



HAL
open science

Classical Core Logic, Relevance, and Substructurality

Pierre Saint-Germier, Peter Verdée

► **To cite this version:**

Pierre Saint-Germier, Peter Verdée. Classical Core Logic, Relevance, and Substructurality. 2024. ⟨hal-04382596⟩

HAL Id: hal-04382596

<https://hal.science/hal-04382596v1>

Preprint submitted on 9 Jan 2024

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.



HAL Authorization

Classical Core Logic, Relevance, and Substructurality

Classical Core Logic (CCL) is a system put forward by Tennant (2017) for relevant deductive reasoning. Like many relevant logics, **CCL** achieves relevance by restricting the structural rule of Weakening. Unlike relevant logics in the **R** family, it blocks the Lewis argument for explosion not by rejecting Disjunctive Syllogism, but by rejecting unrestricted Cut:

$$\frac{A \vdash A \vee B \quad A \vee B, \neg A \vdash B}{A, \neg A \vdash B} \text{ (Cut)}$$

Like **R**, and unlike Meyer's **LR** [2] (often presented as the most natural relevant substructural logic), it validates the distribution of conjunction over disjunction.

Although **CCL** deserves the title of substructural logic, if only because it restricts Weakening and Cut (as pointed out recently by Tennant himself [4]), this arguably remains a somewhat superficial characterization. It is often pointed out that interesting substructural logics need to preserve the operational rules of the logic the structural rules of which are being restricted. From this point of view, **CCL** behaves in a rather unusual way. The goal of this paper is to bring out the unusual features of Classical Core Logic qua substructural logic, and provide a principled explanation for them, based on the specific understanding of the notion of relevance that Classical Core Logic explicates. The upshot is that Classical Core Logic is interesting qua Substructural Logic, although not for the usual reasons.

First, we provide an MSET:MSET sequent calculus for **CCL**, which avoids a non-standard reading of the comma in the published SET:SET and SET:FMLA₀ and presentations of Tennant's [5, 3], and facilitates the comparison with familiar MSET:MSET sequent calculi for Classical Logic (**CL**). One striking feature of this MSET:MSET calculus is that the rules for conjunction and disjunction are *unassorted* in the following sense: disjunction is multiplicative on the left and additive on the right, while conjunction is additive on the left and multiplicative on the right. This unassortedness is in fact crucial to the validation of disjunctive syllogism and the failure of Cut. Another striking feature is that the conditional validates only one half of the residuation property, which suggests that in **CCL** a conditional formula $A \rightarrow B$ is nothing more than an abbreviation for $\neg A \vee B$.

(R \Rightarrow) $A, B \vdash_{CCL} C$ only if $A \vdash_{CCL} B \rightarrow C$

Of course, it is always possible to add a new conditional to **CCL** for which the residuation property holds in both directions, but then one loses the basic property of **CCL** that whenever $\Gamma \vdash_{CL} \Delta$, then $\Gamma' \vdash_{CCL} \Delta'$ for some $\Gamma' \subseteq \Gamma$ and some $\Delta' \subseteq \Delta$. (Counterexample: $\vdash_{CL} p \rightarrow (q \rightarrow p)$.)

Second, building on this insight about the unassortedness of the rules for conjunction and disjunction, we provide a closure frame semantics for **CCL**,

which is essentially a special case of a semantics described in [1] for cognate systems of non-transitive relevant entailment. What is unusual here is that a closure frame semantics is adequate for a *distributive* logic. Once again, the assortedness of the rules for conjunction and disjunction is responsible for this, for what is expressed by the sequent

$$A \wedge (B \vee C) \vdash (A \wedge B) \vee (A \wedge C)$$

may be understood as

$$A \sqcap (B \oplus C) \vdash (A \otimes B) \sqcup (A \otimes C)$$

which is valid in LR^{ND} with additive and multiplicative conjunctions (\sqcap, \otimes) and disjunctions (\sqcup, \oplus).

Third, the general picture that emerges from this proof-theoretic and semantic analysis of **CCL** is a commitment to premise/conclusion relativism, i.e., the view according to which connectives may receive different interpretations depending on whether they occur as premises or conclusions in a sequent. If one takes a classically valid sequent to be relevant whenever all formulas make a contribution to the validity of the sequent, conjunction will have to behave extensionally on the left, but *intensionally* on the right: from the fact all formulas make a contribution in $\Gamma \vdash A \wedge B, \Delta$, it does not follow that all make a contribution in $\Gamma \vdash A, \Delta$ (compare $A, B \vdash A \wedge B$ and $A, B \vdash A$). Dually, disjunction has to behave extensionally on the right, but intensionally on the left: from the fact that all formulas make a contribution in $\Gamma, A \vee B \vdash \Delta$, it does not follow that all make a contribution in $\Gamma, A \vdash \Delta$ (compare $A \vee B \vdash A$ and $A \vdash A, B$). This accounts for the peculiar features of **CCL** when seen as a substructural logic, which makes **CCL** an interestingly peculiar substructural logic, if not an standardly interesting one.

References

- [1] [Anonymized]. “Truthmakers and relevance for FDE, LP, K3, and CL”. In: *Outstanding Contributions to Logic: Kit Fine*. Ed. by Frederick Van De Putte Federico Faroldi. Springer, forthcoming.
- [2] Robert K. Meyer. “Topics in Modal and Many-Valued Logics”. PhD thesis. University of Pittsburgh, 1966.
- [3] Neil Tennant. *Core Logic*. Oxford, England: Oxford University Press, 2017.
- [4] Neil Tennant. “On the Adequacy of a Substructural Logic for Mathematics and Science”. In: *Philosophical Quarterly* 72.4 (2022), pp. 1002–1018. DOI: 10.1093/pq/pqac001.
- [5] Neil Tennant. “Perfect Validity, Entailment and Paraconsistency”. In: *Studia Logica* 43.1-2 (1984), pp. 181–200. DOI: 10.1007/BF00935749.