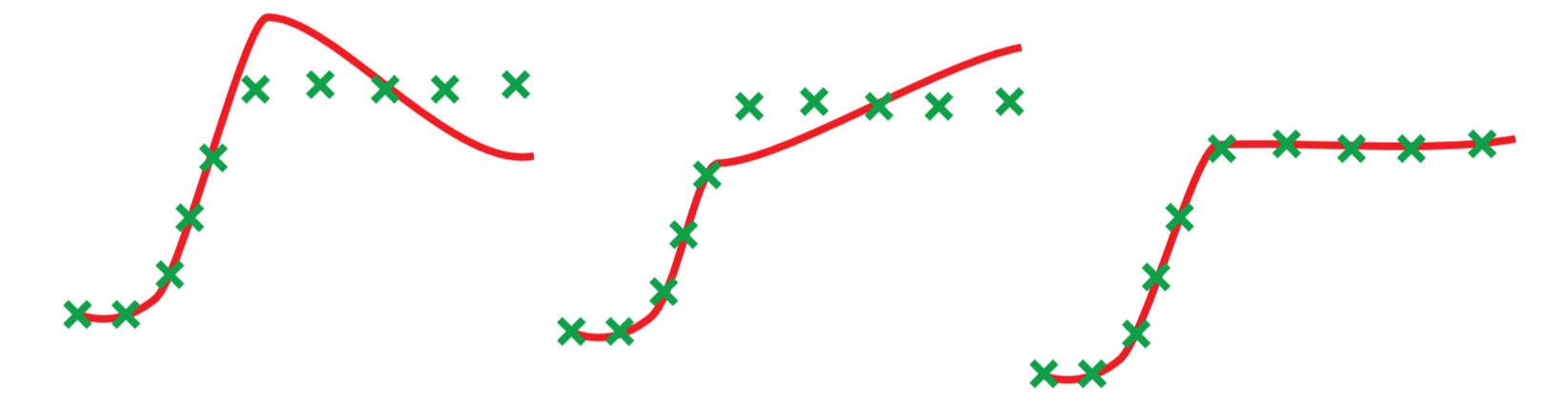
Calibration of 3  $k_n$ :

$$k(t) = \sum_{n=1}^3 k_n \epsilon_n$$



#### Drawbacks of WOF:

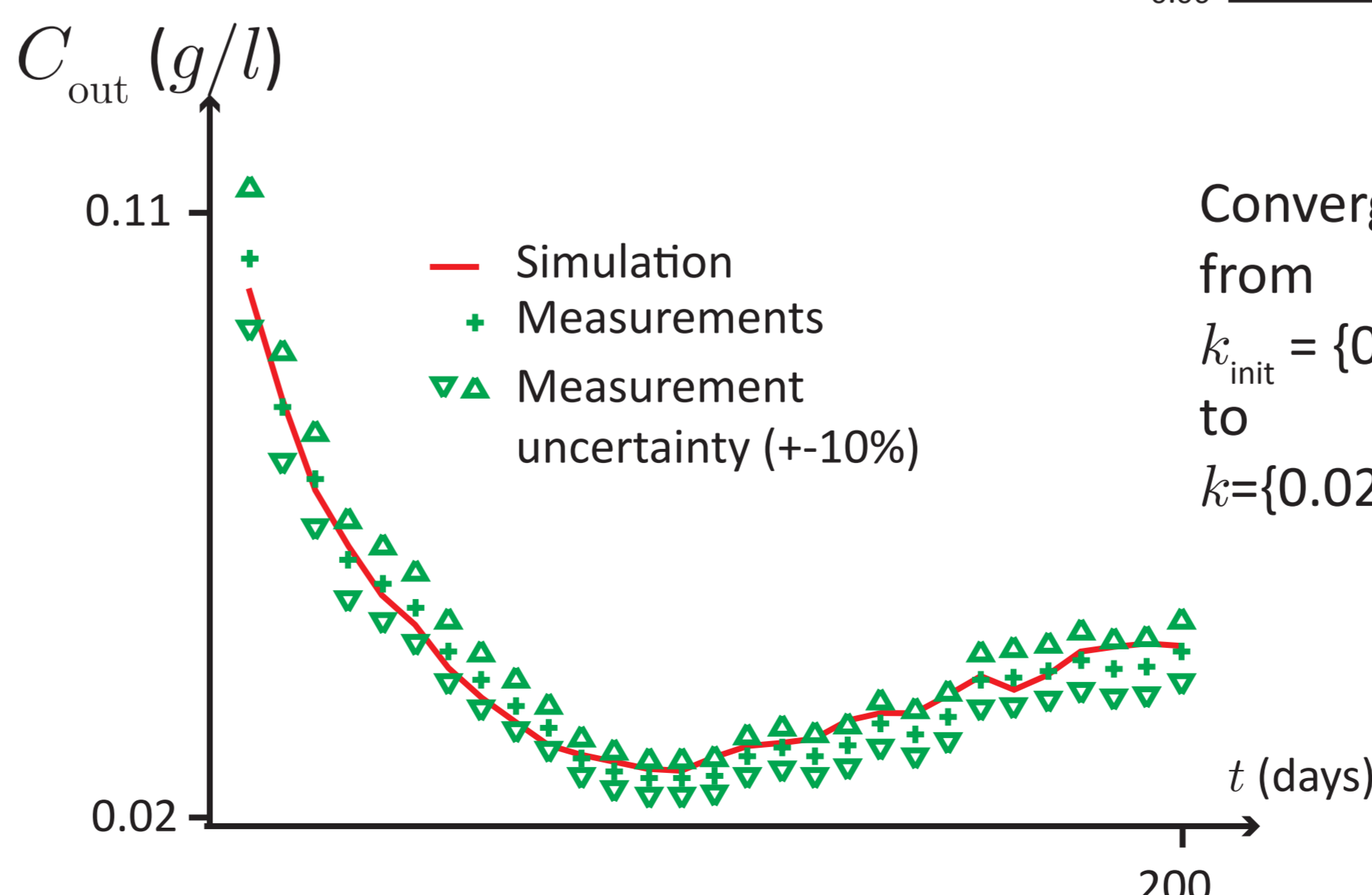
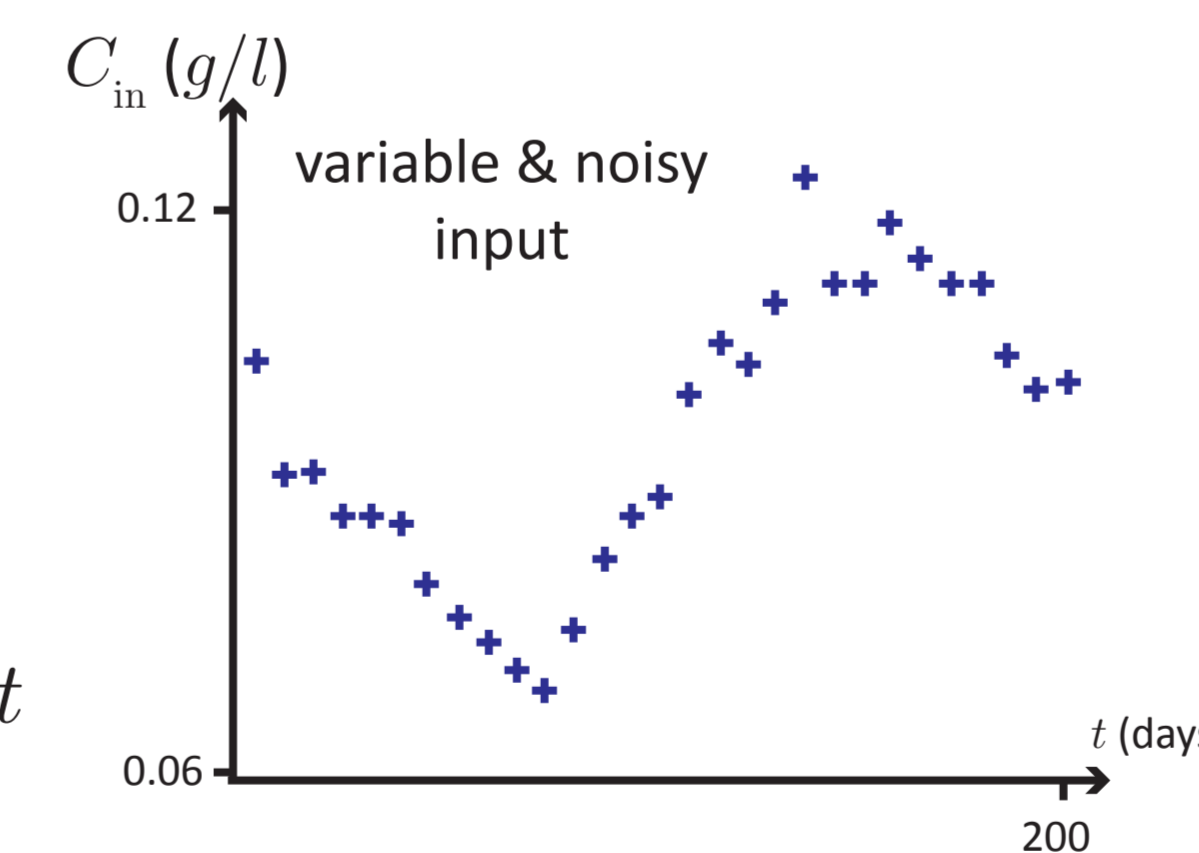
several values of ( $k_1, k_3$ ) give acceptable values of  $W_p$  (error compensation):



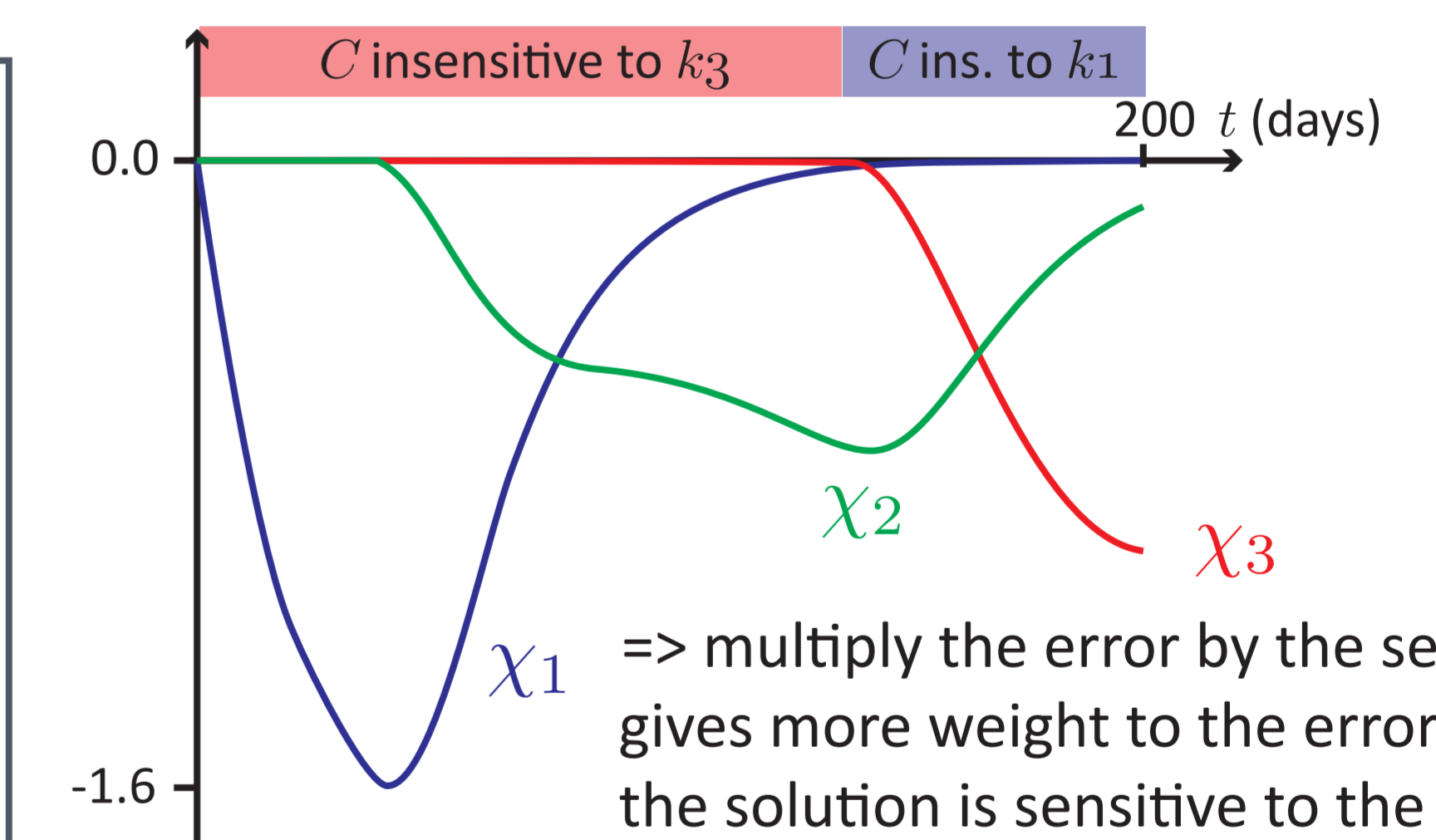
Each parameter calibrated using all the measurements BUT a parameter should not be calibrated using non-sensitive output for that parameter

Test 2 Generation of «observed» values with variable, noisy input and «measurement» uncertainty

$$\begin{aligned} C(x,0) &= 0.1 \text{ g/l} \\ u(x,t) &= 25 \pm 10\% \text{ (m/day)} \\ k_n &= \{0.02; 0.05; 0.06; 0.04\} \end{aligned}$$



Convergence in 12 iterations from  $k_{init} = \{0.042 \ 0.042 \ 0.042 \ 0.042\}$  to  $k = \{0.025 \ 0.048 \ 0.055 \ 0.044\}$



$\Rightarrow$  multiply the error by the sensitivity gives more weight to the error when the solution is sensitive to the parameter and avoid compensations.

### Conclusions

- Weak form-based OF proves easier to use than distance-based OF in practice
- It is well-admitted that calibration and validation should be carried out using different data sets; we may also admit that they should be carried out using different types of OF!
- Sensitivity analysis can help select measurements to be used for calibration of any given parameter

Reference: [1] V. Guinot, B. Cappelaere, C. Delenne and D. Ruelland, Towards improved criteria for hydrological model calibration: theoretical analysis of distance- and weak form-based functions, *Journal of Hydrology*, in press.