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Secrecy energy-efficient multi-user NOMA: closed-form solution to bi-criterion formulation

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Abstract—This paper investigates the secrecy energy-efficiency (SEE) of a multi-user downlink non-orthogonal multiple access (NOMA) transmission in the presence of a passive eavesdropper. The SEE, capturing the trade-off between the achievable secrecy sum-rate and the power consumption, is here formulated as a bi-criterion optimization problem that is shown to be convex. As such, the characterization of the Pareto-boundary boils down to maximizing the scalarized trade-off between the secrecy sum-rate and the power consumption, which is solved in closed-form. Finally, this obtained closed-form is exploited to maximize the ratio between the secrecy sum-rate and the power consumption. Our numerical results highlight a significant relative SEE gain between NOMA and OMA.

I. INTRODUCTION

The unprecedented growth of deployed sensors in homes, cities, etc. turns smart homes and cities into reality. Thus, dealing with reliable denser networks while ensuring high data rates, efficient use of both spectral and power resources as well as security aspects becomes mandatory [1], [2].

Traditionally, *orthogonal multiple access* (OMA) techniques are used, where a single user could be served on each resource block. Despite obviously preventing interference, OMA is not well-adapted to wireless networks dealing with an ever increasing number of users, paving the transition to *non-orthogonal multiple access* (NOMA) techniques [3], [4]. Under NOMA, an arbitrary number of users can be served over the same resource block by using superposition coding at the transmitter side, whereas the receivers handle the interference thanks to successive interference cancellation (SIC) [3].

Further, due to the broadcast nature of the wireless medium, security becomes a vital feature of wireless networks. While cryptography-based security techniques rely on the computational capabilities of the attackers, under physical layer security confidentiality is ensured by exploiting dynamic features of the wireless medium itself, such as interference, noise or fading. In such a framework, the goal is to derive *secrecy rates*, an extension of regular ones measured by the rate gap between the rate achieved by the legitimate user and the one by the eavesdropper to decode the same message [5].

Besides increasing data rates, reducing the power consumption is a crucial requirement of the current and future generations of wireless networks. Since these two are conflicting goals, finding the optimal trade-off, termed as energy-

efficiency (EE), becomes ineluctable [6]. To cope with security aspects, EE has been extended to *secrecy energy-efficiency* (SEE) by considering secrecy sum-rates instead of regular ones [7].

Motivated by the above, this paper focuses on maximizing the SEE of a multi-user downlink NOMA transmission, in the presence of an eavesdropper. The SEE is here formulated as a bi-criterion convex optimization problem, similarly to [8], [9] where no eavesdropper was present. Besides the overall power budget constraint, each of the $K \geq 2$ user is required to meet a minimum Quality of Service (QoS) constraint expressed in terms of its achievable rate.

A. Related works

Recently, both EE and secrecy rate maximization for multi-user $K \geq 2$ downlink NOMA transmission have been widely investigated [10], [9], [11], [12], [13], [14], [15], [16], [17], but very few works consider SEE maximization for such networks.

To the best of our knowledge, the closest works to ours are [9], [18], both exploiting downlink NOMA to serve an arbitrary number K of users under an overall power budget as well as K individual QoS constraints. On the one hand, [9] focuses on the maximization of the EE defined as a bi-objective optimization problem in the absence of eavesdropper. The obtained closed-form solution is then exploited to optimize the more common EE metric defined as the ratio between the sum-rate and the power consumption in a computationally efficient manner requiring only a line search. On the other hand, [18] includes the presence of an eavesdropper and proposes a numerical solution that relies on the dichotomy method to maximize the ratio between the secrecy sum-rate and the power consumption.

Here, we propose to first formulate the SEE as a bi-criterion optimization problem that is solved in closed form, extending both the approach of [9] to include physical layer security considerations, and the metric considered in [18]. Similarly to [9], we then exploit our closed form solution to solve the same SEE metric in terms of the ratio between the secrecy sum-rate and power consumption by the mean of a line search instead of the dichotomy method, as initially proposed in [18].

B. Main contributions

Our main contributions, summarized below, are three-fold:
i) We prove the concavity w.r.t. the power allocation vector of the secrecy sum-rate of a multi-user downlink NOMA system,

This work has been carried in the context of the project Beyond5G, funded by the French government as part of the economic recovery plan, namely “France Relance” and the investments for the future program.

leading to a convex bi-criterion SEE optimization problem.

ii) We derive in closed form all optimal secrecy sum-rate vs. power consumption trade-offs that maximize the SEE defined as a bi-criterion optimization problem.

iii) We exploit this closed form solution to also maximize the ratio between the secrecy sum-rate and power consumption considered in [18] in a computationally efficient manner by requiring only a line search.

II. SYSTEM MODEL AND PROBLEM FORMULATION

This section presents the considered system model and the secrecy energy-efficiency optimization problem.

A. System model

The network under study is composed by a single transmitter (e.g. a base station (BS)), an arbitrary number of legitimate users $K \geq 2$ and a single passive eavesdropper. To serve an arbitrary number of users in a spectral efficient manner, NOMA is considered. As such, the transmitter employs superposition coding and broadcasts $X = \sum_{i=1}^K X_i$, where the message X_i of average power p_i is intended for the i -th legitimate user, $i \in \llbracket 1, K \rrbracket$. Then, the received signals at the k -th legitimate user and at the eavesdropper are respectively given as

$$Y_k = h_k \sum_{i=1}^K X_i + Z_k \text{ and } Y_e = h_e \sum_{i=1}^K X_i + Z_e,$$

where h_k and h_e denote the channel coefficient between the transmitter and the k -th legitimate user and the eavesdropper respectively; $Z_k \sim \mathcal{N}(0, \sigma_k^2)$ and $Z_e \sim \mathcal{N}(0, \sigma_e^2)$ are the Additive White Gaussian Noises (AWGN) at the k -th user and eavesdropper respectively. Throughout this paper, we assume that all noises are of unit variance, or, equivalently, we consider normalized channel gains defined as $\Gamma_k = \frac{h_k^2}{\sigma_k^2}$, $k \in \{\llbracket 1, K \rrbracket, e\}$. Without loss of generality, we assume that the channel gains are ordered as

$$\Gamma_1 \leq \dots \leq \Gamma_{M-1} \leq \Gamma_e < \Gamma_M \leq \dots \leq \Gamma_{K-1} < \Gamma_K, \quad (1)$$

i.e. $M-1$ out of the K legitimate users have weaker channel gains than the eavesdropper.

Following the superposing scheme adopted in NOMA at the transmitter side, the interference at the receiver side is handled via SIC. Thus, each legitimate user k first decodes the messages X_w , $w \in \llbracket 1, k-1 \rrbracket$, corresponding to users with weaker channel gains compared to user k , but suffers the interference of messages X_s , $s \in \llbracket k+1, K \rrbracket$, corresponding to users with stronger channel gains when decoding its own message X_k . Hence, the achievable rate R_k to decode the message X_k at the k -th legitimate user writes as

$$R_k(\mathbf{p}) = \frac{1}{2} \log_2 \left(1 + \frac{\Gamma_k p_k}{\Gamma_k (p_{k+1} + \dots + p_K) + 1} \right),$$

where $\mathbf{p} = (p_1, \dots, p_K)$ denotes the vector collecting the K allocated powers.

In the remaining of the paper, the eavesdropper belongs to the set of users served by the BS, but performs a passive attack undetected by the transmitter. As such, the eavesdropper can access the channel ordering and hence the SIC decoding order.

Hence, the achievable rate R_k^e to decode the message X_k at the eavesdropper writes as

$$R_k^e(\mathbf{p}) = \frac{1}{2} \log_2 \left(1 + \frac{\Gamma_e p_k}{\Gamma_e (p_{k+1} + \dots + p_K) + 1} \right).$$

Further, the considered network is constrained by an overall power budget \bar{P} , such that $\sum_i p_i \leq \bar{P}$, as well as by K individual minimum QoS constraints in terms of individual achievable rates expressed as $R_i(\mathbf{p}) \geq \bar{R}_i$, $i \in \llbracket 1, K \rrbracket$.

In order to simplify the derivations and improve the presentation of our results, we will use the following notations:

$$A_k = 2^{2\bar{R}_k}, \quad \theta_k = \sum_{i=k}^K p_i, \quad k \in \llbracket 1, K \rrbracket \text{ and } \theta_{K+1} = 0.$$

In this notation, the feasible set for satisfying the power budget and QoS constraints can be expressed as

$$\mathcal{P} = \left\{ \mathbf{p} \in \mathbb{R}_+^K \mid \theta_1 \leq \bar{P}, \theta_k \geq A_k \theta_{k+1} + \frac{A_k - 1}{\Gamma_k}, \forall k \in \llbracket 1, K \rrbracket \right\}.$$

B. Problem formulation

Energy-efficiency (EE), which captures the trade-off between the achievable sum-rate and the power consumption of a network, has been defined in several ways in the literature. It has been characterized as a bi-criterion optimization problem, maximizing the achievable sum-rate and the negative power consumption [8], [9]; as a scalarized trade-off between the achievable sum-rate and the power consumption [19], [9], [13] or as the ratio of the two [20], [9], [21].

Although different, all these metrics are linked to one another. Indeed, since the two objectives of the bi-criterion optimization problem are conflicting, i.e. maximizing the sum-rate requires to consume the entire power budget, whereas minimizing the power consumption requires to not transmit at all, its solution lies on the Pareto boundary of the feasible set of rate-power pairs. Further, if the sum-rate is concave w.r.t \mathbf{p} , finding the Pareto boundary reduces to optimize the scalarized trade-off [19] between the achievable sum-rate and power consumption. Finally, if the sum-rate is concave, maximizing the ratio between the sum-rate and the power consumption is a concave-convex fractional problem, whose solution is equivalent to find the unique zero of the scalarized trade-off between the sum-rate and the power consumption, which can be computationally efficiently computed via Dinkelbach algorithm [20], [22].

Since the network under study comprises a passive eavesdropper, we propose to extend the above mentioned EE metrics to SEE ones, by considering *secrecy sum-rate*.

Under passive eavesdropping, the main idea is to exploit the noisy nature of the channels to distinguish between the quality of the signal received by the legitimate users and by the eavesdropper. In fact, as long as the eavesdropper channel quality is inferior to the one of the legitimate users, secret communication can be achieved, where secrecy rates are defined as the difference between the legitimate user rate achieved and the eavesdropper rate to decode the same message. As such, for each legitimate user $k \in \llbracket 1, K \rrbracket$, its

secrecy achievable rate is expressed as

$$R_k^s(\mathbf{p}) = [R_k(\mathbf{p}) - R_k^e(\mathbf{p})]^+, \text{ where } [x]^+ = \max\{0, x\}.$$

Note that, given the considered channel order in (1), the $M-1$ weakest users achieve a zero secrecy rate, whereas all users $X_j, j \in \llbracket M, K \rrbracket$ are able to achieve strictly positive ones.

To summarize, the considered SEE optimization problem writes as a bi-criterion problem consisting in maximizing the secrecy achievable sum-rate $\sum_k R_k^s(\mathbf{p})$ and minimizing at the same time the power consumption, such that both the overall power budget and individual QoS constraints are met:

$$\text{(BiSEE)} \quad \max_{\mathbf{p} \in \mathcal{P}} \left(\sum_{k=1}^K R_k^s(\mathbf{p}); - \sum_{k=1}^K p_k - P_c \right),$$

where P_c is the circuit power consumption accounting for all blocks implemented at the transmitter and receiver sides.

III. CLOSED-FORM OPTIMAL POWER ALLOCATION POLICY

This section presents our main results. We first prove the convex nature of the considered SEE maximization problem expressed as a bi-objective optimization problem. We then provide a feasibility condition allowing to reach the optimal solution, and finally we derive the optimal closed-form power allocation policy, that is also exploited to maximize the SEE metric considered in [18], namely the ratio between the secrecy sum-rate and the power consumption.

A. Concavity of the secrecy sum-rate and equivalent scalarized trade-off maximization

Let us first investigate the optimization problem **(BiSEE)**. Using tools from convex optimization, we prove that the latter is convex, as stated in the following theorem.

Theorem 1. *Following from the concavity of the secrecy achievable sum-rate $\sum_{k=1}^K R_k^s(\mathbf{p})$ w.r.t. \mathbf{p} and from the linearity of the other objective function and constraints, the optimization problem **(BiSEE)** is a convex one.*

Proof: The proof follows similarly to [23] and is omitted here due to space limitation. The main steps are to first derive the Hessian matrix H of the secrecy sum-rate, and then to show that it is semi-definite negative. Finally, exploiting the linearity of the power consumption, as well as from the overall power budget and QoS constraints, the optimization problem under study is a convex one, which concludes the proof. ■

Since the bi-criterion optimization problem **(BiSEE)** is a convex one, finding the Pareto boundary reduces to maximizing the scalarized trade-off of the two objectives defined as

$$\text{(SEE)} \quad \max_{\mathbf{p} \in \mathcal{P}} \sum_{k=1}^K R_k^s(\mathbf{p}) - \alpha \left(\sum_{k=1}^K p_k + P_c \right),$$

where $\alpha \geq 0$ sweeps the entire Pareto boundary and allows to switch between a secrecy-driven optimization problem and a power consumption-driven one, by respectively choosing small or large values of α .

B. Closed form solution of the scalarized energy-efficiency optimization problem

Because of the K individual QoS constraints, the optimization problem may not allow a solution.

Proposition 1. [23, Theorem 2] *The optimization problem **(SEE)** is feasible if and only if the overall power budget of the transmitter \bar{P} is larger than P_{\min} , the overall minimum power required to fulfill the K individual QoS constraints with equality:*

$$\bar{P} \geq P_{\min} = \sum_{k=1}^K \frac{A_k - 1}{\Gamma_k} \prod_{j=1}^{k-1} A_j.$$

Since the SEE optimization problem **(SEE)** is convex, and provided that the above feasible condition is met, it allows a unique solution given in closed-form in the following theorem.

Theorem 2. *Provided that the feasibility condition of Proposition 1 is satisfied, the optimal power allocation policy maximizing the secrecy energy-efficiency **(SEE)** for the considered single eavesdropper downlink multi-user NOMA system is obtained in closed-form as follows:*

$$p_k^*(\alpha) = (A_k - 1) \left(\frac{1}{\Gamma_k} + p_K^*(\alpha) \prod_{i=k+1}^{K-1} A_i + \sum_{i=k+1}^{K-1} \frac{A_i - 1}{\Gamma_i} \prod_{j=k+1}^{i-1} A_j \right), k < K$$

$$p_K^*(\alpha) = \min \left\{ \max \left\{ \hat{p}(\alpha); \frac{A_K - 1}{\Gamma_K} \right\}; u \right\} \text{ with} \quad (2)$$

u given as $u = \frac{1}{\prod_{i=1}^{K-1} A_i} \left(\bar{P} - P_{\min} + \frac{A_K - 1}{\Gamma_K} \prod_{j=1}^{K-1} A_j \right)$ and $\hat{p}(\alpha)$, whose closed-form expression is given in Appendix A, denotes the admissible critical point of the following convex single variable optimization problem

$$\text{(SEE1)} \quad \max_p f(p) \text{ s.t. } \frac{A_K - 1}{\Gamma_K} \leq p \leq u, \text{ with}$$

$$f(p) = \frac{1}{2} \log_2(1 + \Gamma_K p)$$

$$- \frac{1}{2} \log_2 \left(1 + \Gamma_e \left(\prod_{i=M}^{K-1} A_i p + \sum_{i=M}^{K-1} \frac{A_i - 1}{\Gamma_i} \prod_{j=M}^{i-1} A_j \right) \right)$$

$$+ \frac{1}{2} \log_2 \prod_{i=M}^{K-1} A_i - \alpha \left(\prod_{i=1}^{K-1} A_i + \sum_{i=1}^{K-1} \frac{A_i - 1}{\Gamma_i} \prod_{j=1}^{i-1} A_j + P_c \right). \quad (3)$$

Proof: Since the optimization problem is a convex one, the optimal power allocation is obtained thanks to the Karush-Kuhn-Tucker (KKT) optimality conditions. Following a similar approach as [9], one can show that the QoS constraints of all users except the strongest one are active at the solution, meaning that they meet their minimum rate with equality. As such, one can express all the powers allocated to the weakest users as function of the one allocated to the strongest user. The optimization problem **(SEE)** then reduces to the convex single variable problem **(SEE1)**, where the feasible set corresponds to the minimum power required to fulfill the QoS constraint of the strongest user, i.e. $p_K \leq \frac{A_K - 1}{\Gamma_K}$, while the upper-bound

follows from the feasibility condition. The detailed proof can be found in Appendix A. ■

C. Ratio between the secrecy sum-rate and the power consumption

Let us now consider the SEE metric defined as the ratio between the secrecy achievable sum-rate and the power consumption as in [18]: $\zeta_{SEE}(\mathbf{p}) = \frac{\sum_k R_k^s(\mathbf{p})}{\sum_k p_k + P_c}$, whose solution lies also on the Pareto boundary of the bi-criterion optimization problem (**BiSEE**). Although designed for a more general problem, our proposed closed-form solution $\mathbf{p}^*(\alpha)$ in Theorem 2 can also be used to maximize the ratio ζ_{SEE} . Indeed, since the secrecy sum-rate is concave and the power consumption is affine, maximizing the ratio falls into the concave-convex fractional problem family, whose maximization is equivalent to finding the unique zero of scalarized trade-off between the secrecy sum-rate and power consumption, i.e. the objective function of the optimization problem (**SEE**), with respect to α [20]:

$$F(\alpha) = \sum_{k=1}^K R_k^s(\mathbf{p}^*(\alpha)) - \alpha \left(\sum_{k=1}^K p_k^*(\alpha) + P_c \right). \quad (4)$$

The latter can be obtained in a computational efficient manner using Dinkelbach algorithm [22].

IV. NUMERICAL RESULTS

In this section, we compare the SEE of the considered multi-user downlink network in the presence of an eavesdropper exploiting NOMA with its counterpart exploiting OMA. Before delving into numerical simulations, we first present the closed-form optimal power allocation policy under OMA.

A. Optimal closed-form solution under OMA

Under OMA, the closed-form solution maximizing the SEE metric expressed as a bi-objective optimization problem is obtained following the same approach previously presented for NOMA. Under OMA, the achievable rates for decoding the message intended for user k at the k -th legitimate user and at the eavesdropper write as

$$R_k(\mathbf{q}) = \frac{1}{2K} \log_2(1 + \Gamma_k q_k), \quad R_k^e(\mathbf{q}) = \frac{1}{2K} \log_2(1 + \Gamma_e q_k),$$

where $\mathbf{q} = (q_1, \dots, q_K)$ denotes the power allocation vector under OMA. The feasible set in this case is given as

$$\mathcal{Q} = \left\{ \mathbf{q} \in \mathbb{R}_+^K \mid \frac{1}{K} \sum_{k=1}^K q_k \leq \bar{P}, q_k \geq \frac{A_k^K - 1}{\Gamma_k}, \forall k \in \llbracket 1, K \rrbracket \right\},$$

and the scalarized optimization problem can be written as

$$(\mathbf{SEE})^{\text{OMA}} \max_{\mathbf{q} \in \mathcal{Q}} \sum_{k=1}^K R_k^s(\mathbf{q}) - \alpha \left(\frac{1}{K} \sum_{k=1}^K q_k + P_c \right)$$

Theorem 3. *If the following condition, ensuring that each QoS can be met provided the power budget \bar{P} , holds, the optimization problem $(\mathbf{SEE})^{\text{OMA}}$ is feasible:*

$$\frac{A_k^K - 1}{\Gamma_k} \leq \bar{P}, \quad k \in \llbracket 1, K \rrbracket.$$

If feasible, the optimal power allocation policy maximizing $(\mathbf{SEE})^{\text{OMA}}$ for the considered single eavesdropper downlink multi-user OMA system is obtained in closed-form as

$$q_k^*(\alpha) = \begin{cases} \frac{A_k^K - 1}{\Gamma_k}, & k \leq M - 1, \\ \min \left\{ \max \left\{ \hat{q}_k(\alpha); \frac{A_k^K - 1}{\Gamma_k} \right\}, \bar{P} \right\}, & k \in \llbracket M, K \rrbracket, \end{cases}$$

where $\hat{q}_k(\alpha)$ is the following admissible critical point

$$\hat{q}_k(\alpha) = -\frac{(\Gamma_e + \Gamma_k)}{2\Gamma_k\Gamma_e} + \frac{(\Delta^{\text{OMA}})^{1/2}}{4 \ln 2\alpha\Gamma_k\Gamma_e} \text{ with}$$

$$\Delta^{\text{OMA}} = (4 \ln 2\alpha(\Gamma_k - \Gamma_e)) (\ln 2\alpha(\Gamma_k - \Gamma_e) + 2\Gamma_k\Gamma_e)$$

Proof: The proof follows similar steps than the one previously presented for NOMA and is hence omitted. ■

B. Numerical comparison between OMA and NOMA

Let us now numerically compare the performance of NOMA and OMA. In the remaining, the channel gains h_k^2 and h_e^2 are evaluated using the free-space path loss with a carrier frequency $f_c = 3$ GHz, where the distance between the k -th legitimate user and the BS, as well as between the eavesdropper and the BS, is randomly and uniformly drawn between $[0, 1]$ km. We further set the noise variance of all users as well as of the eavesdropper as $\sigma_k = \sigma_e = -60$ dBm and consider a fixed circuitry power consumption of $P_c = 1$ W. Unless otherwise stated, the overall power budget is set to $\bar{P} = 20$ W, all users are required to meet a minimum rate of $\bar{R}_i = 0.05$ bit/s/Hz and the number of users is set to $K = 10$. All the curves are averaged over 10^4 channel realizations satisfying the feasibility conditions of both OMA and NOMA.

In Fig. 1, we show both the optimal sum-rate and the optimal secrecy sum-rate (in bit/s/Hz on the left axis), as well as the optimal sum-power (in W on the right axis), as a function of the trade-off parameter α , where the solid and dashed curves are obtained under NOMA and OMA respectively. Whereas NOMA outperforms its OMA counterpart, irrespective from the value of the trade-off parameter α , in terms of achievable secrecy sum-rate, it may consume more power than OMA. Indeed, for small values of α , i.e. when more emphasis is put on maximizing the secrecy sum-rate, NOMA consumes more power than OMA, but achieves higher secrecy sum-rates; whereas for larger values of α , NOMA consumes less power than OMA while still achieving higher secrecy sum-rates. Furthermore, in the special case $\alpha = 0$, i.e. when the goal is solely to maximize the secrecy sum-rate, NOMA achieves a secrecy sum-rate almost three times larger than the one under OMA. This increase in secrecy sum-rate comes at the cost of the power consumption: while the entire power budget is consumed under NOMA, only a fraction of it is used in OMA. Indeed, under OMA, the $M - 1$ first users are only provided just enough power to meet their QoS, whereas the remaining users are allocated the overall power budget \bar{P} in each of the time slots of duration $1/K$, leading to a sum-power smaller than \bar{P} . Under NOMA, on the other hand, all $K - 1$ first users

are allocated just enough power to meet their QoS, whereas the strongest one is provided all the remaining power budget, leading hence to an overall power consumption of \bar{P} . Finally, when α grows very large, each user is provided just enough power to meet its QoS, such that the sum-rate tends toward the fixed value $\sum_k \bar{R}_k$.

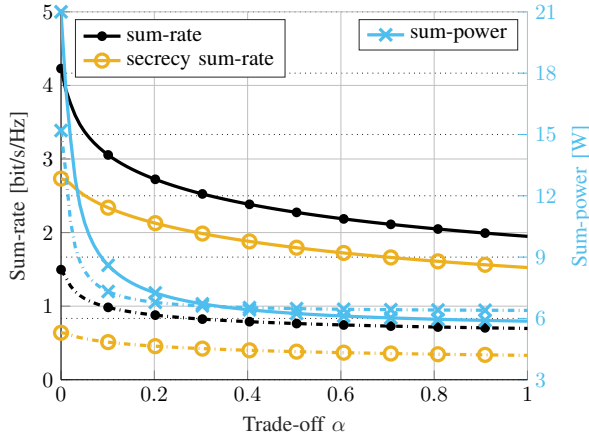


Fig. 1. Sum-rate, secrecy sum-rate (left axis) and sum-power (right axis) vs. trade-off parameter α under NOMA (solid curves) and OMA (dashed curves) for $K = 10$ users.

In Fig. 2, we compare NOMA and OMA in terms of sum-rate, secrecy sum-rate (in bit/s/Hz on the left axis) and sum-power (in W on the right axis) as a function of the number of users K in the network, when the trade-off parameter α is set to $\alpha = 0.5$. The solid curves correspond to NOMA, whereas the dashed ones to OMA. Again, we can see that NOMA outperforms OMA in terms of secrecy sum-rate irrespective from the number of users in the network. Remarkably, this gap increases with the number of users, mainly due to the time-sharing factor $1/K$ present only under OMA. Nonetheless, we can see that for a moderate number of users, NOMA consumes a little more power than OMA, whereas after $K = 9$ users, it consumes less power than OMA.

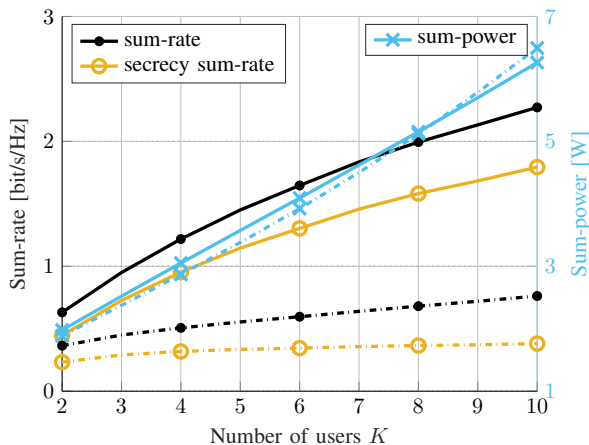


Fig. 2. Sum-rate, secrecy sum-rate (in bit/s/Hz on the left axis) and sum-power (in W on the right axis) vs. number of users K under NOMA (solid curves) and OMA (dashed curves) for $\alpha = 0.5$.

Finally, in Fig. 3, we compare the SEE defined as the ratio ζ_{SEE} under NOMA and OMA as a function of the number of users K in the network. While under OMA the ratio between the secrecy sum-rate and the power consumption decreases as the number of users K increases, the latter reaches its maximum value when the network is composed of $K = 8$ users under NOMA. In this case, the relative gap in terms of SEE between NOMA and OMA reaches 578%.

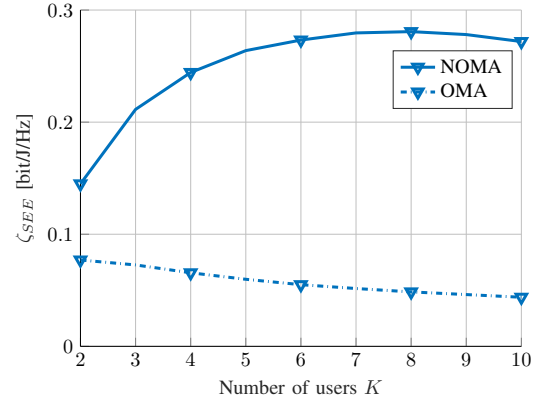


Fig. 3. Ratio between the secrecy sum-rate and the sum-power ζ_{SEE} vs. number of users K under NOMA and OMA.

V. CONCLUSION

In this paper, we investigated the secrecy energy-efficiency of a multi-user downlink NOMA system in the presence of a passive eavesdropper. We here considered a general secrecy energy-efficiency metric defined as a bi-criterion optimization problem consisting in maximizing the secrecy sum-rate and the negative power consumption. Remarkably, we showed that this optimization problem is convex. As such, its Pareto-boundary can be entirely characterized through scalarization, for which we provided a closed-form expression. Furthermore, we exploited this closed-form optimal power allocation policy to also maximize the ratio between the secrecy-sum-rate and power consumption. Our proposed optimal power allocation policy is then compared to OMA as a benchmark, for which we also provided the closed-form optimal power allocation policy. Our numerical results show that NOMA outperforms significantly OMA in terms of secrecy energy-efficiency.

APPENDIX A

OPTIMAL CLOSED-FORM SOLUTION OF (SEE)

Let \mathcal{L} be the Lagrangian of the convex problem (SEE):

$$\begin{aligned} \mathcal{L} = & \frac{1}{2} \log_2 \left(\frac{1 + \Gamma_M \theta_M}{1 + \Gamma_e \theta_M} \right) + \sum_{k=M+1}^K \frac{1}{2} \log_2 \left(\frac{1 + \Gamma_k \theta_k}{1 + \Gamma_{k-1} \theta_k} \right) - \alpha (\theta_1 + P_c) \\ & - \lambda (\theta_1 - \bar{P}) - \sum_{k=1}^K \beta_k \left(A_k \theta_{k+1} + \frac{A_k - 1}{\Gamma_k} - \theta_k \right), \end{aligned}$$

where λ and $\beta = (\beta_1, \dots, \beta_K)$ are the positive Lagrange multipliers for the overall power constraint and the K individual QoS constraints respectively. The KKT optimality conditions

imply that at the solution $\frac{\partial \mathcal{L}}{\partial p_k} = 0, k \in \llbracket 1, K \rrbracket$. As such, the difference between two consecutive Lagrange derivatives also equals zero, i.e. $\frac{\partial \mathcal{L}}{\partial p_k} - \frac{\partial \mathcal{L}}{\partial p_{k-1}} = 0$, leading to

$$\beta_{k-1} A_{k-1} - \beta_k = \frac{1}{2 \ln 2} \left(\frac{\Gamma_k}{1 + \Gamma_k \theta_k} - \frac{\Gamma_{k-1}}{1 + \Gamma_{k-1} \theta_k} \right), k \in \llbracket 2, K \rrbracket.$$

Since $\Gamma_k \geq \Gamma_{k-1}$, the left hand side of the above is positive. Additionally, since $\beta_k \geq 0$ for $k \in \llbracket 1, K \rrbracket$ and $A_{k-1} > 0$, we can deduce that $\beta_{k-1} A_{k-1} > \beta_k \geq 0$. Finally, since $A_{k-1} > 0$, we can conclude that $\beta_{k-1} > 0$ for $k \in \llbracket 2, K \rrbracket$, and therefore $\beta_k > 0$ for $k \leq K-1$. This means that all QoS constraints for the k -th legitimate users with $k < K$ are met with equality:

$$\theta_k = A_k \theta_{k+1} + \frac{A_k - 1}{\Gamma_k}, \quad 1 \leq k < K.$$

By induction, one can show that

$$\theta_k = \prod_{i=k}^{K-1} A_i \theta_K + \sum_{i=k+1}^{K-1} \frac{A_i - 1}{\Gamma_i} \prod_{j=i}^{i-1} A_j + \frac{A_k - 1}{\Gamma_k}, \quad 1 \leq k < K.$$

As such, at the solution, all the power allocated to the weakest users can be expressed as function of the power p_K allocated to the strongest one. Hence, the multi-variable optimization problem under study (**SEE**) reduces to the single variable optimization problem (**SEE1**):

$$(\text{SEE1}) \max_p f(p) \text{ s.t. } \frac{A_K - 1}{\Gamma_K} \leq p \leq u,$$

where the objective function $f(p)$ is given in (3) in Theorem 2. One can first prove that the single variable optimization is a convex one, for which the optimal solution is the feasible critical point given as $p^* = \min \left\{ \max \left\{ \hat{p}(\alpha); \frac{A_K - 1}{\Gamma_K} \right\}; u \right\}$, where

$$u = \frac{1}{\prod_{i=1}^{K-1} A_i} \left(\bar{P} - P_{\min} + \frac{A_K - 1}{\Gamma_K} \prod_{j=1}^{K-1} A_j \right),$$

$$\hat{p}(\alpha) = \frac{- \left(\alpha \gamma_2 \Gamma_K (1 + \gamma_1 \Gamma_e) + \alpha \gamma_2 \Gamma_e \prod_{i=M}^{K-1} A_i \right) + \Delta^{1/2}}{2 \left(\alpha \gamma_2 \Gamma_K \Gamma_e \prod_{i=M}^{K-1} A_i \right)}$$

with

$$\gamma_1 = \sum_{i=M}^{K-1} \frac{A_i - 1}{\Gamma_i} \prod_{j=M}^{i-1} A_j; \quad \gamma_2 = 2 \ln 2 \prod_{i=1}^{K-1} A_i,$$

$$\gamma_3 = \Gamma_K (1 + \gamma_1 \Gamma_e) - \Gamma_e \prod_{i=M}^{K-1} A_i;$$

$$\Delta = (\alpha \gamma_2 \gamma_3)^2 + 4 \alpha \gamma_2 \gamma_3 \Gamma_e \Gamma_K \prod_{i=M}^{K-1} A_i.$$

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