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In this paper we study gender equality in the context of students' group work in mathematics at tertiary level. We use a specific framework, combining the Anthropological Theory of the Didactics and the Joint Action Theory of the Didactics, to examine students' interactions and their roles in a group in relation with the knowledge at stake. Analysing the case of three students in the context of a project-based course, we investigate epistemic symmetry (the equivalence of the roles) according to the students' gender at the macro-level of this course, the meso-level of a session and the micro-level of a given episode. We observe differences at each of these levels in terms of tasks and of gestures performed by each student.

Keywords: Gender equality, students' group work, topos, topogenetic position.

The theme of gender and mathematics has been extensively researched (Leyva, 2017), in particular at tertiary level (e.g., Laursen et al., 2014; Reinholz et al., 2022). Nevertheless, it has received little attention so far in CERME TWG14. One explanation for this scarcity could be that gender equity is considered as a sociopolitical issue, not connected with mathematical knowledge. Indeed many studies investigating gender in mathematics education use quantitative methods with a large number of students, thus they cannot consider precise mathematical contents. However some studies also develop qualitative methods, and identify gendered relationships with the learning of mathematics, linked with the knowledge at stake (Jungwirth, 1991; Barnes, 1998). Our study adopts such a perspective. We focus on students' group work at tertiary level, in the context of a project-based course. Our aim is to investigate and compare the role of each student in relation with the knowledge at stake, to determine whether these roles are equivalent, and whether the differences that emerge may be gender-related.

In the next section, we briefly synthesise background literature and situate our work against this background. We introduce then the theoretical construct we propose, the context of our empirical study and the methods we use. We present our results and discuss these results and the theoretical contribution of our work.

Background

The theme of gender and mathematics has been extensively investigated (see Leyva, 2017 for a recent survey), and the complexity of the gender concept is now acknowledged by research. In this study, we refer to gender as a social construct, and more precisely "a dynamic social construct performed differently across contexts and individuals". (Leyva, 2017, p. 398). We use the expression "gender equality", referring to Collet who considers that there is equality in a teaching-learning situation if the pedagogical environment makes it possible "to establish for everyone a

relationship to knowledge that is as independent as possible of social relationships". (Collet, 2021, p. 5). We claim that specific theoretical and methodological tools are required for analysing the gender equality in a given situation.

Our study belongs to "equity research", defined by Adiredja and Andrews-Larson (2017) as: "research that explicitly focuses on efforts to understand and mitigate systemic differences in ways that people experience and are afforded educational opportunities". (p. 446). This research orientation is developing at university level. In particular, studies about Inquiry-Based Learning or Inquiry-Oriented Instruction (IBL or IOI) approaches investigate a possible gendered impact of student-centred teaching approaches. Laursen et al. (2014) collected data in 100 courses sections, including 42 IBL courses sections. They observed that in the IBL courses, the women's confidence in their ability to do mathematics increased more than the men's. The opposite was observed in the non-IBL courses: women's confidence decreased substantially, compared to a minor decrease for their male classmates. Reinholz et al. (2022) made a statement that can seem to contradict the previous one: comparing IOI classes with non-IOI classes, they found a greater gender inequity-in terms of performance- in favour of men in the IOI classes. Assuming that this may be a consequence of participation patterns in the IOI classes, the authors studied the rate of participation of women and men in these classes, and the nature of this participation. They evidenced that the rate of participation was a predictor of performance; moreover, in some of the classes inequitable opportunities for participation were offered to women and men.

This study evidences the importance for gender equality of the classroom interactions. Jungwirth (1991), using a symbolic interactionism perspective, analysed the teacher-students interactions in secondary school mathematics classes. Her focus was not on the quantity of interactions, but on their nature. She identified gender-specific practices in these interactions, which made it more likely for boys to participate successfully. These practices directly impacted the mathematical activity of the students, and their relation with the mathematics content at stake. Barnes (1998) studied the interactions within groups of students working in an inquiry approach at secondary school. She used a post-structuralist perspective focusing on the roles played by the students in the discourse and on the power relations it produces. Through precise analyses of the discourse within groups of students, she evidenced that the collective inquiry offered opportunities for female students, who were enabled to "position themselves in ways which are empowering for them" (Barnes, 1998).

In our study, like Barnes (1998), we focus on the work of groups of students; the teacher does not intervene. We consider how the work is shared and also analyse the interactions concerning the mathematical knowledge at stake. Considering that the group work takes place within a given institution, and that this institution shapes the mathematical knowledge, we developed a specific framework that we introduce in the next section.

Proposition of a theoretical construct

By networking theoretical elements coming from two germane theories: the Anthropological Theory of the Didactics (ATD, Chevallard, 1999) and the Joint Action Theory of the Didactics

(JATD, Sensevy & Mercier, 2007), we propose a theoretical construct providing tools for identifying and understanding gendered mathematical practices in the context of students' collective work.

ATD considers that any human being is a subject of multiple institutions. These institutions shape the relations of their subjects to knowledge by proposing *praxeologies*. A praxeology comprises a type of tasks (a set of precise tasks) T, a technique τ to accomplish this type of tasks, a technology θ which is a discourse explaining or justifying the technique and a theory Θ. [T, τ] constitute the praxis part of the praxeology and [θ, Θ] the logos part of it. In the context of mathematics at tertiary education, the subjects are students or teachers (our focus being here on the students); the types of tasks can be mathematical, like "proving that a matrix is invertible", but also organisational, like "coordinating the work of a team". In ATD, the set of the tasks that a student performs by themselves is called the *topos* of this student. The concept of *topos* describes the responsibility of a student as a dynamic area (within the different praxeologies present in the classroom). The *topoi* of the students and of the teacher are interdependent, they develop in the classroom along a process named *topogenesis* (Chevallard, 1985). ATD also considers *cooperative* tasks, involving several subjects of the institution. For a cooperative task a subject performs *gestures*, which refer to the set of means that this subject uses to accomplish the task (Chevallard, 1999).

The *topogenesis* concept has been further developed in the Joint Action Theory of the Didactics (JATD, Sensevy & Mercier, 2007). JATD is a socio-cultural theory devoting a central place to the joint construction of knowledge by the teacher and the students in class, interacting within a *milieu*, encompassing material objects, signs, and knowledge. Sensevy and Mercier introduced in particular the concept of *topogenetic position*, described as the symbolic place occupied in the classroom by a student or a teacher, in relation with the knowledge at stake. These authors distinguish between high and low *topogenetic positions*, according to what they call the *epistemic density* of the task performed by a subject, i.e., the potential of the task in terms of modification of the milieu. In our study concerning cooperative tasks, in order to characterise the students' topogenetic positions, we consider the gestures performed by each student to accomplish the task.

We contend that investigating the topoi and the topogenetic positions of female and male students working together can contribute to an analysis of gender equality. We complement these theoretical elements by the concept of *epistemic symmetry* introduced by Gerin (2021). In her investigation of teaching strategies promoting gender equality (in the context of writing at primary school), Gerin, referring to JATD, considers that *epistemic symmetry* is achieved when the female students and the male students working together have the same responsibility towards knowledge. Thus the research question we study here is: In the context of a project-based course, does the students' group work have the characteristics of epistemic symmetry, i.e., are the topoi and topogenetic positions the same for male and female students?

Case study

Context and methods

Our study is situated in France, in the context of the first year of scientific preparatory classes (preparing students for highly selective engineering schools). In the context of a more general project concerning gender equality in such classes, we considered in particular students' cooperation in a project-based course named « supervised personal interest work » (TIPE in French). Teams of students perform during the second semester a collective project involving one or several scientific disciplines (mathematics, physics, engineering, computer science). The teachers do not propose projects, the students are free to choose any topic related to the subjects they are studying. They must then define a problematic and study it both theoretically and experimentally. When the topic is related to mathematics, the experiments usually consist of numerical simulations. For 16 weeks, two hours per week are devoted to this course, the physics and mathematics teachers are present and can be asked to answer a question or explain a new knowledge. At the end of the semester, all groups present their work orally in front of the other students and the teachers, using a slide show.

We presented our research project in two classes (we did not mention our focus on gender equality to avoid influencing the students' behaviours). Four teams of students volunteered for participating in the project; we selected the two teams comprising male and female students, and followed them along the semester. Each team recorded weekly in a logbook shared with the researcher what each of them had done during the session, which resources they had used and possibly what they planned for the next session. Moreover, two sessions were video-recorded and transcribed, the first time after two months of collective work, and the second time one month later. For the sake of brevity, we focus here on one group of three students: two female students, Alice and Clara, and a male student, Thomas (pseudonyms) who chose to work on neural networks and classification. The group was first formed on the basis of a personal affinity between Clara and Alice, then Thomas joined out of interest in the chosen theme. We selected this group for the richness of their notes in the logbook. The first session recorded took place before they reached a clear delimitation of their problematic; for the second session recorded, the problematic was quite clear and the group was working on the numerical experiment.

We distinguish between three levels in our analysis: the macro-level of the whole project-based course; the meso-level of one session; and the micro-level of episodes where the students interact around a type of tasks. At the macro-level, we focus on the students' topoi, and triangulate data from the logbook and the video-recording of the sessions. At the meso-level, we use the video-recording to investigate both the topoi and the topogenetical positions by characterising each student's gestures and classifying them for a given session. We distinguish between high-position gestures (explain, present, fix, solve, articulate with old knowledge), neutral gestures (nod, approve), low-position gestures (ask a question, expose some difficulty). This classification is so far exploratory, it will be refined in further studies. At the micro-level, we analyse the students' discourse and refine the analysis of the topogenetical positions. For each level, we compare the topoi and topogenetical positions according to the gender of the students.

Results

We firstly present our analyses at the macro-level, drawing mainly on the logbook. During the beginning of the course, the students search for resources, explore different directions before choosing a problematic. From that moment on, the types of tasks to be undertaken by the group have a different nature, digital experimentation takes an important part, and the search for resources becomes secondary. Nevertheless we can observe a difference between male and female students' topoi, as illustrated by the following extract of the logbook for May 12, 2022 (our translation).

Thomas:	Creation and implementation of neurons and of the genetic algorithm.
Clara:	I have continued searching how to create generations. The PhD Theses and scientific papers are a bit lacking in concreteness so for the next sessions I will code neural networks with Thomas. However these researches will allow me to mathematically justify the result of the algorithm.
Alice:	research of the optimal number of neurons for the network, result: there is no predefined recipe for the number of neurons, so we try with one hidden layer. + Watching a 'machine learnia' video about programming a two-layer neural network.

Thomas focuses on the praxis part, mainly the implementation of the algorithms, whereas Alice and Clara continue to look for mathematical justifications and optimisation for these algorithms, taking the responsibility for the logos part. This observation also holds for other parts of the logbook, and is confirmed by the video-recording of the second session. A possible explanation is that Thomas is considered an expert in coding, which is close to gender stereotypes. This choice leads Alice and Clara to confront the widely open question of justification and optimisation of these algorithms, which explains the difficulties they encounter. Eventually they give up on justifying the algorithms and choose to help implement them.

We now turn to the meso-level and focus on the first video recorded session. It can be divided into several parts: during the first part, Thomas briefly exposes his difficulties with the notion of gradient, then Alice tells her classmates about a video she watched during the previous session, and presents the numerical tests she implemented. The video and the tests regard the *k-means method* for unsupervised classification. The second part of the session consists of a collective reflection on the directions to give to the work, and the tasks and objectives to be distributed within the group. In the third part, the three students work individually on their own objective, mainly watching or reading resources they found on the internet. At the very end of the session, the students interact with the teacher, explain what they have done and their blocking points.

At the meso-level of the whole session, we listed all the students' gestures and classified them in terms of high-position, neutral or low position. The result is presented in Figure 1.

This figure shows the epistemic asymmetries between male and female students. Although the major episode of part 1 is devoted to Alice explaining the k-means method and presenting her numerical experiments to her classmates, paradoxically her topogenetic position according to her gestures is globally lower than Thomas'. For both female students, 40% of their gestures are associated with a low position, whereas only 10% of Thomas' gestures are low-position gestures.

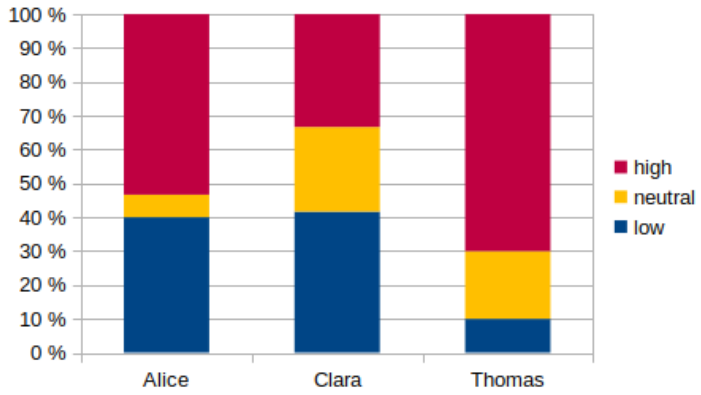

Figure 1: Analysis of gestures with respect to knowledge

In order to refine these observations, and link them with the mathematical content, we now focus at a micro-level on the first part of the session and compare Alice's and Thomas' topogenetic positions when they report about what they did in the previous session.

Thomas exposes his difficulties in terms that seem to discourage his classmates to help.

Thomas: I had begun to compute the gradients, but uh… it… it doesn't lead anywhere in the sense that… it leads to things that are not homogeneous, I mean I multiply matrices but with the wrong sizes and all.

Clara (seems impressed): Oh, it's matrices?

Thomas: Yes indeed, because I compute gradients of matrices with respect to matrices. [Clara nods.] You know, since we vectorialised the equations at the very beginning. You know, you put you know the gradients of capital X, capital W, and all.

The concept of gradient has not been introduced yet by the mathematics teacher, and Thomas presents it as a very complicated concept, whereas in reality he has to compute gradients of a scalar function of two variables. At this stage, Alice gets out of the discussion, she no longer looks at her classmates and tries to focus on her computer's screen. Clara tries to concentrate and understand Thomas' indications but it seems she can't figure out how to help him.

A few minutes later, Alice presents what she has understood of the k-means method. Thomas and Clara listen to her and nod regularly, showing their interest and understanding. Thomas tries to link this method to an algorithm called "nearest-neighbour algorithm", previously encountered in an assignment, Alice confesses she did not envisage this link. She goes on presenting her numerical tests, but her computer is very slow and after a while the results of the test are not displayed, leading Alice to express doubts.

Alice: Normally, normally it displays uh… that's strange, why doesn't it want to display? Ah. It doesn't want to. Because normally it displays the graph with the evolution of inertia with respect to the number of groups you make. […] OK, that's it. Wait, because there… Oh yes. Wait. It mixes both… Uh… OK. Wait, that's strange, it seems it still displays the other one.

When these difficulties occur, Thomas gives instructions to help Alice fix the problem.

Thomas: Suppress the one you have just had. Close it.

His instructions happen to be inefficient and finally Alice modifies her code and finds by herself the solution of the problem.

In this episode we see that Thomas adopts a high topogenetic position, by linking new knowledge with old knowledge, and by trying to fix Alice's difficulties. Thus at this micro-level we also conclude that there is no epistemic symmetry between female and male students for this group.

Discussion and further directions

In the analyses presented above, we evidenced different kinds of epistemic asymmetries in the group of students followed. At the macro-level of the whole course, the male student was in charge of the programming, praxis aspect, while the female students started by working on the justification of the algorithm, the logos aspect, with an *a priori* high epistemic density. Nevertheless it turned out to be so difficult that eventually their actual task became to assist with programming. At the meso-level of the session, we observed the asymmetry in terms of percentages of low-position and high-position gestures. At the micro-level of the episode studied, we identified asymmetries between Alice and Thomas during their discussions about the work each of them has conducted. Thomas adopts from the start a high topogenetic position. When presenting difficulties he met, he emphasises the complexity of the concepts he used: gradient, matrices. Although this might not be conscious, we interpret this emphasis on complexity as a way to discourage the female students to offer their support. When Alice is in a high position, presenting the k-means method, Thomas tries to appear equally high, by foregrounding his understanding of her discourse. He links this method with previous knowledge and offers his help. We do not claim that these observations can be generalised to all the groups in this class or in other classes proposing project-based courses. We do not even claim that gender was the factor explaining the differences observed. Other factors like the perceived ability of the student in the class can impact the interactions during the work in groups. Our aim here was to investigate the epistemic symmetry between male and female students in the case studied. Identifying the causes of the differences observed requires a further study.

Like Jungwirth (1991), we observe that the interactions-between students in our case-are central in the learning of mathematics. We also align with Barnes (1998) about the power relation produced by discourses. The combined use of ATD and JATD provides us with conceptual tools for studying these interactions and power relations, and for linking them with the mathematical content. At the macro-level, a power-relation can result from the fact that some students have a praxis-centred topos, while others are logos-centred, thus with more difficult tasks. This leads to unequal opportunities to develop useful skills. At the micro-level, we noted that a male student can exaggerate the complexity of the mathematical content in order to hinder a cooperation in which his topogenetical position might be low.

In our further work, we intend to investigate further the theoretical construct introduced here, to complement and refine it. One important direction concerns the determination of the topogenetical positions. Focusing on cooperative tasks, we looked at gestures and introduced an exploratory classification, considering for example that "explain" was a high-position gesture and "expose a difficulty" a low-position gesture. Nevertheless a systematic classification is missing, and the link of this classification with the mathematical content at stake also requires a further investigation.

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