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# Comparing Self-Healing Properties of Polarized Weakly-Diffracting Beams

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Abstract—In this work, the self-healing properties of radially and linearly polarized weakly-diffractive beams are compared. The so-called self-healing degree is used as a metric. The studied system is composed of a circular radiating aperture and its radiated fields are perturbed by a circular metallic disk. The analysis is conducted by employing a spectral Green's function approach and a Physical Optics framework to investigate the impact of different obstacle positions and sizes.

Index Terms-component, formatting, style, styling, insert

#### I. INTRODUCTION

Bessel beams [1] are electromagnetic beams that do not undergo diffraction while propagating in free space. These beams, mathematically described by Bessel functions, can be understood as the combination of plane waves with wavevectors lying on a cone [2]. Additionally, Bessel beams present the property of reconstructing their initial intensity profile after encountering an obstacle, a property known as selfhealing [3].

In real life scenarios, weakly-diffractive Bessel beams are generated using finite apertures, which limit their nondiffractive behavior to a defined range, known as the nondiffractive range (NDR). The NDR is situated within the Fresnel region [2] and the self-healing property is also verified within this region. Examples of a Bessel beam launchers were reported in [2], [4].

This research investigates polarized weakly-diffractive Bessel beams radiated by circular apertures, in the case of partial obstruction. Different obstacle positions are considered to provide a comprehensive assessment. The studied vector weakly-diffractive beams [5] have two different polarizations: linear and radial. Considering the coordinate system (x, y, z)and  $\rho = \sqrt{x^2 + y^2}$ , the linearly and the radially polarized field distributions over the apertures can written as equations 1 and 2, respectively [6]:

$$\vec{E}(\rho, z=0) = -iBk_1 J_0(k_1\rho)\hat{x} - B\frac{k_1^2}{k_z} J_1(k_1\rho)\hat{z} \quad (1)$$

$$\vec{E}(\rho, z=0) = i2Ak_1J_1(k_1\rho)\hat{\rho} - A\frac{k_1^2}{k_z}J_0(k_1\rho)\hat{z}$$
(2)

where  $\vec{E}$  is the electric field,  $J_n$  is the Bessel function of first kind and n-th order,  $k_1$  is the transverse propagation constant, and A and B are arbitrary amplitude coefficients.

It is assumed that the beams are transverse magnetic with respect to the z coordinate (TM<sub>z</sub>).

#### **II. THEORETICAL DEVELOPMENT**

A system composed of one radiating aperture and a circular metallic disk is our object of study. The obstacle is placed parallel to the source and our analysis will be restricted to the components of the field that are transverse to the direction of propagation. The problem is analyzed resorting to a scattered-field formulation [5]. Therefore, we divide the fields in three categories: incident  $(\vec{E}^{in})$ , scattered  $(\vec{E}^{s})$ , and total  $(\vec{E}^{tot})$ . While the incident fields are radiated by the apertures with the obstacle absent, the scattered fields are the ones radiated by the currents over the obstacle surface radiating in free space. The total fields are then defined as the sum of incident and scattered fields. Furthermore, the currents induced over the obstacle surface due to the source are derived using a Physical Optics approach [7]. We use the equivalence theorem and image principle to derive the currents over the radiating aperture [8]. The radiated fields, both incident and scattered, are derived using a spectral free space Green function approach [9].

To evaluate the self-reconstruction properties, we employ the self-healing degree as a metric. Aielo et al. [3] were the first to propose a degree of self-healing as a means to quantify the self-reconstruction ability of general beams. In their publication, this metric was used to evaluate the selfreconstruction properties of a Gaussian beam. This approach involves comparing the amplitudes of the original beam with those of the obstructed one, rather than comparing their intensities. This degree, denoted as  $D_{sh}$ , can be understood as a relative distance between the incident and total fields in a defined region, S. This region is limited by the obstacle dimensions and lies on the plane transverse to the direction of propagation. The self-healing degree is written as:

$$D_{sh} = 1 - \frac{||E_t^{in} - E_t^{tot}||}{||E_t^{in}|| + ||E_t^{tot}||}$$
(3)

$$||f|| = \left(\int_{S} f^{*}(x, y, z) f(x, y, z) \, dS\right)^{1/2} \tag{4}$$

where the subscript t indicates the tangential component.

#### **III. NUMERICAL RESULTS**

The source center is positioned on the origin of the coordinate system and the obstruction is placed parallel to the source. The formulation used is frequency independent. The dimensions of the problem are expressed in terms of the wavelength ( $\lambda$ ) and the aperture radius is set to  $25\lambda$ . We define case A as the one in which the aperture radiates a linearly polarized beam while case B is the one related to the radially polarized one. For each case, the obstacle is placed at different positions, whether axially aligned with the radiating apertures or not. Note that the transverse propagation constant is selected to minimize diffraction [10]. As a result,  $k_1$  in case A is different from  $k_1$  in case B.



Fig. 1: Amplitude of the normalized total electric field propagating along z in arbitrary units. The fields are normalized by the maximum amplitude of the incident field. The obstruction is placed at  $(x = 10\lambda, y = 0, z = 57\lambda)$ , the chosen aperture radius is  $25\lambda$ , and the NDR is  $453\lambda$  for (a) and  $385\lambda$  for (b).

The total electric fields along z in both cases is shown in Fig. 1, where we notice a shadow region immediately after the obstruction. The minimum distance required for the beam to reconstruct itself is known as the shadow distance [11]. The self-healing degree as functions of the obstacle radius and the distance of its center from the z-axis is depicted on Fig. 2, where  $r_o$  is the obstruction radius and  $r_a$  is the aperture

radius. The self-healing degree is evaluated at  $z = 226.5\lambda$ and  $z = 453\lambda$  for cases A and B.



Fig. 2: Self-healing degree relation to the obstacle radius  $(r_o)$  and the distance of the obstacle's center to the z-axis is depicted in (a) and (b) respectively. In both cases, the obstacle is placed at  $z = 113\lambda$  and the self-healing degree is evaluated at  $z = 226.5\lambda$  (dashed line) and at  $z = 453\lambda$  (solid line). The chosen aperture radius,  $r_a$ , is  $25\lambda$ .

In Fig. 2a, the obstacle is axially aligned to the aperture and placed  $z = 113\lambda$ . We observe that the self-healing degree decreases as the obstruction size increases. Case B exhibits a higher self-healing degree when compared to A considering the same evaluation distance. This is not valid only for  $0.6r_a \leq r_o \leq 0.85r_a$  at  $z = 226.5\lambda$ , and  $0.6r_a \leq r_o \leq 0.7r_a$  at  $z = 453\lambda$ .

In Fig. 2b, the obstacle is placed  $z = 113\lambda$  and we evaluate its impact on self-healing degree as a function of its distance from the z-axis (offset), with  $r_o = 0.3r_a$ . We observer that, again, Case B exhibits a superior self-healing degree when compared to case A when evaluated at the same distance. Furthermore, there is a common region for both cases, ranging from  $5\lambda \leq \text{offset} \leq 25\lambda$ , where the self-healing degree shows an oscillatory behavior.

#### IV. CONCLUSION

We have reported the evaluation of the self-healing degree of  $TM_z$  linear and radially polarized weakly-diffracting vector beams, studying their response to obstacle radius and distance from the z-axis. As expected, larger obstacles reduced self-healing, while increasing the offset generally improved it, though with oscillations for offsets between  $15\lambda$  and  $25\lambda$ . Finally, its worth mentioning that radial polarization demonstrated a higher self-healing degree in the situations we considered.

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