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# Secrecy energy-efficient multi-user NOMA: closed-form solution to bi-criterion formulation

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**Abstract**—This paper investigates the secrecy energy-efficiency of a multi-user downlink non-orthogonal multiple access (NOMA) transmission in the presence of a passive eavesdropper. The secrecy energy-efficiency, capturing the trade-off between the achievable secrecy sum-rate and the power consumption, is here formulated as a bi-criterion optimization problem that is shown to be convex. As such, the characterization of the Pareto-boundary boils down to maximizing the scalarized trade-off between the secrecy sum-rate and the power consumption, which is solved in closed-form. Finally, this obtained closed-form is exploited to maximize the ratio between the secrecy sum-rate and the power consumption. Our numerical results highlight the relative secrecy energy-efficiency gain between NOMA and OMA that can reach up to 550%.

## I. INTRODUCTION

The unprecedented growth of deployed sensors in homes, cities, wearable, etc. is turning e-health, smart homes and cities into reality. Thus, dealing with denser networks while insuring high data rates, reliable communications, efficient use of both spectral and power resources as well as integrating security aspects becomes mandatory [1], [2].

Traditionally, *orthogonal multiple access* (OMA) techniques are used, where a single user could be served on each resource block. Despite obviously preventing interference, OMA is not well-adapted to wireless networks dealing with an ever increasing number of users, paving the transition to *non-orthogonal multiple access* (NOMA) techniques [3], [4]. Under NOMA, an arbitrary number of users can be served over the same resource block by using superposition coding at the transmitter side, whereas the receivers handle the interference thanks to successive interference cancellation (SIC) [3].

Further, due to the broadcast nature of the wireless medium, security becomes a vital feature of wireless networks. While cryptography-based security techniques rely on the computational capabilities of the attackers, with physical layer security confidentiality is ensured by exploiting dynamic features of the wireless medium itself, such as interference, noise or fading. In such a framework, the goal is to derive *secrecy rates*, an extension of regular ones measured by the rate gap between the rate achieved by the legitimate user and the one by the eavesdropper to decode the same message [5].

Beside increasing data rates, reducing the power consumption is a crucial requirement of the current, as well as of

the future, generation of wireless network. Since these two are conflicting goals, finding the optimal trade-off, termed as energy-efficiency, becomes ineluctable [6]. Finally, following the increasing security concerns, another trade-off metric, called *secrecy energy-efficiency*, has been proposed to extend the energy-efficiency ones by considering secrecy sum-rates instead of regular ones [7].

Motivated by the above, this paper focuses on maximizing the secrecy energy-efficiency of a multi-user downlink NOMA transmission, in the presence of an eavesdropper. The secrecy energy-efficiency is here formulated as a bi-criterion convex optimization problem, similarly to [8], [9] where no eavesdropper was present. Beside the overall power budget constraint, each of the  $K \geq 2$  user is required to meet a minimum Quality of Service (QoS) constraint expressed in terms of its achievable rate.

## A. Related works

Recently, both energy-efficiency and secrecy rate maximization for multi-user  $K \geq 2$  downlink NOMA transmission have been widely investigated [10], [9], [11], [12], [13], [14], [15], [16], [17], but very few works consider secrecy energy-efficiency maximization for such networks.

To the best of our knowledge, the closest works to ours are [9], [18], both exploiting downlink NOMA to serve an arbitrary number  $K$  of users under an overall power budget as well as  $K$  individual QoS constraints. On one hand, [9] focuses on the maximization of the energy-efficiency defined as a bi-objective optimization problem in the absence of eavesdropper. The obtained closed-form solution is then exploited to optimize the more common energy-efficiency metric defined as the ratio between the sum-rate and the power consumption in a computationally efficient manner requiring only a line search. On the other hand, [18] includes the presence of an eavesdropper and proposes a numerical solution that relies on the dichotomy method to maximize the ratio between the secrecy sum-rate and the power consumption.

Here, we propose to first formulate the secrecy energy-efficiency as a bi-criterion optimization problem that is solved in closed form, extending both the approach of [9] to include physical layer security considerations, and the metric considered in [18]. Similarly to [9], we then exploit our closed form solution to solve the same secrecy energy-efficiency metric in terms of the ratio between the secrecy sum-rate and power

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consumption by the mean of a line search instead of the dichotomy method, as initially proposed in [18].

### B. Main contributions

Our main contributions, summarized below, are three-fold:

- i) We prove the concavity of the secrecy sum rate of a multi-user downlink NOMA system with respect to the power allocation vector, leading to a convex bi-criterion secrecy energy-efficiency optimization problem.
- ii) We derive a closed form solution that maximizes the secrecy energy-efficiency defined as a bi-criterion optimization problem. This closed form solution characterizes all optimal pairs of secrecy sum-rate and power consumption, hence encompassing the one maximizing the ratio between the secrecy sum-rate and power consumption.
- iii) We exploit this closed form solution to also maximize the ratio between the secrecy sum-rate and power consumption considered in [18] in a computationally efficient manner by requiring only a line search.

Remarkably, we show that NOMA outperforms OMA in terms of secrecy sum-rate irrespective from the number of users  $K$  in the network as well as from the value of the trade-off parameter that allows to characterize the entire Pareto-boundary of the bi-criterion optimization problem. Nonetheless, for secrecy sum-rate driven optimization, NOMA may consumes more power than OMA. Finally, the relative gap between NOMA and OMA in terms of ratio between the secrecy sum-rate and the power consumption can reach up to 550%, highlighting the interest of our proposed NOMA scheme.

## II. SYSTEM MODEL AND PROBLEM FORMULATION

This section presents the considered system model under study as well as the secrecy energy-efficiency optimization problem.

### A. System model

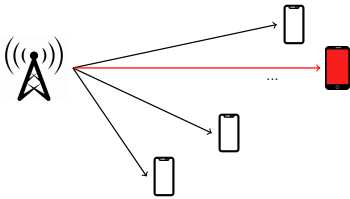


Fig. 1. System model under study: the eavesdropper is highlighted in red, whereas all the other users are considered as legitimate ones.

The network under study is composed by a single transmitter (e.g. a base station), an arbitrary number of legitimate users  $K \geq 2$  and a single passive eavesdropper, as depicted in Fig. 1. To serve an arbitrary number of users in a spectral efficient manner, NOMA is considered. As such, the transmitter employs superposition coding and broadcasts  $X = \sum_{i=1}^K X_i$ , where the message  $X_i$  of average power  $p_i$  is intended for the  $i$ -th legitimate user,  $i \in \llbracket 1, K \rrbracket$ . Then, the received signals at the

$k$ -th legitimate user and at the eavesdropper are respectively given as

$$Y_k = h_k \sum_{i=1}^K X_i + Z_k, \quad (1)$$

$$Y_e = h_e \sum_{i=1}^K X_i + Z_e, \quad (2)$$

where  $h_k$  and  $h_e$  denote the channel coefficient between the transmitter and the  $k$ -th legitimate user and the eavesdropper respectively;  $Z_k \sim \mathcal{N}(0, \sigma_k^2)$  and  $Z_e \sim \mathcal{N}(0, \sigma_e^2)$  are the Additive White Gaussian Noises (AWGN) at the  $k$ -th user and eavesdropper respectively. Throughout this paper, we assume that all noises are of unit variance, or, equivalently, we consider normalized channel gains defined as  $\Gamma_k = \frac{h_k^2}{\sigma_k^2}$ ,  $k \in \llbracket 1, K \rrbracket, e$ . Without loss of generality, we assume that the channel gains are ordered as

$$\Gamma_1 \leq \dots \leq \Gamma_{M-1} \leq \Gamma_e < \Gamma_M \leq \dots \leq \Gamma_{K-1} < \Gamma_K, \quad (3)$$

i.e.  $M - 1$  out of the  $K$  legitimate users have weaker channel gains than the eavesdropper.

Following the superposing scheme adopted in NOMA at the transmitter side, the interference at the receiver side is handled via SIC. Thus, each legitimate user  $k$  first decodes the messages  $X_w$ ,  $w \in \llbracket 1, k - 1 \rrbracket$ , corresponding to users with weaker channel gains compared to user  $k$ , but suffers the interference of messages  $X_s$ ,  $s \in \llbracket k + 1, K \rrbracket$ , corresponding to users with stronger channel gains when decoding its own message  $X_k$ . Hence, the achievable rate  $R_k$  to decode the message  $X_k$  at the  $k$ -th legitimate user writes as

$$R_k(\mathbf{p}) = \frac{1}{2} \log_2 \left( 1 + \frac{\Gamma_k p_k}{\Gamma_k (p_{k+1} + \dots + p_K) + 1} \right), \quad (4)$$

where  $\mathbf{p} = (p_1, \dots, p_K)$  denotes the vector collecting the  $K$  allocated powers.

In the remaining of the paper, and unless otherwise stated, the eavesdropper belongs to the set of users served by the base station, but performs a passive attack undetected by the transmitter. As such, the eavesdropper can access the channel ordering and hence the SIC decoding order. Hence, the achievable rate  $R_k^e$  to decode the message  $X_k$  at the eavesdropper writes as

$$R_k^e(\mathbf{p}) = \frac{1}{2} \log_2 \left( 1 + \frac{\Gamma_e p_k}{\Gamma_e (p_{k+1} + \dots + p_K) + 1} \right). \quad (5)$$

Throughout this paper, the considered network is constrained by an overall power budget  $\bar{P}$ , such that  $\sum_i p_i \leq \bar{P}$ , as well as by  $K$  individual minimum QoS constraints in terms of individual achievable rates expressed as  $R_i(\mathbf{p}) \geq \bar{R}_i$ ,  $i \in \llbracket 1, K \rrbracket$ .

From now on, in order to simplify the derivations and improve the presentation of our results, we will use the following notations:

$$A_k = 2^{2\bar{R}_k}, \quad \theta_k = \sum_{i=k}^K p_i, \quad k \in \llbracket 1, K \rrbracket \text{ and } \theta_{K+1} = 0. \quad (6)$$

In this notation, the feasible set for satisfying the power budget and QoS constraints can be expressed as

$$\Pi = \left\{ \mathbf{p} \in \mathbb{R}_+^K \mid \theta_1 \leq \bar{P}, \theta_k \geq A_k \theta_{k+1} + \frac{A_k - 1}{\Gamma_k} \right\}. \quad (7)$$

### B. Problem formulation

Energy-efficiency, which captures the trade-off between the achievable sum-rate and the power consumption of a network, has been defined in several ways in the literature. It has been characterized as a bi-criterion optimization problem, maximizing the achievable sum-rate and the negative power consumption [8], [9]; as a scalarized trade-off between the achievable sum-rate and the power consumption [19], [9], [13] or as the ratio between the sum-rate and the power consumption [20], [9], [21].

Although different, all these metrics are linked to one another. Indeed, since the two objectives of the bi-criterion optimization problem are conflicting, i.e. maximizing the sum-rate requires to consume the entire power budget, whereas minimizing the power consumption requires to not transmit at all, its solution lies on the Pareto boundary of the feasible set of rate-power pairs. Further, if the sum-rate is concave w.r.t  $\mathbf{p}$ , finding the Pareto boundary reduces to optimize the scalarized trade-off [19] between the achievable sum-rate and power consumption. Finally, if the sum-rate is concave, maximizing the ratio between the sum-rate and the power consumption is a concave-convex fractional problem, whose solution is equivalent to find the unique zero of the scalarized trade-off between the sum-rate and the power consumption, which can be computationally efficiently computed via Dinkelbach algorithm [20], [22].

Since the network under study comprises a passive eavesdropper, we propose to extend the above mentioned energy-efficiency metrics to *secret energy-efficiency* ones, by considering the *secrecy sum-rate*.

Under the framework of physical layer security, the main idea is to exploit the noisy nature of the channels to distinguish between the quality of the signal received by the legitimate users and by the eavesdropper. In fact, as long as the eavesdropper's channel quality is inferior to the one of the legitimate users, secret communication can be achieved, where secrecy rates are defined as the difference between the legitimate user's rate achieved and the eavesdropper's rate to decode the same message. As such, for each legitimate user  $k \in \llbracket 1, K \rrbracket$ , its secrecy achievable rate is expressed as

$$R_k^s(\mathbf{p}) = [R_k(\mathbf{p}) - R_k^e(\mathbf{p})]^+, \text{ where } [x]^+ = \max\{0, x\}. \quad (8)$$

Note that, given the considered channel order in (3), the  $M-1$  weakest users achieve a zero secrecy rate, whereas all users  $X_j, j \in \llbracket M, K \rrbracket$  are able to achieve strictly positive ones.

To summarize, the considered secrecy energy-efficiency optimization problem under study writes as a bi-criterion problem consisting in maximizing the secrecy achievable sum-rate and minimizing at the same time the power consumption,

such that both the overall power budget and individual QoS constraints are met:

$$(\mathbf{BiSEE}) \max_{\mathbf{p} \in \Pi} \left( \sum_{k=1}^K R_k^s(\mathbf{p}); - \sum_{k=1}^K p_k - P_c \right),$$

where  $P_c$  denotes the constant circuit power consumption accounting for all blocks implemented at the transmitter and receiver sides.

### III. CLOSED-FORM OPTIMAL POWER ALLOCATION POLICY

This section presents our main results. We first prove the convex nature of the considered secrecy energy-efficiency maximization problem expressed as a bi-objective optimization problem. We then provide a feasibility condition allowing to reach the optimal solution, and finally we derive the optimal closed-form power allocation policy that maximizes the secrecy energy-efficiency of a multi-user downlink NOMA network in the presence of a passive eavesdropper. This optimal closed-form solution is then exploited to maximize the secrecy energy-efficiency metric considered in [18], namely the ratio between the secrecy sum-rate and the power consumption.

#### A. Concavity of the secrecy sum-rate and equivalent scalarized trade-off maximization

Let us first investigate the optimization problem **(BiSEE)**. Using tools from convex optimization, we prove that the latter is convex, as stated in the following theorem.

**Theorem 1.** *Following from the concavity of the secrecy achievable sum-rate  $\sum_{k=1}^K R_k^s(\mathbf{p})$  w.r.t.  $\mathbf{p}$  and from the linearity of the other objective function and constraints, the optimization problem **(BiSEE)** is a convex one.*

*Proof:* The proof follows similarly to [23]. The main steps are to first derive the Hessian matrix  $H$  of the secrecy sum-rate, and then to show that it is semi-definite negative, which is detailed in Appendix A. Finally, exploiting the linearity of the power consumption, as well as from the overall power budget and QoS constraints, the optimization problem under study is a convex one, which concludes the proof. ■

Since the bi-criterion optimization problem **(BiSEE)** is a convex one, finding the Pareto boundary reduces to maximizing the scalarized trade-off of the two objectives defined as

$$(\mathbf{SEE}) \max_{\mathbf{p} \in \Pi} \sum_{k=1}^K R_k^s(\mathbf{p}) - \alpha \left( \sum_{k=1}^K p_k + P_c \right),$$

where  $\alpha \geq 0$  sweeps the entire Pareto boundary and allows to switch between a secrecy-driven optimization problem and a power consumption-driven one, by respectively choosing small or large values of  $\alpha$ .

#### B. Closed form solution of the scalarized energy-efficiency optimization problem

Because of the  $K$  individual QoS constraints, the optimization problem may not be feasible if the overall power budget does not allow to meet all the QoS constraints with equality. As such, before deriving the optimal power allocation policy,

we first provide the feasibility condition of the optimization problem (SEE).

**Proposition 1.** *The optimization problem (SEE) is feasible if and only if the overall power budget of the transmitter  $\bar{P}$  is larger than  $P_{\min}$ , the overall minimum power required to fulfill the  $K$  individual QoS constraints with equality:*

$$\bar{P} \geq P_{\min} = \sum_{k=1}^K \frac{A_k - 1}{\Gamma_k} \prod_{j=1}^{k-1} A_j. \quad (9)$$

Since the secrecy energy-efficiency optimization problem (SEE) is convex, and provided that the above feasible condition is met, then it allows a unique optimal solution given in closed-form in the following theorem.

**Theorem 2.** *Provided that the feasibility condition of Proposition 1 is satisfied, the optimal power allocation policy maximizing the secrecy energy-efficiency (SEE) for the considered single eavesdropper downlink multi-user NOMA system is obtained in closed-form as follows:*

$$p_k^*(\alpha) = (A_k - 1) \left( \frac{1}{\Gamma_k} + p_K^*(\alpha) \prod_{i=k+1}^{K-1} A_i + \sum_{i=k+1}^{K-1} \frac{A_i - 1}{\Gamma_i} \prod_{j=k+1}^{i-1} A_j \right), \quad k < K$$

$$p_K^*(\alpha) = \min \left\{ \max \left\{ \hat{p}(\alpha); \frac{A_K - 1}{\Gamma_K} \right\}; u \right\}, \quad (10)$$

where  $u$  is given as  $u = \frac{1}{\prod_{i=1}^{K-1} A_i} \left( \bar{P} - P_{\min} + \frac{A_K - 1}{\Gamma_K} \prod_{j=1}^{K-1} A_j \right)$  and

$\hat{p}(\alpha)$ , whose closed-form expression is given in Appendix B, denotes the admissible critical point of the following convex single variable optimization problem

$$(SEE1) \max_p f(p) \text{ s.t. } \frac{A_K - 1}{\Gamma_K} \leq p \leq u, \text{ with}$$

$$f(p) = \frac{1}{2} \log_2(1 + \Gamma_K p) - \frac{1}{2} \log_2 \left( 1 + \Gamma_e \left( \prod_{i=M}^{K-1} A_i p + \sum_{i=M}^{K-1} \frac{A_i - 1}{\Gamma_i} \prod_{j=M}^{i-1} A_j \right) \right) + \frac{1}{2} \log_2 \prod_{i=M}^{K-1} A_i - \alpha \left( \prod_{i=1}^{K-1} A_i + \sum_{i=1}^{K-1} \frac{A_i - 1}{\Gamma_i} \prod_{j=1}^{i-1} A_j + P_c \right).$$

*Proof:* Since the optimization problem is a convex one, the optimal power allocation is obtained thanks to the Karush-Kuhn-Tucker (KKT) optimality conditions. Following a similar approach as [9], one can show that the QoS constraints of all users except the strongest one are active at the solution, meaning that they meet their minimum rate with equality. As such, one can express all the powers allocated to the weakest users as function of the one allocated to the strongest user. The optimization problem (SEE) then reduces to the convex single variable problem (SEE1), where the feasible set corresponds to the minimum power required to fulfill the QoS constraint of the strongest user, i.e.  $p_K \leq \frac{A_K - 1}{\Gamma_K}$ , while the upper-bound follows from the feasibility condition. The detailed proof can be found in Appendix B. ■

*C. Ratio between the secrecy sum-rate and the power consumption*

Let us now consider the secrecy energy-efficiency metric defined as the ratio between the secrecy achievable sum-rate and the power consumption as in [18]

$$\zeta_{SEE}(\mathbf{p}) = \frac{\sum_k R_k^s(\mathbf{p})}{\sum_k p_k + P_c}, \quad (11)$$

whose solution lies also on the Pareto boundary of the bi-criterion optimization problem (BiSEE). Although designed for a more general problem, our proposed closed-form solution  $\mathbf{p}^*(\alpha)$  in Theorem 2 can also be used to maximize the ratio  $\zeta_{SEE}$ . Indeed, since the secrecy sum-rate is concave and the power consumption is affine, maximizing the ratio falls into the concave-convex fractional problem family, whose maximization is equivalent to finding the unique zero of scalarized trade-off between the secrecy sum-rate and power consumption, i.e. the objective function of the optimization problem (SEE), with respect to  $\alpha$  [20]:

$$F(\alpha) = \sum_{k=1}^K R_k^s(\mathbf{p}^*(\alpha)) - \alpha \left( \sum_{k=1}^K p_k^*(\alpha) + P_c \right). \quad (12)$$

The latter can be obtained in a computational efficient manner using Dinkelbach algorithm [22] as

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**Algorithm 1** Dinkelbach algorithm maximizing  $\zeta_{SEE}$

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- 1: Initialization: Fix precision  $\epsilon > 0$ ,  $\alpha = 0$
  - 2: **while**  $F(\alpha) \leq \epsilon$  **do**
  - 3:   Compute opt. power allocation policy  $\mathbf{p}^*(\alpha)$  using (10)
  - 4:   Update  $F(\alpha)$  using (12)
  - 5:   Update  $\alpha = \zeta_{SEE}(\mathbf{p}^*(\alpha))$
  - 6: **end while**
- 

## IV. NUMERICAL RESULTS

In this section, we compare the secrecy energy-efficiency of the considered multi-user downlink network in the presence of an eavesdropper exploiting NOMA with its counterpart exploiting OMA. Before delving into numerical simulations, we first present the closed-form optimal power allocation policy under OMA.

### A. Optimal closed-form solution under OMA

Under OMA, the closed-form solution maximizing the secrecy energy-efficiency metric expressed as a bi-objective optimization problem is obtained following the same approach previously presented for NOMA. Under OMA, the achievable rates for decoding the message intended for user  $k$  at the  $k$ -th legitimate user and at the eavesdropper write as

$$R_k(\mathbf{q}) = \frac{1}{2K} \log_2(1 + \Gamma_k q_k), \quad R_k^e(\mathbf{q}) = \frac{1}{2K} \log_2(1 + \Gamma_e q_k),$$

where  $\mathbf{q} = (q_1, \dots, q_K)$  denotes the power allocation vector under OMA. In this case, the scalarized optimization problem can be written as

$$\begin{aligned} (\text{SEE})^{\text{OMA}} \max_{\mathbf{q}} \quad & \sum_{k=1}^K R_k^s(\mathbf{q}) - \alpha \left( \frac{1}{K} \sum_{k=1}^K q_k + P_c \right) \\ \text{s.t.} \quad & q_k \geq \frac{A_k^K - 1}{\Gamma_k}, \quad k \in \llbracket 1, K \rrbracket, \\ & \frac{1}{K} \sum_{k=1}^K q_k \leq \bar{P}. \end{aligned}$$

**Theorem 3.** *If the following condition on the system parameters hold, the optimization problem  $(\text{SEE})^{\text{OMA}}$  is feasible:*

$$\frac{A_k^K - 1}{\Gamma_k} \leq \bar{P}, \quad k \in \llbracket 1, K \rrbracket.$$

The above ensures that each QoS can be met provided the overall power budget  $\bar{P}$  of the transmitter. Further, if feasible, the optimal power allocation policy maximizing the secrecy energy-efficiency  $(\text{SEE})^{\text{OMA}}$  for the considered single eavesdropper downlink multi-user OMA system is obtained in closed-form as

$$q_k^*(\alpha) = \begin{cases} \frac{A_k^K - 1}{\Gamma_k}, & k \leq M-1, \\ \min \left\{ \max \left\{ \hat{q}_k(\alpha); \frac{A_k^K - 1}{\Gamma_k} \right\}, \bar{P} \right\}, & k \in \llbracket M, K \rrbracket, \end{cases}$$

where  $\hat{q}_k(\alpha)$  is the following admissible critical point

$$\begin{aligned} \hat{q}_k(\alpha) &= -\frac{(\Gamma_e + \Gamma_k)}{2\Gamma_k\Gamma_e} + \frac{(\Delta^{\text{OMA}})^{1/2}}{4 \ln 2\alpha\Gamma_k\Gamma_e} \text{ with} \\ \Delta^{\text{OMA}} &= (4 \ln 2\alpha(\Gamma_k - \Gamma_e)) (\ln 2\alpha(\Gamma_k - \Gamma_e) + 2\Gamma_k\Gamma_e) \end{aligned}$$

*Proof:* The proof follows similar steps than the one previously presented for NOMA and is hence omitted. ■

### B. Numerical comparison between OMA and NOMA

Let us now numerically compare the performance of NOMA and OMA. In the remaining, the channel gains  $h_k^2$  and  $h_e^2$  are evaluated using the free-space path loss with a carrier frequency  $f_c = 3$  GHz, where the distance between the  $k$ -th legitimate user and the base station, as well as between the eavesdropper and the base station, is randomly drawn between  $[0, 1]$  km. We further set the noise variance of all users as well as of the eavesdropper as  $\sigma_k = \sigma_e = -60$  dBm and consider a fixed circuitry power consumption of  $P_c = 1$  W. Unless otherwise stated, the overall power budget is set to  $\bar{P} = 20$  W, all users are required to meet a minimum rate of  $\bar{R}_i = 0.05$  bit/s/Hz and the number of users is set to  $K = 10$ . All the curves are averaged over  $10^4$  channel realizations satisfying the feasibility conditions of both OMA and NOMA.

In Fig. 2, we show both the optimal sum-rate and the optimal secrecy sum-rate (in bit/s/Hz on the left axis), as well as the optimal sum-power (in W on the right axis), as a function of the trade-off parameter  $\alpha$ , where the solid and dashed curves

are obtained under NOMA and OMA respectively. Whereas NOMA outperforms its OMA counterpart, irrespective from the value of the trade-off parameter  $\alpha$ , in terms of achievable secrecy sum-rate, it may consume more power than OMA. Indeed, for small values of  $\alpha$ , i.e. when more emphasis is put on maximizing the secrecy sum-rate, NOMA consumes more power than OMA, but achieves higher secrecy sum-rates; whereas for larger values of  $\alpha$ , NOMA consumes less power than OMA while still achieving higher secrecy sum-rates. Furthermore, in the special case  $\alpha = 0$ , i.e. when the goal is solely to maximize the secrecy sum-rate, NOMA achieves a secrecy sum-rate almost three time larger than the one under OMA. This increase in secrecy sum-rate comes at the cost of the power consumption: while the entire power budget is consumed under NOMA, only a fraction of it is used in OMA. Indeed, under OMA, the  $M-1$  first users are only provided just enough power to meet their QoS, whereas the remaining users are allocated the overall power budget  $\bar{P}$  in each of the time slots of duration  $1/K$ , leading to a sum-power smaller than  $\bar{P}$ . Under NOMA, on the other hand, all  $K-1$  first users are allocated just enough power to meet their QoS, whereas the strongest one is provided all the remaining power budget, leading hence to an overall power consumption of  $\bar{P}$ . Finally, when  $\alpha$  grows very large, each user is provided just enough power to meet its QoS, such that the sum-rate tends toward the fixed value  $\sum_k \bar{R}_k$ .

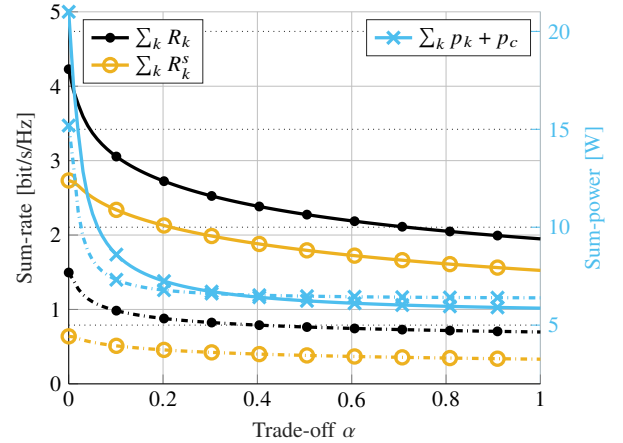


Fig. 2. Sum-rate, secrecy sum-rate (left axis) and sum-power (right axis) vs. trade-off parameter  $\alpha$  under NOMA (solid curves) and OMA (dashed curves) for  $K = 10$  users: NOMA always outperforms OMA in terms of secrecy sum-rate but may consume more power for smaller values of  $\alpha$ , i.e. for secrecy sum-rate driven optimization.

In Fig. 3, we compare NOMA and OMA in terms of sum-rate, secrecy sum-rate (in bit/s/Hz on the left axis) and sum-power (in W on the right axis) as a function of the number of users  $K$  in the network, when the trade-off parameter  $\alpha$  is set to  $\alpha = 0.5$ . The solid curves correspond to NOMA, whereas the dashed ones to OMA. Again, we can see that NOMA outperforms OMA in terms of secrecy sum-rate irrespective from the number of users in the network. Remarkably, this gap increases with the number of users, mainly due to the time-sharing factor  $1/K$  present only under OMA. Nonetheless, we

can see that for a moderate number of users, NOMA consumes a little more power than OMA, whereas after  $K = 10$  users, it consumes less power than OMA.

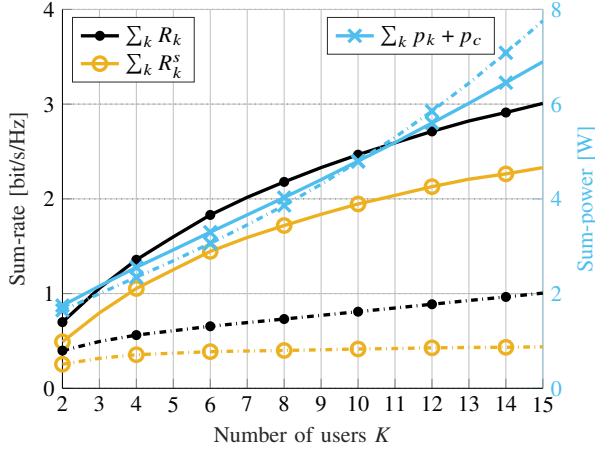


Fig. 3. Sum-rate, secrecy sum-rate (in bit/s/Hz on the left axis) and sum-power (in W on the right axis) vs. number of users  $K$  under NOMA (solid curves) and OMA (dashed curves) for  $\alpha = 0.5$ : NOMA always outperforms OMA in terms of secrecy sum-rate but may consume a little more power for a moderate number of users ( $K \leq 10$ ).

In Fig. 4, we compare NOMA and OMA in terms of sum-rate, secrecy sum-rate (in bit/s/Hz on the left axis) and sum-power (in W on the right axis) as a function of the number of users  $K$  in the network, when the power allocation policy is now optimized to maximize the secrecy energy-efficiency defined as the ratio  $\zeta_{SEE}$ , instead of the scalarized trade-off as presented in Fig. 3. Remarkably, NOMA (solid curves) now outperforms its OMA counterpart (dashed curves) both in terms of secrecy sum-rate and power consumption irrespective from the number of users in the network. Again, the gap in secrecy sum-rate between OMA and NOMA increases with the number of users  $K$  in the network.

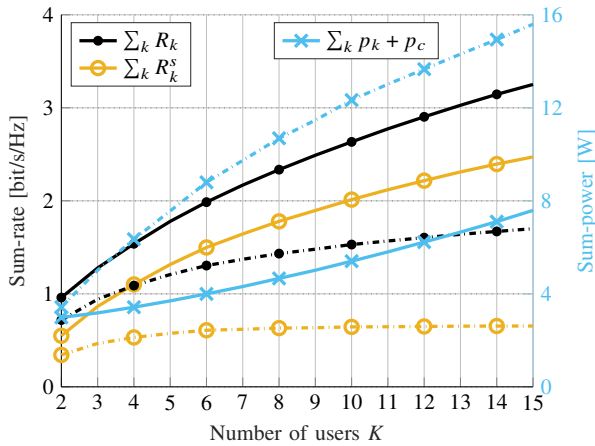


Fig. 4. Sum-rate, secrecy sum-rate (in bit/s/Hz on the left axis) and sum-power (in W on the right axis) vs. number of users  $K$  under NOMA (solid curves) and OMA (dashed curves) when the power allocation policy maximizes the ratio between the secrecy sum-rate and the power consumption  $\zeta_{SEE}$ : NOMA always outperforms OMA in terms of secrecy sum-rate and power consumption, irrespective from the number of users  $K$ .

Finally, in Fig. 5, we compare the secrecy energy-efficiency defined as the ratio  $\zeta_{SEE}$  under NOMA and OMA as a function of the number of users  $K$  in the network. While under OMA the ratio between the secrecy sum-rate and the power consumption decreases as the number of users  $K$  increases, the latter reaches its maximum value when the network is composed of  $K = 7$  users under NOMA. In this case, the relative gap in terms of secrecy energy-efficiency between NOMA and OMA reaches 550%.

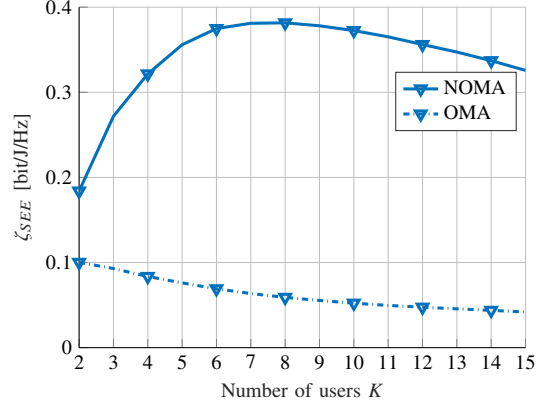


Fig. 5. Ratio between the secrecy sum-rate and the sum-power  $\zeta_{SEE}$  vs. number of users  $K$  under NOMA and OMA: the relative gap between NOMA and OMA can reach up to 550%.

## V. CONCLUSION

In this paper, we investigated the secrecy energy-efficiency of a multi-user downlink NOMA system in the presence of a passive eavesdropper. We here considered a general secrecy energy-efficiency metric defined as a bi-criterion optimization problem consisting in maximizing the secrecy sum-rate and the negative power consumption. Remarkably, we shown that this optimization problem is convex. As such, its Pareto-boundary can be entirely characterized through scalarization, for which we provided a closed-form expression. Furthermore, we exploited this closed-form optimal power allocation policy to also maximize the ratio between the secrecy-sum-rate and power consumption. Our proposed optimal power allocation policy is then compared to OMA as a benchmark, for which we also provided the closed-form optimal power allocation policy. Our numerical results show that NOMA outperforms OMA in terms of secrecy energy-efficiency. Remarkably, the relative gap in terms of secrecy energy-efficiency between NOMA and OMA can reach up to 550%.

## APPENDIX A

### CONCAVITY OF THE SECRECY SUM-RATE

If we denote by  $SSR(\mathbf{p})$  the achievable secrecy sum-rate, we can rewrite it as

$$SSR(\mathbf{p}) = \frac{1}{2} \log_2 \left( \frac{1 + \Gamma_M \theta_M}{1 + \Gamma_e \theta_M} \right) + \sum_{k=M+1}^K \frac{1}{2} \log_2 \left( \frac{1 + \Gamma_k \theta_k}{1 + \Gamma_{k-1} \theta_k} \right).$$



Let us further denote  $\omega_k = \frac{\partial^2 SSR}{\partial p_k^2}$ . The Hessian matrix of the achievable secrecy sum-rate is given as

$$H = \begin{bmatrix} 0 & 0 \\ 0 & \begin{matrix} \omega_M & \omega_M & \omega_M & \cdots & \omega_M \\ \omega_M & \omega_{M+1} & \omega_{M+1} & \cdots & \omega_{M+1} \\ \omega_M & \omega_{M+1} & \omega_{M+2} & \cdots & \omega_{M+2} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \omega_M & \omega_{M+1} & \omega_{M+2} & \cdots & \omega_K \end{matrix} \end{bmatrix}. \quad (13)$$

Consider  $T = -H$ . Showing the concavity of the achievable secrecy sum-rate is equivalent to showing that all principal minors of  $T$  are non-negative. Observe that each principal minor which includes the first row and/or column is always zero. Thus, we look at the determinant of the matrix  $T'$ , obtained from  $T$  by eliminating the first row and first column,

$$|T'| = \begin{vmatrix} -\omega_M & -\omega_M & \cdots & -\omega_M \\ 0 & \omega_M - \omega_{M+1} & \cdots & \omega_M - \omega_{M+1} \\ \vdots & \ddots & \ddots & \vdots \\ 0 & \cdots & \cdots & \omega_{K-1} - \omega_K \end{vmatrix} \\ = -\omega_M \prod_{l=M+1}^K (\omega_{l-1} - \omega_l), \quad (14)$$

which is non-negative since

$$\omega_M = \frac{\partial^2 SSR}{\partial p_M^2} = \frac{1}{2 \ln 2} \left( \frac{\Gamma_e^2}{(1 + \Gamma_e \theta_M)^2} - \frac{\Gamma_M^2}{(1 + \Gamma_M \theta_M)^2} \right) \stackrel{(a)}{<} 0, \\ \omega_{l-1} - \omega_l = -\frac{1}{2 \ln 2} \left( \frac{\Gamma_{l-1}^2}{(1 + \Gamma_{l-1} \theta_l)^2} - \frac{\Gamma_l^2}{(1 + \Gamma_l \theta_l)^2} \right) \stackrel{(b)}{\geq} 0,$$

where (a) and (b) come from the assumed channel order.

We can apply a similar reasoning to all the principal minors of  $T'$ . As such, all principal minors have non-negative determinants, leading to the matrix  $T$  being positive semi-definite, which concludes the proof.

#### APPENDIX B

##### OPTIMAL CLOSED-FORM SOLUTION OF (SEE)

Let  $\mathcal{L}$  be the Lagrangian of the convex problem (SEE):

$$\mathcal{L} = \frac{1}{2} \log_2 \left( \frac{1 + \Gamma_M \theta_M}{1 + \Gamma_e \theta_M} \right) + \sum_{k=M+1}^K \frac{1}{2} \log_2 \left( \frac{1 + \Gamma_k \theta_k}{1 + \Gamma_{k-1} \theta_k} \right) - \alpha(\theta_1 + P_c) \\ - \lambda \left( \theta_1 - \bar{P} \right) - \sum_{k=1}^K \beta_k \left( A_k \theta_{k+1} + \frac{A_k - 1}{\Gamma_k} - \theta_k \right),$$

where  $\lambda$  and  $\beta = (\beta_1, \dots, \beta_K)$  are the positive Lagrange multipliers for the overall power constraint and the  $K$  individual QoS constraints respectively. The KKT optimality conditions imply that at the solution  $\frac{\partial \mathcal{L}}{\partial p_k} = 0, k \in \llbracket 1, K \rrbracket$ . As such, the difference between two consecutive Lagrange derivatives also equals zero, leading to

$$\beta_{k-1} A_{k-1} - \beta_k = \frac{1}{2 \ln 2} \left( \frac{\Gamma_k}{1 + \Gamma_k \theta_k} - \frac{\Gamma_{k-1}}{1 + \Gamma_{k-1} \theta_k} \right), k \in \llbracket 2, K \rrbracket.$$

Since  $\Gamma_k \geq \Gamma_{k-1}$ , the left hand side of the above is positive. Additionally, since  $\beta_k \geq 0$  for  $k \in \llbracket 1, K \rrbracket$  and  $A_{k-1} > 0$ , we can deduce that  $\beta_{k-1} A_{k-1} > \beta_k \geq 0$ . Finally, since  $A_{k-1} > 0$ , we can conclude that  $\beta_{k-1} > 0$  for  $k \in \llbracket 2, K \rrbracket$ , and therefore  $\beta_k > 0$  for  $k \leq K-1$ . This means that all QoS constraints for the  $k$ -th legitimate users with  $k < K$  are active at the solution, hence implying:

$$\theta_k = A_k \theta_{k+1} + \frac{A_k - 1}{\Gamma_k}, \quad 1 \leq k < K.$$

By induction, one can show that

$$\theta_k = \prod_{i=k}^{K-1} A_i \theta_K + \sum_{i=k+1}^{K-1} \frac{A_i - 1}{\Gamma_i} \prod_{j=k}^{i-1} A_j + \frac{A_k - 1}{\Gamma_k}, \quad 1 \leq k < K.$$

As such, at the solution, all the power allocated to the weakest users can be expressed as function of the power  $p_K$  allocated to the strongest one. Hence, the multi-variable optimization problem under study (SEE) reduces to the single variable optimization problem (SEE1):

$$(\text{SEE1}) \max_p f(p) \text{ s.t. } \frac{A_K - 1}{\Gamma_K} \leq p \leq u,$$

with

$$f(p) = \frac{1}{2} \log_2 (1 + \Gamma_K p) \\ - \frac{1}{2} \log_2 \left( 1 + \Gamma_e \left( \prod_{i=M}^{K-1} A_i p + \sum_{i=M}^{K-1} \frac{A_i - 1}{\Gamma_i} \prod_{j=M}^{i-1} A_j \right) \right) \\ + \frac{1}{2} \log_2 \prod_{i=M}^{K-1} A_i - \alpha \left( \prod_{i=1}^{K-1} A_i + \sum_{i=1}^{K-1} \frac{A_i - 1}{\Gamma_i} \prod_{j=1}^{i-1} A_j + P_c \right).$$

One can first prove that the single variable optimization is a convex one, for which the optimal solution is the feasible critical point given as  $p^* = \min \left\{ \max \left\{ \hat{p}(\alpha); \frac{A_K - 1}{\Gamma_K} \right\}; u \right\}$ , where

$$u = \frac{1}{\prod_{i=1}^{K-1} A_i} \left( \bar{P} - P_{\min} + \frac{A_K - 1}{\Gamma_K} \prod_{j=1}^{K-1} A_j \right), \\ \hat{p}(\alpha) = \frac{- \left( \alpha \gamma_2 \Gamma_K (1 + \gamma_1 \Gamma_e) + \alpha \gamma_2 \Gamma_e \prod_{i=M}^{K-1} A_i \right) + \Delta^{1/2}}{2 \left( \alpha \gamma_2 \Gamma_K \Gamma_e \prod_{i=M}^{K-1} A_i \right)}$$

with

$$\gamma_1 = \sum_{i=M}^{K-1} \frac{A_i - 1}{\Gamma_i} \prod_{j=M}^{i-1} A_j; \quad \gamma_2 = 2 \ln 2 \prod_{i=1}^{K-1} A_i,$$

$$\gamma_3 = \Gamma_K (1 + \gamma_1 \Gamma_e) - \Gamma_e \prod_{i=M}^{K-1} A_i;$$

$$\Delta = (\alpha \gamma_2 \gamma_3)^2 + 4 \alpha \gamma_2 \gamma_3 \Gamma_e \Gamma_K \prod_{i=M}^{K-1} A_i.$$



## REFERENCES

- [1] Y. Zhao, W. Zhai, J. Zhao, T. Zhang, S. Sun, D. Niyato, and K.-Y. Lam, "A comprehensive survey of 6G wireless communications," *arXiv preprint arXiv:2101.03889*, 2020.
- [2] L. U. Khan, W. Saad, Z. Han, E. Hossain, and C. S. Hong, "Federated learning for internet of things: Recent advances, taxonomy, and open challenges," *IEEE Commun. Surveys Tuts.*, vol. 23, no. 3, pp. 1759–1799, 2021.
- [3] M. Vaezi, Z. Ding, and H. V. Poor, *Multiple access techniques for 5G wireless networks and beyond*. Springer, 2019, vol. 159.
- [4] Y. Liu, S. Zhang, X. Mu, Z. Ding, R. Schober, N. Al-Dhahir, E. Hossain, and X. Shen, "Evolution of NOMA toward next generation multiple access (NGMA) for 6G," *IEEE J. Sel. Areas Commun.*, vol. 40, no. 4, pp. 1037–1071, 2022.
- [5] A. D. Wyner, "The wire-tap channel," *Bell system technical journal*, vol. 54, no. 8, pp. 1355–1387, 1975.
- [6] C.-X. Wang, X. You, X. Gao, X. Zhu, Z. Li, C. Zhang, H. Wang, Y. Huang, Y. Chen, H. Haas *et al.*, "On the road to 6G: Visions, requirements, key technologies and testbeds," *IEEE Commun. Surveys Tuts.*, 2023.
- [7] M. El-Halabi, T. Liu, and C. N. Georgiades, "Secrecy capacity per unit cost," *IEEE J. Sel. Areas Commun.*, vol. 31, no. 9, pp. 1909–1920, 2013.
- [8] R. Masmoudi, E. V. Belmega, I. Fijalkow, and N. Sellami, "A unifying view on energy-efficiency metrics in cognitive radio channels," in *IEEE EUSIPCO*, 2014, pp. 171–175.
- [9] H. El Hassani, A. Savard, and E. V. Belmega, "A closed-form solution for energy-efficiency optimization in multi-user downlink NOMA," in *IEEE PIMRC*, 2020, pp. 1–5.
- [10] A. A. Nasir, H. D. Tuan, T. Q. Duong, and M. Debbah, "NOMA throughput and energy efficiency in energy harvesting enabled networks," *IEEE Trans. Commun.*, vol. 67, no. 9, pp. 6499–6511, 2019.
- [11] H. El Hassani, A. Savard, E. V. Belmega, and R. C. De Lamare, "Energy-efficient cooperative backscattering closed-form solution for NOMA," in *IEEE GLOBECOM*. IEEE, 2021, pp. 1–6.
- [12] S. Rezvani, E. A. Jorswieck, R. Joda, and H. Yanikomeroglu, "Optimal power allocation in downlink multicarrier NOMA systems: Theory and fast algorithms," *IEEE J. Sel. Areas Commun.*, vol. 40, no. 4, pp. 1162–1189, 2022.
- [13] H. El Hassani, A. Savard, E. V. Belmega, and R. C. de Lamare, "Multi-user downlink NOMA systems aided by ambient backscattering: Achievable rate regions and energy-efficiency maximization," *IEEE Trans. on Green. Commun. Netw.*, vol. 7, no. 3, pp. 1135–1148, 2023.
- [14] J. Tang, J. Luo, M. Liu, D. K. So, E. Alsusa, G. Chen, K.-K. Wong, and J. A. Chambers, "Energy efficiency optimization for NOMA with SWIPT," *IEEE J. Sel. Topics Signal Process.*, vol. 13, no. 3, pp. 452–466, 2019.
- [15] Y. Zhang, H.-M. Wang, Q. Yang, and Z. Ding, "Secrecy sum rate maximization in non-orthogonal multiple access," *IEEE Commun. Lett.*, vol. 20, no. 5, pp. 930–933, 2016.
- [16] G. He, L. Li, X. Li, W. Chen, L.-L. Yang, and Z. Han, "Secrecy sum rate maximization in NOMA systems with wireless information and power transfer," in *IEEE WCSP*, 2017, pp. 1–6.
- [17] M. Tian, Q. Zhang, S. Zhao, Q. Li, and J. Qin, "Secrecy sum rate optimization for downlink MIMO nonorthogonal multiple access systems," *IEEE Signal Process. Lett.*, vol. 24, no. 8, pp. 1113–1117, 2017.
- [18] R. Yao, L. Yao, X. Zuo, N. Qi, Y. Liu, and J. Xu, "Secrecy energy efficiency maximization in a NOMA system," in *IEEE ICCSN*, 2019.
- [19] S. P. Boyd and L. Vandenberghe, *Convex optimization*. Cambridge university press, 2004.
- [20] A. Zappone, E. Jorswieck *et al.*, "Energy efficiency in wireless networks via fractional programming theory," *Foundations and Trends® in Communications and Information Theory*, vol. 11, no. 3–4, pp. 185–396, 2015.
- [21] H. El Hassani, A. Savard, E. V. Belmega, and R. C. De Lamare, "Energy-efficient solutions in two-user downlink NOMA systems aided by ambient backscattering," in *IEEE GLOBECOM*, 2022, pp. 1673–1678.
- [22] W. Dinkelbach, "On nonlinear fractional programming," *Management science*, vol. 13, no. 7, pp. 492–498, 1967.
- [23] Z. Yang, W. Xu, C. Pan, Y. Pan, and M. Chen, "On the optimality of power allocation for NOMA downlinks with individual QoS constraints," *IEEE Commun. Lett.*, vol. 21, no. 7, pp. 1649–1652, 2017.