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Teoría Constructiva de Tipos.

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Lógica Dialógica y Teoría Constructiva de Tipos.

Nuevas Exploraciones en la Genealogía y Arquitectura
del Significado y de la Lógica como Interacción.

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Juan Redmond

Rodrigo López Orellana

a Leone Gazziero
por tantas horas pensando juntos
Shahid Rahman

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Prefacio

" En estos tiempos, en estos días inciertos, en los que la vida se hace cada vez más pequeña, osemos volver a las raíces dialógicas de la filosofía.

En efecto, la filosofía nos enseña que el ser racional es una labor que emprendemos juntos.

La emergencia del pensamiento conceptual, la constitución del conocimiento y del sentido son el resultado de un acto plural de generosidad hacia las perspectivas del otro".

Shahid Rahman
mensaje online, Universidad Japonesa de Egipto, 22 de
Noviembre 2022, *Día de la filosofía*.

En los albores del siglo XX, cuando la formalización de la matemática y la lógica alcanza su cúspide, dos perspectivas epistemológicas de raíces venerables agitaron vivas discusiones en torno a su naturaleza, a saber,

- Tanto las matemáticas como la lógica se caracterizan por ser ciencias en las cuales conocimiento y significado se adquieren por demostración.
- La lógica se distingue como una *teoría* normativa, es decir como una ciencia que proporciona los fundamentos conceptuales, los fundamentos filosóficos, de la ciencia demostrativa.¹

Dichas perspectivas dieron y siguen dando lugar a posiciones antagónicas. Sobre toda la segunda perspectiva, cuya fuente principal sea muy probablemente el texto de los *Analytica Posteriora*. A los extremos de tales controversias encontramos posiciones que o bien niegan la distinción entre lógica y matemáticas, o entienden *teoría* en un sentido puramente sintáctico y formal o en un sentido totalmente informal, empírico y desprovisto de toda normatividad.

Hacia 1960, Paul Lorenzen, al que pronto se unió Kuno Lorenz, puso en marcha en Erlangen el proyecto, llamado el *Constructivismo de Erlangen*, desarrollado en torno a su núcleo conceptual, la *Lógica Dialógica*, que cuestionaba las principales ideas históricas y sistémicas subyacentes a las polémicas mencionadas. En efecto Lorenzen recordó que el proyecto aristotélico debía leerse sobre el trasfondo de la teoría dialéctica de los *Tópica*, tan descuidada, salvo contadas excepciones, por las principales corrientes tanto de los lógicos como de los historiadores de la filosofía de la época.

De hecho, según el punto de vista dialógico, el nacimiento de la lógica como teoría de la demostración, la aparición misma de pensamiento conceptual, es inherente a la noción de argumentación crítica, en el curso de la cual afirmaciones, particularmente en contextos éticos y epistemológicos, tienen que someterse al juego de pedir y dar razones frente al tribunal del espacio público. Uno de los principales resultados de este enfoque es que ser racional significa asumir la responsabilidad de las consecuencias teóricas y prácticas de nuestras afirmaciones y acciones.

El volumen, presenta de forma autocontenida los desarrollos más recientes del Marco Dialógico, y por una parte, discute la contribución de la perspectiva Dialógica al concepto inferencial de conocimiento y significado, y en particular respecto a los fundamentos

¹ Véase Martin-Löf (1993, Lecture 1)

constructivos de la lógica y las matemáticas desarrollados en el seno de la Teoría Constructiva de Tipos (TCT) de Per Martin-Löf; y por otra parte muestra nuevos resultados en historia y filosofía de la lógica.

Nuestro estudio incluye, diálogos materiales -adversariales y colaborativos, con contenido matemático y empírico, nuevos cuadros para el desarrollo de lógicas modales, temporales y déonticas, y nuevos cuadros para la teoría de la argumentación. El objetivo principal de nuestra discusión de diálogos con contenido matemático – que se reduce a tratar operaciones elementales en el conjunto de los números naturales y conjuntos finitos – ,es de preparar el terreno para exponer el enfoque dialógico a la operación de evaluación: según este enfoque la operación de evaluación articula el pasaje de diálogos adversariales a colaborativos.

Respecto a los resultados en lógica modal y temporal y en teoría de la argumentación, mostraremos cuanto podemos aún aprender de la tradición árabe, particularmente en relación con los desarrollos posteriores a Avicena, lamentablemente descuidados en la mayor parte de los estudios contemporáneos sobre lógica y teoría de la argumentación. En efecto, tales desarrollos de la tradición árabe, asombrosamente ricos en extensión y profundidad, propusieron nuevos y originales enfoques que invitan a explorar nuevos caminos para la comprensión del rol de la perspectiva dialógica en la constitución de la lógica, el conocimiento y el significado.

El texto, que abunda en ejemplos y ejercicios resueltos, está dividido en dos partes principales. A saber:

I La primera parte, que lleva el título *Razonamiento Inmanente II* (RI-II), iniciado por Rahman & Gallois (2024), incluye una introducción somera pero autónoma a la lógica dialógica básica que hace hincapié en los principios lógicos y filosóficos que fundamentan el marco dialógico– para una lectura detallada véase el volumen de Redmond & López Orellana (2023) y un estudio pormenorizado de RI- II.

RI- II, , profundiza en un desarrollo reciente de las reglas dialógicas para teoría de la prueba escritas por Ansten Klev (2023). Tales reglas tienen su origen en la interpretación dialógica de la noción Kantiana de conocimiento asertórico propuesta por Per Martin-Löf (2017), a fin de obtener un criterio no circular de corrección para aserciones (assertions) en teoría de la prueba y más precisamente en Teoría Constructiva de Tipos (TCT). Según la TCT, la aplicación más fundamental de la noción de conocimiento asertórico es proporcionar el criterio de corrección para una aserción J:

J es correcta si el que afirma *J* la conoce asertóricamente.

Una inferencia es válida si y solamente si preserva la corrección desde las premisas hasta la conclusión.

Conocimiento asertórico es explicado en términos dialógicos: justificar una inferencia basta con suponer que el interlocutor haya afirmado las premisas (en lugar de haberlas demostrado) – cfr. Martin-Löf (2024).

RI-II, integra por un lado la nueva formulación de las reglas dialógicas para TCT concebidas por Martin-Löf y desarrolladas por Klev que simplifican la notación de la primera versión de *Razonamiento Inmanente* – cfr. Rahman et al. (2018), pero por el otro propone una perspectiva que acentúa el fundamento dialógico de la teoría del significado y de la teoría de la prueba.

Es así que, a diferencia del enfoque de Klev, RI-II contiene reglas que indican cómo desarrollar partidas y estrategias ganadoras en torno a la afirmación de una tesis, de modo

que una demostración resulta de la construcción de una estrategia ganadora a partir de las *explicaciones dialógicas del significado*.

Más aún, la interpretación dialógica de la TCT, dado que no distingue el nivel estratégico del nivel de partidas, no proporciona los fundamentos dialógicos últimos de conocimiento asertórico. En efecto, según el cuadro de *Razonamiento Inmanente* la interpretación dialógica de conocimiento asertórico no está fundada en la afirmación de premisas de complejidad arbitrarias, pero en la Regla Socrática que gobierna la justificación de afirmaciones elementales. Afirmaciones que involucran proposiciones complejas “heredan” durante el desarrollo de una partida, el conocimiento asertórico de las afirmaciones elementales que resultan del análisis de las afirmaciones complejas.

II La segunda parte, *La Sabiduría de Nuestros Mayores: Lecturas Contemporáneas*, contiene estudios recientes en lógica Aristotélica y post-Aristotélica, que subraya una de las aplicaciones más fructíferas del marco dialógico, a saber: la reconstrucción dialógica de ciertos enfoques en la historia y filosofía de la lógica que propone nuevos desarrollos en lógica temporal-modal, déontica y teoría de la argumentación.

Nuestras propias reconstrucciones representan apenas una pequeñísima muestra de lo que podemos, y quizá incluso debemos, aprender de la literatura en Lógica y Dialéctica árabes. Las ideas fundamentales que guían nuestros desarrollos en el marco de IR-II están basadas en los siguientes principios:

1. La modalidad es entendida bajo la óptica de la noción aristotélica de predicable de los *Topica*, es decir, los diferentes modos mediante los cuales el sujeto – el dominio (restringido) de cuantificación – se relaciona con el predicado (definido sobre el dominio). Los predicables se declinan en el modo de relación por definición (humanos son animales racionales), por género (humanos son animales), por propio (humanos pueden leer) o accidente (humanos pueden caminar). Mientras que los primeros modos establecen una relación necesaria, los otros dos modos establecen una relación de posibilidad (más precisamente de contingencia).
2. La relación de necesidad, requiere que el predicado sea atribuido *actualmente* a cada presencia o instancia del sujeto. La relación de contingencia, requiere que el predicado, un proceso que expresa una capacidad (por ej. leer), o disposición (por ej. respirar o reír) o esquema de acción (por ej. caminar) o evento (por ej. llover), sea atribuido *potencialmente* en lugar de actualmente, a cada instancia del dominio de cuantificación.
3. Potencialidad y actualidad son comprendidos dialéctica y temporalmente. En el caso de una cuantificación universal, por cada instancia *a* del sujeto producida por el interrogador, el defensor debe producir una presencia del predicado atribuida a la instancia *a* del sujeto en al menos un momento (si no la relación sería imposible) y una ausencia en al menos otro momento (si no la relación sería necesaria). En el caso de capacidades, basta una instancia genérica del sujeto (en lugar de individual): basta que un hombre alguna vez actualice la capacidad atribuida para hacer constar que los hombres tienen tal capacidad.
4. Dado que momentos no son portadores de propiedades (Aristóteles nos enseña que el tiempo no es una substancia), temporalidad no puede implementarse como un cuantificador más una función proposicional – el momento 14hs no es portador de la capacidad de *frenar delante del semáforo rojo*. Tampoco como un operador temporal al modo de Prior. Lo que parece mucho más apropiado y simple es concebir la temporalidad como afectando ejecuciones de capacidades o acciones o disposiciones: mi propia ejecución de frenar a tal hora y lugar.
5. Momentos deben ser concebidos como constitutivos de *historias* o intervalos de

tiempo que expresan duraciones. Historias que no implican determinismo requieren una estructura de tiempo bifurcado. Una tal estructura permite indeterminismo local con futuro abierto pero pasado cerrado o irrevocable. Tal estructura es también compatible, es decir combinable, con la noción de la consciencia o experiencia personal y subjetiva de la temporalidad.

6. En contextos éticos y/o jurídicos las modalidades déonticas presuponen que el agente puede elegir no ejecutar la acción prescrita o ejecutar una iniquidad. En ese sentido, obligación presupone, más que implica, posibilidad (de ejecución). Responsabilidad ética y jurídica constituye de este modo un elemento fundamental de la significación de las modalidades déonticas y en ella reside la distinción kantiana entre necesidad causal y necesidad moral.

Los estudios que desarrollan tales reconstrucciones son versiones en español de artículos escritos en colaboración y en su mayor parte, ya publicados. La lista de co-autores es la siguiente: Profesores M. Crubellier (Lille), M. Marion (UQAM), W. E. Young (McGill), F. Zidani (Alger II), C. Lion (Lille), Z. McConaughy (Lille), Doctorandos: R. Boussad (Lille), M. Drissi (Lille), L. Gallois (Lille), A. Seck (Lille).

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I RAZONAMIENTO INMANENTE II

La *lógica dialógica* es un marco de diálogo para el significado, el conocimiento y la lógica arraigado en una tradición sobre la teoría y práctica de la argumentación que se remonta a la dialéctica en la Antigüedad griega, cuando las cuestiones semánticas, epistémicas y éticas se abordaban a través de debates en los que partes opuestas discutían una tesis a través de preguntas y respuestas.²

La lógica dialógica contemporánea fue concebida por Paul Lorenzen en 1958, el año de su conferencia *Logik und Agon* celebrada en Venecia en el Duodécimo Congreso Mundial de Filosofía y fue desarrollada posteriormente por Kuno Lorenz. Inicialmente la Lógica Dialógica apuntaba a superar algunas limitaciones de la *Lógica Operativa* del propio Lorenzen (1955).³

Los trabajos de Lorenzen y Lorenz y la *Escuela Constructivista de Erlangen* tomó más tarde la forma de un marco dialógico general pragmático y lúdico para la filosofía del lenguaje, la lógica y las ciencias. Este marco fue el resultado del entrelazamiento de las matemáticas y la lógica constructivas,⁴ de una lectura dialéctica del surgimiento de la filosofía del lenguaje y la lógica en la tradición griega Antigua,⁵ y de la noción de juegos de lenguaje de Wittgenstein;⁶ con especial énfasis en el fundamento ético del pensamiento conceptual.⁷

Lorenz añadió tres perspectivas metodológicas importantes al proyecto⁸, a saber

- a) la reconstrucción de teorías del significado y el conocimiento de la historia de la filosofía que incluyen no sólo a Platón sino también las tradiciones indias y el pensamiento chino,
- b) una comprensión semiótica, peirceana, de la noción Wittgensteniana de juegos de lenguaje, y
- c) el uso explícito de la teoría matemática de juegos en lógica para el estudio de las propiedades metalógicas de los diversos sistemas lógicos desarrollados en el seno del marco dialógico.

Como mencionado en la introducción, el present volumen focaliza en un nuevo desarrollo del marco dialógico, llamado *Razonamiento Inmanente II*,⁹ que extiende el poder expresivo de tal marco incorporando ciertas propiedades de la teoría constructiva de tipos (TCT) de Per Martin-Löf. En efecto, en sus trabajos más recientes Martin-Löf (2017,2019a,b) destaca la importancia de la perspectiva dialógica para elucidar las raíces deónticas del concepto de contenido de juicio y proporcionar una teoría no circular de inferencia.¹⁰

I.1 Nociones Básicas de Lógica Dialógica

² Aunque ha habido enfoques paralelos, a menudo independientes, en otras tradiciones y períodos, particularmente en el subcontinente indio – cfr. Lorenz (1998) –, y también en el contexto de las teorías de la argumentación jurídica islámica medieval, que se desarrollaron mucho antes de la recepción de la lógica aristotélica – véase Young (2016, 2022), Miller (2020).

³ Cfr. Lorenz (2001); Schroeder-Heister (2008).

⁴ Cfr. Lorenzen (1978).

⁵ Cfr. Ebbinghaus (1964), Lorenz y Mittelstrass (1966, 1967), Lorenz (2011).

⁶ Cfr. Kamlah y Lorenzen (1967), Lorenz (1970); Lorenz (2008, 2011).

⁷ Cfr. Lorenzen (1969), Lorenzen y Schwemmer (1973), Lorenz (2021).

⁸ Cfr. Lorenz (1961, 1998, 2008, 2010, 2015, 2021).

⁹ Cfr. Rahman et al. (2018).

¹⁰ Cfr. Klev (2022, 2023).

La lógica dialógica de Lorenzen y Lorenz estudia diálogos; pero también toma la forma de diálogos. En un diálogo, dos partes (jugadores) debaten sobre una tesis (una determinada afirmación que otorga el tema principal del debate). El debate se desarrolla de acuerdo a ciertas reglas que fijan el marco formal para su desarrollo. El desarrollo del debate involucra enunciados declarativos, llamados *afirmaciones* (*statements*) y enunciados interrogativos llamados solicitudes o preguntas. Afirmaciones, dejando de lado la tesis, constituyen o bien respuestas a preguntas o bien ataques – como discutiremos más adelante en el marco dialógico es crucial distinguir entre *afirmación* (*statement*) y *aserción* (*assertion*).

El jugador que expone la tesis es el Proponente (**P**), y su interlocutor, el jugador que cuestiona la tesis, es el Oponente (**O**). Al cuestionar la tesis del Proponente, el Oponente exige al Proponente que defienda su afirmación mediante una secuencia de jugadas, gobernadas por las reglas adoptadas para el desarrollo de un diálogo.

Las reglas que dan forma a un diálogo se dividen en dos tipos:

1. Reglas de **partículas** – en lógica de primer orden son las reglas que determinan como cuestionar y cómo defender conectivas y cuantificadores. Las reglas de partículas otorgan la explicación dialógica del **significado local** de una expresión;
2. Reglas **estructurales**, también llamadas reglas de desarrollo o procedurales – ellas determinan como comenzar y terminar el diálogo, quién gana y quién pierde, etc. Las reglas estructurales otorgan la explicación dialógica del significado **global** de tal expresión

El marco dialógico, incluye un tercer nivel de significado: el nivel de estrategias que otorga la noción de inferencia válida. Sin embargo, como discutiremos más adelante, a diferencia de los inferencialistas, la noción de estrategia ganadora no es suficiente: saber ganar no es ciertamente lo mismo que saber cómo jugar.

En realidad, la noción fundamental del marco dialógico es la de la preservación de la **explicación dialógica del significado**. Efectivamente, una de las motivaciones históricas principales de la invención del marco dialógico es de desarrollar un marco en donde la noción de *admisibilidad*, introducida por primera vez por la *Lógica Operativa* de Lorenzen (una lógica en la cual reglas de inferencia fijan el significado), es substituida por la noción de *preservación del significado local*.

De hecho, como señaló Schroeder-Heister (2008), admisibilidad fue la vía por la cual Lorenzen otorgó significado a la teoría de la prueba y lo que distinguió su enfoque del formalismo sintáctico. La idea es que admisibilidad nos permite identificar **un núcleo de significado invariante** en relación con un conjunto de reglas: una regla es admisible si conserva el significado de las expresiones fijadas por ciertas reglas introducidas en un sistema inicial declarado como sistema de referencia.¹¹

Después del *giro dialógico*, la identificación de un núcleo invariante de significado se redujo a identificar el significado local puesta a la obra tanto en las reglas estructurales

¹¹ La noción de admisibilidad sigue siendo uno de los principios principales de los enfoques teóricos de la prueba contemporáneos; sobre todo después que Dag Prawitz vinculó atribuciones de admisibilidad y procedimientos de normalización – véase Lecomte y Quatrini (2011a,b).

como en las estratégicas.¹² Atribuciones de admisibilidad respecto a una inferencia (o reglas de inferencia) son ahora reemplazadas por atribuciones de *definibilidad dialógica* (*dialogue- definiteness*):

- Una expresión disfruta de la propiedad de *definibilidad dialógica*, si puede afirmarse como tesis en una partida individual y finita de suma cero, que termina con pérdida o pérdida para uno de los jugadores y que está regulada por reglas estructurales que preservan las explicaciones dialógico del significado.¹³
- Más aún, en el contexto dialógico la justificación de reglas de inferencia equivale a mostrar que preservan el significado local, es decir, que preservan un núcleo de explicación dialógica del significado invariante respecto a los jugadores y las reglas estructurales.¹⁴
- En resumen, desde la perspectiva del dialógico, la raíz de las reglas de inferencia, expresadas por reglas para construir una estrategia ganadora, es la explicación dialógica del significado local puesto en funcionamiento mediante reglas estructurales que las preservan. Las reglas dialógicas no son una interpretación dialógica de reglas de inferencia sino que las justifican.

El primer punto es crucial para cualquier forma de pluralismo dialógico, ya que elegir entre diferentes patrones de razonamiento supone un núcleo invariante respecto al cual deben constituirse las normas para determinar tanto el desarrollo de una partida como el de una estrategia ganadora. El núcleo invariante en sí es el resultado de un proceso semiótico dinámico previo mediante el cual se ha fijado el significado local, aunque el proceso puede iniciarse nuevamente más adelante. Según este punto de vista, las opciones inherentes al pluralismo dialógico comienzan antes de que la discusión sobre las reglas estructurales pase a primer plano – cfr. Lion (2020, pp. 125-159; 265-310).

Como señalan Clerbout y McConaughy (2022) en su artículo en la *Stanford Encyclopedia of Philosophy*, el desarrollo del Marco Dialógico experimenta actualmente un creciente interés especialmente

- en el campo de la teoría de la argumentación;¹⁵
- en historia y filosofía de la lógica y matemáticas;¹⁶
- en lógicas no clásicas;¹⁷
- en informática;¹⁸

¹² Presumiblemente la noción de admisibilidad de Lorenzen fue inspirada por la noción Aristotélica de reducción a las figura perfectas – cfr. Ebbinghaus (1964). Así, según la interpretación de Lorenzen, las reducciones al silogismo perfecto, si tienen éxito, muestran que las reglas reducidas preservan el significado local establecido por las reglas dialécticas para los cuantificadores codificados por las reglas para la demostración por éctesis – cfr. y Crubellier et al. (2019).

¹³ Lorenz (2001).

¹⁴ Esta es la razón por la que los operadores tipo *tonk* pueden descartarse en el marco dialógico – cfr. Rahman y Redmond (2016).

¹⁵ Cfr. – véase Gethmann (1982), Barth y Krabbe (1982), Walton (1984), Johnson (1999), Woods y Walton (1989), Woods y cols. (2002), van Eemeren y Grootendorst (2004), Prakken (2005), Vaidya (2013), Novaes (2015, 2020), Dutilh Novaes y French (2018), French (2019).

¹⁶ Cfr. Ebbinghaus (1964), Lorenz y Mittelstrass (1966, 1967), Molinero (2020), Keffer (2001), Yrjönsuuri (2001), Hintikka (2006), Novaes (2007), Castelnérac y Marion (2009), Crubellier (2011), Clerbout et al. (2011), Marion y Ruckert (2015), Gorisse (2017, 2018), Crubellier et al. (2019), Uckelman (2013); Young (2016, 2022), McConaughy (2021); Iqbal (2022).

¹⁷ Cfr. Rahman y Rückert (1999), Keiff (2007, 2010); Rückert (2011); Alama et al. (2011), Uckelman (2013).

¹⁸ Cfr Blass (1992), Lecomte y Quatrini (2011a,b), Fermüller (2003).

- en lingüística aplicada, razonamiento jurídico, inteligencia artificial y teoría de juegos;¹⁹
- En lógica y matemáticas constructivas.²⁰

I.1.1 Reglas de Partículas; Explicaciones Dialógicas del Significado Local

Es importante de recalcar que las reglas de partículas explican el significado de una expresión independientemente de las reglas estructurales que fijan el desarrollo de un diálogo (también llamado *partida*) e independientemente de quién de los interlocutores es el que fija la tesis, es decir, estas reglas prescriben cómo cuestionar una afirmación y cómo defenderla, pero no fijan quién es el que reta y quién el defensor.

Las explicaciones dialógicas del significado local de las constantes lógicas estándar de primer orden, en el contexto original de Lorenzen y Lorenz, son las siguientes (las líneas punteadas indican opciones del jugador):²¹

REGLAS DE PARTICULAS		
	Pregunta	Respuesta
<p>$X ! AVB$</p> <p>X afirma la disyunción A o B</p>	<p>$Y ?\vee$</p> <p>Y le pide a X de elegir uno de los componentes de la disyunción</p>	<p>$X ! A$</p> <p>X elige afirmar el componente izquierdo de la disyunción</p> <p>-----</p> <p>$X ! B$</p> <p>X elige afirmar el componente derecho de la disyunción</p>
<p>$X ! A \wedge B$</p> <p>X afirma la conjunción A y B</p>	<p>$Y ?\wedge_1$</p> <p>Y elige preguntar por el componente izquierdo de la conjunción</p> <p>-----</p> <p>$Y ?\wedge_2$</p> <p>Y elige preguntar por el componente derecho de la conjunción</p>	<p>$X ! A$</p> <p>X afirma el componente izquierdo de la conjunción</p> <p>$X ! B$</p> <p>X afirma el componente derecho de la conjunción</p>
<p>$X ! A \supset B$</p> <p>X afirma la implicación <i>Si A entonces B</i></p>	<p>$Y ! A$</p> <p>Y concede en afirmar el antecedente, a condición que X afirme el consecuente</p>	<p>$X ! B$</p> <p>X afirma el consecuente</p>
<p>$X ! \neg A$</p> <p>X afirma la negación A</p>	<p>$Y ! A$</p> <p>Y afirma en contrario, que A</p>	<p>—</p> <p>No hay defensa, pero durante el desarrollo del juego X intentará de forzar a Y de afirmar una contradicción o usar la afirmación de Y para sus propios propósitos</p>

¹⁹ Cfr. Ranta (1988, 1994), Hintikka (1996), Hintikka y Sandu (1997), Prakken (2005), Ginzburg (2012).

²⁰ Cfr. Coquand (1995), Felscher (1985), Sørensen y Urzyczyn (2006); Martin-Löf (2015, 2017, 2019a,b), Klev (2022, 2023), Sterling y Angiuli (2021).

²¹ Véase Rückert (2011); Clebout (2014c,b); Clerbout & McConaughey (2022).

$X ! \exists xA(x)$	$Y ?_{\exists}$	$X ! A(a)$
X afirma que hay al menos un elemento en el dominio que es <i>A</i>	Y pregunta: Quién es <i>A</i> ?	X elige el elemento <i>a</i> y afirma que ése <i>a</i> es uno de los elementos del dominio que es <i>A</i>
$X ! \forall xA(x)$	$Y ?_{\forall a}$	$X ! A(a)$
X afirma que todos los elementos en el dominio son <i>A</i>	Y elige el elemento <i>a</i> y pregunta: Es <i>a</i> uno de aquellos que son <i>A</i> ?	X afirma que en efecto <i>a</i> es uno de los elementos del dominio que es <i>A</i>

I.1.2 Reglas Estructurales: Explicaciones Dialógicas del Significado Global

Las reglas estructurales regulan el uso de las explicaciones dialógicas del significado local en el contexto del desarrollo de una partida. Una regulación tal establece la explicación dialógica del significado global de las expresiones que ocurren durante el desarrollo de la partida. La elección entre diferentes reglas de desarrollo posibles, determina qué patrones de razonamiento cuentan como significativos y cuáles no: por ejemplo, la elección entre las reglas RS1i y RR1c determina si el razonamiento se desarrolla bajo el enfoque de una teoría del significado intuicionista o más bien bajo el enfoque de una teoría clásica del significado.

Las reglas que presentamos a continuación son básicas en el sentido de que diferentes lógicas y enfoques de significado podrían requerir ampliarlas agregando nuevas reglas o agregando condiciones adicionales a las reglas existentes.

Reglas Estructurales para Lógica Intuicionista y Clásica

RS0 (Comienzo)

Una partida comienza cuando uno de los jugadores afirma una proposición llamada tesis; ese jugador adopta ahora el rol de Proponente (**P**) y esa jugada lleva el número 0.²²

El otro jugador, el Oponente (**O**), elige un rango de repetición que determina cuántas veces puede desafiar o defender la misma jugada durante una partida.

Generalmente es suficiente que **O** elija un rango de repetición de 1 (*m*:= 1), denominada jugada 1 y que **P** elija un rango de repetición de 2 (*n*:= 2), denominada jugada 2.²²

RS1i (Regla de desarrollo intuicionista de partidas)

²² **Observaciones a propósito de RS0.** Esta regla asegura que las partidas sean finitas (aunque pueda haber un número infinito de ellas). De hecho, una característica importante de la concepción de proposición y afirmación de Lorenzen-Lorenz es su finitud: para que una expresión cuente como una proposición *A* debe existir una partida individual con una tesis en la que se afirma *A* y que alcanza una posición final después de un número finito. de jugadas de acuerdo con las reglas estructurales y con las reglas de partículas definidas para la partida. Lorenz (2001). En efecto, si los juegos de lenguaje deben concebirse como mediadores de significado, estos juegos de lenguaje deben ser juegos que realmente podamos realizar. Lorenz (2001) denomina una tal concepción de proposición, proposición *dialogo-definida*. Clerbout (2014c,b,a) profundizó en las consecuencias metalógicas del rango de repetición y mostró cómo se relaciona con la semiindecidibilidad de la lógica de primer orden. Un resultado filosófico general importante del trabajo de Clerbout es que, aunque cada partida debe ser finita, esto no impide la existencia de un número infinito de ellas.

Una vez elegidos los rangos de repetición, cada jugada es una cuestión o una defensa en reacción a una jugada anterior, de acuerdo con las reglas de partículas.

Cada jugador puede cuestionar la misma jugada como máximo n veces, donde n es el rango de repetición del jugador, o defenderse de la última cuestión del adversario aún no respondida.²³

RS1c (Regla de desarrollo clásico de partidas))

Una vez elegidos los rangos de repetición, cada jugada es una cuestión o una defensa en reacción a una jugada anterior, de acuerdo con las reglas de partículas.

Cada jugador puede cuestionar la misma jugada o defenderse de una cuestión como máximo n veces, donde n es el rango de repetición de la jugada, aunque la defensa no sea una respuesta a la última cuestión del adversario.²⁴

RS2.1 (Regla formal)

P no puede afirmar una proposición elemental a menos que **O** la haya afirmado primero. Las proposiciones elementales no pueden cuestionarse. Esta regla produce *diálogos formales* o *puramente asertóricos* e impide que la tesis consista en afirmar una proposición elemental.

RS2.2 (Regla Socrática)

P no puede afirmar una proposición elemental a menos que **O** la haya afirmado primero. Las proposiciones elementales de **P** pueden cuestionarse pero no las de **O**. Esta regla produce *diálogos materiales* o *materialmente asertóricos*.²⁵

Si la tesis consiste en la afirmación de la proposición A , y **O** no afirmó la misma proposición antes, **O** puede cuestionar la tesis con la pregunta $?A$. Las proposiciones elementales de los juegos materiales tienen contenido.

Esto requiere una regla específica a cada una de las proposiciones elementales involucradas en una partida. Por ejemplo, en el caso de afirmaciones sobre números naturales, la Regla Socrática estipula que **P** siempre puede pedirle a **O** que afirme que 0 es un número natural.

²³ Esta numeración tiene el efecto que los números pares son jugadas de P y los números impares son jugadas de O.

²⁴ Esta regla establece que el deber de defender primero el último desafío tiene prioridad sobre el rango de repetición para defender. Es de tenerse en cuenta que todo ataque a una negación siempre quedará sin respuesta, ya que no hay defensa contra ella. Por lo tanto, tal ataque siempre contará como el último ataque.

La posibilidad de elegir algunas reglas estructurales sobre otras (como la elección entre SR1i y SR1c) pone de relieve el poder explicativo del marco dialógico. Los conectivos clásicos e intuicionistas comparten el mismo significado local, pero difieren sólo en una regla global. En efecto, en un marco dialógico, si hay algún terreno común entre los enfoques clásico e intuicionista del significado, este terreno común debe encontrarse en las explicaciones dialógicas locales del significado. En 1996, la posibilidad de desarrollar una pluralidad de sistemas dialógicos se generalizó y dió lugar al acimiento del pluralismo dialógico concebido por S. Rahman y colaboradores – véase, Keiff (2007) y Rückert (2011).

²⁵ Discutimos *diálogos materiales* más adelante cuando presentamos las reglas dialógicas para los números naturales, conjuntos finitos, funciones y proposiciones con contenido empírico.

Esta regla se combina con una regla de significado local para números naturales que establece si **X** afirma que el sucesor de n es un número natural, **Y** puede solicitar a **X** que afirme además que n también es un número natural.

En el caso de proposiciones elementales empíricas, algún proceso de verificación determinado por el contexto epistemológico vigente determinará qué proposiciones elementales pueden ser afirmadas por **O** sin cuestionamiento.²⁶

RS3 (Finalizar y Ganar una Partida)

La partida termina cuando es el turno de un jugador de hacer una jugada pero no le queda ninguna jugada disponible. Ese jugador pierde y el otro jugador gana.

Ejemplo de partida ganada para una tesis no válida

O		P	
		$\neg A \vee B \supset B \wedge A$	0
1	$m:=1$	$n:=2$	2
3	$\neg A \vee B$	$\neg B \wedge A$	4
5	$? \wedge_1$	$\neg B$	8
7	$\neg B$	3 $? \vee$	6
		P gana	

Numeración: En la columna de **P** mientras que los números a la derecha representan el número de la jugada de **P**, los números a la izquierda (en este caso uno solo) indican la jugada de **O** atacada. En la columna de **O** los números a la izquierda representan el número de la jugada de **O**, los números a la derecha indican la jugada de **P** atacada.

Cuestiones (ataques) se formulan en una nueva línea, defensas afirman en la misma línea que en la que formuló la cuestión.

Descripción informal de la partida

P afirma la tesis $A \vee B \supset B \wedge A$ (jugada 0)

O y **P** eligen respectivamente el rango de repetición 1 y 2 (jugadas 1 y 2)

O cuestiona la implicación concediendo el antecedente (jugada 3) a condición que **P** afirme el consecuente.

P afirma el consecuente (jugada 4)

O pregunta por el componente izquierdo de la conjunción (jugada 5)

P Dado RS2, **P** no puede responder aún, y por tanto opta por iniciar un contrataque preguntando a **O** que elija qué componente de la disyunción está dispuesto a afirmar (jugada 6)

O elige (erróneamente) de responder afirmando el componente derecho de la disyunción (jugada 7)

P responde ahora a la cuestión que **O** formuló en la jugada 5, es decir B (jugada 8), y gana, pues **O** ya la había afirmado antes, y **O** no tiene otra jugada a su disposición.

²⁶ **Observaciones a propósito de RS2.1 y RS2.21.** Estas reglas, ausentes en otras semánticas de juego como la GTS de Jaakko Hintikka (1973 pp. 77-82), constituyen una de las principales características del Marco Dialógico. De hecho, como ya mencionado, mientras elaboraba un nuevo concepto de contenido juicio, estrechamente vinculado al concepto de *supuesto epistémico* (y de conocimiento *asertórico* codificado en tales supuestos) de Goran Sundholm (1997, 2012, 2019), Martin-Löf (2015, 2017, 2019a,b) propuso una interpretación dialógica de tal concepto que hace uso de estas reglas en una forma que no se restringe a proposiciones elementales (como discutiremos más adelante). La regla socrática debe su nombre a Marion y Rückert (2015), quienes indican como fuente histórica algunos textos cruciales de la Dialéctica de Platón y Aristóteles (ver también Crubellier et al. (2019); McConaughy (2021)). La regla socrática extiende la regla formal de Lorenzen & Lorenz (1976) a los diálogos materiales.

Obviamente tal ejemplo muestra que ganar un partida no es lo mismo que probar que la tesis es válida. La jugada 7 es una jugada débil. Una tesis es válida si resiste a todos las opciones de **O**. En este caso, la jugada que representa el test mas sólido, es que **O** opte por afirmar A (no B).

O		P	
		$!A \vee B \supset B \wedge A$	0
1	$m:=1$	$n:=2$	2
3	$!A \vee B$	$!B \wedge A$	4
5	$? \wedge_1$		4
7	$!A$	3	$? \vee$
O gana			

En suma, la tesis no es válida pues la opciones de **O** dan lugar a cuatro partidas, en dos gana y en dos pierde, a saber:

1. pregunta por la izquierda y defiende la izquierda (**O** gana),
2. pregunta por la izquierda y defiende la derecha (**O** pierde),
3. pregunta por la derecha y defiende la derecha (**O** gana),
4. pregunta por la derecha y defiende la izquierda (**O** pierde),

Como son opciones de **O**, **P** tiene que ganar en todas. En efecto, una tesis es válida si **P** pueden ganar cualquiera sean las jugadas elegidas por **O**. Esto sugiere que necesitamos definir la noción de estrategia ganadora y de validez como estrategia ganadora de **P**.

I.1.3 Estrategia Ganadora y Validez

Existen varias formas de definir una estrategia ganadora dentro de un Marco Dialógico – véase por ejemplo Krabbe (1985); Clerbout (2014c,b,a); Rahman et al. (2018); León (2023). En aras de una presentación simple, ofreceremos una variación de Felscher (1985), que también se puede encontrar en las tablas dialógicas con ramas de Redmond y Fontaine (2011), y en Piecha (2015) – Rahman et al. (2018), págs. 90-109 describen un método para encontrar una estrategia ganadora a partir de una sucesión de partidas.

- El jugador **X** tiene una estrategia ganadora si, para cada partida relevante, por cada jugada del otro jugador **Y**, **X** tiene a su disposición una jugada que finaliza la partida.

Puesto que validez se define respecto a la existencia de una estrategia ganadora para **P**, restringimos la noción de estrategia ganadora al caso del Proponente:

Definición. Una estrategia ganadora para **P** que afirma la tesis A es un árbol \mathcal{F} definido por:

- La raíz (nodo de profundidad 0) de \mathcal{F} es la jugada $\mathbf{P} ! A$
- Si el nodo es una jugada de **O** (es decir, si el número del nodo es impar), entonces tiene exactamente un nodo sucesor (que es una jugada de **P**). Si **P** tiene varias opciones, solo una será retenida. Esto tiene como consecuencia que la estrategia ganadora se construye a partir de una de las opciones de **P** (una es suficiente)
- Si el nodo es una jugada de **P** (es decir, si el número de un nodo es par), entonces tiene tantos nodos sucesores como jugadas posible tenga **O** en esa

posición. Cada una de esas opciones abre una rama del árbol
 Una estrategia es una estrategia ganadora para **P** si todas las ramas terminan con una jugada por la cual **P** afirma una proposición elemental.

Validez. Una proposición es válida si **P** tiene una estrategia ganadora para una tesis que la afirme.

Metalógica. Se puede mostrar que tal definición con las nociones de validez para las lógicas clásica e intuicionista de primer orden - véase Felscher (1985); Clebout (2014c,b,a).

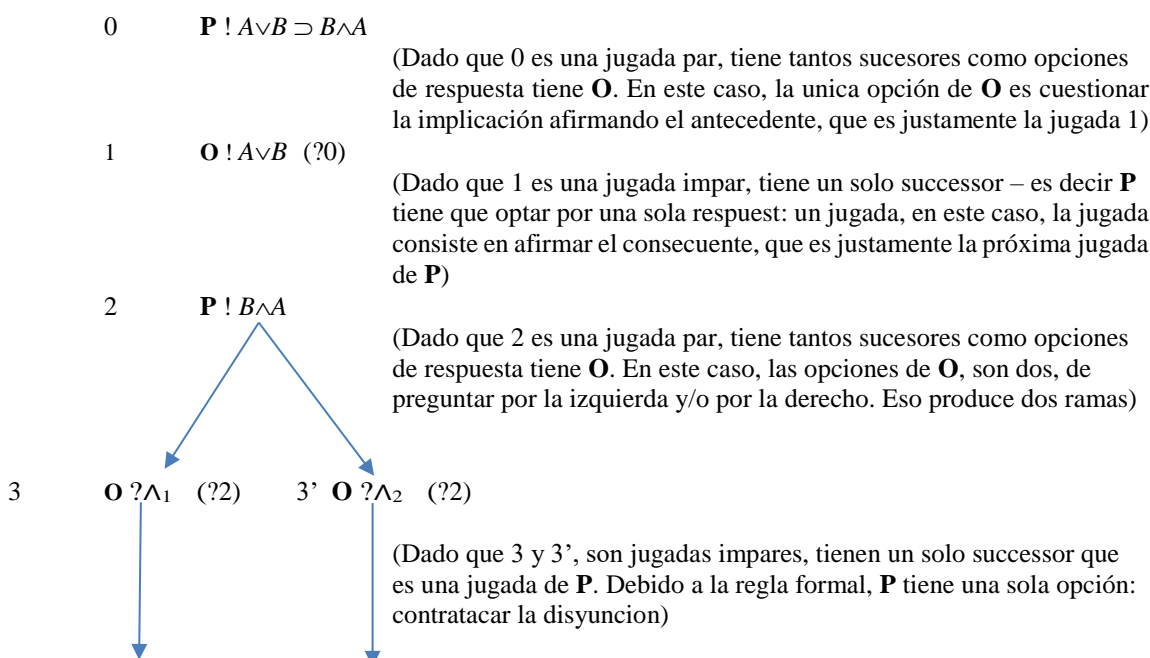
Arboles finitos. Las estrategias ganadoras para una tesis de primer orden pueden, en general, dar lugar a árboles con un número infinito de ramas finitas (cada rama es una obra de teatro). Se puede evitar tener que desarrollar un árbol infinito para una estrategia ganadora para **P** introduciendo las siguientes instrucciones basadas en las restricciones prescritas por la Regla Formal (Socrática):

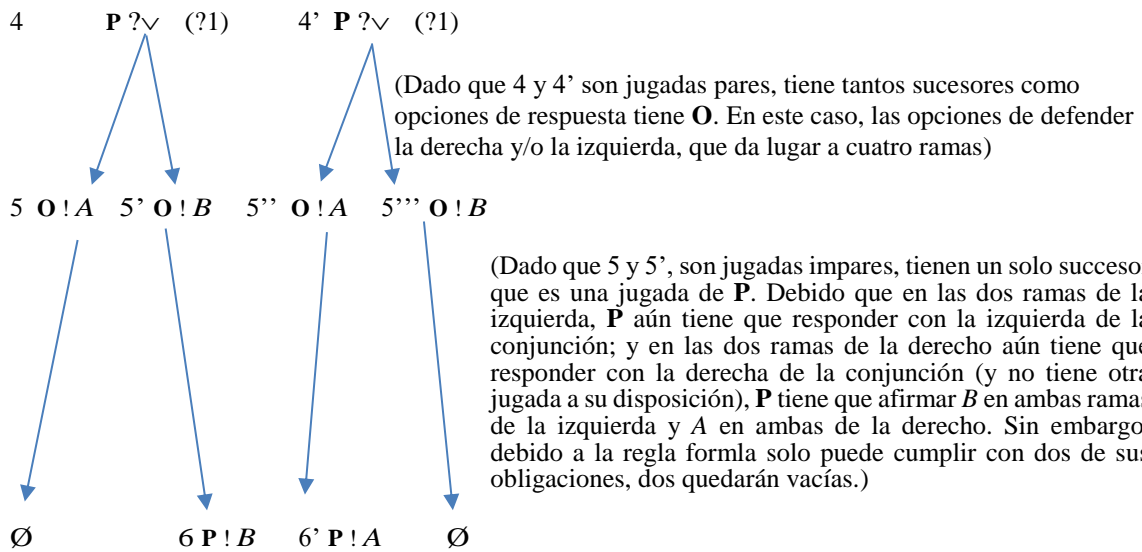
Quando **O** tiene la oportunidad de elegir cómo atacar un universal o defender un existencial, su jugada óptima es elegir siempre un *a* que sea nuevo respecto al desarrollo de la rama (partida). Por el contrario **P**, que necesita que **O** afirme las proposiciones elementales que **O** solicita; cuando ataca un universal de **O**, o se defiende de un existencial ocurriendo en una rama (partida), copiará las propias elecciones de **O** -siempre y cuando **O** haya ya hecho antes tales elecciones en esa rama (partida).

Ejemplo de un árbol generado por una estrategia ganadora

En el siguiente ejercicio podremos ver la diferencia entre opciones y ramificaciones. En efecto, ambos jugadores tienen muchas opciones de juego y van prefiriendo unas u otras en sus búsquedas de una estrategia efectiva para ganar. De entre esas opciones hay un conjunto que corresponden a opciones del Oponente. Estas opciones del Oponente desencadenan juegos paralelos (ramificaciones) y el Proponente debe ganar todos ellos para poder afirmar que posee una estrategia ganadora para la tesis.

Obviamos aquí las jugadas en donde los jugadores elijen el rango de repetición.





Puesto que en una de las dos ramas de la izquierda, la última jugada es la jugada 5 de **O**, **O** gana en la partida representada por esa rama. Análogamente, en una de las ramas de la derecha la última jugada es la jugada 5''' de **O**, **O** gana en la partida representada por esa rama. En conclusion, **P** no posee una estrategia ganadora, pues no toda rama termina con una jugada de **P**, y por ende la tesis no es válida. En realidad es **O**, que posee una estrategia ganadora que se resume en lo siguiente:

- o bien, responder con la izquierda y preguntar por la izquierda;
- o bien responder con la derecha y preguntar por la derecha

Si reconstruimos cada una de las partidas expresada por el árbol obtenemos cuatro partidas (reincorporamos las jugadas correspondientes al rango de repetición), cada una corresponde a una rama del árbol:

I
(corresponde a la primera rama del árbol)

O			P		
			$!A \vee B \supset B \wedge A$	0	
1	$m:=1$		$n:=2$	2	
3	$!A \vee B$	0	$!B \wedge A$	4	
5	$? \wedge_1$	4			
7	$!A$		3	$? \vee$	6
O gana					

II
(corresponde a la segunda rama del árbol)

O			P		
			$!A \vee B \supset B \wedge A$	0	
1	$m:=1$		$n:=2$	2	
3	$!A \vee B$	0	$!B \wedge A$	4	
5	$? \wedge_1$	4	$!B$	8	
7	$!B$		3	$? \vee$	6
			P gana		

III
(corresponde a la tercera rama del árbol)

O			P		
			$!A \vee B \supset B \wedge A$	0	
1	$m:=1$		$n:=2$	2	
3	$!A \vee B$	0	$!B \wedge A$	4	

5	$? \wedge_2$	4	$! A$	8
7	$! A$		3	$? \vee$
			P gana	

IV
(corresponde a la cuarta rama del árbol)

O			P	
			$! A \vee B \supset B \wedge A$	0
1	m:=1		n:=2	2
3	$! A \vee B$	0	$! B \wedge A$	4
5	$? \wedge_2$	4		
7	$! B$		3	$? \vee$
O gana				

I.2 Razonamiento y Significado Inmanentes

Los diálogos son juegos de dar y pedir razones y es en el seno de esa interacción que los conceptos son constituidos; sin embargo, en el marco dialógico estándar, las razones de cada afirmación quedan implícitas. El marco teórico de *Razonamiento Inmanente* (RI) contiene una forma de afirmación inspirada por la Teoría Constructiva de Tipos (TCT) de Martin-Löf para hacer explícitas estas razones.²⁷ Las afirmaciones de RI toman así la siguiente forma— véase Rahman et al. (2018):

$$\mathbf{X} ! a \varepsilon B^{28}$$

donde B es una proposición y a su *razón local*, es decir, la razón circunstancial particular que da derecho a afirmar B . Por lo tanto, $\mathbf{X} ! a \varepsilon B$ expresa que X puede producir una razón local para justificar su afirmación.

De esta manera, las razones que tienen los interlocutores para hacer una afirmación se especifican en el nivel del lenguaje objeto.

Como es de esperarse, en RI se distinguen **razones locales** y **razones estratégicas**:

razones locales ocurren en partidas,
razones estratégicas codifican una recapitulación de todas las jugadas relevantes para constituir una estrategia ganadora.

Es así que RI nos permite enriquecer el marco dialógico estándar dentro de una teoría interactiva del significado que hace uso de la expresividad de los lenguajes totalmente interpretados propios de la TTC. Esta expansión del marco dialógico no sólo permite integrar los análisis de la anáfora proporcionados por la TTC, incluyendo cuantificadores ramificados y cuantificadores generalizados – véase Sundholm (1986, 1989); Ranta (1994) – pero también proporciona un marco para estudios de historia y filosofía de la lógica.²⁹

I.2.1 La Perspectiva Dialógica

Mientras elaboraba un nuevo concepto de contenido de juicio, Martin-Löf (2015, 2017, 2019a,b) destacó la contribución del Marco Dialógico a la noción de *conocimiento asertórico* expresado por *supuestos epistémicos*.³⁰ Tal interpretación dialógica ofrece en el seno de una teoría de la prueba del significado, una explicación de validez inferencial que no cae en el bien conocido círculo de justificación.³¹

²⁷ Véase en Martin-Löf y Sambin (1984).

²⁸ La notación “ ε ” significa relación elemento de un conjunto intensional. La notación más reciente en TTC es “ \cdot ”, sin embargo, utilizamos “ ε ”, que fue utilizada por Lorenz y Mittelstrass (1966, 1967) para la relación de instanciación dentro de su análisis dialógico de la teoría del significado de Platón

²⁹ Véase Crubellier et al. (2019) quienes usan el marco para estudiar Aristóteles, Iqbal (2022) quien ofrece un lúcido análisis del razonamiento jurídico medieval, y Rahman y Zarepour (2019) quienes utilizan un tal marco para reconstruir una de las formas principales de la lógica temporal de Avicena.

³⁰ Debemos la noción de supuesto epistémico Göran Sundholm (1997, 2012, 2019).

³¹ La concepción habitual en matemáticas de *juicio conocido* (judgement known) como *juicio demostrado*, produce un círculo cuando se utiliza para justificar la inferencia. En efecto, en matemáticas, es natural considerar un juicio como conocido si ha sido demostrado. Bajo esta explicación de juicio conocido, la

Según este punto de vista, a fin de justificar una inferencia basta con suponer que alguien ha afirmado las premisas (en lugar de haberlas demostrado) y, bajo ese supuesto, demostrar que se puede afirmar la conclusión: en otras palabras la validez de la inferencia se justifica poniendo en juego el conocimiento asertórico atribuido a las premisas.

La idea subyacente es que el conocimiento asertórico expresado por un supuesto epistémico es más débil que el conocimiento apodíctico: el último, pero no el primero, requiere demostración. Sundholm (2019) produjo la siguiente formulación de tal idea

Cuando digo **por lo tanto**, otorgo a otros mi autoridad para afirmar la conclusión, dada la autoridad prestada a aquellos otros para afirmar las premisas.

Más aún, en una serie de conferencias y textos, que discuten el fundamento deóntico de la lógica, Martin-Löf señala que el principio central filosófico del enfoque dialógico es que la noción de contenido de juicio equivale a la noción pragmática y normativa de una tarea por resolver. Así, un juicio es correcto si y sólo si el agente que lo hace sabe (es capaz de, puede) realizar la tarea que constituye su contenido – véase Klev (2024).

Esto nos remonta a los inicios de la Lógica Dialógica y al trabajo de Lorenz (2010) y a su formulación del significado como dominio de un esquema de acción. El principio filosófico fundamental del enfoque dialógico, observa Lorenz, es el rechazo de las formas reduccionistas habituales de entender la interfaz entre semántica y pragmática. Según la concepción de Lorenz (2011) la interfaz entre semántica y pragmática no debe entenderse

ni como el resultado de una *semantización de la pragmática* como la practicada por diferentes variantes de la semántica formal, donde aspectos deónticos, epistémicos, ontológicos y temporales se convierten en conectivas veritativo-funcionales,

ni como el resultado de una *pragmatización de la semántica* como la practicada por las teorías de los actos de habla, donde un núcleo proposicional, cuando es puesto en uso, se complementa con modos para producir afirmaciones, preguntas, órdenes, etc.

Más bien, la visión de Lorenz (2010) es que cada enunciado pone en obra simultáneamente ambos aspectos, el pragmático y el semántico, ya que un enunciado significa (estrato semántico) y comunica (estrato pragmático) a la vez. La articulación que los diferencia emerge durante una fase posterior del proceso dialógico básico, a saber, el enseñar y el aprender mediante la interacción.

A pesar de que las teorías inferencialistas en general como la de Brandom (1994) y el de la TCT en particular comparten con el Marco Dialógico un enfoque pragmatista del significado, la Dialógica no se reduce a una interpretación

explicación de la validez de la inferencia es la siguiente: la conclusión J puede demostrarse bajo el supuesto de que las premisas J_1, \dots, J_n han sido demostradas. Sin embargo, que un juicio haya sido demostrado significa por ende que es la conclusión final de una cadena de inferencias válidas (incluyendo el caso de axiomas concebidos como conclusiones de inferencias válidas). Al menos para el discurso matemático, la explicación de la validez inferencial es, por tanto, circular: apela a la noción de juicio demostrado, que a su vez se explica en términos de validez” – cfr. Klev (2024).

dialógica de un sistema deductivo predefinido.

En efecto, desde el punto de vista dialógico, las unidades básicas del significado y el conocimiento son afirmaciones (*statements*) no aserciones (*assertions*). Mientras que **afirmaciones** constituyen el contenido puesto en juego durante una partida, **aserciones**, constituyen estrategias. Estrategias corresponden a las inferencias que ocurren en sistemas deductivos, pero ellas no originan la construcción del significado sino las partidas son las que llevan a cabo tal tarea.

Así, mientras que afirmaciones ocurren en una partida y son el resultado de la aplicación de las reglas anónimas que fijan el significado local; aserciones codifican las tareas que ambos jugadores deben resolver para construir una estrategia ganadora para el jugador **P**.

Como ilustrado más arriba, desde el punto de vista dialógico, las estrategias están constituidas por partidas, de modo que los objetos estratégicos ganados mediante un proceso de “recapitulación” en tales partidas codifican la secuencia de jugadas que deben realizarse para cumplir la tarea asociada.

Por ende, aunque la perspectiva dialógica comparte con la TCT la tesis de que en el nivel estratégico el contenido de una aserción no es la proposición misma sino el contenido es que la proposición sea verdadera, la dialógica se aparta de la TCT al fundamentar el conocimiento, sobre la noción más básica de afirmación (en una partida)

En otras palabras, la interacción de afirmaciones que constituyen una partida, desarrollada por las reglas que fijan las explicaciones dialógicas de significado, y en particular por la Regla Socrática, constituye la raíz de lo que en el nivel estratégico produce la interpretación dialógica de *conocimiento asertórico* esgrimida por Martin-Löf para fundamentar el concepto de inferencia correcta. La interpretación dialógica de la TCT, dado que no distingue el nivel estratégico del nivel de partidas, no proporciona los fundamentos dialógicos últimos de conocimiento asertórico.

En suma, las tres nuevas innovaciones principales de la lógica dialógica son:

- La introducción de un nivel de explicaciones de significado invariante en relación con los jugadores, el desarrollo de una partida y en relación a la construcción de una estrategia.
- La introducción de un nivel de partida que incluye un mecanismo de verificación decidible de definibilidad dialógica que proporciona la noción dialógica de proposición y que substituye el mecanismo de verificación de tipos asumido por la TCT.
- La interpretación dialógica de conocimiento asertórico no está fundada en la afirmación de premisas de complejidad arbitrarias, pero en la Regla Socrática que gobierna la justificación de afirmaciones elementales. Afirmaciones que involucran proposiciones complejas “heredan” durante el desarrollo de una partida, el conocimiento asertórico de las afirmaciones elementales que resultan del análisis de las afirmaciones complejas.

I.2.2 Explicaciones Dialógicas del Significado Local en IR-II

La explicación dialógica del significado de una expresión está definida por reglas que prescriben cómo impugnar una aserción que involucra una tal expresión y cómo defenderla durante la interacción de dos (o más) interlocutores.

Se trata de una *explicación de significado*, ya que, al identificar la razón local canónica necesaria para defender su afirmación, las reglas explican *por qué* tenemos derecho a

afirmar lo que se afirma.

I.2.2.1 Disyunción, Conjunción y Cuantificador Existencial

Las explicaciones dialógicas del significado local para la disyunción, la conjunción y el existencial son casi las mismas que Klev (2022, 202x) desarrolló para la TCT.

La principal diferencia es que en marco de RI-II las reglas se formulan de forma independiente al jugador.

La doble flecha \Rightarrow que aparece en las tablas siguientes debe entenderse como el análogo del nivel de juego de la flecha de evaluación utilizada en Klev (2022, 202x).

Como discutiremos más adelante la tarea asociada a una afirmación de la forma $d \Rightarrow c \in C$ se resuelve mediante cierta forma de cálculo que muestra, cuando tiene éxito, cómo obtener c a partir de d . El cálculo en sí, esencialmente un proceso que Klev llama *desenvolvimiento definicional* (*definitional unfolding*), también puede moldearse mediante reglas de interacción dialógica.

Disyunción		
	Pregunta	Respuesta
$\mathbf{X} ! d \in AVB$ Asumiendo ³² $A, B \in prop$	$\mathbf{Y} ?_{\vee d}$	$\mathbf{X} ! d \Rightarrow I_{z^{\vee}}(a) \in AVB$ ----- $\mathbf{X} ! d \Rightarrow D_{r^{\vee}}(b) \in AVB$
$\mathbf{X} ! d \Rightarrow I_{z^{\vee}}(a) \in AVB$ ----- $\mathbf{X} ! d \Rightarrow D_{r^{\vee}}(b) \in AVB$	$\mathbf{Y} ?_{\vee}$	$\mathbf{X} ! I_{z^{\vee}}(a) \in AVB$ ----- $\mathbf{X} ! D_{r^{\vee}}(b) \in AVB$
$\mathbf{X} ! I_{z^{\vee}}(a) \in AVB$ ----- $\mathbf{X} ! D_{r^{\vee}}(b) \in AVB$	$\mathbf{Y} ?_{I_{z^{\vee}}(a)}$ $\mathbf{Y} ?_{D_{r^{\vee}}(b)}$	$\mathbf{X} ! a \in A$ $\mathbf{X} ! b \in B$

Notación:

La expresión $I_{z^{\vee}}(a)$ indica que la razón que justifica la afirmación de la disyunción es la razón local para su componente izquierdo.

La expresión $D_{r^{\vee}}(b)$ indica que la razón que justifica la afirmación de la disyunción es la razón local para su componente derecho.

Conjunción		
	Pregunta	Respuesta

³² Esta forma de asunción puede también formularse mediante reglas dialógicas de la manera siguiente : si \mathbf{X} afirma una disyunción, una conjunción o una implicación, \mathbf{Y} puede preguntar por la formación de la primera componente con la preguntas $\mathbf{Y} ? Form_1$, $\mathbf{Y} ? Form_2$, \mathbf{X} dará las respuestas $A \in prop$, $B \in prop$ (o $\perp = \emptyset \in prop$) respectivamente ; si \mathbf{X} afirma un cuantificador, la respuestas al mismo tipo de preguntas será $A \in prop$, $B(x) \in prop$ ($a: A$).

$\mathbf{X} ! d \varepsilon A \wedge B$ Asumiendo $A, B \varepsilon prop$	$\mathbf{Y} ? \wedge d$	$\mathbf{X} ! d \Rightarrow \langle a, b \rangle \varepsilon A \wedge B$
$\mathbf{X} ! d \Rightarrow \langle a, b \rangle \varepsilon A \wedge B$	$\mathbf{Y} ? \wedge$	$\mathbf{X} ! \langle a, b \rangle \varepsilon A \wedge B$
$\mathbf{X} ! \langle a, b \rangle \varepsilon A \wedge B$	o bien $\mathbf{Y} ? \wedge 1$ o bien $\mathbf{Y} ? \wedge 2$	$\mathbf{X} ! a \varepsilon A$ $\mathbf{X} ! b \varepsilon B$

Existencial		
	Pregunta	Respuesta
$\mathbf{X} ! d \varepsilon (\exists x \varepsilon A)B(x)$ Asumiendo $A \varepsilon prop$ $B(x) \varepsilon prop (a: A)$	$\mathbf{Y} ? \exists d$	$\mathbf{X} ! d \Rightarrow \langle a, b \rangle \varepsilon (\exists x \varepsilon A)B(x)$
$\mathbf{X} ! d \Rightarrow \langle a, b \rangle \varepsilon (\exists x \varepsilon A)B(x)$	$\mathbf{Y} ? \exists$	$\mathbf{X} ! \langle a, b \rangle \varepsilon (\exists x \varepsilon A)B(x)$
$\mathbf{X} ! \langle a, b \rangle \varepsilon (\exists x \varepsilon A)B(x)$	$\mathbf{Y} ? \exists 1$ ----- $\mathbf{Y} ? \exists 2$	$\mathbf{X} ! a \varepsilon A$ $\mathbf{X} ! b \varepsilon B(a/x)$

1.2.2.2 Implicación, Negación y Cuantificador Universal

Es en la implicación donde se manifiesta la diferencia con las reglas de diálogo de la teoría constructiva de tipos. En efecto, la explicación dialógica del significado de la implicación, requiere distinguir:

1. Las reglas para el significado local que son independientes de quién es el jugador que tiene el rol de Proponente y quién el rol de Oponente. Rahman & Redmond (2016) las denominaron *reglas para jugadores anónimos*.
2. Los efectos de las reglas globales, que ponen en funcionamiento las reglas para el significado local en el contexto del desarrollo de una partida.
3. La explicación estratégica del significado que resulta de abstraer la función que establece la dependencia del significado del consecuente con respecto al antecedente determinado por las reglas globales del nivel de juego. Como discutiremos en próxima sección, desde el punto de vista dialógico **la función es un método** que prescribe como el Proponente puede obtener una razón para el consecuente **para toda razón** que el Oponente produzca para justificar el antecedente.

En cuanto a 1 recordemos que las explicaciones dialógicas de significado locales no sólo son reglas para jugadores anónimos, sino que también establecen lo que vale como una jugada legítima independientemente de consideraciones de optimalidad. En el caso de la implicación, las reglas locales simplemente determinan que impugnarla equivale a que el retador proporcione una razón local para el antecedente y el defensor proporcione una

razón local para el consecuente **durante una partida**. La notación $(IzY^\supset)DrX^\supset \varepsilon A \supset B$ para la razón local de la implicación no se lee como una función; ella simplemente indica que la defensa de la implicación requiere que el defensor, **X**, defienda el consecuente (el componente derecho Dr de la implicación) afirmando $b \varepsilon B$ después de que el retador **Y** ataque la implicación afirmando $a \varepsilon A$ (el componente izquierdo Iz de la implicación). Para simplificar la notación, en las tablas siguientes eliminaremos los índices **X** e **Y** que aparecen en $(IzY^\supset)DrX^\supset$ y en $(IzY^\forall)DrX^\forall$. Quién afirma el antecedente y quién el consecuente queda claro de todos modos cuando se cuestiona la implicación.

Implicación		
	Pregunta	Respuesta
$\mathbf{X} ! d \varepsilon A \supset B$ Asumiendo $A, B \varepsilon prop$	$\mathbf{Y} ? \supset_d$	$\mathbf{X} ! d \Rightarrow (Iz^\supset)Dr^\supset \varepsilon A \supset B$
$\mathbf{X} ! d \Rightarrow (Iz^\supset)Dr^\supset \varepsilon A \supset B$	$\mathbf{Y} ? \supset$	$\mathbf{X} ! (Iz^\supset)Dr^\supset \varepsilon A \supset B$
$\mathbf{X} ! (Iz^\supset)Dr^\supset \varepsilon A \supset B$	$\mathbf{Y} ! a \varepsilon A$	$\mathbf{X} ! b \varepsilon B$

Negación		
	Pregunta	Respuesta
$\mathbf{X} ! d \varepsilon A \supset \perp$ Asumiendo $A \varepsilon prop$ $\perp = \emptyset \varepsilon prop$	$\mathbf{Y} ? \supset_d$	$\mathbf{X} ! d \Rightarrow (Iz^\supset)Dr^\supset \varepsilon A \supset \perp$
$\mathbf{X} ! d \Rightarrow (Iz^\supset)Dr^\supset \varepsilon A \supset \perp$	$\mathbf{Y} ? \supset$	$\mathbf{X} ! (Iz^\supset)Dr^\supset \varepsilon A \supset \perp$
$\mathbf{X} ! (Iz^\supset)Dr^\supset \varepsilon A \supset \perp$	$\mathbf{Y} ! a \varepsilon A$	$\mathbf{X} ! \perp$

N.B. En el marco dialógico, el jugador que afirma \perp abandona y el otro jugador gana. Esto requiere la siguiente modificación de la regla estructura **RS3**

RS*3 (Finalizar y Ganar una Partida)

La partida termina cuando es el turno de un jugador de hacer una jugada pero no le queda ninguna jugada disponible o solo le queda afirmar \perp . Ese jugador pierde y el otro jugador gana.

Universal		
	Pregunta	Respuesta
$\mathbf{X} ! d \varepsilon (\forall x \varepsilon A)B(x)$ Asumiendo $A \varepsilon prop$ $B(x) \varepsilon prop (a: A)$	$\mathbf{Y} ? \forall_d$	$\mathbf{X} ! d \Rightarrow (Iz^\forall)Dr^\forall \varepsilon (\forall x \varepsilon A)B(x)$
$\mathbf{X} ! d \Rightarrow (Iz^\forall)Dr^\forall \varepsilon (\forall x \varepsilon A)B(x)$	$\mathbf{Y} ? \forall$	$\mathbf{X} ! (Iz^\forall)Dr^\forall \varepsilon (\forall x \varepsilon A)B(x)$

$X ! (Iz^\forall)Dr^\forall \varepsilon (\forall x \varepsilon A)B(x)$	$Y ! a \varepsilon A$	$X ! b \varepsilon B(a/x)$

I.2.3 Estrategias

I.2.3.1 Estrategias y Demostraciones

A la ocasión de recibir el premio Rolf Schock, Martin-Löf (2022) resumió su propia lectura de la interpretación dialógica de la noción de inferencia con las siguientes palabras:

El *concluyente* recibe las premisas de los *premiseros*, y a su vez proporciona, o transmite, la conclusión. Como resultado de recibir las premisas de los premiseros, el concluyente obtiene el derecho, o permisión, de solicitar a los premiseros que realicen sus respectivas tareas C_1, \dots, C_n . La validez de la regla equivale a la capacidad del concluyente para realizar C cuando recibe esta ayuda de los premiseros. Por lo tanto, el efecto es que los premiseros junto con el concluyente pueden realizar la tarea de conclusión C . Se ve aquí la innovación ausente en las explicaciones habituales de inferencia y de reglas de inferencia: la novedad es que en una inferencia, el concluyente tiene el derecho de pedir a los premiseros que lleven a cabo sus respectivas tareas C_1, \dots, C_n , lo que significa que el concluyente recibe ayuda de los premiseros para realizar estas tareas. La validez de la regla equivale a que el concluyente sea capaz de hacer C dada la ayuda de los premiseros para hacer C_1, \dots, C_n . Esta formulación es menos elegante, pero tiene la ventaja de poner de manifiesto el carácter interactivo de la inferencia con más claridad que la formulación de Sundholm.

De hecho, estas palabras subrayan la característica principal de la postura interactiva. Sin embargo, el término "ayudar" utilizado por Martin-Löf que en el contexto de fundamentar un nuevo concepto de contenido de juicio y de inferencia correcta por medio de una interpretación dialógica de conocimiento asertórico, tiene perfecto sentido, también evoca una discusión tan desafortunada como infructuosa respecto a si la lógica dialógica de Lorenzen y Lorenz es solamente adversarial y que por ende necesita una extensión capaz de incluir interacción colaborativa o si la dialógica es colaborativa desde sus orígenes.

Efectivamente, por una parte, Hodges (2001) acusa a la dialógica de describir las jugadas de la explicación del significado local de la implicación como ataque y defensa, cuando, según su lectura, el retador *ayuda* al conceder el antecedente.

Por otra parte, Dutilh Novaes (2015, 2020) acusa a la lógica dialógica de ser sólo o principalmente adversarial, dado que todo diálogo termina en victoria o pérdida para uno de los jugadores.

Dejando de lado la confusión, entre ganar o perder una partida que, como ya hemos indicado varias veces, es compatible con el hecho que una tesis no sea ni verdadera ni falsa – basta recordar que, cuando se juega con la regla estructural par lógica intuicionista, no hay estrategia ganadora ni para el tercero excluido ni para su negación – la verdad es que, cuando el foco es conocimiento asertórico, la interacción generada por el marco dialógico es colaborativa y adversarial a la vez.

Examinemos una vez más el caso de $AVB \supset B \wedge A$, pero consideremos ahora un diálogo material concreto como, digamos, $(1=1 \vee 1=0) \supset (1=0 \wedge 1=1)$, con rango de repetición 1. Supongamos además que **O**, tras conceder el antecedente y tras que **P** afirme el consecuente, **O**, que está en disposición de querer “ayudar” a su interlocutor decide pedir la derecha de la conjunción. Ciertamente, se trata de una opción legítima y conforme a la explicación dialógica del significado de la implicación, pero ¿está **O** colaborando realmente? Por un lado, **O** ayuda a **P** a ganar la partida; por otro, no colabora al no someter la tesis al escrutinio más duro posible.

Sea como fuere, llevemos a cabo un diálogo para demostrar la conmutación de la disyunción. Comenzamos con el desarrollo de una de las partidas que constituyen la estrategia ganadora y proseguimos con construcción de la estrategia ganadora que proporciona la demostración.

Ejemplo

Se demuestra $e \varepsilon B \vee A$, dada la premisa $d \varepsilon A \vee B$

La idea es clara: sea lo que sea lo que **O** elija para resolver la tarea vinculada a la premisa, **P** puede utilizarlo para resolver su propia tarea, dependiendo de la elección de **O**. Así pues, el oponente está ayudando al proponente a resolver su propia tarea. Esta es la forma en que un dialógico entiende una demostración dialógica. En aras de la simplicidad, dejamos tácita la elección de los rangos de repetición.

Convención notacional: En el contexto de RI la notación $\mathbf{P} ! e \varepsilon B \vee A \llbracket d \varepsilon A \vee B \rrbracket$ indica que **P** se compromete a construir una estrategia ganadora para $e \varepsilon B \vee A$, siempre que **O** conceda $d \varepsilon A \vee B$. Así, un desafío sobre $\mathbf{P} ! e \varepsilon B \vee A \llbracket d \varepsilon A \vee B \rrbracket$, equivale a que **O** afirme $d \varepsilon A \vee B$. De hecho, esta notación expresa la contrapartida dialógica de la barra de inferencia por la que los corchetes dobles encierran las premisas y la expresión a la izquierda de esos corchetes es la conclusión.

O		P	
		$! B \vee A \llbracket d \varepsilon A \vee B \rrbracket$	0.0
0.1	$! d \varepsilon A \vee B$	$! e \varepsilon B \vee A$	0.2
1	$? \vee e$	$! e \Rightarrow Dr^v(a) \varepsilon B \vee A$	8
3	$! d \Rightarrow Iz^v(a) \varepsilon A \vee B$	0.1	$? \vee d$ 2
5	$! Iz^v(a) \varepsilon A \vee B$	3	$? \vee$ 4
7	$! a \varepsilon A$	5	$? Iz^v(a)$ 6
9	$? \vee$ 8	$! Dr^v(a) \varepsilon B \vee A$	10
11	$? Dr^v(a)$	$! a \varepsilon A$	12
		P gana	

0.0. **P** me comprometo a demostrar $e \varepsilon B \vee A$, dada la premisa $d \varepsilon A \vee B$

0.1. **O** De acuerdo, sea $d \varepsilon A \vee B$

0.2. **P** pues estoy dispuesto a demostrar $e \varepsilon B \vee A$

1. **O** Qué componente de la disyunción estás dispuesto a elegir a fin de resolver la tarea asociada a esa disyunción?
2. **P** Elije tú primero como es que resolverás la tarea asociada a $d \varepsilon A \vee B$.
3. **O** Pues bien, de la evaluación de d resulta una razón local para la izquierda.
4. **P** Prosigamos entonces con la componente izquierda de $d \varepsilon A \vee B$.
5. **O** En efecto la razón local de mi afirmación es, de acuerdo a lo que vengo de afirmar, $Iz^v(a)$.
6. **P** Como resuelves la tarea $Iz^v(a) \varepsilon A \vee B$.
7. **O** Afirmando $a \varepsilon A$.

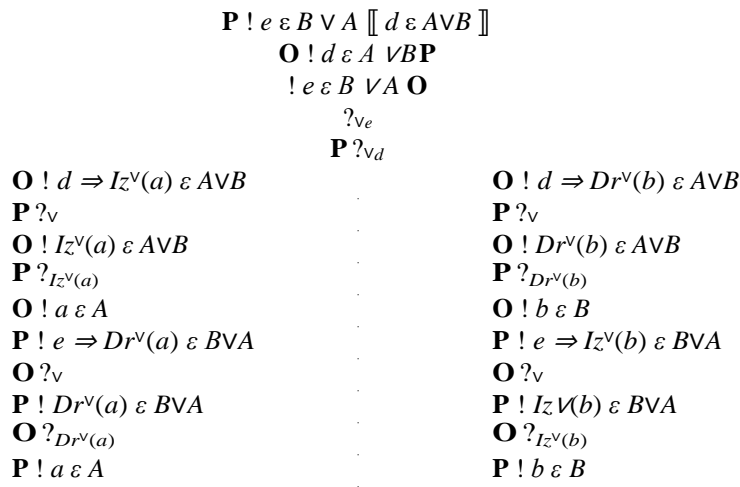
8. **P** Ahora estoy dispuesto a responder to primera pregunta 1. Mi respuest es $e \Rightarrow Dr^v(a) \varepsilon BVA$.
9. **O** Jugemos entonces de acuerdo a esa elección tuya.
10. **P** Sí, estoy dispuesto a defender mi afirmación $Dr^v(a) \varepsilon BVA$.
11. **O** Cómo es que resuelves la tarea asociada a $Dr^v(a) \varepsilon BVA$.
12. **P** Afirmando $a \varepsilon A$: cuando en tu jugada 7 afirmaste $a \varepsilon A$ me autorizaste a afirmarla yo mismo.

Obviamente, la secuencia de jugadas de esta partida no constituye aún una estrategia ganadora para la tesis.

Una estrategia ganadora, recordemos, es aquella según la cual **P** gana elija lo que elija **O**. Así que ahora tenemos que examinar la partida en la que **O** elija la derecha con su jugada 3.

Una estrategia ganadora para tal tesis resulta al componer dos partidas, tale que cualquiera que sea la forma que **O** elija para defender la disyunción $d \varepsilon AVB$, la estrategia proporcionará una partida para $e \varepsilon BVA$ ganada por **P**.

Conforme a la definición de estrategia, una estrategia ganadora generada por una sucesión de partidas relevantes produce el siguiente árbol, con la tesis en la raíz del árbol (profundidad 0), y donde asumimos que los rangos de repetición ya han sido fijados como $m := 1$ para **O** y $n := 2$ para **P**:



I.2.3.2 Afirmaciones efectuadas y estrategias

I.2.3.3 Objetos estratégicos para una familia C

En el ejemplo anterior el desarrollo de las partidas que constituyen la estrategia ganadora no utiliza otras reglas que la explicación dialógica del significado de la disyunción y las reglas estructurales básicas. En efecto, el operador $\mathbf{D}(c, x.d, y.e)$ definido por las reglas de eliminación, requerida para demostrar la conmutatividad de la disyunción en el marco de la TCT, no ocurre en ninguna de esas partidas.

Desde un punto de vista dialógico, las reglas estratégicas, correspondientes a las reglas de eliminación, aplican las explicaciones dialógicas del significado local a

aquellas afirmaciones de **O** a partir de las cuales se construirán estrategias ganadoras para **P**.

Inspirados por Crubellier (2011,2014), Rahman et al. (2018) (capítulo 7) conciben las estrategias como una *recapitulación* de las interacciones que dan forma a las jugadas relevantes y, por lo tanto, sus objetos estratégicos, en su forma abstracta más general, codifican las jugadas relevantes para constituir una estrategia.

Más precisamente, la contraparte dialógica de los operadores que en la TCT ocurren en la conclusión de reglas de eliminación son operadores compuestos por explicaciones dialógicas del significado indexadas por el jugador, que llamamos objetos *C*-estratégicos, que indican cómo reconstruir las partidas que conducen a una estrategia **P**-ganadora para una tesis *C*, una familia de conjuntos, definida sobre la disyunción, la implicación / el cuantificador universal o la conjunción / el cuantificador existencial.³³

Discutamos entonces brevemente los objetos *C*-estratégicos para las conectivas básicas.

El objeto *C*-estratégico para la disyunción

Supongamos que **P** tiene una estrategia ganadora para la tesis *C*, que es una familia de conjuntos definida sobre la disyunción *AVB*, y supongamos además que

- **O** afirmó $d \in AVB$,
- para cualquier $x \in A$ elegida por **O**, **P** tiene un método m para desarrollar una partida para $C[Iz^v(x)]$ ganada por él mismo,
- para cualquier $y \in B$ elegida por **O**, **P** tiene un método m' para desarrollar una partida en $C[Dr^v(y)]$ ganada por él mismo.

Por tanto la familia $C[d]$, es dependiente de ambos componentes de la disyunción – i.e. *C* depended en la izquierda ($C[Iz^v(x)]$) y de la derecha ($C[Dr^v(y)]$). **P** debe construir su estrategia ganadora para ambos de tales miembros de la familia *C*.

Entonces, el objeto *C*-estratégico para la tesis $C[d]$ tiene la siguiente forma, que por supuesto, sigue la anotación TCT para el selector para la disyunción:

$$\mathbf{P} ! \mathbf{D}(d, x.m, y.m') \in C[d].$$

La forma de evaluar el objeto estratégico para obtener las jugadas relevantes codificadas por este objeto *C*-estratégico viene determinada por las reglas de igualdad. Supongamos ahora que

- **O** afirmó $a \in A$,
- para cualquier $x \in A$ elegida por **O**, **P** tiene un método m para desarrollar una partida para $C[Iz^v(x)]$ ganada por él mismo,
- para cualquier $y \in B$ elegida por **O**, **P** tiene un método m' para desarrollar una partida para $C[Dr^v(y)]$ ganada por él mismo.

Dado que **P** posee un método para ganar una partida para $C[Iz^v(x)]$ pa cualquier x

³³ Es una familia, pues *C* puede ser cualquier conjunto dependiente, que constituye la tesis. En otras palabras, *C* es un conjunto arbitrario dependiente de la conjunción, implicación, etc – por ejemplo, si la tesis *C* tiene la forma *A*, y ha sido definida como dependiente de la conjunción $A \wedge B$ entonces la estrategia ganadora se construye eligiendo la izquierda de la conjunción.

εA , ese método puede usarse en el caso que $a/x \varepsilon A$ y obtener:

$$\mathbf{P} ! \mathbf{D}(Iz^{\vee}(a), x.m, y.m) = m[a] \varepsilon C[Iz^{\vee}(a)]$$

De forma análoga, si

- **O** afirmó $b \varepsilon B$
- para cualquier $x \varepsilon A$ elegida por **O**, **P** tiene un método m para desarrollar una partida para $C[Iz^{\vee}(x)]$ ganada por él mismo,
- para cualquier $y \varepsilon B$ elegida por **O**, **P** tiene un método m' para desarrollar una partida para $C[Dr^{\vee}(y)]$ ganada por él mismo.

Dado que **P** posee un método para ganar una partida para $C[Dr^{\vee}(y)]$ pa cualquier $x \varepsilon B$, ese método puede usarse en el caso que $b/y \varepsilon B$ y obtener:

$$\mathbf{P} ! \mathbf{D}(Dr^{\vee}(b), x.m, y.m) = m'[b] \varepsilon C[Dr^{\vee}(b)]$$

Las igualdades instruyen de construir de la estrategia ganadora de tal modo que

Si d es $Iz^{\vee}(a)$, la afirmación $C[d]$ se gana empleando $m[a]$, es decir el **segundo** miembro del triplo que ocurre en **D**.

Si d es $Dr^{\vee}(b)$, la afirmación $C[d]$ se gana empleando $m'[b]$, es decir el **tercer** miembro del triplo que ocurre en **D**

En breve las igualdes indican cual de los dos últimos miembros en **D** emplear en el caso que d sea o bien $Iz^{\vee}(a)$ o $Dr^{\vee}(b)$.

Dadas estas reglas, es sencillo demostrar que el objeto estratégico **D** preserva la explicación dialógica del significado local de la disyunción. En efecto,

supongamos que **P** afirmó $\mathbf{D}(d, x.m, y.m') \varepsilon C[d]$ y que **O** concedió $d \varepsilon AVB$.

P responderá al desafío de **O** sobre **D** pidiéndole que elija qué lado de la disyunción está dispuesta a defender, antes de comprometerse a defenderse del desafío sobre **D**.

Si la elección de **O** es $Iz^{\vee}(a) \varepsilon AVB$ y el desarrollo de la partida sobre esta afirmación de **O** se desarrolla de tal manera que ella (**O**) termina afirmando $a \varepsilon A$; entonces, si **P** tiene una estrategia ganadora, ganará con la jugada $\mathbf{P} ! m(a) \varepsilon C[Iz^{\vee}(a)]$. Esta última jugada es la defensa de **P** contra el desafío sobre **D**.

Lo mismo ocurre si la jugada sobre **O** ! $Dr^{\vee}(b) \varepsilon AVB$. se desarrolla de tal manera que **O** termina afirmando $b \varepsilon B$.

Nótese que lo que son m y m' depende de las explicaciones dialógicas designificado locale para C . Así, las partidas que construyen la estrategia ganadora codificada por **D** desarrollan las explicaciones dialógicas del significado local para AVB y C . Se puede dar una "justificación" dialógica similar para todos los objetos estratégicos C que se discuten a continuación.

Observación

Con respecto a la igualdad, Rahman et al. (2018) señalaron que, desde una perspectiva dialógica, la igualdad entre razones locales expresa el resultado, a nivel de objetos, del acto de **P** de enunciar una razón local apelando a la autoridad de **O**. En otras palabras, en cuadro de RI-I todo uso de la Regla Socrática a nivel de razones locales da lugar a una igualdad de juicio, que expresa el resultado de justificar el uso de una razón local apelando a “tú la has usado de la misma manera”. A diferencia de IR-I, aquí no “objetivamos” sistemáticamente a nivel de una partida el resultado de aplicar la Regla Socrática. En IR-II el papel fundamental de las igualdades de juicio es indicar cómo obtener aquellas partidas (gobernadas por la Regla Socrática) que construyen una estrategia ganadora a partir del objeto estratégico resultante.

El objeto *C*-estratégico para la conjunción

Supongamos que **P** tiene una estrategia ganadora para la tesis *C*, que es una familia de conjuntos definida sobre la conjunción $A \wedge B$, y supongamos además que

- **O** afirmó $d \in A \wedge B$,
- para cualquier $x \in A$ y cualquier $y \in B$ elegido por **O**, **P** tiene un método m para desarrollar una partida en $C[\langle x, y \rangle]$ ganada por él mismo. El método es una función $(x,y)m[x,y]$ que toma argumentos x, y y que instruye como ganar $[d]$

Entonces, el objeto *C*-estratégico para la tesis $C[d]$ tiene la siguiente forma:

$$\mathbf{P} ! \mathbf{E}(d, (x,y)m[x,y]) \in C[d].$$

La forma de computar el objeto estratégico para obtener las partidas relevantes codificadas por este objeto *C*-estratégico viene determinada por las reglas de igualdad.

Supongamos que

- **O** afirmó tanto $a \in A$ como $b \in B[a]$,
- para cualquier $x \in A$ y cualquier $y \in B$ elegida por **O**, **P** tiene un método m para desarrollar una partida para $C[\langle a, b \rangle]$ ganada por él mismo,

Entonces, el objeto estratégico para $C[\langle a, b \rangle]$ es:

$$\mathbf{P} ! \mathbf{E}(\langle a, b \rangle, (x,y)m[x,y]) = m[a, b] \in C[\langle a, b \rangle].^{34}$$

Para identificar aquellas partidas que constituyen una estrategia ganadora para una familia *C* dependiente de una conjunción, es útil tener a mano los operadores $Iz^\wedge(d)$, y $Dr^\wedge(d)$. Estos operadores, que son la contrapartida dialógica de las proyecciones izquierda y derecha dentro de TCT, son funciones de la forma $(x,y)x$ y $(x,y)y$ que toman el lugar de m , y “extraen” el componente izquierdo y derecho de d respectivamente. Las proyecciones pueden definirse en términos de **E** de la siguiente manera:

- $Iz^\wedge(d) = \mathbf{E}(d, (x, y)x) \in A$ para $d \in A \wedge B$

³⁴ Nótese que d , el primer miembro del operador **E**, a diferencia de d que ocurre en **D**, es fijo. Por ello es que la estrategia **P**-ganadora para una familia definida sobre la disyunción necesita de dos ramas que corresponden a los dos modos posibles de evaluar d , mientras que para construir una estrategia **P**-ganadora para una familia definida sobre la conjunción basta con una sola rama.

- $Dr^\wedge(d) = \mathbf{E}(d, (x,y)y) \varepsilon B$ para $d \varepsilon A \wedge B$

En tal caso la familia $C[d]$ puede definirse como $C[Iz^\wedge(d)]$ y como $C[Dr^\wedge(d)]$. \mathbf{P} puede elegir construir su estrategia ganadora para uno o ambos de tales.

Obsérvese que si \mathbf{O} afirmó $\langle a, b \rangle \varepsilon A \wedge B$, las afirmaciones adicionales $Iz^\wedge(\langle a, b \rangle) \varepsilon A$, $Iz^\wedge(\langle a, b \rangle) = a \varepsilon A$, $Dr^\wedge(\langle a, b \rangle) \varepsilon B$ y $Dr^\wedge(\langle a, b \rangle) = b \varepsilon B$ pueden introducirse como respuestas específicas a la peticiones de $\mathbf{P} ?_{Iz^\wedge 1}$, $\mathbf{P} ?_{Dr^\wedge 2}$.

El objeto C-estratégico para la implicación

Abstraer la función

La explicación dialógica del significado local de la implicación, a saber $(Iz^\supset)Dr^\supset \varepsilon A \supset B$, indica que el deber de producir una razón local par el conseqente por parte del jugador que afirmó la implicación depende de que el retador, proporcione una razón local para el antecedente cuando lanza su desafío a la implicación. Retomando la terminología libro *Immanent Reasoning* de 2018, podemos denominar expresiones de las forma $((Iz^\supset)Dr^\supset$, *instrucciones*.

A partir tales instrucciones se puede abstraer una función, tal que el retador proporciona un argumento para la función cuando produce una razón local para el antecedente, de manera tal que

si $b \varepsilon B$ ($x \varepsilon A$) entonces
 $(x)b \varepsilon (x \varepsilon A)B$

si $b \varepsilon B(x \varepsilon A)$ $a \varepsilon A$ entonces
 $(x)b(a) = b(a/x) \varepsilon B(a/x)$

si $(Iz^\supset)Dr^\supset \varepsilon (x \varepsilon A)B$ entonces
 $(Iz^\supset)Dr^\supset = (x)(Iz^\supset)Dr^\supset(x) \varepsilon (x \varepsilon A)B$.³⁵

Pero esta abstracción, tiene lugar a nivel de estrategia no a nivel de las partidas. La función, que tiene ahora el rol de una razón estratégica que justifica la aserción de la implicación, expresa que \mathbf{P} tiene una estragie ganadora para tal implicación, que consiste en una función que tiene como argumento *toda razón* local que producida por \mathbf{O} para el antecedente, y que tiene como valor una razón local para el conseqente

Así por ejemplo, si la implicación $d \varepsilon A \vee B \supset B \vee A$ constituye la tesis de un diálogo, y si d se evalúa como $(Iz^\supset)Dr^\supset \varepsilon A \vee B \supset B \vee A$, entonces habrá dos partidas para esa tesis y que pueden ser ganadas por \mathbf{P} , dependiendo de la elección de \mathbf{O} :

Si \mathbf{O} elige el lado izquierdo para defender la disyunción en el antecedente, \mathbf{P} puede ganar eligiendo el lado derecho para defender la disyunción en el conseqente.

³⁵ En TCT estas reglas son justificadas mediante la regla de β -conversión. Nótese que esto sugiere que en realidad estrategias dialógicas corresponden con tipos más que conjuntos. Así mientras que una regla de introducción en TCT solo involucra object canónicos, la operación de abstracción en tipos (el análogo de la regla de introducción para conjuntos), no está restringido a objetos canónicos : las expresiones $(x)b$ o incluso $(Iz^\supset)Dr^\supset$ son algunas de las formas que elementos del tipo $(x \varepsilon A)B$ pueden tener. Son las igualdades que indican qué forma tienen los objetos de tipo $(x \varepsilon A)B$.

Si **O** elige el lado derecho para defender la disyunción en el antecedente, **P** puede ganar eligiendo el lado izquierdo para defender la disyunción en el consecuente.

A partir del desarrollo de esas partidas, obtenemos el abstracto de la función $\mathbf{f} \in AVB \supset BVA$ – la función es del tipo $(AVB)BVA$. El resultado de tal abstracción es el objeto estratégico (o razón estratégica) $\lambda(\mathbf{f}) \in AVB \supset BVA$ – siempre que se haya verificado que argumentos idénticos dan valores idénticos, y que se dé un criterio de identidad para las funciones.

El objeto estratégico $\lambda(\mathbf{f})$ codifica la información de que cualquiera sea la razón **O** produzca para afirmar le antecedente, **P** puede utilizarla como argumento de la función **f** que justifica la afirmación del consecuente.³⁶

Aplicación y el objeto C-estratégico

Supongamos que **P** tiene una estrategia ganadora para la tesis *C*, que es una familia de conjuntos definida sobre la implicación $A \supset B$, y supongamos también que **O** afirmó $d \in A \supset B$.

Esto requiere que la estrategia ganadora para *C* se construya a partir de la función abstraída de la razón local $(Iz \supset)Dr \supset \in A \supset B$, siempre que se dé un criterio de identidad, y que la razón local cumpla la condición de funcionalidad.³⁷

Más precisamente, la razón *d* de **O** se evalúa como el abstracto $\lambda(\mathbf{f})$, donde **f** es un elemento del tipo función $(A)B$. Supongamos además, que **P** tiene un método *m* para desarrollar una partida para $C[\lambda(\mathbf{f})]$ ganada por él mismo (**P**), para **f** en $(A)B$.

En otras palabras, supongamos que

- **P** tiene una estrategia ganadora para la tesis *C*, que es una familia de conjuntos definida sobre la implicación $A \supset B$, y supongamos además que
- **O** afirmó $d \in A \supset B$, y que
- Siempre que **O** produzca una **f**, tal que $\mathbf{f} \in (A)B$, **P** tiene un método *m*(**f**) para desarrollar una partida sobre $C[\lambda(\mathbf{f})]$, tal que $\mathbf{P} ! m(\mathbf{f}) \in C[\lambda(\mathbf{f})]$ es ganada por el propio **P**,

Por tanto, la familia $C[d]$, contiene, conjuntos de la forma $C[[\lambda(\mathbf{f}_i)]]$ para cualquiera de las funciones \mathbf{f}_i producida por **O**. Entonces, el objeto estratégico para $C[d]$ es:

$$\mathbf{P} ! \mathbf{F}(d, m) \in C[d]$$

Recurriendo a una notación de orden superior, podemos ahora introducir el operador de aplicación **Ap** como sigue

$$\mathbf{Ap}(d, a) := \mathbf{F}(d, (y)y(a))$$

donde *y* es una variable de función ligada.

³⁶ Nótese que este procedimiento de abstracción es similar al descrito por Sundholm (2013) para generalizar una demostración con contenido concreto y obtener así una demostración de validez lógica.

³⁷ Como se señala en Klev (2022, 202x), dialógicamente hablando, la funcionalidad requiere que el defensor de $\mathbf{f} \in (\alpha)\beta$ produzca $\mathbf{f}(a) = \mathbf{f}(a') \in \beta$ siempre que el retador proporcione $a = a' \in \alpha$. En el cuadro dialógico, el criterio de identidad para las funciones equivale a lo siguiente 1) el defensor de $\mathbf{f} \in (\alpha)\beta$ debe producir $\mathbf{f} \in (\alpha')\beta'$ siempre que el retador proporcione $(\alpha)\beta = (\alpha')\beta' \in \text{tipo}$; 2) el defensor de $\mathbf{f} = \mathbf{f}' \in (\alpha)\beta$ debe producir $\mathbf{f} = \mathbf{f}' \in (\alpha')\beta'$ siempre que el retador proporcione $(\alpha)\beta = (\alpha')\beta' \in \text{tipo}$.

Entonces obtenemos

$$\mathbf{P} ! \mathbf{Ap}(d, a) \varepsilon B[a].$$

Dialógicamente hablando, la aplicación $\mathbf{Ap}(d, a)$ – es decir $\mathbf{Ap}(\lambda(\mathbf{f}), a)$ – expresa que \mathbf{P} posee una estrategia ganadora para $B[a]$ cuando d , es una razón local para $A \supset B$ y a constituye una razón local para A .

I.2.4 Diálogos Materiales Desarrollos Colaborativos y Adversariales

Las críticas usuales al marco dialógico pueden acaso verse como distintas variaciones de una misma observación general:

- La Dialógica de Lorenzen y Lorenz carece de la dinámica necesaria para expresar modos de interacción argumentativa mediante los cuales se constituye conocimiento y significado más allá del involucrado en la semántica de las partículas lógicas. Si la Dialógica se reduce a validez lógica no es claro en qué consiste su contribución aparte de proporcionar otro cuadro formal para lógicas conocidas.³⁸

Declinemos esta observación general en tres puntos:

1. La Dialógica de Lorenzen y Lorenz solo contempla juegos adversariales, y por tanto no da lugar a diálogos en los que ambos interlocutores colaboran en aras de obtener un fin común, que no se reduce a ganar o perder.
2. La Dialógica de Lorenzen y Lorenzen solo proporciona el significado de constantes lógicas, y por tanto no constituye una teoría general del significado, para proposiciones que expresan contenido material, ya sea matemático o empírico.
3. Dado el punto anterior la Dialógica carece de la dinámica necesaria para incluir argumentos derrotables o no monotónicos.

Respecto a la distinción entre juegos colaborativos y adversariales, como ya indicamos en un texto anterior, la distinción presta muchas veces a equívocos, aún en el caso restringido de validez lógica. Tal vez valga recalcar que, en efecto, tales críticas, porque descuidan la distinción entre nivel de partida y nivel estratégico, ignoran el hecho que, si bien cada partida es un juego de suma 0, eso no conduce a estrategias ganadoras para la negación de proposiciones veritativo-funcionalmente indecidibles como el tercero excluido (en el sentido de intuicionistas o constructivistas).

Más aún, en el contexto de diálogos materiales se puede diseñar el desarrollo de una partida de modo de producir ambos juegos colaborativos y adversariales. Como veremos más adelante, en el caso de operaciones aritméticas (en el conjunto de números naturales) y algebraicas (en el conjunto **Bool**), parece más natural de desarrollar partidas en las que ambos interlocutores se esfuerzan por un objetivo común, en este caso, el resultado de una operación aritmética.

En verdad, el objetivo principal de nuestra discusión de diálogos con contenido matemático es de proporcionar ciertas nociones preliminares al enfoque dialógico sobre la noción de evaluación empleada en TCT: según este enfoque la operación de

³⁸ Véase Hodges (2001), Dutilh Novaes (2015, 2020), Hodges y Väinänen (2019).

evaluación articula el pasaje de diálogos adversariales a colaborativos.

Observemos además, *derrotabilidad* es en principio una propiedad de partidas no de estrategias – i.e. no es una propiedad de estructural que afecta la relación de inferencia lógica, particularmente en diálogos en los que se pone en juego la constitución del significado de una expresión, y ambos interlocutores colaboran para constituir una base común de entendimiento.³⁹ Para ponerlo en términos de la TCT, derrotabilidad afecta las reglas de formación durante el desarrollo de partidas mediante las cuales la formación de una expresión adquiere una modificación que o bien extiende o cambia radicalmente el significado de tal expresión. Decimos en *principio*, pues el cuadro dialógico también permite variaciones de reglas estructurales. Tal perspectiva dio lugar al nacimiento del *pluralismo dialógico* iniciado por Rahman y Rückert en la Universidad del Saarland y desarrollado ulteriormente en la Universidad de Lille,⁴⁰ pero el cuadro no trata de la dinámica que puede acaecer a afirmaciones elementales.

Respecto a la contribución del marco Dialógico y contenido, ya observamos que compartimos con Martin-Löf (2024) la afirmación que la perspectiva dialógica proporciona los fundamentos de conocimiento asertórico a la obra en razonamientos tanto formales como materiales.

Sin embargo, disentimos de Martin-Löf, respecto a lo fundamentos últimos de tal conocimiento. En efecto, según el cuadro de *Razonamiento Inmanente* la interpretación dialógica de conocimiento asertórico no está fundada en la afirmación de premisas de complejidad arbitrarias, pero en la Regla Socrática que gobierna la justificación de **afirmaciones elementales**.

Afirmaciones que involucran proposiciones complejas “heredan” **durante el desarrollo de una partida no de una inferencia**, el conocimiento asertórico de las afirmaciones elementales que resultan del análisis de las afirmaciones complejas.

Esto requiere el estudio de reglas especiales de interacción para la afirmación de proposiciones elementales que expresan contenido matemático o empírico. Técnicamente hablando debemos establecer como atacar una afirmación elemental con contenido concreto y cuántas veces puede llevarse a cabo un tal ataque. En efecto, como veremos, cada tipo de proposición elemental tiene un rango específico al tipo de contenido que expresa.

Los tipos de contenidos que discutiremos en el presente capítulo, refieren al contexto del conjunto de los números naturales, del conjunto **Bool**, y de conjuntos finitos. Incluimos, también secciones sobre el conjunto de identidad proposicionales (ya que afirmar una identidad proposicional es de hecho una afirmación elemental), sobre universos, y sobre proposiciones empíricas. Cerramos el capítulo con un estudio somero sobre derrotabilidad y modalidad temporal, que será desarrollado histórica y sistemáticamente en diversos capítulos de la segunda parte de nuestro volumen.

I.2.4.1 Números Naturales

I.2.4.1.1 La Regla Socrática y Número Naturales

Como ya mencionado en el contexto de diálogos materiales, la regla Socrática permite a **O** de atacar afirmaciones elementales sobre números naturales, ssi **O** no hizo la

³⁹ Véase Rahman & Iqbal & Soufi (2019), Rahman & Iqbal (2021), Iqbal (2022), Martínez Cazalla et al. (2020), Lion (2023).

⁴⁰ Véase, Rahman & Keiff (2004), Keiff (2007) y Rückert (2011).

misma afirmación antes.

En el caso particular de afirmaciones elementales sobre números naturales

- **P** puede también atacar afirmaciones elementales de **O** a excepción de la afirmación $O ! 0 \in \mathbb{N}$

Las reglas de ataque y defensa son las siguientes

Números Naturales		
	Pregunta	Respuesta
$X ! d \in \mathbb{N}$ Asumiendo $\mathbb{N} \in set$	$Y ?_{Nd}$	$X ! d \Rightarrow 0 \in \mathbb{N}$ ----- $X ! d \Rightarrow s(n) \in \mathbb{N}$
$X ! d \Rightarrow 0 \in \mathbb{N}$ ----- $X ! d \Rightarrow s(n) \in \mathbb{N}$	$Y ?_{No}$ $Y ?_{Ns(n)}$	$X ! 0 \in \mathbb{N}$ ----- $X ! s(n) \in \mathbb{N}$
$X ! s(n) \in \mathbb{N}$	$Y ?n$	$X ! n \in \mathbb{N}$

Más aún,

- **P** puede requerir de **O** de afirmar que **0** es un número natural de la manera siguiente
 $P ? 0 \in \mathbb{N}$
 $O ! 0 \in \mathbb{N}$
- Desde un punto de vista estrictamente material, **X** puede también requerir de **Y** de mostrar que una afirmación de la forma $s(n) \in \mathbb{N}$ aplicando el ataque $?n$, hasta llegar a la afirmación $! 0 \in \mathbb{N}$. Sin embargo, en el práctica, solo dejamos a **O** a emplear tal ataque.
- Ambos jugadores pueden requerir del interlocutor de aplicar definiciones nominales de modo de obtener que una cifra sea reescrita en forma canónica, por ej.
 $X ! 1 \in \mathbb{N}$
 $Y ?:=1$
 $X ! s(s(0)) \in \mathbb{N}$
- En el caso de una afirmación de la forma $C[n]$ (asumiendo $C[x] \in prop (x \in \mathbb{N})$ – discutido en la próxima sección –, el número de ataques es igual al número de sucesores de **0** que definen n más un último ataque involucrando $C[0]$.

En el contexto de los números naturales, la operación de evaluación permite articular el pasaje de partidas (o subpartidas) colaborativas a adversariales. Así es que mientras que durante una partida colaborativa ambos jugadores buscan juntos de constatar, empleando la regla descrita en la tabla, que cierta expresión en constituye en efecto la forma canónica de un número natural, durante una partida adversarial, el atacante comienza por jugar la partida respecto a d , y luego respecto al número canónico, a la derecha de la flecha de evaluación: si la expresión canónica del número natural es igual al resultado de la operación que constituye d , entonces **P** gana sino pierde. De alguna manera se puede decir que un diálogo adversarial está constituido por dos subpartidas colaborativas.

Esto asume que d expresa alguna operación aritmética – si d no expresa una operación aritmética pero una cifra decimal, el proceso es inmediato, basta emplear la definición nominal. Dado que las explicaciones dialógicas de las operaciones aritméticas son discutidas en la próxima sección, esperamos hasta entonces para detallar diálogos la articulación entre diálogos adversariales y colaborativos.

I.2.4.1.2 Afirmar Propiedades de los Números Naturales

Supongamos que X afirma $C[n]$ donde n es un número natural arbitrario. Entonces, Y puede atacar la afirmación eligiendo un número natural k .

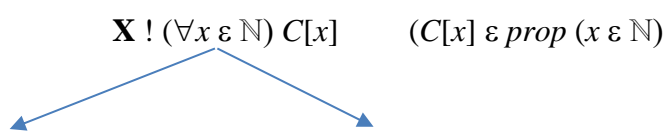
Para el caso que Y elija algún número natural k diferente de 0, X tendrá que afirmar que

1. hay un número natural m , tal que $s(m)$ es la forma canónica de k , y
2. que $C[s(m)]$.

- La afirmación $C[s(m)]$ supone que X puede afirmar también $C[m]$. Tal suposición motiva otro ataque mediante el cual Y requiere que X de afirmar $C[s(m')]$ tal que $s(m')$ es la forma canónica de m y así sucesivamente hasta $C[0]$.

Afirmación de C sobre los Números Naturales		
	Pregunta	Respuesta
$X ! C[n]$ Asumiendo $\mathbb{N} \varepsilon set$ $C \varepsilon set (x \varepsilon \mathbb{N})$	$Y ! 0 \varepsilon \mathbb{N}$ ----- $Y ! k \varepsilon \mathbb{N}$ (k es diferente de 0)	$X ! C[0]$ $X ! C[s(m)]$ ($s(m)$ es la forma canónica de k)
$X ! C[s(m)]$	$Y ! m \varepsilon \mathbb{N}$	$X ! s(m') \varepsilon \mathbb{N}$ ($s(m')$ es la forma canónica de m)
$X ! s(m') \varepsilon \mathbb{N}$... $X ! s(0) \varepsilon \mathbb{N}$	$Y ! m' \varepsilon \mathbb{N}$... $Y ! 0 \varepsilon \mathbb{N}$	$X ! s(m'') \varepsilon \mathbb{N}$ ($s(m'')$ es la forma canónica de m') ... $X ! C[0]$

Si se busca construir una estrategia ganadora, X deberá poder ganar una partida para $C[0]$ y toda otra partida para un número natural distinto de 0. En otras palabras, si Y elije n (diferente de 0) el número de ataques sobre $C[x]$ es igual al número de sucesores de 0 que define n más un último ataque mediante le cual O elije 0.



Ataque: $\mathbf{Y} ! 0 \in \mathbb{N}$
 Defensa: $\mathbf{X} ! C[0]$

Ataque: $\mathbf{Y} ! k \in \mathbb{N}$ (k es diferente de 0)
 Defensa: $\mathbf{X} ! C[s(m)]$ ($s(m)$ es la forma canónica de k)

Ataque: $\mathbf{Y} ! m \in \mathbb{N}$
 Defensa: $\mathbf{X} ! C[s(m')]$ ($s(m')$ es la forma canónica de m)

...

Ataque: $\mathbf{Y} ! 0 \in \mathbb{N}$
 Defensa: $\mathbf{X} ! C[0]$

- La razón local para $C[n]$ depende de la estructura interna de C y se manifiesta durante el desarrollo de las partidas sobre la afirmación $C[n]$.
- En realidad, desde el punto de vista puramente de las partidas, sería mejor formular los nuevos ataques como dando lugar a nuevas partidas. Un árbol para una estrategia ganadora constituida por tales partidas las incorpora como ramas. Sin embargo, para tener una visión de conjunto de las diferentes opciones, hemos preferido permitir **nuevos retos en lugar de nuevas partidas**, e integrar de este modo todas las opciones dentro de la misma partida – véase ejemplo 3 en la sección I.4.3.

N.B. La regla preserva por una parte la explicación dialógica del significado local de los operadores en C y por otra parte la regla Socrática para afirmaciones expresa la prueba por inducción matemática (o recursión). En efecto, basta observar que una estrategia ganadora para la afirmación que $C[n]$ es el caso para todo número natural n requiere afirmar que C es el caso para 0, y que C es el caso para el sucesor de m a condición que C sea el caso para m .

Esto puede ser generalizado por medio del concepto de objeto C -estratégico para \mathbb{N} , cuya formulación discutimos en la próxima sección.

I.2.4.1.3 Objetos Estratégicos para una Familia C sobre \mathbb{N}

Supongamos una vez más que \mathbf{P} tiene una estrategia ganadora para la tesis

- $\mathbf{P} ! C[n]$ donde n es un número natural arbitrario

Supongamos además que

- \mathbf{O} elige un número k y afirma $k \in \mathbb{N}$. mediante esa afirmación requiere que la tesis se jugada en torno a $C[k]$
- \mathbf{P} puede poseer un método d para ganar una partida sobre $C[0]$, cuando k es 0
- para $x \in \mathbb{N}$, \mathbf{P} tiene un método e para ganar una partida para $C[s(x)]$, si \mathbf{P} posee también un método y para defender la afirmación $C[x]$ – en breve, e está constituido por los argumentos x e y .

Así, e es una función de la forma $(x, y)e[x, y]$, tal que si x es un número natural e y un método para defender $C[x]$, e proporciona un método para defender $C[s(x)]$ – o $C[s(m)]$ si $s(m)$ es la forma canónica del número k elegido por \mathbf{O} .

Por tanto, la familia $C[n]$, es dependiente de las dos opciones principales de \mathbf{O} , elegir 0 y elegir un número natural diferente de 0 expresado en forma canónica – i.e., en forma de

sucesor.

P debe construir su estrategia ganadora para ambas opciones,

Si **O** elige 0 entonces **P** puede construir (esa rama de) su estrategia ganadora aplicando el método d , cuya forma depende de la estructura interna de C

Si **O** elige un número natural diferente de 0 que es el sucesor de algún m , entonces la segunda rama de la estrategia ganadora para **P** está constituida por el método e , que proporciona una partida ganadora para **P** para la $C[s(m)]$ a partir de m (cuya forma canónica es $s(m)$) de y de una nueva aplicación de e pero ahora respecto a $s(m)$. Aquí también la forma del método e y del método y depende de la estructura interna de C .

Esto puede codificarse mediante el operador **R** (que sugiere recursividad)

$$\mathbf{P} ! \mathbf{R}(n, d, (x, y)e(x, y)) \in C[n].$$

La forma de evaluar el objeto estratégico **R** para obtener las opciones que constituyen la ramas principales de la estrategia ganadora está determinada por las reglas de igualdad.

- $\mathbf{R}(0, d, (x, y)e(x, y)) = d \in C[0]$
- $\mathbf{R}(s(m), d, (x, y)e[(x, y)] = e[m, \mathbf{R}(m, d, (x, y)e(x, y))] \in C[s(m)]$

Nótese que los argumentos x, y de la función e ocurriendo a la derecha de la igualdad, están constituidos por m/x y $\mathbf{R}(m, d, (x, y)e(x, y))/y$

Las igualdades instruyen de construir de la estrategia ganadora de tal modo que

Si n es **0**, la afirmación $C[n]$ se gana empleando d , es decir el **segundo** miembro del tripo que ocurre en **R**.

Si n tiene la forma $s(m)$ la afirmación $C[n]$ se gana empleando e , es decir el **tercer** miembro del tripo que ocurre en **R**.

En breve las igualdes indican cual de los dos últimos miembros en **R** emplear en el caso que n sea o bien **0** o diferente de **0**.

Tomemos como ejemplo de familia C definida sobre los números naturales la propiedad

Ser idéntico a **0** o al sucesor de un número natural.

Eso permite afirmar la tesis

*Todo número natural n es o bien idéntico a **0** o bien idéntico al sucesor de un número natural*

P puede construir una estrategia ganadora con el objeto estratégico $\mathbf{R}(n, d, (x, y)e(x, y)) \in C[n]$ de la siguiente manera:

El método d consiste en **P** eligiendo la izquierda de la disyunción cuando el número natural elegido por **O** es **0** e instruye de cómo defender victoriosamente la afirmación de que la elección de **O** satisface la identidad requerida (afirmando trivialmente que **0** es en efecto idéntico a **0**)

El método e consiste en elegir la derecha de la disyunción cuando el número natural elegido por \mathbf{O} es diferente de $\mathbf{0}$ – en el ejemplo, el número 2.

La aplicación del método e instruye de cómo desarrollar una partida mediante la cual \mathbf{P} puede defender victoriosamente la afirmación que 2 es igual al sucesor de 1, aplicando un método y (en donde x es igual a 1) que permite defender la afirmación que $1/x$ es idéntico al sucesor de $\mathbf{0}$, y la defensa victoriosa de esta última afirmación requiere otro desarrollo de e que permite a \mathbf{P} defender la afirmación que 0 es idéntico a 0 (ahora d ocupa el lugar de y).

En el ejemplo asumimos que \mathbf{O} elige $\mathbf{0}$ y 2. Pero el método es fácilmente generalizable para cualquier número natural: es suficiente de remplazar 2 por sucesor de m , y m por sucesor de 0 o por algún sucesor de un sucesor ... sucesor de $\mathbf{0}$.

I.2.4.1.4 Adición y Multiplicación Diálogos Colaborativos y Adversariales

En el contexto de diálogos para la adición y la multiplicación se puede diseñar el desarrollo de una partida de modo de producir o bien juegos colaborativos o adversariales. De hecho, en el caso de operaciones aritméticas (en el conjunto de números naturales) parece más natural de desarrollar partidas en las que ambos interlocutores se esfuerzan por un objetivo común, en este caso, el resultado de una operación aritmética.

I.2.4.1.4.1 Adición

Comenzemos con las reglas determinan la explicación del significado local de la adición que provee además una definición recursiva de tal operación y que constituye una extensión de la Regla Socrática, i.e., indica qué preguntas y qué respuestas están asociadas a afirmaciones elementales sobre adiciones.

Adición de Números Naturales		
	Pregunta	Respuesta
$\mathbf{X} ! a+b \in \mathbb{N}$ Asumiendo $a, b \in \mathbb{N}$	$\mathbf{Y} ? a+0$ (si b es 0)	$\mathbf{X} ! a$
	----- $\mathbf{Y} ? a+s(m)$ (si $s(m)$ es la forma canónica de b)	$\mathbf{X} ! s(a+m) \in \mathbb{N}$

Reglas estructurales RSAd1 y RSAd2 para la adición

- RSAd1.** \mathbf{Y} puede formular la preguntas $\mathbf{Y} ? a+0$, $\mathbf{Y} ? a+s(m)$ aún en el caso de afirmaciones de la forma $\mathbf{X} ! s(a+0)$, $\mathbf{X} ! s(a+s(m))$. En efecto, la regla para números naturales que prescribe como atacar afirmaciones de la forma $s(n)$, requiere que n tenga la forma canónica de un número natural, no de una operación.

- RSAd2.** El defensor **encuentra** el resultado, si tras la aplicación sucesiva de la regla, la respuesta a la pregunta sobre la afirmación $a+0$, en ambas de las opciones de **Y**, expresa la forma canónica de un número natural: el número natural que, resulta de la adición. Indicamos con una estrella a la derecha del signo de exclamación que el resultado ha sido encontrado. En principio la partida puede proseguir para verificar si el número es o no un número natural, aplicando la regla para números naturales, hasta que el proceso de análisis termina con la afirmación de que **0** es un número natural. Destacamos la serie de jugadas asociada con la verificación poniéndola en gris.

Partidas colaborativas y adversariales

Mientras que durante una partida colaborativa ambos jugadores buscan juntos el resultado de una adición, durante una partida adversarial, el atacante fuerza al defensor de anticipar una respuesta, que luego se somete a verificación aplicando la regla de la adición sobre la tesis.

Diseñamos las partidas de tal modo que es **O** quién decide si la partida será de colaboración o adversarial

- RSAd3.** Dada la afirmación $\mathbf{P} ! a+b \in \mathbb{N}$, puede elegir entre pedir a **P** que anticipe una respuesta o colaborar con la construcción del resultado usando la regla de la adición. Ver la tabla sobre partidas colaborativas y adversariales concerniendo la adición.
- RSAd4.** En el curso de una partida adversarial, si **P** afirma haber encontrado el resultado (en el sentido de RSAd2) **O** puede requerir de **P** que afirme la igualdad entre el resultado anticipado y el resultado obtenido formulando la pregunta:

$\mathbf{O} ? n =$ (donde n es el resultado anticipado por **P**).

Si **P** no puede afirmar la igualdad entre el resultado anticipado y el obtenido la partida finaliza y **P** pierde, si **P** puede afirmar la igualdad la partida prosigue verificando que el resultado obtenido es efecto un número natural y **P** gana.

Adición de Números Naturales Partida Colaborativa y Adversarial		
	Pregunta	Respuesta
$\mathbf{P} ! a+b \in \mathbb{N}$	Partida Colaborativa $\mathbf{O} ? a+0$ (si b es 0) ----- $\mathbf{O} ? a+s(m)$ (si $s(m)$ es la forma canónica de b)	$\mathbf{P} ! a$ $\mathbf{P} ! s(a+m) \in \mathbb{N}$
$\mathbf{P} ! a+b \in \mathbb{N}$	Partida Adversarial $\mathbf{O} ? n \in \mathbb{N}$	$\mathbf{P} ! k \in \mathbb{N}$
$\mathbf{P} ! a+b \in \mathbb{N}$ $\mathbf{P} ! k \in \mathbb{N}$	$\mathbf{O} ? a+0$ (Pregunta dirigida a	$\mathbf{P} ! a$

	$\mathbf{P} ! a+0 \in \mathbb{N}$ <hr style="border-top: 1px dashed black;"/> $\mathbf{O} ? a+s(m)$ (Pregunta dirigida a $\mathbf{P} ! a+ s(m) \in \mathbb{N}$)	$\mathbf{P} ! s(a+m) \in \mathbb{N}$

Ejemplo de Diálogo colaborativo

O			P	
			$! 2+1 \in \mathbb{N}$	0
1	$? 2+s(0)$	0	$! s(2+0) \in \mathbb{N}$	2
3	$? 2+0$	2	$! s(2) \in \mathbb{N}$	4
5	$? :=2$	4	$! * sss(0) \in \mathbb{N}$	6
7	$? ss(0)$	6	$! ss(0) \in \mathbb{N}$	8
9	$? s(0)$	8	$! 0 \in \mathbb{N}$	12
11	$! 0 \in \mathbb{N}$		$? 0 \in \mathbb{N}$	10
			<p>P encuentra el resultado de la operación en la jugada 6</p> <p>En la jugada 10 P hace uso de la regla Socrática para \mathbb{N}, que permite a P de forzar a O a afirmar $0 \in \mathbb{N}$.</p>	

Ejemplo de Diálogo adversarial perdido

O			P	
			$! 2+1 \in \mathbb{N}$	0
1	$? n$	0	$ss(0) \in \mathbb{N}$	2
3	$? 2+s(0)$	0	$! s(2+0) \in \mathbb{N}$	4
5	$? 2+0$	4	$! s(2) \in \mathbb{N}$	6
7	$? :=2$	6	$! * sss(0) \in \mathbb{N}$	8
9	$? n =$	2,8		
			<p>P pierde pues no puede afirmar la igualdad requerida</p>	

Ejemplo de Diálogo adversarial ganado

O			P	
			$! 2+1 \in \mathbb{N}$	0
1	$? n$	0	$! sss(0) \in \mathbb{N}$	2
3	$? 2+s(0)$	2	$! s(2+0) \in \mathbb{N}$	4
5	$? 2+0$	4	$! s(2) \in \mathbb{N}$	6
7	$? :=2$	6	$! * sss(0) \in \mathbb{N}$	8
9	$? n =$	2,8	$! sss(0) = sss(0) \in \mathbb{N}$	10
11	$? ss(0)$	8	$! ss(0) \in \mathbb{N}$	12
13	$? s(0)$	12	$! 0 \in \mathbb{N}$	16
15	$! 0 \in \mathbb{N}$		$? 0 \in \mathbb{N}$	14
			P gana	

	puesto que las afirmaciones 2 y 8 son iguales, y el desarrollo de la partida verifica que el resultado es un número natural
--	---

Si permitimos partidas en las que P es forzado a anticipar el resultado de una adición, tal anticipación también puede entenderse como la afirmación

$$\mathbf{P} ! a+b \Rightarrow n \in \mathbb{N},$$

es decir, la evaluación de $a+b$ es n . Esta afirmación produce un juego adversarial compuesto por dos juegos colaborativos, uno a la izquierda y uno a la derecha de flecha de evaluación. Ganar y perder está determinado por la regla RSAd4 que consiste en comparar el resultado de $a+b$, obtenido usando las reglas de adición, con n .

Las reglas que prescriben como desarrollar una partida que emplea la flecha de evaluación son las siguientes:

Adición y Evaluación		
	Pregunta	Respuesta
$\mathbf{P} ! a+b \in \mathbb{N}$	$\mathbf{O} ?_{\mathbb{N}a+b}$	$\mathbf{P} ! a+b \Rightarrow n \in \mathbb{N}$
$\mathbf{P} ! a+b \Rightarrow n \in \mathbb{N}$	Subpartida por la derecha de \Rightarrow $\mathbf{O} ?_{\mathbb{N}n}$	$\mathbf{P} ! 0 \in \mathbb{N}$ ----- $\mathbf{P} ! s(n) \in \mathbb{N}$
	----- Subpartida por la izquierda de \Rightarrow $\mathbf{O} ? a+0$ (si b es 0) ----- $\mathbf{O} ? a+s(m)$ (si $s(m)$ es la forma canónica de b)	$\mathbf{P} ! a$ $\mathbf{P} ! s(a+m) \in \mathbb{N}$

Ejemplo de Diálogo adversarial con flecha de evaluación ganado

O			P	
			$! 2+1 \in \mathbb{N}$	0
1	$?_{\mathbb{N}2+1}$	0	$! 2+1 \Rightarrow n \in \mathbb{N}$	2
3	$?_{\mathbb{N}n}$	2	$! sss(0) \in \mathbb{N}$	4
5	$? 2+s(0)$	0	$! s(2+0) \in \mathbb{N}$	6
7	$? 2+0$	6	$! s(2) \in \mathbb{N}$	8
9	$? :=2$	8	$!* sss(0) \in \mathbb{N}$	10
11	$? n =$	4,10	$! sss(0) = sss(0) \in \mathbb{N}$	12

	P gana No desarrollamos aquí las jugadas que verifican $sss(0) \in \mathbb{N}$
--	---

El objeto estratégico para la Adición

Para construir una estrategia ganadora podemos recurrir al operador **R** para los números naturales. El operador puede emplearse para dar una definición inductiva de la adición, de modo que C es \mathbb{N} mismo, e es $s(y)$, b es o bien **0** o el número natural $s(m)$, e y es $a+m$:

$$a+b = \mathbf{R}(b, a, (x, y)s(y)) \in \mathbb{N}$$

Es decir,

si b es **0**, **P** debe construir su estrategia ganadora en torno a la afirmación que el segundo componente de **R**, es decir a , es un número natural
 si b tiene la forma $s(m)$, y por tanto y tiene la forma $a+m$, **P** debe construir su estrategia ganadora en torno a la afirmación que el tercer componente de **R**, es decir $s(y)$, $s(a+m)$ evalúa en un número natural.

En breve:

$$\mathbf{R}(0, a, (x, y)e(x, y)) = a \in \mathbb{N}$$

$$\mathbf{R}(s(m), a, (x, y) s(y)) = (x, y)s(y) = (x, y)s(a+m). \in \mathbb{N}$$

I.2.4.1.4.2 Multiplicación

$$a \times 0 = 0 \text{ and } a \times S(b) = (a \times b) + a.$$

ejemplo

$$4 \times s(2) = (4 \times 2) + 4$$

$$(4 \times s(1) + 4) = (4 \times 1 + 4) + 4$$

$$(4 \times s(0) + 4) + 4 = (4 \times 0) + 4 + 4 + 4$$

I.2.4.2 el Conjunto **Bool**

I.2.4.3 Conjuntos Finitos

I.2.4.4 Funciones y Afirmaciones Hipotéticas

I.2.5 Identidad Proposicional

1.2.6 Universos

I.2.7 Proposiciones Empíricas

I.3 Evaluar en un Marco Dialógico

Según las reglas dialógicas para TCT (DTCT) discutidas en Klev (202x), afirmaciones de evaluación se resuelven mediante actos de cálculo⁴¹. Sin embargo, en una conferencia reciente Klev (2022) ha desarrollado reglas para impugnar y responder evaluaciones en el seno de un diálogo de tal modo que evaluar consiste en aplicar definiciones.

Como discutiremos en secciones posteriores, mientras que *evaluar como calcular* es una perspectiva útil en el contexto de proposiciones empíricas o de operaciones matemáticas, la concepción de *evaluar como desarrollo definicional* parece ser más fructífera cuando el objetivo es fijar el núcleo de significado de una expresión que ocurre durante una interacción dialógica.

De hecho, en lugar de emplear *estipulaciones*, como las empleadas por Klev en su concepción de desarrollo definicional, es más natural al cuadro dialógico de entender tal desarrollo como vinculado a un proceso de transmisión de la propiedad de *invariancia de partida* respecto al significado de una expresión fijado por la Regla Socrática.

Más aún, en la práctica, en ciertos contextos podemos en general prescindir de evaluaciones.

I.3.1 Evaluar como Calcular

Desde un punto de vista dialógico, las respuestas $c \in C$ (donde c representa un elemento canónico) y $d \Rightarrow c \in C$ a la petición sobre la resolución de la tarea expresada por una afirmación de la forma $d \in C$, se entienden como compromisos, a saber,

- Hacer explícito cómo se resuelve la tarea expresada por la afirmación cuestionada. Este compromiso equivale a elegir un elemento canónico.
- Mostrar que d evalúa al elemento canónico previamente elegido.⁴²

Si deseamos integrar cálculos en el seno de un diálogo, podemos situar esos actos en **subjuegos, de forma que su resultado influya en el desarrollo del diálogo principal:**

1. se cuestiona la evaluación,
2. la evaluación se lleva a cabo fuera de la obra principal,
3. el resultado se introduce en la obra principal; si es positivo, la obra continúa; si es negativo, la obra se detiene y gana el antagonista.

Con respecto al desafío, adoptamos la siguiente respuesta a la petición $O ? \Rightarrow | \neq$:⁴³

$$P \boxed{!} \Rightarrow d c \in C \text{ o } P \boxed{!} \neq \text{abandono}$$

⁴¹ Si una afirmación implica una cantidad puramente matemática, su valor se determina mediante el cálculo. Sin embargo, si una afirmación implica una cantidad empírica, su valor se determina mediante la experimentación - Martin-Löf (2014) p. 5.

⁴² Por lo tanto, ambas partes para resolver la tarea deben cumplirse en interdependencia. Así, supongamos que se afirma que $2 + 2 \in \mathbb{N}$ y como respuesta a la petición, se producen las afirmaciones $2 \in \mathbb{N}$ y $2 + 2 \Rightarrow 2 \in \mathbb{N}$.

Claramente la primera respuesta es correcta y la segunda, aplica ciegamente la regla, pero la segunda respuesta afirma que un cálculo imposible es realizable.

⁴³ Esta forma de formular una pregunta es una adaptación de la notación que usa Ranta (1994, capítulo 6.11) para preguntas.

La casilla con el signo de exclamación indica que el enunciado está asociado a un acto de cálculo que tiene lugar en una subdiálogo. Obsérvese que, desde el punto de vista dialógico, los cálculos también pueden integrarse en el marco dialógico, como una secuencia de peticiones y respuestas determinadas por las explicaciones dialógicas del significado de las expresiones implicadas.

Pongamos esto junto en una tabla

\Rightarrow	Preguntat	Respuesta
$\mathbf{P} ! d \Rightarrow c \varepsilon C$	$\mathbf{O} ? \Rightarrow / \not\Rightarrow$	$\mathbf{P} \boxed{!} d \Rightarrow c \varepsilon C$ o $\mathbf{P} \boxed{!} \not\Rightarrow$ -abandono

I.3.2 Evaluación como Desarrollo Definicional

La segunda formulación de las reglas de evaluación de Klev se inspira en la definición inductiva de la relación de evaluación dada por Martin-Löf (2021, p. 206). Según esta formulación, la evaluación se entiende como un *desarrollo definicional*, es decir, la sustitución repetida del definiendum por el definiens correspondiente hasta que se alcanza un objeto primitivo.

Cada sustitución puede entenderse como un paso de evaluación. Si se cuestiona, una sentencia de evaluación, $d \Rightarrow c \varepsilon C$, se defiende

- O bien afirmando : $\vdash d = c \varepsilon C$ – afirmando que según la estipulación ($: \vdash$) prescrita por la Regla Socrática para C , c es el definiens del definiendum d ;
- O bien, se afirma $d' \varepsilon C$ y $: \vdash d' = d \varepsilon C$ y $: \vdash d' = c \varepsilon C$. Si d' no es directamente c , en lugar de $: \vdash d' = c \varepsilon C$. se afirma $d' \Rightarrow c \varepsilon C$, y el proceso comienza de nuevo.⁴⁴

Klev presenta tal proceso mediante la siguiente regla en donde la línea punteada indica la opción de terminar la evaluación si d' puede definirse como c en un único paso:

$$\begin{array}{l}
 \vdash d \Rightarrow c \varepsilon C \\
 \hline
 \vdash d' \varepsilon C \\
 : \vdash d = d' \varepsilon C \\
 d' \Rightarrow c \varepsilon C \\
 \text{-----}
 \end{array}$$

⁴⁴ En RI-I, como ya se ha mencionado, utilizamos razones locales primitivas e *instrucciones* para justificar afirmaciones. Instrucciones se *resuelven* en razones locales primitivas de un modo similar empleado en un desarrollo definicional. Sin embargo, los procedimientos de resolución empleados en RI-I son más bien complejos. Las nuevas reglas de evaluación de Klev (2022), son más sencillas.

$$: \vdash d' = c \varepsilon C$$

De hecho, desde el punto de vista dialógico, la noción clave con respecto a las identidades evocadas por una evaluación es la de un proceso de transmisión de la propiedad de *invariancia de partida* respecto al significado de una expresión fijado por la Regla Socrática en diálogos materiales o la Regla Formal en diálogos formales.

Así, cuando **X** afirma $d \Rightarrow c \varepsilon C$, el jugador afirma que en una partida en la que él defiende C y en la que se emplea d en lugar de c , produce una partida que no afecta el resultado respecto a una partida en la que ocurre c pero no d . La idea subyacente es de requerir del jugador que afirma $d \Rightarrow c \varepsilon C$ de establecer un vínculo con la jugada $c \varepsilon C$

Por ejemplo, dado $d \Rightarrow I_z^V(a): A \vee B$, si se puede substituir d por $I_z^V(a)$ para defender $A \vee B$, entonces decimos que la identidad $d = I_z^V(a) \varepsilon A \vee B$ ha sido verificada *módulo invariancia de partida* (módulo-ip).

En este ejemplo, si el diálogo es material, la regla Socrática establece que tanto d como $I_z^V(a)$ pueden usarse para defender $A \vee B$, dado que ambas razones locales producen la jugada ulterior $a \varepsilon A$.

Claramente este procedimiento puede generalizarse para una cadena de identidades módulo-ip

\Rightarrow	Pregunta	Respuesta
X ! $d \Rightarrow c \varepsilon C$	Y ? \Rightarrow	X ! $d =_{mpi} c \varepsilon C$ or X ! $d =_{mpi} d' \varepsilon C$
X ! $d =_{mpi} d' \varepsilon C$	Y ? $\Rightarrow_{d'}$	X ! $d' \Rightarrow c \varepsilon C$
X ! $d' \Rightarrow c \varepsilon C$	Y ? \Rightarrow	Las mismas alternativas que al comienzo

- Es important notar que el requerimiento de satisfacer la propiedad de módulo-ip introduce materialidad incluso en diálogos fomales. En efecto, supongamos que **O** afirmó

$$2+2 \Rightarrow 1 \varepsilon \mathbb{N}$$

Tal afirmación obliga al oponente a vincular $2+2$ y 1 por medio de una identidad modulo-pi. Pero las Reglas Socráticas para \mathbb{N} y para la adición, hacen que tal identidad, aún concediendo que sea puramente formal o estipulativa, no conserva la significación de número natural y adición.

- En cuanto a las reglas estructurales para RI-II son básicamente las mismas que para la Dialógica.

En la próxima sección proponemos como desorrallar partidas sin recurrir a la

flecha de evaluación.

I.3.3 Ejercicios

Las soluciones pueden construirse a partir de los ejemplos desarrollados en la sección I.4.3. Sin embargo, en tal sección no ocurren afirmaciones de evaluación. Desarrollar jugadas que ponen en obra el proceso de evaluación es simple: en el caso que la expresión d a la izquierda de la evaluación tenga un *contenido concreto explícito*, se trata de o bien calcular o bien buscar una cadena de definiciones pertinentes; en el caso que d tenga un contenido *implícito* podemos simplemente asumir que un tal cálculo ha sido realizado. El lector podrá ensayar en distintos ejercicios alternativas con d explícitos o implícitos

Ej. 1 Identidad en **Bool**.

Desarrollar un estrategia ganadora para
 $(\forall x \in \text{Bool}) [\mathbf{Id}(\text{Bool}, x, \text{yes}) \vee \mathbf{Id}(\text{Bool}, x, \text{no})]$.

Ej. 2 Identidad en \mathbb{N} .

Desarrollar un estrategia ganadora para
 $(\forall x \in \mathbb{N}) [\mathbf{Id}(\mathbb{N}, x, 0) \vee (\exists y \in \mathbb{N}) \mathbf{Id}(\mathbb{N}, x, s(y))]$.

Ej. 3 Identidad y Función.

Desarrollar un estrategia ganadora para $\mathbf{Id}(A, a, a') \supset \mathbf{Id}(B, f(a), f(a'))$,
dado los conjuntos A y B y la función $f(x) \in B (x \in A)$.

Ej. 3 Identidad y Producto Cartesiano.

Desarrollar un estrategia ganadora para $\mathbf{Id}(A \times B, \langle a, b \rangle, \langle a', b' \rangle)$
dado los conjuntos A y B , y las premisas $\mathbf{refl}(A, a, a') \in \mathbf{Id}(A, a, a')$,
 $\mathbf{refl}(B, b, b') \in \mathbf{Id}(B, b, b')$ y la afirmación hipotética
 $\mathbf{refl}(A \times B, \langle x, y \rangle, \langle x, y \rangle) \in \mathbf{Id}(A \times B, \langle x, y \rangle, \langle x, y \rangle) (x \in A, y \in B)$.

I.4 Anticipar en Lugar de Evaluar

Un modo de entender la tesis que desencadena el desarrollo de una partida es como una afirmación por la cual el proponente se compromete a producir una razón local para la proposición en juego.

La afirmación no consiste en sostener que **P** posee ya actualmente tal razón local, pero más bien que el proponente prevee que una tal razón puede producirse.

Este modo de entender la afirmación de una tesis, pone de relieve su aspecto potencial y temporal. Más aún, inspirándonos en Husserl, podemos distinguir *expectativa* de *anticipación*. Mientras que la expectativa asociada a la afirmación de una tesis no indica explícitamente cuál es la razón local prevista, *la anticipación*, ofrece, no la razón actual, pero más bien un procedimiento asociado a la conectiva principal de la tesis

La idea Husserliana de anticipación o pro-tensión es que ahora en el presente, en el ahora, experimentamos el futuro. El futuro anticipado habita el presente.

Nos permitimos aquí de extrapolar este concepto y asociarlo con afirmación de una tesis. Dado que no se trata de tener ya ni la razón local ni el objeto estratégico final que son producto del desarrollo de las partidas, parece natural de comprender la anticipación como una *instrucción* o procedimiento a seguir para producir la actualización de la razón local en el seno de una partida.

La idea subyacente es que proposiciones son tareas a realizar o más generalmente tipos de acciones a ejecutar; Una anticipación es formulada entonces como una instrucción de como ejecutar la acción. Al final de la partida, se pueden recapitular todos los pasos, que actualizan la ejecución y en un paso aún mas general reformular las instrucciones como objetos estratégicos que indican como construir una estrategia de ganancia (si existe).

De hecho, la idea es de emplear instrucciones solamente cuando se afirma la tesis, y solamente respecto a la conectiva principal, usar expectativas para el ataque y la defensa. En Husserl una anticipación requiere cierta retención del pasado que prefigura la forma de la anticipación. En nuestro contexto, lo que prefigura la forma de la anticipación son las reglas locales

Expectativas son menos precisas. En efecto si asociamos una tesis a una expectativa, no hay una instrucción que indique como obtenerla razón local buscada, pero simplemente que hay una tal razón – notacionalmente una afirmación que solo involucra expectativas está constituida por un signo de exclamación y la proposición afirmada, por ejemplo la afirmación **X ! A** se entiende aquí como que el jugador afirma que prevee poder producir una razón que sostiene *A*.

I.4.1 Anticipaciones, Instrucciones y Expectativas

Si la conectiva principal de la tesis es *AVB*, **P** “anticipa” que se puede producir una razón local para tal afirmación produciendo una razón local para uno de los miembros de la disyunción. Expresamos la forma de anticipación de una tesis con la disyunción como operador principal mediante la siguiente instrucción en donde la barra indica que **P** puede producir una razón local para tal tesis escogiendo entre realizar la tarea a asociada con el componente izquierdo o con el componente derecho. Cada una de estas realizaciones produce una razón local que sostiene la afirmación de la tesis

$$P! I_z^\vee \mid Dr^\vee \varepsilon AVB$$

Si desarrollamos esta idea para todas las conectivas lógicas obtenemos

Anticipación	Pregunta	Respuesta
$\mathbf{P} ! I_z^\vee Dr^\vee \varepsilon AVB$ Asumiendo $A, B \varepsilon prop$	$\mathbf{O} ?\vee$	$\mathbf{P} ! A$ ----- $\mathbf{P} ! B$
$\mathbf{P} ! \langle I_z^\wedge, Dr^\wedge \rangle \varepsilon A \wedge B$ Asumiendo $A, B \varepsilon prop$	$\mathbf{O} ?\wedge_1$ ----- $\mathbf{O} ?\wedge_2$	$\mathbf{P} ! A$ $\mathbf{P} ! B$
$\mathbf{P} ! \langle I_z^\exists, Dr^\exists \rangle \varepsilon (\exists x \varepsilon A)B(x)$ Asumiendo $A \varepsilon prop$ $B(x) \varepsilon prop (a: A)$	$\mathbf{O} ?\exists_1$ ----- $\mathbf{O} ?\exists_2$	$\mathbf{P} ! a \varepsilon A$ $\mathbf{P} ! B(a)$
$\mathbf{P} ! (I_z^\supset)Dr^\supset \varepsilon A \supset B$ Asumiendo $A, B \varepsilon prop$	$\mathbf{O} ! A$	$\mathbf{P} ! B$
$\mathbf{P} ! (I_z^\supset)Dr^\supset \varepsilon A \supset \perp$ Asumiendo $A \varepsilon prop$ $\perp = \emptyset \varepsilon prop$	$\mathbf{O} ! A$	$\mathbf{P} ! \perp$
$\mathbf{X} ! (I_z^\forall)Dr^\forall \varepsilon (\forall x \varepsilon A)B(x)$ Asumiendo $A \varepsilon prop$ $B(x) \varepsilon prop (a: A)$	$\mathbf{O} ! a \varepsilon A$	$\mathbf{P} ! B(a)$

Una vez que la tesis ha sido atacada la partida se desarrolla siguiendo las siguientes reglas que en el caso que las afirmaciones de ataque y defensa involucren componentes complejos, siguen los preceptos estándar de la explicación dialógica del significado local.

Si los componentes son elementales las afirmaciones están compuestas por razones locales. Ellas ponen de manifiesto que la tarea prevista ha sido realizada.

	Pregunta	Respuesta
$\mathbf{X} ! AVB$ Asumiendo $A, B \varepsilon prop$	$\mathbf{Y} ?\vee$	$\mathbf{X} ! A$ Si A es compleja $\mathbf{X} ! a \varepsilon A$ Si A es elemental

		<p>-----</p> <p>X ! B Si B es compleja</p> <p>X ! b ∈ B Si A es elemental</p>
<p>X ! A ∧ B</p> <p>Asumiendo A, B ∈ prop</p>	<p>Y ? ∧1</p> <p>-----</p> <p>Y ? ∧2</p>	<p>X ! A Si A es compleja</p> <p>X ! a ∈ A Si A es elemental</p> <p>X ! B Si B es compleja</p> <p>X ! b ∈ B Si B es elemental</p>
<p>X ! (∃x ∈ A)B(x)</p> <p>Asumiendo A ∈ prop B(x) ∈ prop (a: A)</p>	<p>Y ? ∃1</p> <p>-----</p> <p>Y ? ∃2</p>	<p>X ! A Si A es compleja</p> <p>X ! a ∈ A Si A es elemental</p> <p>X ! B(a_i) Si A y B son complejas, a_i ∈ A_i y A_i es el componente elemental que resulta del análisis de A, relevante para la atribución de B.</p> <p>X ! e ∈ B(a) Si A y B son elementales</p> <p>X ! e ∈ B(a_i) Si A es compleja y B elemental</p>
<p>X ! A ⊃ B</p> <p>Asumiendo A, B ∈ prop</p>	<p>Y ! A Si A es compleja</p> <p>Y ! a ∈ A Si A es elemental</p>	<p>X ! B Si A es compleja</p> <p>X ! b ∈ B Si A es elemental</p>
<p>X ! A ⊃ ⊥</p> <p>Asumiendo A ∈ prop ⊥ = ∅ ∈ prop</p>	<p>Y ! A Si A es compleja</p> <p>Y ! a ∈ A Si A es elemental</p>	<p>X ! ⊥</p>
<p>X ! (∀x ∈ A)B(x)</p> <p>Asumiendo A ∈ prop B(x) ∈ prop (a: A)</p>	<p>Y ! A Si A es compleja</p>	<p>X ! B(a_i)</p> <p>Si A y B son complejas, a_i ∈ A_i y A_i es el componente elemental que resulta del análisis de A, relevante para la atribución de B.</p>

		$\mathbf{X} ! e \in B(a_i)$ Si A es compleja y B elemental
	$\mathbf{Y} ! a \in A$ Si A es elemental	$\mathbf{X} ! e \in B(a)$ Si A y B son elementales

- **N.B. 1** Las reglas para cuantificadores En el caso de cuantificadores, están diseñadas de tal manera que, si el primer componente es complejo, el defensor no puede afirmar el segundo componente hasta que el primer componente no haya sido descompuesto (mediante interacción dialógica) en sus subcomponentes elementales. Es así que el contenido de afirmaciones sobre proposiciones cuantificadas se obtiene una vez que las anticipaciones han sido verificadas.
- **N.B. 2** En algunos contextos, afirmaciones complejas pueden formularse agregando instrucciones, en lugar de expresarlas como meras expectativas.

I.4.2 Expectativas para Identidad, Bool y Números Naturales

El uso de anticipaciones y expectativas conduce a cierta pérdida de contenido, y en otros contextos un lenguaje más rico como el que desarrollamos en las secciones precedentes será preferible. Particularmente eso es el caso cuando la tesis es una afirmación elemental⁴⁵.

Sin embargo, esta forma de diálogos, pone de relieve

- Que el contenido de una tesis es propagado por las proposiciones elementales que la constituyen – de forma similar al concepto leibniziano de "pensamiento ciego", según el cual un cálculo se concibe como un mecanismo sintáctico hasta llegar a la conclusión que, una vez alcanzada, recupera el contenido de la tesis que se busca demostrar. Más aún, como mencionamos en N.B.2 nada impide de agregar anticipaciones en la forma de instrucciones en toda jugada en la que se afirma una proposición compleja.
- Que el contenido de una tesis es una tarea u acción *a realizar* mediante la interacción dialógica. Según esta lectura, las razones locales que resultan de la interacción, pueden entenderse como realizaciones que justifican la afirmación que la tarea expresada por la tesis puede ser llevada a cabo. En suma, el contenido de una tesis es concebido como un *esquema de acción* – en términos de Lorenz(2010).

Más generalmente, anticipaciones que toman la forma de instrucciones, son extremadamente útiles en el caso que el primer componente de un cuantificador sea complejo. En tales casos, que no tienen que necesariamente reducirse a afirmaciones de la tesis, la anticipación toma la forma de una instrucción con rol anafórico. Por ejemplo:

$$\mathbf{X} ! (\forall x \in (\exists y \in D)A(y))B(Iz^{\exists})$$

Una glosa posible en lenguaje natural es

Para todo objeto de aquellos D que son A , **tales objetos** (es decir, aquellos objetos

⁴⁵ En efecto si la tesis es una afirmación elemental, la anticipación consiste en afirmar que hay cierta razón local de cuya evaluación resulta una razón local canónica asociada a la tesis.

que producen una razón local para la **izquierda** del existencial) son B

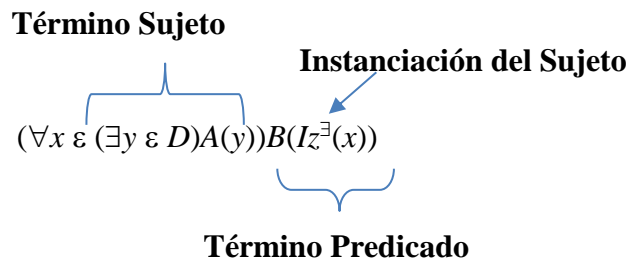
En efecto cuando Y ataca el universal, ella debe producir primero alguna razón local x para el dominio del cuantificador. Como el dominio es un existencial, x es complejo, y está constituido por un par tal que el componente izquierdo, es un y que es D , y un componente derecho, que atribuye A a y (por tanto, el existencial expresa el conjunto de todos los D que son A). Bajo estas circunstancias, X tiene el deber de atribuir A al componente izquierdo de todo x .

Dado que es claro que x anticipa una razón local para un existencial podemos emplear la notación alternativa siguiente que es en realidad una versión muy próxima de la habitual en TCT:

$$X ! (\forall x \varepsilon (\exists y \varepsilon D)A(y))B(Iz^{\exists}(x))$$

En suma, la instrucción $Iz^{\exists}(x)$ anticipa que cualquiera sea la razón local que realiza uno los D que son A , y que constituyese así la cabeza de la anáfora, la realización de tal instrucción (la cola de la anáfora) es un B .

En verdad, como desarrollado por McConaughey (2021) y discutido en la segunda parte principal del presente volumen, $(\forall x \varepsilon (\exists y \varepsilon D)A(y))B(Iz^{\exists}(x))$ manifiesta la forma lógica general de las premisa de un silogismo aristotélico:



En efecto la premisa de un silogismo en *Barbara* **no** expresa la implicación

Si todos los individuos (del dominio) son hombres, entonces son mortales

sino más bien

Todos aquellos individuos (del dominio) que son hombres, (esos) son mortales

I.4.3 Ejemplos

En los ejercicios siguientes desarrollamos las partidas relevantes y delineamos como construir una estrategia ganadora, sin embargo; dejamos al lector, la tarea de desarrollar el árbol resulante y la tarea de agregar los objetos estratégicos como una recapitulación final.

Ejemplo 1 Silogismo en *Bárbara*

En este ejemplo la conclusión constituye la tesis principal del proponente y las premisas P1 y P2, afirmaciones del oponente

O	P
---	---

			$!(Iz\forall)Dr^\forall \varepsilon (\forall x \varepsilon (\exists y \varepsilon D)A(y))C(Iz^\exists(x))$	0
P1	$!(\forall x \varepsilon (\exists y \varepsilon D)A(y))B(Iz^\exists(x))$			
P2	$!(\forall x \varepsilon (\exists y \varepsilon D)B(y))C(Iz^\exists(x))$			
1	$!(\exists y \varepsilon D)A(y)$	0	$!c \varepsilon C(a)$	18
3	$!a \varepsilon D$		1 $? \exists_1$	2
5	$!e \varepsilon A(a)$		1 $? \exists_2$	4
11	$!b \varepsilon B(a)$		P1 $!(\exists y \varepsilon D)A(y)$	6
7	$? \exists_1$	6	$!a \varepsilon D$	8
9	$? \exists_2$	6	$!e \varepsilon A(a)$	10
17	$!c \varepsilon C(a)$		P2 $!(\exists y \varepsilon D)B(y)$	12
13	$? \exists_1$	12	$!a \varepsilon D$	14
15	$? \exists_2$	12	$!b \varepsilon B(a)$	16
			P gana	

Ejemplo 2

Todo elemento de **Bool** es o bien idéntico a **sí** o bien idéntico a **no**

Desarrollar un estrategia ganadora para $(\forall x \varepsilon \text{Bool}) [\mathbf{Id}(\text{Bool}, x, \mathbf{yes}) \vee \mathbf{Id}(\text{Bool}, x, \mathbf{no})]$

Puesto que el operador principal de la tesis es un universal **O** puede elegir un elemento arbitrario de Bool, incluso un elemento no canónico. Si b , es efecto no canónico, de acuerdo a las reglas locales para Bool, **P** puede requerir de **O** de afirmar que de la evaluación de b resulta uno de los canónicos.

En orden de abreviar la partida asumimos directamente que **O** elije un canónico. De todos modos, si **O** elijiese un b que no evalúa como uno de los canónicos, **O** perdería la partida inmediatamente. Sin embargo, contravendría el objetivo de encontrar una estrategia ganadora. En efecto, la construcción de una estrategia ganadora asume que **O** hace la jugada óptima con el fin de examinar la tesis.

O			P	
			$!(Iz\forall)Dr^\forall \varepsilon (\forall x \varepsilon \text{Bool}) [\mathbf{Id}(\text{Bool}, x, \mathbf{no}) \vee \mathbf{Id}(\text{Bool}, x, \mathbf{sí})]$	0
1	$!\mathbf{no} \varepsilon \text{Bool}$	0	$!\mathbf{Id}(\text{Bool}, \mathbf{no}, \mathbf{no}) \vee \mathbf{Id}(\text{Bool}, \mathbf{no}, \mathbf{sí})$	2
3	$? \forall$	2	$!Iz^\forall(\mathbf{refl}(\text{Bool}, \mathbf{no}) \varepsilon \mathbf{Id}(\text{Bool}, \mathbf{no}, \mathbf{no}))$	4
5	$?Iz^\forall$	2	$!\mathbf{refl}(\text{Bool}, \mathbf{no}) \varepsilon \mathbf{Id}(\text{Bool}, \mathbf{no}, \mathbf{no})$	6
7	$? \mathbf{Id}_1$	6	$!\mathbf{no} = \mathbf{no} \varepsilon \text{Bool}$	8
9	$? \mathbf{Id}_2$	6	$!\mathbf{no} \varepsilon \text{Bool}$	10
			P gana	

La segunda partida relevante para la construcción de la estrategia ganadora se desarrolla de forma análoga

O			P	
			$!(Iz\forall)Dr^\forall \varepsilon (\forall x \varepsilon \text{Bool}) [\mathbf{Id}(\text{Bool}, x, \mathbf{no}) \vee \mathbf{Id}(\text{Bool}, x, \mathbf{sí})]$	0
1	$!\mathbf{sí} \varepsilon \text{Bool}$	0	$!\mathbf{Id}(\text{Bool}, \mathbf{sí}, \mathbf{no}) \vee \mathbf{Id}(\text{Bool}, \mathbf{sí}, \mathbf{sí})$	2
3	$? \forall$	2	$!Dr^\forall(\mathbf{refl}(\text{Bool}, \mathbf{sí}) \varepsilon \mathbf{Id}(\text{Bool}, \mathbf{sí}, \mathbf{sí}))$	4
5	$?Dr^\forall$	2	$!\mathbf{refl}(\text{Bool}, \mathbf{sí}) \varepsilon \mathbf{Id}(\text{Bool}, \mathbf{sí}, \mathbf{sí})$	6
7	$? \mathbf{Id}_1$	6	$!\mathbf{sí} = \mathbf{sí} \varepsilon \text{Bool}$	8
7	$? \mathbf{Id}_2$	6	$!\mathbf{sí} \varepsilon \text{Bool}$	10
			P gana	

Ejemplo 3

Todo número natural n es o bien idéntico a **0**

o bien idéntico al sucesor de un número natural

Desarrollar un estrategia ganadora para $(\forall x \in \mathbb{N}) [\mathbf{Id}(\mathbb{N}, x, 0) \vee (\exists y \in \mathbb{N}) \mathbf{Id}(\mathbb{N}, x, s(y))]$

Obviamente hay al menos dos partidas posibles dependiendo si **O** se decide por 0 o por otro número natural. Una estrategia ganadora requiere que **P** gane la tesis en cada una de las opciones

Recordemos que si **O** elije un número n distinto de 0, el número de ataques sobre la tesis principal es igual al número de sucesores de 0 que define n más un último ataque mediante le cual **O** elije 0.

Comenzamos con la primera opción, en la que **O** elije 0 para desafiar la tesis

O			P	
			$!(Iz\forall)Dr^{\forall} \varepsilon (\forall x \varepsilon \mathbb{N}) [\mathbf{Id}(\mathbb{N}, x, 0) \vee (\exists y \varepsilon \mathbb{N}) \mathbf{Id}(\mathbb{N}, x, s(y))]$	0
1	$! 0 \varepsilon \mathbb{N}$	0	$! \mathbf{Id}(\mathbb{N}, 0, 0) \vee (\exists y \varepsilon \mathbb{N}) \mathbf{Id}(\mathbb{N}, 0, s(y))$	2
3	$? \forall$	2	$! Iz^{\forall}(\mathbf{refl}(\mathbb{N}, 0)) \varepsilon \mathbf{Id}(\mathbb{N}, 0, 0)$	4
5	$? Iz^{\forall}$	2	$! \mathbf{refl}(\mathbb{N}, 0) \varepsilon \mathbf{Id}(\mathbb{N}, 0, 0)$	6
7	$? \mathbf{Id}_1$	6	$! 0 = 0 \varepsilon \mathbb{N}$	8
9	$? \mathbf{Id}_2$	8	$! 0 \varepsilon \mathbb{N}$	10
			P gana	

En efecto, si **O** ataca el universal con la afirmación $0 \varepsilon \mathbb{N}$, **P** gana elijiendo el componente izquierdo de la disunción. El ataque de **O** a la afirmación de identidad, se gana, afirmando que la reflexidad $\mathbf{refl}(\mathbb{N}, a) \varepsilon \mathbf{Id}(\mathbb{N}, 0, 0)$ se sigue de que $0 \varepsilon \mathbb{N}$, y de que **O** ha hecho tal afirmación cuando atacó el universal.

- Recordemos que de acuerdo a la Regla Socrática, **O** puede atacar una afirmación elemental únicamente si **O** no hizo la misma afirmación en una jugada previa.
- Recordemos también que un ataque $? \mathbf{refl}$ a una afirmación de Identidad $\mathbf{refl}(A, a) \varepsilon \mathbf{Id}(A, a, a)$ se responde con la afirmación $a \varepsilon A$.

Examinemos la segunda opción de **O** por la cual se decide por atacar el universal con un número natural arbitrario, digamos $2 \varepsilon \mathbb{N}$, para no prolongar demasiado la partida.

Recordemos que según las reglas para deducción natural,

- Las cifras pueden reescribirse en notación canónica cuando se pregunta por ellas: en este ejemplo la pregunta $? := 1$ se responde con $s(0)$ y $? := 2$ se responde con $s(s(0))$
- **P** no puede atacar la afirmación $\mathbf{O} ! 0 \varepsilon \mathbb{N}$, pero puede pedirle a **O** de afirmarla con la pregunta $? 0 \varepsilon \mathbb{N}$
- El ataque $? n$ a una afirmación de la forma $s(n) \varepsilon \mathbb{N}$, se responde con $n \varepsilon \mathbb{N}$

La construcción de una estrategia ganadora, requiere responder a los ataques $\mathbf{O} ! 2 \varepsilon \mathbb{N}$, $\mathbf{O} ! 1 \varepsilon \mathbb{N}$ y $\mathbf{O} ! 0 \varepsilon \mathbb{N}$

Afin de hace evidente el desarrollo principal de la partida obviamos los pasos en los que se verifica que los números afirmados son en efecto números naturales. Dejamos al lector incorporar tales jugadas en la partida

O	P
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			$!(Iz\forall)Dr^{\forall} \varepsilon (\forall x \varepsilon \mathbb{N}) [\mathbf{Id}(\mathbb{N}, x, 0) \vee (\exists y \varepsilon \mathbb{N}) \mathbf{Id}(\mathbb{N}, x, s(y))]$	0
1	$! 2 \varepsilon \mathbb{N}$	0	$! \mathbf{Id}(\mathbb{N}, s(s(0)), 0) \vee (\exists y \varepsilon \mathbb{N}) \mathbf{Id}(\mathbb{N}, s(s(0)), s(y))$	4
3	$! s(s(0)) \varepsilon \mathbb{N}$	1	$?:=2$	2
5	$? \forall$	4	$(\exists y \varepsilon \mathbb{N}) \mathbf{Id}(\mathbb{N}, s(s(0)), s(y))$	6
7	$? \exists_1$	10	$! 1 \varepsilon \mathbb{N}$	8
9	$? \exists_2$	10	$! Dr^{\forall}(\mathbf{refl}(\mathbb{N}, s(s(0)))) \varepsilon (\mathbf{Id}(\mathbb{N}, s(s(0)), s(1)))$	10
11	$? Dr^{\forall}$	14	$! \mathbf{refl}(\mathbb{N}, s(s(0))) \varepsilon (\mathbf{Id}(\mathbb{N}, s(s(0)), s(1)))$	12
13	$?:=1$	12	$! s(0) \varepsilon \mathbb{N}$	14
15	$? \mathbf{Id}_1$	12	$! s(s(0)) = s(s(0)) \varepsilon \mathbb{N}$	16
17	$? \mathbf{Id}_2$	16	$! s(s(0)) \varepsilon \mathbb{N}$	18
19	$! s(0) \varepsilon \mathbb{N}$	0	$! \mathbf{Id}(\mathbb{N}, s(0), 0) \vee (\exists y \varepsilon \mathbb{N}) \mathbf{Id}(\mathbb{N}, s(0), s(y))$	20
21	$? \forall$	20	$! (\exists y \varepsilon \mathbb{N}) \mathbf{Id}(\mathbb{N}, s(0), s(y))$	22
23	$? \exists_1$	22	$! 0 \varepsilon \mathbb{N}$	24
25	$? \exists_2$	22	$! Dr^{\forall}(\mathbf{refl}(\mathbb{N}, s(0))) \varepsilon (\mathbf{Id}(\mathbb{N}, s(0), s(0)))$	26
27	$? Dr^{\forall}$	26	$! \mathbf{refl}(\mathbb{N}, s(0)) \varepsilon (\mathbf{Id}(\mathbb{N}, s(0), s(0)))$	28
29	$? \mathbf{Id}_1$	28	$! 0 = 0 \varepsilon \mathbb{N}$	30
31	$? \mathbf{Id}_2$	28	$! 0 \varepsilon \mathbb{N}$	32
33	$! 0 \varepsilon \mathbb{N}$	0	$\mathbf{Id}(\mathbb{N}, 0, 0) \vee (\exists y \varepsilon \mathbb{N}) \mathbf{Id}(\mathbb{N}, 0, s(y))$	34
35	$? \forall$	34	$! Iz^{\forall}(\mathbf{refl}(\mathbb{N}, 0)) \varepsilon \mathbf{Id}(\mathbb{N}, 0, 0)$	36
37	$? Iz^{\forall}$	36	$! \mathbf{refl}(\mathbb{N}, 0) \varepsilon \mathbf{Id}(\mathbb{N}, 0, 0)$	38
39	$? \mathbf{Id}_1$	38	$! 0 = 0 \varepsilon \mathbb{N}$	40
41	$? \mathbf{Id}_2$	38	$! 0 \varepsilon \mathbb{N}$	42
			P gana	

Hay diversas maneras las partidas, pero la idea general pare construir una estrategia ganadora debiera estar clara: cualquiera sea el número natural n **O** elija, **P** gana o bien con la identidad de 0, o si n es diferente de 0, **P** debe encontrar un m , tal que su sucesor sea idéntico a n .

Esto requiere

1. verificar que n y m sean en efecto número naturales – i.e. que pueden ser descompuestos de tal manera que son algunos de los sucesores s_i de 0.
2. encontrar un m' tal que su sucesor sea idéntico a m (si m es diferente de 0)
3. mostrar que m es idéntico a 0, si m es 0

Ejemplo 4

La identidad entre los argumentos a y a' de la función f implica la identidad entre $f(a)$, $f(a')$.

Desarrollar una estrategia ganadora para

$$\mathbf{Id}(A, a, a') \supset \mathbf{Id}(B, f(a), f(a')),$$

dados los conjuntos A y B y la función $f(x) \varepsilon (x \varepsilon A)B$

La función $f(x) \varepsilon B (x \varepsilon A)$ satisface la identidad que satisface todo elemento de un conjunto consigo mismos. Obtenemos así la afirmación hipotética

$$\mathbf{AH} \quad \mathbf{X} \quad ! \mathbf{refl}(B, f(x)) \varepsilon \mathbf{Id}(B, f(x), f(x)) (x \varepsilon A)$$

Una estrategia ganadora se puede obtener empleando substitución de idénticos y AH

- Recordemos que un ataque a una afirmación hipotética de **X** consiste en **Y** eligiendo los argumentos para substituir las variables. En nuestro ejemplo la afirmación

$$\mathbf{O} ! \text{refl}(B, f(x)) \varepsilon \mathbf{Id}(B, f(x), f(x)) (x \varepsilon A)$$

se ataca con la afirmación

$$\mathbf{P} ! a \varepsilon A$$

- N.B. **P** requiere hacer explícita la razón local, solo cuando **O** es forzada a afirmar la identidad expresada en la tesis.
- Como señalado en la sección sobre identidad, a nivel de partidas dejamos la razón que justifica la regla de sustitución de idénticos tácita.

O		P	
P	$! f(x) \varepsilon B (x \varepsilon A)$		
			$! (Iz \supset Dr \supset \varepsilon \mathbf{Id}(A, a, a') \supset \mathbf{Id}(B, f(a), f(a'))) \quad 0$
AH	$! \text{refl}(B, f(x)) \varepsilon \mathbf{Id}(B, f(x), f(x)) (x \varepsilon A)$		
1	$! \text{refl}(D, d) \varepsilon \mathbf{Id}(A, a, a') \quad 0$ (que asume $D = A \varepsilon \text{set}$)		$! \text{refl}(D', d') \varepsilon \mathbf{Id}(B, f(a), f(a')) \quad 8$
3	$! \text{refl}(B, f(a)) \varepsilon \mathbf{Id}(B, f(a), f(a))$	AH	$a \varepsilon A \quad 2$ ($a \varepsilon A$ ha sido asumida en la formación de la identidad afirmada en 1)
5	$! \mathbf{Id}(B, f(a), f(a'))$	1,3	$? a'x \varepsilon A \quad 4$ ($a' \varepsilon A$ ha sido asumida en la formación de la identidad afirmada en 1)
7	$! \text{refl}(D', d') \varepsilon \mathbf{Id}(B, f(a), f(a'))$ (que asume $D' = B \varepsilon \text{set}$)	5	$? \text{refl} \quad 6$
			P gana (pues O ha sido forzada a afirmar en la jugada 5 el consecuente de la implicación afirmada en 0)

Una estrategia alternativa puede obtenerse como sigue:

Si **O** afirma la identidad

$$\mathbf{O} ! \text{refl}(D, d) \varepsilon \mathbf{Id}(A, a, a'), \mathbf{O} \text{ debe también también afirmar}$$

$$\mathbf{O} ! a = a' \varepsilon A \text{ (empleando simetría y transitividad a las afirmaciones } d = a \varepsilon A, d = a' \varepsilon A)$$

P puede ahora afirmar

$$\mathbf{P} ! a = a' \varepsilon A$$

y aplicando la regla de funciones a la premisa $f(x) \varepsilon B (x \varepsilon A)$, obligar a **O** a afirmar $f(a) = f(a') \varepsilon B$. Es decir

$$\mathbf{O} ! f(x) \varepsilon B (x \varepsilon A)$$

$$\mathbf{P} ! a = a' \varepsilon A$$

$$\mathbf{O} ! f(a) = f(a') \varepsilon B$$

Esto permite a **P** afirmar el consecuente y responder con $f(a) = f(a') \varepsilon B$ cuando **O** ataca tal consecuente, asumiendo $D' = B \varepsilon \text{set}$

Ejemplo 5

Identidad de pares respecto al producto Cartesiano

Dados los conjuntos A, B

Dada la premisa que a y a' son idénticas respecto al conjunto A
 Dada la premisa que b y b' son idénticas respecto al conjunto A
 Dada la afirmación hipotética que los pares $\langle x, y \rangle$ y $\langle x, y \rangle$ son idénticos respecto al producto cartesiano entre A y B , asumiendo $x \in A, y \in B$
 Demuestre que los pares $\langle a, b \rangle$ y $\langle a', b' \rangle$ son idénticos respecto al producto cartesiano entre A y B

La regla de formación del producto cartesiano requiere que los dos componentes del producto sean conjuntos, y que sus elementos sean pares. Esto indica que una afirmación hipotética del producto cartesiano toma la forma siguiente:

$$\langle x, y \rangle \in A \times B \quad (x \in A, y \in B),$$

Dado que cada elemento de un conjunto satisface identidad respecto a sí mismo, esto también vale para los $\langle x, y \rangle$ en $A \times B$:

$$\text{AH} \quad \text{refl}(A \times B, \langle x, y \rangle) \in \text{Id}(A \times B, \langle x, y \rangle, \langle x, y \rangle) \quad (x \in A, y \in B).$$

Así que podemos reformular el ejercicio como la tarea de desarrollar una estrategia ganadora para $\text{Id}(A \times B, \langle a, b \rangle, \langle a', b' \rangle)$

dado los conjuntos A y B ,
 las premisas $\text{refl}(A, a, a')$ $\text{Id}(A, a, a')$, $\text{refl}(B, b, b')$ $\in \text{Id}(B, b, b')$ y
 la afirmación hipotética AH
 $\text{refl}(A \times B, \langle x, y \rangle) \in \text{Id}(A \times B, \langle x, y \rangle, \langle x, y \rangle) \quad (x \in A, y \in B).$

- Recordemos que, si la tesis tiene la forma de una identidad y es afirmada como una expectativa, el primer ataque consiste en preguntar por la razón local.
- Recordemos que, a nivel de partidas dejamos la razón que justifica la regla de sustitución idénticos tácita.
- Recordemos que un ataque a una afirmación hipotética de \mathbf{X} consiste en \mathbf{Y} eligiendo los argumentos para sustituir las variables. En nuestro ejemplo la afirmación (con razón local tácita)

$$\mathbf{O} \quad \text{Id}(A \times B, \langle x, y \rangle, \langle x, y \rangle) \quad (x \in A, y \in B),$$

se ataca con las afirmaciones

$$\mathbf{P} \quad \begin{aligned} & a \in A, y \\ & b \in B. \end{aligned}$$

- Como en el ejemplo anterior, \mathbf{P} requiere hacer explícita la razón local, solo cuando \mathbf{O} es forzada a afirmar la identidad expresada en la tesis

\mathbf{O}		\mathbf{P}	
		$\text{Id}(A \times B, \langle a, b \rangle, \langle a', b' \rangle)$	0
P1	$\text{refl}(A, a) \in \text{Id}(A, a, a')$		
P2	$\text{refl}(B, b) \in \text{Id}(B, b, b')$		
AH	$\text{Id}(A \times B, \langle x, y \rangle, \langle x, y \rangle) \quad (x \in A, y \in B).$		
1	?refl	0	$\text{refl}(D_1 \times D_2, \langle d_1, d_2 \rangle) \in \text{Id}(A \times B, \langle a, b \rangle, \langle a', b' \rangle)$
3	$\text{Id}((A \times B, \langle a, y \rangle, \langle a, y \rangle) \quad (y \in B)$	AH	$a \in A$ ($a \in A$ ha sido asumida en la formación de la identidad afirmada en P1)
5	$\text{Id}((A \times B, \langle a, y \rangle, \langle a', y \rangle) \quad (y \in B)$	P1,3	$? a' \in A$ ($a' \in A$ ha sido asumida en la formación de la identidad afirmada en P1)

7	Id (($A \times B$, $\langle a, b \rangle$, $\langle a', b' \rangle$)	5	$b \in B$	6
9	Id (($A \times B$, $\langle a, b \rangle$, $\langle a', b' \rangle$)	7	$b' \in B$	8
11	refl ($D_1 \times D_2$, $\langle d_1, d_2 \rangle$) ε Id (($A \times B$, $\langle a, b \rangle$, $\langle a', b' \rangle$) (esta respuesta fuerza a O a las afirmaciones adicionales $D_1 \times D_2 = A \times B$, $\langle d_1, d_2 \rangle = \langle a, b \rangle \varepsilon A \times B$ $\langle d_1, d_2 \rangle = \langle a', b' \rangle \varepsilon A \times B$)	11	?refl	10
		P gana		

- **Conclusiones de la Primera Parte: El Auténtico Dialógico**

Desde la perspectiva de la Teoría Constructiva de Tipos (TCT) de Per Martin-Löf, el objetivo principal de las reglas dialógicas es de proporcionar un marco formal para el desarrollo de las formas de contenido asertórico empleadas en la teoría de tipos.

Desde la perspectiva de Built-In-Opponent (BIO) de Catarina Dutilh Novaes (2020), la interpretación dialógica de procesos inferenciales tiene como objetivo principal persuadir y convencer un destinatario (a veces plural) sobre el razonamiento que conduce a la conclusión obtenida.

Desde la perspectiva inferencialista de Robert Brandom (1994), el objetivo principal de la interpretación dialógica es de proporcionar un marco formal para poner en obra las componentes déonticas de un sistema dado de reglas de inferencia comprendidas como la interacción de derechos y obligaciones en el seno de juegos de pedir y dar razones.

Los tres enfoques enfatizan ideas importantes del Marco Dialógico:

- el entretrejo de los componentes deónticos y epistémicos del significado, en el caso de la TCT, y
- el rol de los aspectos psicológicos y sociológicos en el contexto de una explicación, en el caso del enfoque BIO.

Sin embargo, en contraste con estas interpretaciones, los creadores de la dialógica no concibieron su enfoque como una interpretación dialógica de un sistema deductivo ya dado, sino como un marco dentro del cual un sistema deductivo resulta de aquellas reglas de desarrollo de una estrategia ganadora, que preservan las explicaciones dialógicas del significado local y global determinado por las reglas de partículas y estructurales.

Según la perspectiva del viejo enfoque que nos permitimos llamar la perspectiva del *auténtico dialógico*, el nivel de partida es el nivel donde se forja el significado. El significado dialógico tiene lugar en el nivel de partida. Más aún, la posibilidad de seleccionar no sólo aquellas partidas que mejor cumplen la tarea explicativa, sino también la posibilidad de seleccionar las normas para el desarrollo de tales partidas, es de hecho una de las virtudes más salientes del Marco Dialógico.

En efecto, el surgimiento de conceptos, afirmamos, no resulta sólo de juegos de dar y pedir razones (juegos que involucran preguntas- de por qué): son también juegos que apuntan a mostrar cómo es que la razón invocada cumple una tarea explicativa respecto a cómo es que la conclusión ha sido obtenida. Más aún, en tales juegos dialógicos el por qué y el cómo resultan de un proceso mutuo y alterno de actualizaciones y esquematizaciones.

Esta característica del constructivismo dialógico tiene sus raíces en Herder (2016), para quién el proceso cultural es un proceso de educación, en el que enseñar y aprender siempre ocurren juntos. En este sentido, como señala Lorenz (2010), la situación dialógica de enseñanza-aprendizaje es donde interactúan la competencia (la perspectiva del yo) y la cooperación (la perspectiva del tú).

Las ideas de Paul Lorenzen y Kuno Lorenz sobre la lógica dialógica como restablecimiento de los vínculos históricos entre, la ética y la lógica proporcionan una respuesta clara al escepticismo de Wilfried Hodges respecto a la contribución del marco dialógico – véase Hodges y Väänänen (2019).

Sin embargo, la crítica de Hodges parece apuntar al interés matemático de una concepción

dialógica de la lógica, más que a un interés filosófico, que después de todo no atrae mucho de su interés.

Los profundos vínculos entre la lógica dialógica, la teoría de tipos constructivos y los fundamentos constructivos de las matemáticas acaso constituyan una respuesta suficiente a las preguntas de Hodges sobre el interés matemático o lógico del marco dialógico.

De todas formas en lugar de desarrollar una respuesta más extensa y en lugar de hacer un comentario general sobre la contribución filosófica del Marco Dialógico a los fundamentos de la lógica y las matemáticas,— véase Lion (2023) —, resaltemos cuatro puntos que resultan de las observaciones anteriores:

1. El enfoque dialógico ofrece un marco robusto y sólido para el desarrollo de fundamentos de la lógica basados en la inferencia.
2. La interacción de nociones epistémicas y deónticas arraigadas en explicaciones de significados dialógicos locales establece nuevas formas de implementar la interfaz pragmática/semántica en el seno del lenguaje, la lógica, y la epistemología.
3. La interpretación dialógica de conocimiento asertórico no está fundada en la afirmación de premisas de complejidad arbitrarias, pero en la Regla Socrática que gobierna la justificación de afirmaciones elementales. Afirmaciones que involucran proposiciones complejas “heredan” durante el desarrollo de una partida, el conocimiento asertórico de las afirmaciones elementales que se obtienen del análisis de las complejas.
4. La perspectiva dialógica es una característica irreductible del significado y el conocimiento.

La interpretación dialógica de la TCT, dado que no distingue el nivel estratégico del nivel de partidas, no proporciona los fundamentos dialógicos últimos de conocimiento asertórico. La semántica formal al estilo Tarski es ciega al primer punto, malinterpreta la naturaleza de la interfaz involucrada en el segundo e ignora el tercero y cuarto puntos.

Por supuesto, esto representa una forma radical de dialogicismo que profesamos y promovemos. Sin embargo, la presente propuesta también puede verse como una invitación a la reflexión sobre la construcción y arquitectura de una racionalidad plural y dinámica en aras del entendimiento mutuo, y el saber compartido que constituyen la noción misma de ser humano.

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II LA SABIDURÍA DE NUESTROS MAYORES: LECTURAS CONTEMPORÁNEAS

Esta segunda parte del volumen contiene estudios recientes en lógica Aristotélica y post-Aristotélica, que subraya una de las aplicaciones más fructíferas del marco dialógico, a saber: la reconstrucción dialógica de ciertos enfoques en la historia y filosofía de la lógica que propone nuevos desarrollos en lógica temporal-modal, déontica y teoría de la argumentación.

Nuestras propias reconstrucciones representan apenas una pequeñísima muestra de lo que podemos, y quizá incluso debemos, aprender de la literatura en Lógica y Dialéctica árabes. Las ideas fundamentales que guían nuestros desarrollos en el marco de IR-II están basadas en los siguientes principios:

1. La modalidad es entendida bajo la óptica de la noción aristotélica de *predicable* de los *Topica*, es decir, los diferentes modos mediante los cuales el sujeto – el dominio (restringido) de cuantificación – se relaciona con el predicado (definido sobre el dominio). Los predicables se declinan en el modo de relación por definición (humanos son animales racionales), por género (humanos son animales), por propio (humanos pueden leer) o accidente (humanos pueden caminar). Mientras que los primeros modos establecen una relación necesaria, los otros dos modos establecen una relación de posibilidad (más precisamente de contingencia). Mientras que el modo de relación por definición es simétrica, la relación por género no la es. Así también la relación por propio admite simetría, pero no la relación de accidente.
2. La relación de necesidad, requiere que el predicado sea atribuido *actualmente* a cada presencia o instancia del sujeto. La relación de contingencia, requiere que el predicado, un proceso que expresa una capacidad (por ej. leer), o disposición (por ej. respirar o reír) o esquema de acción (por ej. caminar) o evento (por ej. llover), sea atribuido *potencialmente* en lugar de actualmente, a cada instancia del dominio de cuantificación.
3. Potencialidad y actualidad son comprendidos dialécticamente y temporalmente. En el caso de una cuantificación universal, por cada instancia *a* del sujeto producida por el interrogador, el defensor debe producir una presencia del predicado atribuida a la instancia *a* del sujeto en al menos un momento (si no la relación sería imposible) y una ausencia en al menos otro momento (si no la relación sería necesaria). En el caso de capacidades, basta una instancia genérica del sujeto (en lugar de individual): basta que un hombre alguna vez actualice la capacidad atribuida para hacer constar que los hombres tienen tal capacidad.
4. Dado que momentos no son portadores de propiedades (Aristóteles nos enseña que el tiempo no es una substancia), temporalidad no puede implementarse como un cuantificador más una función proposicional – el momento 14hs no es portador de la capacidad de *frenar delante del semáforo rojo*. Tampoco como un operador temporal al modo de Prior. El imperativo *frenar delante del semáforo rojo*, parece expresar un esquema de acción a realizar sin indicación explícita de tiempo – a menos que haya un “siempre” tácito, pero entonces hay que proporcionar un medio de indicar que la obligación ha sido transgredida o no en un momento dado: no siempre cumplimos con nuestras obligaciones. Lo que parece mucho más apropiado y simple es concebir la temporalidad como afectando ejecuciones de capacidades o acciones o disposiciones: mi propia ejecución de frenar a tal hora y lugar.
5. Momentos deben ser concebidos como constitutivos de *historias* o intervalos de tiempo que expresan duraciones. Historias que no implican determinismo requieren una estructura de tiempo bifurcado. Una tal estructura permite

indeterminismo local con futuro abierto pero pasado cerrado o irrevocable. Tal estructura es también compatible, es decir combinable, con la noción de la consciencia o experiencia personal y subjetiva de la temporalidad.

6. En contextos éticos y/o jurídicos las modalidades déonticas presuponen que el agente puede elegir no ejecutar la acción prescrita o ejecutar una iniquidad. En ese sentido, obligación presupone, más que implica, posibilidad (de ejecución). Responsabilidad ética y jurídica constituye de este modo un elemento fundamental de la significación de las modalidades déonticas y en ella reside la distinción kantiana entre necesidad causal y necesidad moral.

La mayor parte de los estudios son versiones de artículos ya publicados – en la sección correspondiente proporcionamos las referencias precisas. Tales textos son autónomos y por ende contienen algunos contenidos que se repiten, pero eso los hace autocontenidos.

Esta parte incluye

1. una presentación compacta de diversos textos que muestran como Razonamiento Inmanente elucida los fundamentos dialécticos de la silogística;
2. una versión en español de un artículo sobre la reconstrucción pragmática de N. Belnap del ejemplo Aristotélico sobre la batalla naval de mañana.
3. una versión en español de un artículo que también sugiere una nueva forma de abordar la lógica modal;
4. una versión en español de un artículo que presenta acaso una sino la primera lógica temporal de predicados, y que también propone una nueva forma de abordar la lógica modal;
- 5.

II.1 Releyendo Aristóteles.

II.1.1 La Lectura Dialéctica del Silogismo.

The text is a simplified version of work developed in
M. Crubellier, Z. McConaughey, M. Marion, S. Rahman (2018) : "Dialectic, The Dictum de Omni and Ecthesis". *History and Philosophy of Logic.*, vol. 40, No. 3, pp. 207-233.,
Z. McConaughey (2021): *Aristotle, Science and the Dialectician's Activity. A Dialogical Approach to Aristotle's Logic.* PhD-Université de Lille and
S. Rahman et A. Seck (2022) : Suhrawardī's Stance on Modalities and his Logic of Presence. Talk presented at the *Workshop on Arabic Logic in honour of Tony Street* University of California, Berkeley, 24-25 April 2022.

II.1.1.1 Introduction

Given that demonstrations can be analyzed into inferences, Aristotle is naturally understood as having constructed the theory of inference (or deduction) of *Prior Analytics* as a tool for his theory of demonstrative science in *Posterior Analytics*,⁴⁶ and there is a long tradition of commentators, harking back at least to Pacius, according to which Aristotle's belief that there is no demonstrative knowledge of singulars terms⁴⁷ entails that inferences in *Prior Analytics* could not involve such

⁴⁶ After all, Aristotle himself tells us in the very first sentence of *Prior Analytics* that his treatise is about 'demonstration' and that its object is 'demonstrative science' (*An. Pr.* A1, 24a10-11).

⁴⁷ This claim is based on Aristotle's argument according to which individuals cannot be predicated of other things, and the concomitant claim at *An. Pr.* A27, 43a42-43 that "arguments and inquiries are almost always

terms.⁴⁸ Thus, the typical syllogism ‘Humans are mortal, Socrates is human, therefore Socrates is mortal’ could not be truly Aristotelian.⁴⁹ Still, there are a number of proofs within Aristotle’s own presentation of his theory of inference that appear at first sight to involve singular terms such as the proof of the convertibility of universal negatives or ‘*e*-conversion’, that have been called ‘proofs by ecthesis’:

Now, if *A* belongs to none of the *Bs*, then neither will *B* belong to any of the *As*. For if it does belong to some (for instance to *C*), it will not be true that *A* belongs to none of the *Bs*, since *C* is one of the *Bs*.⁵⁰

Although he does not use that word in this very passage, Aristotle calls the selection of a *C* ‘ecthesis’ (ἐκθεσις) – translated by Robin Smith and others before him as ‘setting out’.⁵¹

In this paper, we shall propose a new perspective on ecthesis, presenting it as a *procedure* such that (answering the second question) it will be seen as fully pertaining to Aristotle’s theory of inference, and (answering the first one) as involving *both* singular and general terms

But we should state at the outset that, according to our perspective, although it is part of the theory of inference, ecthesis is not at the same level, so to speak, as that of the rules of syllogisms and of conversion. With the *dictum de omni* one can recover the meaning explanation of the main building blocks of Aristotle’s theory of inference, the universal affirmative (*AaB*), universal negative (*AeB*), particular affirmative (*AiB*), and particular negative (*AoB*) propositions, and we see ecthesis as a procedure implementing the *dictum*,⁵² that allows one to prove the *admissibility* of the basic rules of his theory, i.e., rules of the first figure (*Barbara*, *Celarent*, *Darii* and *Ferio*), and the three conversion rules (for propositions *a-e-i*).

While our perspective involves a bit of ‘formalism’, it is meant to be more historically sensitive than is usually the case in the secondary literature on logical aspects of Aristotle, as it relies on the claim that dialectic, far from being simply discarded by Aristotle when he wrote *Prior Analytics*, actually forms its historical context.⁵³

In this we follow E. W. Beth, Kurt Ebbinghaus, as well as Mathieu Marion & Helge Rückert in claiming that the *dictum de omni* at *An. Pr.* A 2, 24b28-29 originates in a *dialectical* rule in the *Topics* (in *tTop.* Θ 2, 157a34-37),⁵⁴

This dialectical rule involves one of the players, in their terminology (taken from Aristotle), *Questioner* getting the other player *Answerer*, to concede a few instances before she can introduce a universal affirmative proposition such ‘*A* belongs to all *B*’ (*AaB*), and ask *Answerer* for a counterexample: if unable to provide one, *Answerer* must then concede it.

To argue their point, Marion & Rückert followed a suggestion by Jan von Plato⁵⁵ in using Martin-Löf’s Constructive Type Theory⁵⁶ to read *AaB* as meaning that no *c* of type *B* – or no ‘*c* : *B*’ – can be found for which it is not the case that *A(c)*.

We shall here travel further along that path, using a dialogical take on CTT that yields an

chiefly concerned with” things that are, as explained a few lines above, “both predicated of others and have others predicated of them” (*An. Pr.* A27, 43a29-31).

⁴⁸ See Lukasiewicz 1957, p. 4-7.

⁴⁹ For a modern statement, see Ross *ad An. Pr.* A1, 24a17 in Ross 1949a, p. 289.

⁵⁰ *An. Pr.* A2, 25a15-17. Unless indicated, we use Robin Smith’s translation of *Prior Analytics* in Smith 1989.

⁵¹ See Smith 1982, p. 113, 1983, p. 225, 1989, p. xxiii. For earlier precenses, see Heath 1921, I, p. 370, II, p. 533 and Ross 1949b, p. 36 n. 2. Jules Tricot *ad An. Pr.* A2, 25a15-17 in Tricot 1971 p. 8 & p. 332, used ‘ecthèse’, which hardly counts as translation.

⁵² It is also linked nowadays with the notion of downwards and upwards monotony developed during the Middle Ages. See, e.g., Parsons 2014, p. 45-48.

⁵³ This claim is not new, see, e.g., Kapp 1942.

⁵⁴ Beth linked ecthesis to the no counterexample reasoning embodied in his rules for semantic tableaux in Beth 1969, p. 35 & 37 (originally published in 1955), so he stands at the origin of the viewpoint developed here, although he did not notice the link with dialectic. This link was first noted in Ebbinghaus 1964, p. 57 n. 1 and it was fully argued for in Marion & Rückert 2016. Jaakko Hintikka had also linked ecthesis to the rule of existential instantiation in, e.g., Hintikka 1973, p. 111, or Hintikka 1991, p. 175, and recognized his debt to Beth in Hintikka 2006, p. 10. But, despite some perceptive remarks in, e.g., Hintikka 1993, p. 14-19, to our knowledge he never developed a dialectical account of ecthesis of the sort we set forth here.

⁵⁵ von Plato 2009, 2013, sec. 14.1.

⁵⁶ See, e.g., Martin-Löf 1984 or Granström 2011.

interactive logical framework called ‘immanent reasoning’,⁵⁷ which we will adapt to Aristotle’s syllogistic.

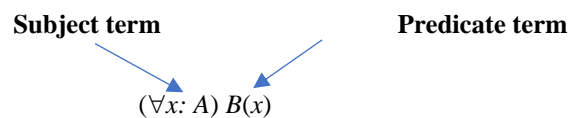
II.1.1.2 Subject and Predicate and The Dialogical Meaning Explanations of Quantifiers

If we integrate the instances of the Subject and Predicate terms into the encoding the dialectical setting the *dialogical meaning explanation* of the universal quantifier amounts to the following:

- If player **X** states a universal quantifier he has to be able to associate a suitable presence of the **consequent** for any arbitrary presence of the **antecedent** chosen by the challenger **Y**.

Statement	Challenge	Defence
$X \ ! \ (\forall x: A) B(x)$	$Y \ a: A$	$X \ ! \ b[a]: B(a)$
X states the universal <i>Every A is B</i>	Y chooses an arbitrary instance <i>a</i> of A	X associates <i>a</i> with a instance of <i>B(a/x)</i>

Notice that this notation follows the idea of traditional logic that predication amounts to stating that the Predicate-Term applies to any instance of the Subject Term:



The dialogical meaning explanation for an existential, leaves the choice of the instance to the defender:

Statement	Challenge	Defence
$X \ ! \ (\exists x: A) B(x)$	Challenge1 $Y \ ? \ \mathcal{L}^{\exists}$	Defence1 $X \ a: A$
X states the existential <i>Some A is B</i>	Y asks for the left: Which/ Who of the A’s are B?	X responds that <i>a</i> is one of those A’s
	----- Challenge2 $Y \ ? \ a/x: A \ \mathcal{R}^{\exists}$	Defence2 $X \ b(a): B(a)$
	Y asks for the right: Show me that this choice of yours (namely, <i>a: A</i>) is indeed a B	X associates the instance of <i>a</i> with a instance of <i>B(a/x)</i>

Notice that this allows an existential assertion to have the same subject-predicate structure as the one for the universal quantifier. Moreover, it allows expressing the Subject-Term as a restricting an underlying domain. Let us take Ibn Sīnā’s example

Some poets are good

Which, as pointed by Ibn Sīnā in the *al-Iṣārāt* (1983, Chapter 10.1, pp. 501-502) does not support the inference *There is someone, say Imra’ a al-Qays, who is good and a poet.*⁵⁸ Clearly what is asserted is

Some poets are good as poets $(\exists x: Poets) Good(x)$

In other word: *Within the domain restricted by the the subject, namely, poets, some are good.* As mentioned in our discussion on ecthesis, the existential expresses a set, in our example the set

⁵⁷ See Rahman et al. to appear.

⁵⁸ In fact, by Ibn Sīnā’s precise counterexample is that from the premises *Imra’ a al-Qays is good* and *Imra’ a al-Qays is a poet*, *Imra’ a al-Qays is a good poet* does not follow.

of

Those instances of poet that are good (as poets) {x: poet | Good(x)}

Since this set is what the meaning of the existential amounts to, the dialogical meaning explanation is the same as the one of the existential:

Statement	Challenge	Defence
<p style="text-align: center;">X ! {x: A B(x)}</p> <p>X states that the set of those A's that are B can be defined over A</p>	<p>Challenge1 Y ? \mathcal{L}^o \emptyset</p> <p>Y asks for the left: Which/ Who of the A's are B?</p> <hr style="border-top: 1px dashed black;"/> <p>Challenge2 Y ? $a/x: A \mathcal{R}^o$</p> <p>Y asks for the right: Show me that this choice of yours (namely, $a: A$) is indeed a B</p>	<p>Defence1 X a: A</p> <p>X responds that a is one of those A's</p> <hr style="border-top: 1px dashed black;"/> <p>Defence2 X b[a]: B(a)</p> <p>X associates the instance of a with a instance of $B(a/x)$</p>

Once the logical constants have been constituted out of its dialectical meaning explanations, those moves can be identified, allowing a higher level – namely, the strategic level.⁵⁹

⁵⁹. At the strategic level the Proponent, has a winning strategy for a a universal iff for *any* instance of the Subject the challenger can show produce an instance of the Predicate for this choice of the Opponent. The way to implement this, is to allow the Opponent to choose always a **new** instance

Statement	Challenge	Defence
<p style="text-align: center;">P ! ($\forall x: A$) B(x)</p> <p>P states that he has a winning strategy for the universal</p>	<p style="text-align: center;">O a: A</p> <p>O chooses the new instance a of A, and requests P to show that it witnesses a instance of $B(x)$</p>	<p style="text-align: center;">P ! b(a): B(a)</p> <p>P associates a with a instance of $B(a/x)$</p>

Thus, at the strategic level, the truth of the universal requires **P** to be able to associate instances of the Subject with instances of the Predicate, by means of substituting x in $b(x)$ for any instance of the Subject **O** might choose. In other words, the truth of the universal ($\forall x: A$) $B(x)$ requires the Proponent

- a) to build out of the associations triggered by whatever instances of A **O** might choose, witnesses of $B(a/x)$ an association-procedure $b(x)$,
 - b) to build the abstract construct of this procedure, namely $(\lambda x)b(x)$, called the lambda-abstract of $b(x)$. The lambda-abstract indicates that for whatever choice of the Opponent the association-procedure $b(x)$ can be executed in order to yield an instance of $B(a/x)$ for that choice, and
 - c) **P**'s execution of $b(a/x)$ is justified by **O** being forced to state himself $B(a)$: this actually is the core of proving the validity (building a winning-strategy) of a syllogism involving universals.
- Thus, the canonical form of a winning strategy for a universal has the form:

$$\mathbf{P} (\lambda x)b(x): (\forall x: A) B(x)$$

Any c witnessing the universal is a construct equal to the lamda-abstract $(\lambda x)b(x)$, built out of plays whereby **P** can execute the association-procedure $b(a/x)$ witnessing $B(a/x)$, for any $a: A$ chosen by **O**. Notice the proviso, **new**, implements the idea that the winning-strategy is not dependent upon one particular choice of **O**. Indeed, if the winning-strategy is a robust one, it will yield the same result for any alphabetic variant of instances of the antecedent.

Statement	Challenge-Defence	Strategic object
<p style="text-align: center;">P ! ($\forall x: A$) B(x)</p> <p>P states that he has a winning strategy for the universal</p>	<p style="text-align: center;">Challenge O a: A</p> <p>O chooses the new instance a of A, and requests P to show that it witnesses a instance of $B(x)$</p> <hr style="border-top: 1px solid black;"/> <p style="text-align: center;">Defence P ! b(a): B(a)</p> <p>P associates a with a instance of $B(a/x)$, and O is forced to state himself $B(a)$</p>	<p style="text-align: center;">P $(\lambda x)b(x)=c: (\forall x: A) B(x)$</p>

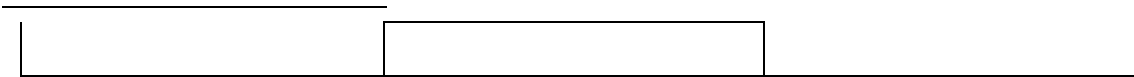
Let us now observe that within a syllogism, premises and conclusion share a common domain, over which the Subject term and the Predicate term have been defined. This is what allows the middle term, to occur as the Predicate in one premise and as Subject in the other. Moreover, since as we will see when we introduce modalities into the universal; the Subject term can be also modalized, we need a first term to build the modal relation : as in *All Humans who are necessarily Contingently literate Beings, are necessarily necessary Rational*. Let for the moment ignore the modality in order to stress the underlying structure of the Subject – the “Subject” denominates here the Subject term + the domain upon which the Subject term has been defined:

All Humans who are Literate Beings are Rational

Should be understood as

Rational *All instances z of the set of all those Humans who are Literate Beings , are*

Domain Subject term specifying the Domain Predicate term



In relation to an existential, such as $(\exists x: A) B(x)$, at the strategic level, the truth of this existential requires **P** to be able to state some instance $a: A$ chosen by **P** himself, as response to the first challenge, and to state $B(a)$, as a response to the second challenge, by building the association procedure $b(a/x)$.

Statement	Challenges	Defence
P $!(\exists x: A) B(x)$ P states that he has winning-strategy for the existential	Challenge1 O ? \mathcal{L} \exists	Defence1 P $a: A$ P responds by choosing one a and that it is one these instances that witnesses A
	----- Challenge2 O ? $a/x:$ $A \mathcal{R}^{\exists}$	----- Defence2 P $b(a): B(a)$ P associates the instance of a with a instance of $B(a/x)$

In other words, the truth of the existential requires the Proponent

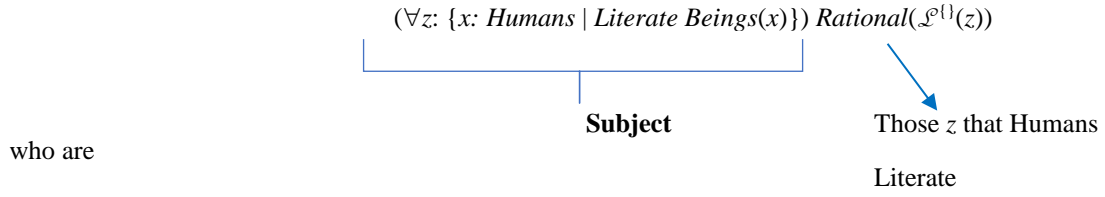
- to state that some instance a , chosen by **P** himself witnesses A , and build out of the association of a with a instance of the $B(a/x)$ an association-procedure $b(a/x)$
- to build the complex construct $\langle a, b(x) \rangle$, which results from **P**'s responses to both challenges:
- P**'s choice of some $a: A$, and the execution of $b(a/x)$ is justified by **O** being forced to state himself both $a: A$ and $B(a)$: this actually is the core of proving the validity (building a winning-strategy) of a syllogism involving existentials

Thus, the canonical form of a winning strategy for a universal has the form:

$$\mathbf{P} \langle a, b(x) \rangle: (\exists x: A) B(x)$$

So, any c witnessing the existential can be analysed as the pair $\langle a, b(x) \rangle$.

Statement	Challenges-Defences	Strategic Object
P $!(\exists x: A) B(x)$ P states that he has winning-strategy for the existential	Challenge1 O ? \mathcal{L} \exists	P $\langle a, b(x) \rangle = c: (\exists x: A) B(x)$
	Defence1 P $a: A$ ----- Challenge2 O ? $a/x: A \mathcal{R}^{\exists}$ Defence2 P $b[a]: B(a)$	



Notice that in the context of a dialectical interaction, it is necessary to indicate that the “testing-instances” for a universal stated by **X**, are chosen by the challenger **Y**. In our example it amounts to the indication that **Y**’s choice of an instance witnessing the subject, is *an animal who is human*. Since *Animal* is the left component of every instance z witnessing the Subject, we adopt the notation “ $\mathcal{L}^{\{1\}}(z)^Y$ ”, that indicates that **Y** choses some animal who is rational, as his (**Y**’s) case to build a counterexample to the universal – cf. Crubellier et al. (2019) and McConaughey (2021, chapter 4):

Notice that this analysis reflects the Subject-Predicate form of traditional syllogism.

From this perspective the semantic and logical analysis is a consequence of the dialectical interaction underlying a debate on meaning and its possible extensions. As discussed below, the dialectical framework, allows what we call a dynamic encoding. In other words, an encoding where the instances witnessing the Subject and Predicate terms are made explicit during the challenge-defence interaction. Thus, before the interaction the assertions in a syllogism have the following forms:

	Traditional Form Before Interaction	Dynamic Encoding Before Interaction	Explicit Dialectical Encoding Before Interaction
Universals	X ! Every (D who are) S is P X ! No (D who are) S is P	X ! (Every S_D) P X ! (Every S_D)no- P	X ! ($\forall z: \{x: D \mid S(x)\}) P(\mathcal{L}^{\{1\}}(z)^Y)$ X ! ($\forall z: \{x: D \mid S(x)\}) \sim P(\mathcal{L}^{\{1\}}(z)^Y)$
Particulars	X ! Some (D who are) S is P X ! Some (D who are) S is not P	(Some S_D) P X ! (Some S_D)no- P	X ! ($\exists z: \{x: D \mid S(x)\}) P(\mathcal{L}^{\{1\}}(z)^Y)$ X ! ($\exists z: \{x: D \mid S(x)\}) \sim P(\mathcal{L}^{\{1\}}(z)^Y)$

	Traditional Form During Interaction	Dynamic Encoding During Interaction	Explicit Dialectical Encoding During Interaction
Universals	X ! Every (D who are) S is P Y ! d_i is S X ! d_i is P	X ! (Every S_D) P Y $S_D(d_i)$ X ! $P(d_i)$	X ! ($\forall z: \{x: D \mid S(x)\}) P(\mathcal{L}^{\{1\}}(z)^Y)$ Y $S_D(d_i)$ X ! $P(d_i)$ Given Y $\mathcal{L}^{\{1\}}(z)=d_i: D$
	X ! No (D who are) S is P Y d_i is S X ! d_i is not P	X ! (Every S_D)no- P Y $S_D(d_i)$ X ! $\sim P(d_i)$	X ! ($\forall z: \{x: D \mid S(x)\}) \sim P(\mathcal{L}^{\{1\}}(z)^Y)$ Y $S_D(d_i)$ X ! $\sim P(d_i)$ Given Y $\mathcal{L}^{\{1\}}(z)=d_i: D$
Particulars	X ! Some (D who are) S is P Y which S ? X ! d_i is S Y which P ? X ! d_i is P	X ! (Some S_D) P Y ? $\mathcal{L}^{\{1\}}$ X ! $S(d_i)$ Y ? $d_i: D \mathcal{R}^{\{1\}}$ X ! $P(d_i)$	X ! ($\exists z: \{x: D \mid S(x)\}) P(\mathcal{L}^{\{1\}}(z)^Y)$ Y ? $\mathcal{L}^{\{1\}}$ X ! $S(d_i)$ Y ? $d_i: D \mathcal{R}^{\{1\}}$ X ! $P(d_i)$
	X ! Some (D who are) S is not P Y which S ? X ! d_i is S Y which P ? X ! d_i is not P	X ! (Some S_D)no- P Y ? $\mathcal{L}^{\{1\}}$ X ! $S(d_i)$ Y ? $d_i: D \mathcal{R}^{\{1\}}$ X ! $\sim P(d_i)$	X ! ($\exists z: \{x: D \mid S(x)\}) \sim P(\mathcal{L}^{\{1\}}(z)^Y)$ Y ? $\mathcal{L}^{\{1\}}$ X ! $S(d_i)$ Y ? $d_i: D \mathcal{R}^{\{1\}}$ X ! $\sim P(d_i)$

Many contemporary reconstructions that appeal to relevant, modal, or other forms of so-called non-classical logics, seem to overlook the fact that it is the dialectical constitution of meaning that leads the way in the logical shaping of dialectics. As dialogicians often put it: the dialectical conception of logic is not logic plus dialectical interaction, but it is dialectical interaction that shapes both meaning and logical reasoning.

II.3 Syllogism: Dialectical Meaning Explanations at Work

Here we present only some examples of each figure

II.3.1 First figure

Let us now argument in the form of a dialogue that implements these rules, focusing on an example that occurs very often in the commentators (since we restrict to categorical logic, we leave here unanalyzed the modal component),

All humans have the capacity of laughing
 All those who have the capacity of laughing, have the capacity of reading
 Therefore, all humans have the capacity of reading

- **N.B.** We use here the **re-ordering** of the premises within the Arabic tradition, which dominated post-Aristotelian and Medieval Philosophy. This makes it closer to the logic form of contemporary quantification and of course does not change the validity of the moods involved but it does not coincide with the Latin medieval denominations of the valid forms of each figure. A lucid presentation of this point has been provided by Street (2008), let us quote his excellent summary: *The first mood of the first figure is Barbara, given by Aristotle in the form: A belongs to all B (major premise), B belongs to all C (minor premise), therefore A belongs to all C (conclusion). The major premise is so-called because it provides the predicate of the conclusion, whereas the minor premise provides the subject. Arabic logicians stated Barbara differently in two respects. First, they put the subject of the premise before the predicate, and secondly, they put the minor premise before the major: every C is B, every B is A, therefore every C is A, which is just as obvious—or perfect—an inference as it is when stated in Aristotle’s fashion. [...]. The second mood, Celarent, has different vowels to show that the major premise and the conclusion are E-propositions, that is, of the form ‘no C is B’. But now the order of the premises as stated in the Arabic will be out of step with the vowels in the Latin mood name: every C is B (a-proposition), no B is A (e-proposition), therefore no C is A (e-proposition). Still, we should refer to this as Celarent because we can then compare it easily to the analyses of the same inference by Aristotle and the medieval Latin authors.* Street (2008, pp. 176-177).

If we place a syllogism within a dialogue, then, the idea is that the proponent **P**, claims that that the conclusion holds if the opponent **O**, concedes to state the premises. This yields the notation:

$$\begin{array}{ll}
 \mathbf{O} ! (\text{Every}J_D)B & \mathbf{O} ! (\forall z: \{x: D \mid J(x)\}) B(\mathcal{L}^{\{1\}}(z)^{\mathbf{P}}) \\
 \mathbf{O} ! (\text{Every}B_D)A & \mathbf{O} ! (\forall z: \{x: D \mid B(x)\}) A(\mathcal{L}^{\{1\}}(z)^{\mathbf{P}}) \\
 \hline
 \mathbf{P} ! (\text{Every}J_D)A & \mathbf{P} ! (\forall z: \{x: D \mid J(x)\}) A(\mathcal{L}^{\{1\}}(z)^{\mathbf{O}})
 \end{array}$$

Recall that explicit dialectical notation, also indicates which player is in charge of the substituting the variables. Since the premises are stated by the opponent, and these are universals, it is the proponent who will choose the presence of the subject of which the predicate is requested to be stated. The dual is the case of the universal in the conclusion: since the conclusion is stated by the Proponent, it is the Opponent who will choose the presence of the subject of which the predicate is requested to be stated. In fact, a consequence of the Socratic rule is that a winning strategy for **P** should follow the idea to leave **O** choose first and the copy-cat this choice for his own challenges to the premises. Notice that **O** is forced to choose if he challenges a universal (stated by **P**) or defends an existential stated by himself.

0. **P** ! I can show that every element of domain of discourse D , who is a J is possibly A follows from the premises: I), Every element of D who is J , is possibly B and II) Every element of D who is B is A .
1. **O** ! Fine, I give you the premises. Show me now, that the consequent of the conclusion follows from some d_i in the domain of discourse D who is J : **O** challenges the conclusion with the move **O** ! $J(d_i)$.
2. **P** ! What I will do is to show you that the endorsement of the premises will force you to assert this consequent. Let us start with the first premise. Since the first premise states that every element of the domain of discourse who is J has the capacity B , and you just chose d_i in the domain of discourse with your first move (sic move 1), B should also hold of d_i , right?: **P** challenges the first premise with the move **P** $you_1 J(d_i)$; whereby “ you_1 ” stands for the indication “you just stated the same at move 1).
3. **O** ! Indeed, I have to assume that d_i is one of those in the domain of discourse that are B .
4. **P** ! Ok, however, premise II states that every element of the domain of discourse who is B , is A , this must include all those elements in D who are B , so let us again take precisely this d_i that

- you just conceded with your move 3 as being a B . This instance of B , must be also an instance of A , right?: \mathbf{P} challenges the second premise with the move $\mathbf{P} \text{ you}_3: B(d_i)$.
5. $\mathbf{O} !$ Right. This element of the domain D must be A : \mathbf{O} defends the second premise with the move $\mathbf{O} ! A(d_i)$.
 6. $\mathbf{P} !$ But this is exactly what you asked me to show. You just conceded it with your move 5: \mathbf{P} defends the conclusion with the move $\mathbf{P} \text{ you}_5: A(d_i)$.

This development is based on the following rules, that we take from Rahman et al. (2018, p. 62), adapted to syllogism in McConaughy (2021, chapter 4.2, table 4.9) and present in a simplified form:

1. Starting rule

The player who states the conclusion move 0 is the proponent \mathbf{P} .

2. Development rule

Once the starting rule has been implemented, each player in turn plays a move according to the dialectical meaning explanations for quantifiers, connective, modalities and the other structural moves.

3. Socratic rule

Some specific propositions, we call them *unanalysable constituents*, may not be stated by \mathbf{P} , unless \mathbf{O} stated them before. \mathbf{O} can state such propositions when required. When \mathbf{P} states such a proposition, he will justify it with the indication *you_n*, which indicates that his statement is backed by \mathbf{O} 's endorsement of it at move n , and that he (\mathbf{P}) adheres himself to the knowledge conveyed by \mathbf{O} 's endorsement.

In the context of Aristotle's logic unanalysable propositions are elementary propositions.

4. Pragmatic coherence rule (concerns mainly the third figure)

When the conclusion Proponent defends is particular and all the premises Opponent defends are universal, Proponent may request the Opponent to instantiate the subject of a premise with the instance d_i , chosen by \mathbf{P} , **provided** d_i is **new**: challenge: $\mathbf{P} ?_{J(d_i)}$; defence: $\mathbf{O} ! J(d_i)$ (for a universal with $\{x: D \mid J(x)\}$ as subject, and J as Subject-Term), (this prevents \mathbf{O} to state $J(d_i)$ when he endorsed before some $J^*(d_i)$ whereby J and J^* are incompatible).

5. Ending rule

The player who states \perp give-up, immediately loses. Otherwise, the player who has no available move left at this turn loses.⁶⁰

O		P	
I	$!(\forall z: \{x: D \mid J(x)\}) B(\mathcal{L}^{\{1\}}(z))$	$!(\forall z: \{x: D \mid J(x)\}) A(\mathcal{L}^{\{1\}}(z))$	0
II	$!(\forall z: \{x: D \mid B(x)\}) A(\mathcal{L}^{\{1\}}(z))$		
1	$! J(d_i)$	<i>you₅</i> : $A(d_i)$	6
?0	$\mathcal{L}^{\{1\}}(z)=d_i: D$		
3	$! B(d_i)$?I $! J(d_i)$ $\mathcal{L}^{\{1\}}(z)=d_i: D$	2
5	$! A(d_i)$?II <i>you₃</i> : $B(d_i)$	4
Proponent wins			

- The dialogue ends since 6 is an unanalysable constituent of the conclusion, namely $A(d_i)$, that cannot be challenged, since \mathbf{O} himself endorsed it with move 5.

\mathbf{P} can repeat the same sequence of moves for any arbitrary element of discourse \mathbf{O} happens to choose to challenge the universal in the conclusion. In other words, \mathbf{P} has a winning strategy for this syllogism, and therefore it is valid.

⁶⁰ Comments on the Rules.

- i) The rationale behind the *Socratic rule* is that proving the conclusion of syllogism within a dialectical framework amounts to analysing the premises in such a way that the resulting statements are those that constitute the conclusion. Since the premises are stated by \mathbf{O} and the conclusion by \mathbf{P} the later but not the former is committed to justify the conclusion by justifying each of the constituents of the conclusion, by grounding them on statements of \mathbf{O} involving constituents of the premises. In other words, the use of the Socratic rule allows defining a winning strategy for \mathbf{P} (the dialectical way to proof validity) as a sequence of moves that force \mathbf{O} to state those constituents of the premises that should provide a justification of the conclusion.
- ii) The rationale behind the pragmatic coherence rule, will be commented when we discuss the third figure: it concerns the way to deal with ontological assumptions such as those required by *Darapti*.
- iii) The prescription on giving up in the ending rule concerns the dialogical interpretation of negation. When challenging a negation such as $\sim A$ stated by player \mathbf{X} , the challenger \mathbf{Y} must now overtake the burden of the proof and state A . The defender of the negation has two options, either counterattack A , or simply give up and concede. The latter is indicated by the move $\mathbf{X} ! \perp$ give-up. In the Aristotelian texts the move \perp give-up, corresponds to the dialectical use of the term *ἀδύνατον*.

The strategy can be seen as a “recapitulation” and generalization that produces an algorithm for winning.⁶¹ In our case, informally the winning strategy amounts to the following:

- 1) Let **O** choose any arbitrary instance of the universal in the conclusion
 - 2) **P** should use exactly this instance, *whichever* this instance chosen by **O** is, to challenge the first premise, and force **O** to predicate *B* of it
 - 3) **O** predicates *B* of it
 - 4) **P** should use exactly this endorsement of **O** (that *B* can be predicated of the instance at stake), to challenge the second premise
 - 5) **O** is forced to predicate *A* of it
 - 6) **P** can now use this last endorsement to respond to the challenge to the conclusion
- **Apply this sequence for any d_i chosen by **O** at move 1**

The emergent winning strategy can be represented as a sequent calculus, where **P**’s assertions are to translated as assertions at the right of the turn-style and **O**’s assertions at the left \sim . However, the point is that this sequent calculus has been generated by the winning strategy produced by the dialogue: it is the interaction between players that puts the dialogical meaning explanation of the modalities at work, by fleshing out the meaning of each constituent. The sequent-calculus is only the abstract outcome based on a *pensée aveugle*, to put in Leibniz’s words.

II.3.2 Second Figure

Again a frequent example for this figure is

All humans have the capacity of laughing
 No stone has the capacity of laughing
 Therefore, no human is a stone

$\mathbf{O} ! (\text{Every}J_D)B$ $\mathbf{O} ! (\text{Every}A_D)\sim B$ <hr style="width: 100%;"/> $\mathbf{P} ! (\text{Every}J_D)\sim A$	$\mathbf{O} ! (\forall z: \{x: D \mid J(x)\}) B(\mathcal{L}^{\{1\}}(z)^{\mathbf{P}})$ $\mathbf{O} ! (\forall z: \{x: D \mid A(x)\}) \sim B(\mathcal{L}^{\{1\}}(z)^{\mathbf{P}})$ <hr style="width: 100%;"/> $\mathbf{P} ! (\forall z: \{x: D \mid J(x)\}) \sim A(\mathcal{L}^{\{1\}}(z)^{\mathbf{O}})$
--	--

The dialogue presented in the form of the dynamic encoding of the moves is the following:

O		P		
I	$! (\text{Every}J_D)B$	$! (\text{Every}J_D) \sim A$		0
II	$! (\text{Every}A_D) \sim B$			
1	$! J(d_i)$	$! \sim A(d_i)$		2
?0				
3	$! A(d_i)$			
5	$! B(d_i)$?I	$! \text{you}_1: J(d_i)$	4
7	$! \sim B(d_i)$?II	$\text{you}_3: A(d_i)$	6
9	$! \perp \text{give-up}$?7	$\text{you}_5: B(d_i)$	8
		Proponent wins		

- The dialogue ends since **O** gives up with move 9, after **P** challenges moves 7 that contradicts **O**’s own endorsement of $B(d_i)$ at move 5.

II.3 Third Figure

The third figure has some difficulties linked to the existential in the conclusion in *Darapti*:

All humans are non-birds
 All humans are non-horses
 Therefore, some non-birds are non-horses

⁶¹ As pointed out by McConaughey (2021, p. 140), Kapp (1942, pp. 14-16 & 71) points out the importance of two stages in a dialectical context, anticipation and recapitulation. These are indeed, the elements that allow to build a winning strategy. See too Crubellier (2011) who indicates that one of the first meanings of *syllogismos* is precisely recapitulation.

This example also confronts us with the question: Should we keep the subject-predicate structure or rather link in the conclusion the minor and major terms with a conjunction?

In relation to the second question, logically speaking we can adopt both of the options. However, the conjunction-analysis seems to contravene Aristotle's own Subject-Object formulation of the conclusion, namely:

Thus, the conclusion of this negative form of *Darapti* is encoded as:

Explicit Encoding	Dynamic Encoding
$(\exists z: \{x: D \mid \sim A(x)\}) \sim B(\mathcal{L}^{\text{ol}}(z))$	$(\text{Some } \sim A_D) \sim B$

In our framework such a reading yields

$\mathbf{O} ! (\text{Every } J_D) \sim B$	$\mathbf{O} ! (\forall z: \{x: D \mid J(x)\}) \sim B(\mathcal{L}^{\text{ol}}(z))^{\mathbf{P}}$
$\mathbf{O} ! (\text{Every } J_D) \sim A$	$\mathbf{O} ! (\forall z: \{x: D \mid J(x)\}) \sim A(\mathcal{L}^{\text{ol}}(z))^{\mathbf{P}}$
-----	-----
$\mathbf{P} ! (\text{Some } \sim A_D) \sim B$	$\mathbf{P} ! (\exists z: \{x: D \mid \sim A(x)\}) \sim B(\mathcal{L}^{\text{ol}}(z))^{\mathbf{P}}$

The usual way to deal with the ontological commitment required by *Darapti* is to add as a third premise an existential that assures the non-emptiness of the subject of the minor and the major premises or instead of a premise the non-emptiness of the Subject-Term, is added as an Existential occurring in the consequent of the universals or as an Existential Predicate. .

However, this does not render justice to the fact that ontological commitment is to understood as part of the contextually bounded meaning of the definitely necessary propositions. Stating a necessary universal commits \mathbf{O} to presences of the subject, however, in a context where \mathbf{O} stated two universals with the same Subject-terms, then once \mathbf{O} endorsed that one individual witnesses, the Subject-Term, \mathbf{P} can now use this endorsement in order to challenge the second universal: \mathbf{O} stated after all that every witness of the Subject-term enjoys the property expressed by the Predicate-term.

Moreover, the precise specification of the set that constitutes the conclusion, has to be constructed out of \mathbf{O} 's endorsements during the dialogical interaction.

DARAPTI AND THE PRAGMATIC COHERENCE RULE

Let us recall here the *pragmatic coherence rule* introduced by Zoe McConaughey (2021, chapter 4.2, table 4.9), that implements both Aristotle's view that ontological commitments are contextually-bounded commitments:

Pragmatic Coherence Rule:

When the conclusion Proponent defends is particular and all the premises Opponent defends are universal, Proponent may request the Opponent to instantiate the subject of a premise with the instance d_i , chosen by \mathbf{P} , **provided** d_i is **new**: challenge: $\mathbf{P} ?_{J(d_i)}$; defence: $\mathbf{O} ! J(d_i)$ (for a universal with $\{x: D \mid J(x)\}$ as subject, and J as Subject-Term), (this prevents \mathbf{O} to state $J(d_i)$ when he endorsed before some $J^*(d_i)$ whereby J and J^* are incompatible)

The dialogue, again, presented in the form of the dynamic encoding of the moves is the following. Here we considered the negative elementary propositions $\sim A(d_i)$ and $\sim B(d_i)$, and unanalysable constituents, but as commented below this does not change the logical result:

	\mathbf{O}	\mathbf{P}	
I	$! (\text{Every } J_D) \sim B$	$! (\text{Some } \sim A_D) \sim B$	0
II	$! (\text{Every } J_D) \sim A$		
1	$? \mathcal{L}^{\text{ol}}$	$you_5: \sim A(d_i)$	8
?0			

3	! $J(d_i)$?II	? $J(d_i)$ (use of the pragmatic coherence rule)	2
5	! $\sim A(d_i)$?II	<i>you</i> ₃ : $J(d_i)$	4
7	! $\sim B(d_i)$?I	<i>you</i> ₃ : $J(d_i)$	6
9	? R^\exists		<i>you</i> ₇ : $\sim B(d_i)$	10
?0				
Proponent wins				

- Instead of choosing immediately one animal that is not a bird, the Proponent, deploys the pragmatic coherence rule and asks **O** to endorse that d_i is one of the humans who are no birds. Then **P** asks, **O** to endorse that this very human is also no-horse. Only after these endorsements, has **P** now the knowledge of the presence of a particular animal, namely a human; that witness both being a non-bird and non-horse.
- The dialogue ends since 10 is an unanalysable constituent of the conclusion, namely the negative literal $\sim B(d_i)$, that cannot be challenged, since **O** himself endorsed it with move 7.
- Notice that if we prefer not to consider negations of elementary propositions to be unanalysable the end-result of the dialogues will not change. It will only a bit longer: as soon as **O** challenges $\sim A(d_i)$, **P** will do the same, similarly with $\sim B(d_i)$.

III.4 Indirect Proof

As pointed out by Zoe McConaughey (2021, 4.4.1) an indirect proof amounts to adding the following structural rule

<p>1*. Indirect reduction <i>This rule modifies the Starting rule by giving Proponent an alternative way to play.</i> Proponent may start an indirect deduction play by by stating the negation of the contradictory of the conclusion.</p> <p>This requires the thesis to have the form: P Conclusion [Premise1, Premise2] This admits the Gloss: P: <i>I state the Conclusion C, provided the Premises P1 and P2</i> The first move of O will start by endorsing the Premises.</p> <p>Thus, if the conclusion is Universal affirmative, move 2 of P will be stating the negation of its negative particular form Universal negative, move 2 of P will be stating the negation of its affirmative particular form Particular affirmative, move 2 of P will be stating the negation of its negative universal form Particular negative, move 2 of P will be stating the negation of its affirmative particular form</p>

Let us show how to run an dialogue for *Barbara* by indirect deduction.

O		P	
		! (Every J_D)A [(Every J_D)B, (Every B_D)A]	0
1.1	! (Every J_D)B		0
1.2	! ($\forall z: \{x: D \mid B(x)\} A(\mathcal{L}^{\exists}(z))$)		0
3	! (Some J_D) \sim A	indirect: \sim (Some J_D) \sim A	2
5	$J(d_i)$?3 ? \mathcal{L}^{\exists}	4
7	$\sim A(d_i)$?5 ? \mathcal{R}^{\exists}	6
9	! $B(d_i)$?1.1 <i>you</i> ₅ : $J(d_i)$	8
11	! $A(d_i)$?1.2 <i>you</i> ₉ : $B(d_i)$	10
13	! \perp give-up	?7 <i>you</i> ₁₁ : $A(d_i)$	12
Proponent wins			

- The dialogue ends since **O** gives up with move 13, after **P** challenges moves 12 that contradicts **O**'s own endorsement of $A(d_i)$ at move 11. By these means **P** shows that, given the premises, **O**'s endorsing of the contradictory of the conclusion (move 3) leads to contradiction

By means of indirect reductions the following conversions can dialectically be justified – we leave the dialogues to the reader:⁶²

⁶² We adapted here McConaughey's (2021, 4.4.1) notation to our own.

Statement	Simple Conversion	Conversion by Subalternation
$(EveryA_D)Non-B$	e-conv: $(EveryB_D)Non-A$	e-sub-conv: $(SomeB_D)Non-A$
$(EveryA_D)B$	\emptyset	a-sub-conv: $(SomeB_D)A$
$(SomeA_D)B$	i-conv: $(SomeB_D)A$	\emptyset

The expressions at the left of the colon encode the strategic moves that provide a winning strategy for the corresponding conversion deploying the indirect reduction rule. More generally, they convey that there is a winning strategy for such a conversion.

II.5 Reduction to the first figure⁶³

Reductions to the first figure can be declined in three main steps

Reduction to the first figure
<p>a) After O's concession of the premises, given P's starting move P ! Conclusion [Premise1, Premise2], after, the Proponent, will not state the conclusion in move 2, but he might use one or more of the conversion challenges (see table below) to the premises according to the table of conversions below</p> <p>b) Once the conversions have been stated by O, P might be able to force O to conclude, using the moods of the first figure according to the table of first figure rules below</p> <p>c) Once O concluded as required by the preceding step, P will state the conclusion of the thesis as follows:</p> <p style="padding-left: 40px;">P you(n)-barbara: Conclusion, or P you(n)-celarent: Conclusion, or P you(n)-darii: Conclusion, or P you(n)-ferio: Conclusion.</p>

Statement	Conversion-Challenge	Conversion-Defence
O ! $(EveryA_D)Non-B$	P ? e-conv or P ? e-sub-conv	O ! $(EveryB_D)Non-A$ ----- O ! $(SomeB_D)Non-A$
O ! $(EveryA_D)B$	P ? a-sub-conv	O ! $(SomeB_D)A$
O ! $(SomeA_D)B$	P ? i-conv	O ! $(SomeB_D)A$

In the following table the moods of the first figure are used as (already justified) rules – the justification can be carried out by ecthesis or by indirect reduction as discussed above.

Statement	Reduction-Challenge	Reduction-Defence
O ! $(EveryA_D)B$	P ? barbara	O ! $(EveryA_D)C$
O ! $(EveryB_D)C$		
O ! $(EveryA_D)B$	P ? Celarent	O ! $(EveryA_D)Non-C$
O ! $(EveryB_D)Non-C$		
O ! $(SomeA_D)B$	P ? darii	O ! $(SomeA_D)C$
O ! $(EveryB_D)C$		
O ! $(SomeA_D)B$	P ? ferio	O ! $(SomeA_D)Non-C$
O ! $(EveryB_D)Non-C$		

N.B. Recall that we are deploying the Arabic and post-Aristotelian re-ordering of the premises.

Let us run a play for the reduction of *Festino* (second figure) to the mood *Ferio* of the first figure:

⁶³ The following method and auxiliary rules are a slight variant of McConaughey (2021, 4.5)

O		P	
		! (Some _{A_D})Non-C 0	[(Some _{A_D})B, Every _{C_D})Non-B]
1.1	! (Some _{A_D})B 0		
1.2	! Every _{C_D})Non-B 0	you(5)-ferio: (Some _{A_D})Non-C	6
3	! Every _{B_D})Non-C	?1.2 ? e-conv	2
5	(Some _{A_D})Non-C	?1.1, 3 ? ferio	4
Proponent wins			

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II.1.2 Dialógica, la Batalla Naval del Mañana y Tiempo Ramificado Según Belnap

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Abstract. In several texts, some authored by himself alone and some in collaboration N. D. Belnap proposed a pragmatist approach to predictions and further speech acts such as promising, betting, and wondering, in an indeterministic setting within a branching structure that shapes the future course of events. In the joint paper “Future Contingents and the Battle Tomorrow”, M. Perloff and N. E. Belnap discuss Aristotle’s famous example in the context of STIT-logic. In particular, the paper studies the pragmatics of predictions of future contingents under the background of the 480 BCE battle of Salamis: the general commanding the Greek Athenian fleet predicts that a battle will be fought on the sea while the Spartan general denies it. According to Perloff and Belnap’s analysis, this kind of prediction is neither true nor necessary at the moment of utterance, but can be *vindicated* or *impugned*, retrospectively. The central thought of the present paper is that *vindication* is a dialogical process associated to *statements made*. In the particular case that the statement expresses a prediction, the dialogical process involves plays on the *settled past truth* of the predicted contingent future. This way of analysing Tomorrow’s Sea Battle highlights the interplay of the ontological perspective and the linguistic perspective, where vindication or impugment of the predictions are under scrutiny.

1 In honour of Göran Sundholm

It is a pleasure to dedicate the present paper to Göran Sundholm. This paper touches historical and philosophical topics that gave rise to longstanding and prolific conversations on knowledge, meaning, and logic between him and the dialogical community.

As *Dialogicians* of Lille are fond of recalling, such conversations have been flowing since at least 1998, when Shahid Rahman met Göran Sundholm for the first time in a train for Greifswald taking them to a workshop on Hugh McColl, organized by Stephen Read and Michael Astroh. The dialogue enriched substantially through the years not only by sharing advisory duties of master and PhD students, by participating at common seminars and workshops – particularly during Sundholm’s visiting Professorship at the University of Lille in 2012 –, but also by the joining of junior and senior colleagues, whose inputs led to a new direction for the development of dialogical logic known as *Immanent Reasoning*.⁶⁴ A public acknowledgement of the academic debt the group of Lille owes to Sundholm has recently been secured through his nomination as *honorary professor* of the University of Lille.

In fact, Sundholm’s historical and philosophical insights on Proof Theory, and his reflections on assertion and judgment in particular, influenced new developments on the dialogical stance in the crossing of epistemology, philosophy of language, and argumentation that reaches far beyond the initial work at Erlangen and Saarbrücken.⁶⁵

⁶⁴ See in particular Rahman et al. (2018).

⁶⁵ Cf. among others, Keiff (2007), Marion (2010), Redmond (2011), Ginzburg (2012), Fontaine (2013), Marion & Rückert (2016), Shafiei (2018), Dutilh Novaes & French (2018), Dutilh Novaes (2021), Rückert (2019), Crubellier et al. (2019), Fermüller (2021), McConaughey (2021), Sterling (2021), Clerbout & McConaughey (2022), Iqbal (2022), Dutilh Novaes (2022), Lion (2023), Klev (2023)-forthcoming.

2 Introduction

It is a quite common conception that while the past is fixed and determined, the future is open: while only one set of events actually takes place, before it becomes fixed in the actual course of events, there are many real possibilities that could take place. Thus, if Themistocles and Eurybiades argue over there being or not a sea battle the next day, at the moment of utterance there are two real possibilities for the following day: either there will be a sea battle or there will not. Only one will happen, but on the previous day, the two options are equally possible. While the past is fully determined, the future is open – at least to a degree compatible with free will and some form of causal indeterminism.

Although such a conception seems quite natural to our common sense, the analysis of assertions involving future contingents in such a setting gives rise to challenging logical and philosophical questions which have triggered, from the days of ancient philosophy to our contemporary developments, heated debates regarding the nature of time and of temporality in diverse fields of knowledge.⁶⁶

In several texts, N. D. Belnap and co-authors have strived towards answering these challenges, focusing mainly on assertions involving future events, such as predictions, promises and bets. Their work combines speech-act theory with a model-theoretical semantics congenial with *local* indeterminism, i.e., the indeterminism that results from considering only the transition between one initial set of events to a second one, disregarding past transitions.⁶⁷

As we will see, regarding predictions, their core idea is that, since the future is open, predictions cannot be assessed at the moment of utterance, they require a second time reference for their assessment. At that second time reference, the prediction can be vindicated or impugned, depending on the actual course of events.

In this paper, we pursue Belnap et al.'s approach to predictions but we contend that the pragmatic and normative dimensions of the speech acts of predicting and vindicating are best rendered in a dialogical framework. Taking Perloff and Belnap's (2011) interpretation of the sea battle prediction, we first recall the main lines of their construal before developing a dialogical reconstruction of their scenario.

3 Belnap et al.'s STIT framework for vindicating predictions

3.1 The sea battle scenario

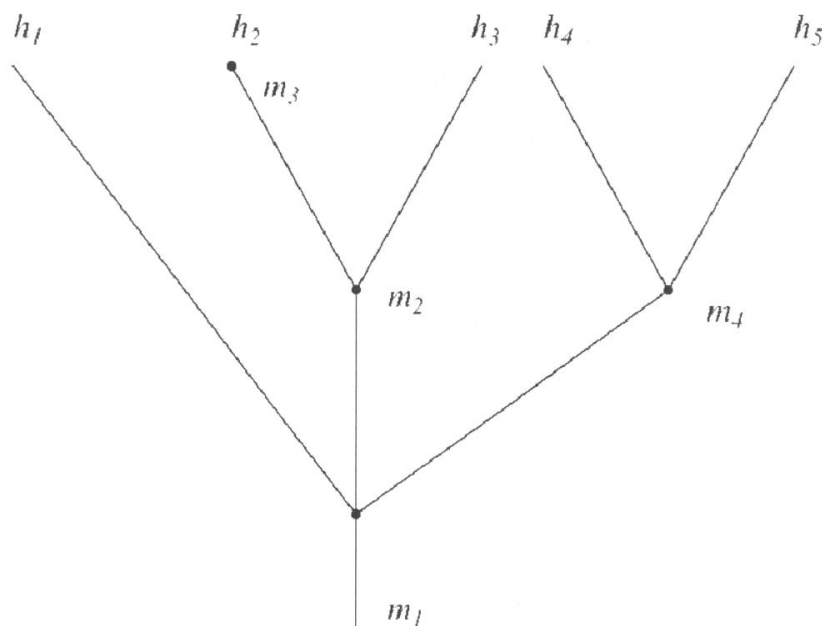
In their 2011 paper "Future Contingents and the Battle Tomorrow", M. Perloff and N. D. Belnap focus on predictions. Taking inspiration from Aristotle's problem regarding future contingents (*De Interpretatione*, 9), they provide a scenario to flesh out their theory: in 480 BCE, the Athenian Themistocles, in command of the Greek fleet fighting the Persians, predicts that a battle will be fought the next day at sea, while the Spartan Eurybiades denies it. Under the conception of an indeterminate future, this kind of prediction is neither true nor necessary at the moment of utterance. In order to be able to evaluate such a prediction, a later moment is required at which the outcome of the prediction has been settled; the pragmatics of predictions thus requires a "double time reference" (Perloff and Belnap 2011, p. 593; Belnap 2002). Indeed, predictions cannot be assessed on the spot, as a mere description could, since they involve something that has not yet happened. Predictions thus call for a second moment, the moment of assessment, which comes once

⁶⁶Just to mention a few from a very large list, see Prior (1967), Thomason (1984), van Benthem (1991), Bobzien (1993), Gaskin (1995), Crubellier (2010, 2011), Coope (2005), Øhrstrøm & Hasle (2015), Goranko & Rumberg (2020). In the context of his analysis of the Master argument, Vuillemin (1984) even classifies philosophical systems with regard to the endorsement or rejection of the principles involved in the future contingents.

⁶⁷See Belnap and Green (1994), Perloff and Xu (2001), Belnap (2002).

it is possible to determine whether or not the prediction was fulfilled. Thus, the prediction can, retrospectively, be vindicated or impugned.

Building on the branching tree account of indeterminism provided by Prior (1967) and Thomasson (1970, 1984), Perloff and Belnap (2011, pp. 597–588) propose a general model for speaking of future contingents, predictions, and assessments. This model is a branching tree in which histories branch to the future but not to the past; every node is a moment m_i that occurs, and an uninterrupted branch of moments constitutes a history h_i (a particular course of events). In the figure they provide, reproduced below in Figure 1, there are four (decisive) moments, m_1 – m_4 , which mark a branching in space-time between at least two different histories or alternative possible courses of events. In Figure 1, there are five histories, h_1 – h_5 .



branching histories from Perloff and Belnap (2011, p. 587)

Figure 1: Tree of

The tree structure entails that a particular moment can be shared by many histories: m_1 is shared by all five histories, while m_2 is shared by h_2 and h_3 but by none of the others, and so on. What is more, “ m_2 is in the future of possibilities of m_1 , not simply in its future. A future of possibilities, unlike a future portion of a history, can contain incompatible moments” (Perloff and Belnap 2011, p. 587). In order to indicate that moment m_i occurs on history h_j , Perloff and Belnap use the notation “ m_i/h_j ”.

If we take a simplified version depicting the sea battle prediction, we obtain just one branching (Figure 2).

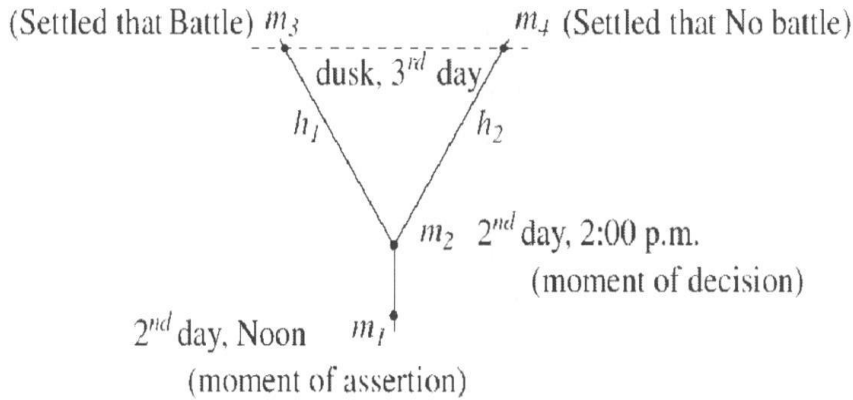


Figure 2: The sea battle prediction from

Perloff and Belnap (2011, p. 589)

Under this representation m_1 is the moment of assertion, where Themistocles claims that there will be a sea battle the next day while Eurybiades claims that there will not be a sea battle – thus, m_1/h_1 and m_1/h_2 , or more briefly, $m_1/(h_1, h_2)$. The moment m_2 (also shared by both histories) is the moment when the Persians decide whether or not to engage battle the next day, thus branching into two possible courses of events, one in which a sea battle occurs (branching towards the history h_1), the other in which no sea battle takes place (branching towards the history h_2). So at the moment of assertion m_1 , it is *possible* that there will be a sea battle (at m_3/h_1) or that there will not be a sea battle (at m_4/h_2). According to Perloff and Belnap, these are *real, objective* possibilities, they are not just linguistic or epistemic possibilities.

3.2 Truth and settled truth

An important notion in this construal is the distinction between mere truth and settled truth (Perloff and Belnap 2011, p. 591): a settled truth is what is true no matter what the future brings. Thus, at the time of assertion, it is only *possible* that there will be a sea battle since it is *also* possible that no sea battle will take place (we are dealing with future contingents). None of the two possibilities is a *settled truth*, none is true no matter what the future brings, they both depend on the future courses of events. However, at m_3 and m_4 , the matter is settled: it is a settled truth that there was a sea battle at m_3 , in history h_1 , while it is a settled truth that there was no sea battle at m_4 , in history h_2 . While retrospectively, since h_1 actually happened, it is correct to say that Themistocles was right, that it was true there would be a sea battle the next day, it is incorrect to say that it was a *settled* truth at that time. For there to be a settled truth of future contingents, the event must have occurred, we must be in a history in which the event occurred.

This notion of *settled truth* thus involves the use of histories. That p is a settled truth at a moment (i.e., that p is true in all the histories that cross the moment) is written **Sett:** p . This notion validates the implication from a fact p to its settled truth **Sett:** p :

$$(1) p \supset \mathbf{Sett}:p \quad \text{for elementary } p$$

As the authors point out, the notion of settled truth is not the notion of necessity in contemporary modal logic; it just amounts to the idea that the truth of p at a moment does not depend on what happens later. In other words, if p is elementary and true at m_1 then being settled true does not say more than that it is true when it is.

As a matter of fact, the notion of *settled truth* comes close to Aristotle’s notion of the actual necessity of events: if something has happened, then it is necessary; but we cannot infer from that that it was absolutely necessary since at some point (e.g. the previous day), it was not necessary for the event to happen.

What is, necessarily is, when it is; and what is not, necessarily is not, when it is not. But not everything that is, necessarily is; and not everything that is not, necessarily is not. For to say that everything that is, is of necessity, when it is, is not the same as saying unconditionally that it is of necessity. Similarly, with what is not. And the same account holds for contradictories: everything necessarily is or is not, and will be or will not be; but one cannot divide and say that one or the other is necessary. I mean, for example: it is necessary for there to be or not to be a sea-battle tomorrow; but it is not necessary for a sea-battle to take place tomorrow, nor for one not to take place – though it is necessary for one to take place or not to take place. (Aristotle, *De Interpretatione*, 19a23–32; transl. Ackrill)

Such a principle is known by Al-Fārābī (*Al-Taḥṣīl al-sa’āda*), and Ibn Sīnā (*Al-Shifa’: al-Ilahiyyat*) after him, as *conditional necessity*.⁶⁸ The implication (1) can be understood as a (propositional) interpretation of this Aristotelian notion of conditional necessity in a setting for models with branching histories.⁶⁹

The framework developed by Perloff and Belnap (2011) includes two monadic propositional time operators, **Was** and **Will**, used to express respectively that something was true and that something will be true.⁷⁰ Thus, given suitable definitions of these operators, the framework also validates another Aristotelian principle, namely, the necessity of the past, as opposed to the indeterminacy of the future.⁷¹

(2) **Was**: $p \supset$ **Sett**:**Was**: p for elementary p

Note that the restriction to an elementary p prevents substituting a present or past p with a future p , which would destroy the necessity of the past and the indeterminacy of the future which the theory is precisely trying to safeguard.⁷² For instance, in (1) and (2), if

⁶⁸See Miller (1986) and Adamson (2006). Ibn Sīnā introduces explicit intervals of time in order to shape modal syllogisms where conditional necessity occurs (Street 2006, 2008, 2013; Strobino 2015, 2016). William of Ockham later dubs this kind of necessity “accidental necessity”, which he thus justifies in his *Ordinatio*, I, Prologue, q. 6: “I add this because of propositions that are necessary *per accidens*, as is the case with many past tense propositions. They are necessary *per accidens*, because it was contingent that they be necessary, and because they were not always necessary” (quoted from Plantinga 1986, p. 245). This passage is discussed by Plantinga (1986, sections III and IV).

⁶⁹The link between $p \supset$ **Sett**: p and Aristotle’s notion of conditional necessity was already established by Jules Vuillemin (1977, 1979, 1984), who introduces conditional necessity as an axiom in his reconstruction of the *Master Argument* of Diodorus Cronus, using the Polish notation $p \supset \mathbf{L}p$. However, since his formal rendering is not that accurate and the semantic is missing, Vuillemin’s reconstruction has triggered a harsh criticism; see Denyer (1998, 2009).

⁷⁰“The truth-value of future-tensed sentences at a moment depends on what course history takes. [...] **Will**: A is true at a moment and history with respect to a model and a moment of use $(M, m_c, m/h \vdash \mathbf{Will}:A)$ just in case there is a moment m_i on h_i later than m at which A is true $(M, m_c, m_i/h \vdash A)$. With respect to the past tense, we have **Was**: A is true at a moment and history with respect to a model and a moment of use $(M, m_c, m/h \vdash \mathbf{Was}:A)$ just in case there is a moment m_i prior to m at which A is true $(M, m_c, m_i/h \vdash A)$. The idea is to shift along the present history, h_i , to earlier moments to check for A ” (Perloff and Belnap 2011, p. 592). Analogously one can define **Will-always**: A , and **Was-Always**: A , by quantifying universally over the moments in the relevant history.

⁷¹Besides the passages about future contingents in the *De Interpretatione*, the notion of deliberation shows how important the indeterminacy of the future is for Aristotle: as his saying goes, we do not deliberate on what cannot be otherwise, that is, since the past is determinate, we deliberate only on “what is future and contingent” (see *Nichomachean Ethics*, 1139b5–11).

⁷²The restriction of the necessity to the past, to assertions that are “strictly” about the past, has been explicitly formulated by Ockham. For a thorough discussion on Ockham’s views on this restriction see Plantinga (1986).

we substitute p by **Will**: p , we obtain these two implications for which it is easy to find countermodels:

(1*) **Will**: $p \supset$ **Sett**: **Will**: p

(2*) **Was**:**Will**: $p \supset$ **Sett**:**Was**:**Will**: p

Less intuitive is the operator for possibility **Poss**, which, if defined as the dual of **Sett**, yields: **Poss**: p is true at m iff for at least one history that passes through m , p is true at m (Belnap, Perloff & Xu 2001, p. 32). This validates, for elementary p , the implication **Poss**: $p \supset p$, which amounts to stating that if the possibility of p is true at m on h , then p has been actualized at m (on h); this possibility then trivially implies p at the same moment on h . Thus, for elementary p , the truth of **Poss**: p follows the principle of plenitude.⁷³ Moreover, in such a setting, for elementary p , **Poss**: p , **Sett**: p , and p , are indistinguishable.

However, in a framework for an open future, an alternative notion of possibility gives up the duality with **Sett**, defining the possibility **fA** as being true iff there is a history h passing through m such that A is true at some non-actual future n on h , $n > m$. See Øhrstrøm & Hasle (2015, section 4d, definition g) who add it to the Peircean framework for indeterminism.⁷⁴ In fact, their notation indicates that it constitutes an alternative to the notion of open future **F**, rather than an alternative way to define possibility. Accordingly, such a notion of possibility excludes not only the past (the possibility cannot be actualized in the past) but also the present time of utterance.⁷⁵

3.3 Vindicating predictions and double time reference

An account of vindication and of impugment can be provided using (i) the double time reference embedded in the speech act of predictions, which distinguishes the moment of utterance and the moment of assessment, together with (ii) the semantic definitions of settled truth (**Sett**), of a future occurrence (**Will**), and of a past occurrence (**Was**). A first account is provided in the following way, which is however deemed inadequate because restricted to the sea battle example and not generalizable to cases of embedded future operators (Belnap 2002, p. 49).

(A) At the later moment, m_3 , Themistocles' assertion *There will be a sea battle tomorrow* made to Eurybiades at m_1 is *vindicated* just in case it is *settled* at m_3 that *There is a sea battle is true* (paraphrase of Perloff and Belnap 2011, p. 594).

Perloff and Belnap however propose this second account of vindication, which they consider to be the correct one.

⁷³Vuillemin (1984) also attributes this principle to Aristotle, obtaining it from the dual of Aristotle's accidental necessity. This, however, seems to contradict Aristotle's explicit rejection of plenitude.

⁷⁴ As discussed in Rahman & Seck (forthcoming), a precursor of such an alternative approach to possibility can be found in the modal logic of the influential post-Avicennian Islamicate thinker Shihāb al-Dīn Suhrawardī (549 H/1155 CE) – cf. *al-Ishrāq* 1999, p. 16-17.

⁷⁵A further notion of possibility within a framework for indeterminacy, based on identifying stretches of time across histories, was indicated to Rahman in a personal email by Tero Tulenheimo (Tampere). Tulenheimo drew our attention to Belnap and Green's (1994, p. 372) notion of *instant t*, i.e. an equivalence class of moments that results by correlating moments m' , totally disjoint from the moment m on the actual flow of time h . Then, for this other notion of possibility labeled \langle **Poss** \rangle , \langle **Poss** \rangle : A is true at m iff there is at least an alternative flow of time whose now-point m' is correlated with m , and A is true at m' . Given this primitive, it makes sense to ask what might have happened at this very instant. However, eventually, Belnap and Green (1994) reject correlating moments across histories for being scientifically and intuitively unclear.

(B) At the later moment, m_3 , Themistocles' assertion made to Eurybiades at m_1 is vindicated just in case it is settled at m_3 that his assertion, (*) "There will be a sea battle tomorrow" was true – but by no means was it settled true – at m_1 . (Perloff and Belnap 2011, p. 595).

The predictions made by Themistocles and Eurybiades – and, as a matter of fact, any prediction whatsoever – are assertions that a certain event *will* or *will not* occur. The first account of vindication only considers whether or not the predicted event did occur in the course of events, i.e., whether or not it is a *settled truth*: the prediction says there *will* be such an event, the vindication says that it is *settled* there *was*. The second account of vindication does not bear on the fact as much as on the prediction: the prediction is vindicated at the moment of assessment when it is *settled* at that moment that it *was* the case that the event *will indeed* take place or not, depending on the prediction. This is not the same as saying that, say, Themistocles's, prediction *is true* when he makes his prediction, since such accounts generate paradoxes paving the way to determinism or merely epistemological open futures. It also differs from saying that at the time of prediction, Themistocles's prediction is *settled true*, since at that time the possible future branches towards h_1 or h_2 . The account states that *in the future* (the moment of assessment) it is settled that in the past it *would indeed* be the case. In the dialogical reconstruction below, we will keep this second account of vindication of predictions.

3.4 Recapitulation of Perloff and Belnap's construal

To sum up, according to Perloff and Belnap's analysis of the open future, predictions of the kind "There will be a sea battle tomorrow" are neither settled true nor necessary at the moment of utterance; they however call for a later moment of assessment at which such speech acts can be vindicated or impugned. Vindication and impugnement are thus produced in relation to (i) the original utterance moment, (ii) the actualized history h_i , and (iii) an assessment moment within h_i , at which the taking place or not of the predicted event has been *settled true* – i.e., true whatever the future brings.

This perspective, which still endorses the validity of the Excluded Middle in relation to the predictions, concurs in this respect with the family interpretations of Aristotle's view on the matter called, after Rescher (1963) and Adamson (2006), Al-Fārābī's solution-B.⁷⁶

What characterizes Belnap et al.'s proposal is (a) the conception of a genuinely dynamic open future that precludes attaching to each utterance of a prediction a privileged history, known as the actual future or the thin red-line,⁷⁷ which fixes the ways the future course of

⁷⁶Belnap's approach contrasts on one side with the so-called *standard interpretation* of Aristotle's solution, according to which the law of Excluded Middle fails, and on the other with the *non-standard interpretation*, which actually consists in a family of several variants that endorse the validity of that law. The main point of the non-standard interpretations is that the components of the disjunction of future contingents are true or false *indefinitely* – in the sense that for the moment, no state of affairs obtains that makes it have this or that truth-value. Such interpretations that hold the validity of Excluded Middle have been attributed, among others, to Boethius and Abelard (Mignucci 1998; Seel 2001a, b, c). Tulenheimo (2020a,b) penned a formal semantics for the Boethius–Abelard approach. More precisely, as Tulenheimo (2001, p. 66) already pointed out, his own approach can be understood as a model-theoretic interpretation of the position that Vuillemin (1984, chapter 8) and Seel (2001, p. 28) identify with one of Carneades (214–129/8 BCE), who seems to have been the first philosopher to have held the view that future contingent propositions are made true by future facts, not by present facts about a future event – the latter might be attributed to Ammonius (Seel 2001c, pp. 239–246; cf. Seel (ed.) 2001; 2001a, p. 36; 2001b, p. 233). Belnap and Green (1994) also accept the law of Excluded Middle, but contest the non-standard solutions known as *Ockhamist Theories*. Ockhamist theories assume a branching course of events but either do not distinguish actual courses of events from purely possible ones, or take it that each utterance time brings with it one privileged actual future – conceived in a rather static manner –, called the *thin-red-line* (Øhrstrøm&Hasle 2015, section 5.3).

⁷⁷Cf. Øhrstrøm (1981), Malpass & Waver (2012), Borghini & Torrenco (2013).

events will go; (b) the speech-act analysis of the notion of vindication and its role for assessing predictions. In relation to the latter, a predictive assertion has deontic consequences for the speaker: the speaker deserves credit if the prediction obtains, and discredit if it does not obtain.

3.5 An issue with the semantization of pragmatics in Perloff and Belnap

The issue we have with Belnap et al.'s construal is that they separate the pragmatist analysis from the semantic levels, so that the normative dimension of the speech acts they study (predictions, promises, bets) is either studied from the pragmatist perspective of the speech acts, or from the perspective of semantics, which then seeks to reduce the pragmatic and normative dimension to the ontological model expressing the prediction situation.⁷⁸ The study of vindication and impugment, we submit, should be rooted in the intertwining of normativity and meaning.

A clear case of the pragmatization strategy is the following text, where the authors suggest to leave the semantics of operators unchanged but switch to a pragmatist perspective that stops the syntactic analysis as soon as assertions of the form **Will:** α are reached.

We are now in a position to see that on the present account of assertion, it makes sense to talk of asserting “*Will:(the coin lands heads)*” exactly because assertion constitutes a way of closing the history parameter—not indeed semantically (the semantics of the asserted sentence is unchanged), but pragmatically, by the very act of assertion. (Belnap et al. 2001, p. 174).

The main motivation of this strategy is that it also allows to deal with embedded temporal operators:

There is, however, a more general and more satisfactory account of vindication, one which makes even better use of double time references within the doctrine of branching histories and its accompanying semantics. This account no longer requires syntactic unpacking of the assertion. We use, instead, the rich semantic content of the assertion (*) at the moment of utterance $m_i (= mc)$. Since we are committed to the relativization of future-tensed assertions to moment/history pairs, the question becomes, for which moments and for which histories should we be evaluating Themistocles' assertion? (Perloff and Belnap 2011, p. 594)

However, the latter text announces a second strategy, that proposes to give a semantic account of the speech acts under consideration, where, according to the authors, the introduction of a “double time reference” and a model theoretical rendering of *settled truth* allows a general approach on speech acts in the presence of indeterminism. They recapitulate the merits of their model in the following fashion.

Among the advantages to be gained in employing a semantics that treats assertion as publicly normative, calling for vindicating or impugning, we

⁷⁸Taking up the terminology introduced by Lorenz (1987, pp. 65–66; reprint 2010, pp. 71–72), the first pitfall consists in a pragmatization of semantics, while the second, which concerns Perloff and Belnap's (2011) construal, consists in a semantization of pragmatics. Because the semantic perspective is not generalizable for cases with infinite chains, at that moment they switch to the purely pragmatic perspective, without bringing the two perspectives in one unified theory of meaning.

count our ability to understand assertion as one of a variety of speech acts, including asserting, betting, promising, all of which, in the presence of indeterminism, are illuminated by similar considerations. Just as an assertion about an undetermined future is vindicated or impugned at a later moment, when the assertion is settled true or false, a promise to see to something is either kept or not kept only at a later moment, when the matter is settled. In like manner, a wager made about an undetermined future cannot come due for settlement until a later moment. (Perloff and Belnap 2011, p. 596; our italics)

And later, on the next page,

We suggest that a semantic account invoking double time references is applicable, with suitable adjustments, for dealing with the whole range of speech acts whose assessment must wait, in the presence of indeterminism, for an appropriate moment. Betting, promising, and even the speech-act-like case of wondering, which might be thought of as wanting to know about a not yet settled future, *all require for satisfactory assessment the earlier moment of the original speech act as well as the later moment of assessment.* On this account, every history through the moment of assertion receives equal treatment. *Assertions are vindicated or impugned, promises fulfilled or not, bets resolved one way or the other, and wonderings laid to rest at the later moment, a moment when the matter is settled one way or the other.* (Perloff and Belnap, 2011, p. 597; our italics)

In these passages, a descriptive shift occurs: the normative dimension of speech acts, clearly recognized (p. 590, again p. 596), is transformed into semantic characteristics within the model. The first step is to reduce speech acts to acts, and thus to moments within the model, on the same level as any other moment; the second step is to recognize the normative aspect of speech acts depending on future contingents, which calls for a later moment of assessment, and to semanticize this aspect through the double time reference: a speech act such as a prediction is linked to a second moment in the model, the moment of assessment. The normative dimension of speech acts is thus reduced to the semantic model, in what can be called after Lorenz (1987, p. 65; reprint 2010, p. 71) a semanticization of pragmatics. Yet, the vindication of a prediction must consist in a public recognition that the prediction was correct; there is no private vindication, the moment must be public and made public through a speech act. There is thus an intimate link between the speech act of prediction and the speech act of vindication, a link that cannot be reduced to a reference (the assessment refers to the prediction and to the content of the prediction): these two “moments” are of the same nature, and they are of a different nature from other “moments” such as the moment of decision (the Persians decide to attack) or the moment of happening (there is a sea battle). The semantic model thus captures the *meaning* embedded in the normative dimension of predictions and their vindication or impugment, but this semanticization loses the *normative* aspect.

This paper’s proposal consists in adopting the framework of dialogical logic in order to study the link between a prediction made and its vindication or impugment, while keeping fully alive the normative and pragmatic dimension of these speech acts. What needs to be explicitly added in a *descriptive* manner to the model-theoretic construct is already embedded in the dialogical framework in a *normative* way, i.e., without transforming a normative dimension into a descriptive one (which reduces the dynamics of norms to a static description of them). While the semantic model expresses the ontology but semanticizes (and thus loses) the normative aspect, as will be shown below, the

dialogical approach to the vindication of predictions immediately carries a normative aspect in the dialogue between the two players while still allowing to recuperate the ontological model of indeterminate futures.

4 Vindicating predictions in the dialogical approach

The dialogical approach to predictions in the spirit of Belnap et al.'s construal consists in a public assessment of a prediction depending on the actual course of events. A *prediction made* is a statement expressing a future contingent, thus taking the form **Will:** A or **Will:** $\neg A$, made by an agent at some point in time. This disjunction does not express the law of Excluded Middle, but in the particular case of *tomorrow's* sea battle, the internal or external negation does not make a difference. In order to focus on predictions, we will adopt the internal negation (the dialogues can however be adapted to the external one for a focus on the Excluded Middle).

The public assessment compares the merits of an opposite pair of predictions made depending on the actual course of events. In the confrontation between the opposite predictions and the actual course of events, the prediction that stands the test is vindicated, while the other is impugned. The dialogical framework provides a natural setting for this confrontation between predictions and facts: in the public assessment of predictions, someone, called the Proponent (he, by convention), will argue for one of the predictions while someone else, called the Opponent (she, by convention), engages her responsibility regarding what the actual course of events has been. In this setting, sharing the burden of proof warrants the impartiality of the assessment: the person who is responsible for vindicating a prediction and impugning its opposite is not the person responsible for determining what the actual course of events was.

In this framework, the Proponent will have to defend a prediction made depending on the course of events provided by the Opponent. The “second time reference” does not have to be a particular time of assessment defined in the model (such as m_3 or m_4), since the assessment time is *de facto* given by carrying out the public assessment in the dialogical form: the “second time reference” that is required in order to assess the prediction can occur, in the dialogical setting, at any time once the outcome of the prediction has been settled. Thus, in the case of the sea battle prediction, m_1 is the moment of utterance, m_3 is the moment from which it is *settled true* that a sea battle took place the day after m_1 , and m_4 is the moment from which it is *settled true* that no sea battle took place on that day. What is required at the time of assessment, whatever that time may be, is a reference to a time where the outcome has been settled, so m_3 or m_4 or any time after that in the ensuing histories (thanks to the principle of the determinate past). If the Proponent is capable of defending one of the predictions made regardless of the way the Opponent defends which course of events took place and regardless of the way she challenges his claims (we can imagine a series of Opponents defending and challenging the Proponent in all possible ways), then the Proponent is said to have a “winning strategy” and the prediction made is vindicated while the other prediction made is impugned.

4.1 An informal presentation of the dialogue and key concepts

Adopting Perloff and Belnap's second account of vindication of a prediction, vindicating requires that *it is settled* that *there was a moment* at which it is true that *there will (or will not) be* a certain event. The dialogical approach makes this requirement the Opponent's responsibility. The Proponent will have to vindicate one of the two predictions based on the course of events warranted by the Opponent. Informally, the Proponent starts the public assessment (or play), by claiming that *either the prediction of Themistocles is vindicated, according to which there will be A the next day, or the prediction of Eurybiades is vindicated, according to which there will not be A the next day, provided that the Opponent determines whether the next day it is settled that it was the case at the moment*

of prediction that *A* would happen, or that it would not happen. This claim is the object of the public assessment and can be expressed in the following way, which will be detailed below.

$$! \quad \text{Will:}A@t_{mr}(m_1) \quad \vee \quad \text{Will:}\neg A@t_{mr}(m_1) \quad [\text{f:}(\text{Sett:Was:Will:}A@t_{mr}(m_1)) \quad \vee \quad \text{f:}(\text{Sett:Was:Will:}\neg A@t_{mr}(m_1))]$$

If the Proponent can defend Themistocles's statement that a sea battle would be fought the next day by using the Opponent's claim that the course of events is such that it is settled that a sea battle would be fought the next day, then Themistocles's prediction is vindicated and Eurybiades's is impugned. The same goes, *mutatis mutandis*, for vindicating Eurybiades's prediction.

The evaluation starts with Opponent accepting her role, i.e., accepting to defend either that it is a settled truth that it was the case there will be *A* the next day or the other disjunct; the Proponent then accepts his own role, i.e., to defend the first disjunction and thus to vindicate one of the two predictions. Once the roles have been clearly distributed, the Opponent brings up a challenge to the Proponent's claims; he can then either respond to the challenge, or challenge one of the Opponent's own claims. The public assessment thus proceeds with the Opponent and the Proponent reacting in turn to the other's actions by either challenging a claim made by the other or answering to a challenge raised against one of their own claims. The way the challenges and answers can be carried out is defined by the *particle rules*. These rules determine how any logical constant stated can be challenged and how such a challenge can be answered. The particle rules thus define, at once, what kind of action and reaction is appropriate in a dialogue game (here, the public assessment), and what the meaning of the logical constants is. The dialogical framework is thus a pragmatist framework whereby syntax and semantics are set at the same time.

The particle rules (specified below for **Will**, **Was**, **Sett** and **f**) define the appropriate challenges and answers for each logical constant; they go from what is more complex, the statement with the logical constant, to what is less complex, what is required in order to be entitled to make a claim. There is thus a contextual notion of assertion at play, which is rooted in Wittgenstein's language games and has been one of the main tenets of Paul Lorenzen, who conceived dialogical logic by the turn of 1958, and of Kuno Lorenz after him.⁷⁹ As Mathieu Marion (2010) has shown, the core principles of the dialogical framework come very close to Robert Brandom's (2017, 2002) inferentialism, which provides rich insights on assertion understood as the most basic move in the public space for conceptual thinking enacted by games of giving and asking for reasons. More precisely, such a move (an assertion) engages further moves following the interaction of obligations and rights this move commits and entitles to.

What is more, the fact that in a dialogue game the interaction between the players breaks a complex claim down into simpler claims (of which the burden of proof may be distributed between the two players) conforms with the process of vindicating a prediction: the entire prediction made is taken as object of scrutiny, and the public assessment consists in making sure the Proponent is indeed warranted to hold each of the claims that must be held for the prediction to be vindicated depending on the course of events warranted by the Opponent's role in the exchange. The evaluation starts with the predictions made in their entirety and goes down to check the details.

4.1.1 An informal dialogical account of vindication

In the case of the sea battle scenario, we obtain two rounds for a full public assessment. We will first develop one round in an informal fashion, point out how the second round

⁷⁹ See the seminal paper of Lorenzen (1960) and further developments in Lorenzen & Lorenz (1978).

proceeds, and then introduce the formal notation in the dialogical framework before carrying out the plays using the formal notation.

The public assessment really starts when the Opponent has claimed that *it is settled that it was the case that there will be A the next day or it is settled that it was the case that there will not be A the next day* (move 1), and the Proponent has claimed in turn that *there will be A the next day (Themistocles) or there will not be A the next day (Eurybiades)* (move 2). A visual representation of the first round is provided in Figure 3 below. The answers to the challenges are written on the same line as the challenge, the order of the moves needs to be followed using the indications in the outer columns.

Move 3. It is the Opponent's turn, she challenges Proponent's claim of move 2 by asking who is right, i.e., which of the two disjuncts Proponent will try to vindicate.

Move 4. It is the Proponent's turn, instead of answering the Opponent immediately, he first challenges her claim of move 1. He will come back to her challenge only much later, at move 12. Challenging claim 1, the Proponent asks the Opponent to choose which disjunct she claims to be true.

Move 5. The Opponent answers the challenge by choosing the first disjunct (in this round; the second disjunct is chosen in the second round), i.e., she claims that at m_3/h_1 , it is settled that it was the case that there will be *A* the next day.⁸⁰

Move 6. The Proponent challenges the previous move, asking whether it was settled for the history (or course of events) h_1 .

Move 7. The Opponent answers by saying yes, allowing her to delete the settled clause and simply claim that at m_3/h_1 it was the case that there will be *A* the next day.

Move 8. The Proponent challenges the previous move by asking when it was the case.

Move 9. The Opponent answers by saying that at the moment m_1 common to the histories h_1 and h_2 ($m_1/(h_1, h_2)$),⁸¹ there will indeed be *A* the next day.

Move 10. The Proponent challenges that claim by asking when it will be the case.

Move 11. The Opponent answers by saying that at the moment m_3 in the history h_1 (m_3/h_1), *A* happened.

Move 12. The Proponent now knows the actual course of events. He answer the pending challenge on his claim 2, and chooses which prediction he will vindicate. Here, since the Opponent gave her word that *A* happened at m_3 , actualizing history h_1 , the Proponent decides to vindicate Themistocles's prediction. He thus answers the challenge by saying that at $m_1/(h_1, h_2)$, Themistocles is right that there will be *A* the next day.

Move 13. The Opponent challenges this claim by asking when it will happen.

Move 14. The Proponent can then recall what the Opponent said move 11: that *A* happened at m_3/h_1 .

The Opponent has nothing more to say, so the Proponent has successfully vindicated the prediction he chose, namely, in this case, Themistocles' prediction. However, if we look at the choices the Opponent made during the public assessment, we can notice that she had to make a decision for move 5: she had to choose which disjunct was actually the case, and she chose the first disjunct. Thus, a second round of public evaluation can take place in which the Opponent chooses the second disjunct. In that case, the Proponent would not be able to vindicate Themistocles's prediction, he would not have the means to ground this vindication in the actual course of events guaranteed by the Opponent. However, if the Opponent chooses the second disjunct, by choosing to defend Eurybiades's prediction at move 12, the Proponent can successfully vindicate that prediction and impugne Themistocles's.

4.1.2 Normative dimension and dialogical meaning explanations

It is now clear that the dialogical approach to the vindication of predictions is set in a pragmatic and normative exchange which however reflects the ontological structure

⁸⁰ We adopt here Perloff and Belnap's notation m_i/h_j mentioned above in section 3.1 for indicating that moment m_i occurs on history h_j .

⁸¹ As mentioned above in section 3.1 $m_i/(h_j, h_k)$ stands for m_i/h_j and m_i/h_k .

represented in the tree structure of Perloff and Belnap's (2011) model: the indeterminacy of the future is embedded in the choice the Opponent must make, but the determinacy of the present and past is embedded in the fact that the Opponent must actually choose one alternative. Her *various options* correspond to the branching possibilities, but her actual *choices* correspond to the course of events. The normative dimension is safeguarded (i) in the public dimension of dialogues, (ii) in the fact that the meaning of the logical constants is defined through the particle rules defining the appropriate interaction, and especially (iii) in the fact that the Proponent can try to vindicate a prediction that has not been fulfilled (eg. Eurybiades's prediction when the Opponent gave her word the sea battle did take place), in which case he will be unable to ground his choice of prediction, thus losing the play and failing to successfully vindicate his choice of prediction.

The level of plays, i.e., the level of actual dialogues between the players, reflects the public recognition of a vindication when it is successful: the Proponent wins the play, making it clear to all that the vindication was successful. When he does not win, it is clear that there is no public recognition of his claim. The distinction between the play level and the strategy level, which takes all the relevant plays into consideration, is a characteristic feature of the dialogical framework that makes it fall neither in the pragmatization of semantics, nor the semantization of pragmatics flagged out by Lorenz (1987, 2010).⁸²

move	Opponent		Proponent	move
		Challenged move	Thesis for vindicating a prediction	0
1	Claim at $m_1/(h_1, h_2)$ It is settled that it was the case that there will be A the next day OR it is settled that it was the case that there will not be A the next day	0	Claim at $m_1/(h_1, h_2)$ There will be A the next day (Themistocles) OR there will not be A the next day (Eurybiades)	2
3	Challenge at $m_1/(h_1, h_2)$ Who is right?	2	Claim at $m_1/(h_1, h_2)$ Themistocles is right, there will be A the next day (choice of the first disjunct)	12
5	Claim at m_3/h_1 It is settled that it was the case that there will be A the next day (choice of the first disjunct)	1	Challenge at $m_2/(h_1, h_2)$ Which of the two disjuncts claimed at 1 is the case?	4
7	Claim at m_3/h_1	5	Challenge at m_3/h_1 Is this the case at h_1 ?	6

⁸²If in the standard dialogical framework the normative dimension is present but implicit, it can be explicitly expressed in the object language in the variant Immanent Reasoning.

Yes, it was the case (at this moment) that there will be A the next day.			
9	Claim at $m_1/(h_1, h_2)$	7	Challenge at m_3/h_1
	There will be A the next day		When was this the case?
11	Claim at m_3/h_1	9	Challenge at $m_1/(h_1, h_2)$
	A		When will this be the case?
13	Challenge at $m_1/(h_1, h_2)$	12	
	When will this happen?		You said with your own move (11) that this A will happen at m_3/h_1

Figure 3: Informal dialogue for vindicating Themistocles's prediction

4.2 Vindicating predictions made in the dialogical framework

Let us now express this informal exchange in a formal way, using the tools of the dialogical framework. We will assume to some extent a basic knowledge of the dialogical framework, as presented for instance in the first sections of the *Stanford Encyclopedia of Philosophy* entry on “Dialogical Logic” (Clerbout, McConaughey 2022) or in the present volume’s contribution by Rahman and Gallois. Before carrying out the play and building the strategy, we will present elements required for the vindication of predictions which are not present in the standard framework.

4.2.1 Time and Modality

We will present the following three key elements:

1. adverbials for the indexical tomorrow,
2. dialogical contexts,
3. moves involving the operators **Sett**, **f**, **Was**, and **Will**.

Adverbials

As we have seen, Perloff and Belnap express predictions with the monadic operator **Will**. We wish here to express the fact that the prediction refers to *the day after* the enunciation, i.e., to “tomorrow”. Departing from Perloff and Belnap’s (2011) work, as well as the more technical formulation of Belnap (2002), we will use a notation for *tomorrow* based on Aarne Ranta (1994, chapter 5.9) for temporal indexical expressions.

One way to deal with temporal conditions shaping statements is to introduce them as contextual parameters that can be also made explicit at object-language level by means of adverbial expressions in order to *enrich* an assertion that has already content.⁸³ This contrasts not only with the view that temporal operators are first-order quantifiers with metalogical truth-conditions, but also with the model-theoretical approaches to temporal logic of the Priorian style.

In our notation, the expression “@” occurring in “@*tmr*(m_1)”, stands for Ranta’s (1994, p. 103 & p. 106) monadic propositional operator **AT**, used for rendering temporal

⁸³ We owe the expression *enrichment* to Recanati (2017).

(locative) adverbial constructions such as *Fighting on 24 Feb. 2023*, *Fighting now*, *Fighting tomorrow*. The expression “*tmr(m₁)*” within the scope of the adverbial “*@tmr(m₁)*” stands for the span $\langle m_1, l \rangle$ which begins with the moment $m_1 \in Day$ and has 1-day-unit as length.⁸⁴ In the present paper we will use adverbials only for the indexical *tomorrow*. Notice that, though m_3 (on history 1) and m_4 (on history 2) both refer to the day after m_1 , which we would expect to be the same day, in Perloff and Belnap’s (2011) construal of indeterminate futures, they are strictly speaking different: according to the authors, a moment is a set of events, so that the set m_3 , which includes the taking place of the sea battle, differs from the set m_4 , which precludes it.

The main use of temporal parameters introduces them as *dialogical contexts* as in the dialogical framework for standard modal logic (see Rahman & Rückert 1999, rep. 2011). The point is that this stresses Perloff and Belnap’s own use of the *context of use*, which in fact privileges the Priorian analysis of temporal operators. Of course, the price to pay is that the semantics of the temporal operators shifts to the metalanguage. In a dialogical setting, however, temporal contexts are made explicit during the development of the vindication process, while defending or challenging modal and temporal operators. The operators *Was*, *Will*, and *Sett*, involving temporal dialogical contexts, can be introduced in a quite straightforward manner.

Temporal Dialogical Contexts

Temporal Dialogical Contexts follow the notation of modal dialogical contexts (see for instance the entry “Dialogical Logic” of the SEP). Moreover, every statement in a play will be assumed to have been set in a dialogical temporal context of the form m_i/h_j (with i and j standing for natural numbers). In the dialogues below

- The thesis stated by the Proponent is set at the temporal dialogical context $m_1/(h_1, h_2)$, i.e., at the context of the predictions’s utterance.

We combine the use of temporal dialogical contexts with adverbials in the following way

- When the actual course of events is known, and the outcome of the adverbial indexical and the temporal context become identical, we substitute the indexical *tmr(m₁)* occurring in $A@tmr(m_1)$ (stated in the dialogical context m_3/h_1) or $\neg A@tmr(m_1)$ (stated in the dialogical context m_4/h_2) by the exact moment after m_1 referred to by that indexical, namely, $A@m_3/h_1$ or $\neg A@m_4/h_2$ and we delete in the dialogue the dialogical temporal context.

The Operators Sett, f, Was, Will

Sett. From a dialogical point of view, the operator *Sett* has universal force on the set of histories passing through a moment m_i . Thus, the particle rule prescribes that it is *the challenger* who can choose any one of those histories, say h_j , and require that the defender states the rest of the proposition precisely at the same moment m_i but on the history h_j , i.e. at m_i/h_j .

Statement	Challenge	Defence
m_i/h_i X ! Sett:A	m_i/h_i Y ? Sett h_j	m_i/h_j X ! A

In the sea battle scenario there is no real choice since only one history, h_1 , passes through m_3 , and only one other, h_2 , passes through m_4 .

f. The thesis for the sea battle scenario includes the operator *f*. This is the possibility operator *f* of Øhrstrøm & Hasle (2015, section 4d). From the dialogical point of view, **f**

⁸⁴In fact, if we introduce the expression *tmr(m_i)* within the framework of Ranta’s *Type Theoretical Grammar* (1994, p. 115), it can be postulated as being some element d of the set of spans $span(Day) = Day \times \mathbb{N} \in set$, whereby Day is a finite set defined as a scale on *Time*. So, $d \in span(Day)$ is a pair where a moment $m_i \in Day$ is defined as its beginning and a natural number n indicates its length.

has existential force on both the set of histories passing through a moment m_i and on the sequence of moments occurring on those histories. Thus, the particle rule prescribes that it is *the defender* who can choose a moment *after* the moment of utterance along the chosen history.

Statement	Challenge	Defence
m_i/h_i $X ! f:A$	m_i/h_i $Y ?_f$	m_j/h_j $X ! A$ Where $m_j > m_i$

In the given scenario, the choices are reduced to m_3/h_1 if the left of the disjunction is chosen, or m_4/h_2 if the right one is. Actually, in the dialogue, a kind of shortcut is made. In order to avoid prolonging an already long dialogue, instead of stating one side of the disjunction first and then defending the **f** operator, we simply require that the defender of a disjunction composed of settled truths chooses the dialogical context for stating the chosen disjunct.

Was. From the dialogical point of view, the **Was** operator has existential force on the backward sequence of moments occurring in a given history. Thus, the particle rule prescribes that it is the defender who can choose a moment before the moment of utterance of **Was**, along the given history.

Statement	Challenge	Defence
m_i/h_i $X ! Was:A$	m_i/h_i $Y ?_{was}$	m_j/h_i $X ! A$ Where $m_i > m_j$

In the sea battle scenario, the moment in the past of **Was**, stated at either at m_3/h_1 or m_4/h_2 , is the time of utterance of the predictions under the scope of this operator, i.e., $m_1/(h_1, h_2)$ – recall that in the sea battle scenario m_3 and m_4 share the same past, thus, m_1 is on both, h_1 and h_2 .

Will. From the dialogical point of view, the **Will** operator also has existential force but on the forward sequence of moments occurring in a given history. Thus, the particle rule prescribes that it is the defender who can choose a moment after the moment of utterance of **Will**, along the relevant history.

Statement	Challenge	Defence
m_i/h_i $X ! Will:A$	m_i/h_i $Y ?_{will}$	m_j/h_i $X ! A$ Whereby $m_j > m_i$

In the sea battle scenario, the moment in the future of **Will** stated at $m_1/(h_1, h_2)$ is either m_3/h_1 or m_4/h_2 .

4.2.2 Repetition Ranks and Plays

As mentioned above, the main thesis of the dialogue can be formulated as the following thesis

$$! Will:A@t_{mr}(m_1) \vee Will:-A@t_{mr}(m_1)$$

$$[f:(\text{Sett:Was:Will}:A@tmr(m_1)) \vee f:(\text{Sett:Was:Will}:\neg A@tmr(m_1))]^{85}$$

The dialogical (structural) rule for challenging a thesis with premises, which are in square brackets, prescribes that the Opponent states the premises, i.e., $f:(\text{Sett:Was:Will}:A@tmr(m_1)) \vee f:(\text{Sett:Was:Will}:\neg A@tmr(m_1))$, to which the Proponent has to answer by stating the first part of the thesis, i.e., $\text{Will}:A@tmr(m_1) \vee \text{Will}:\neg A@tmr(m_1)$.

According to the standard (structural) rules of the dialogical framework, after setting the thesis the players have to choose a repetition rank, which bounds the number of times a player can challenge a claim made by the interlocutor, and the number of times a player can answer to the same challenge made by the interlocutor. In the dialogue below, the Opponent chooses rank $\rho:= 1$, so she can challenge any claim only once and, more importantly, she can also defend a disjunction only once during a play.

Notice that if the defence of **Will** requires considering an infinite chain of moments (clearly excluded in case of tomorrow's sea battle scenario) two options are available:

1. follow the standard procedure for dealing with them in the dialogical setting, namely, build a strategy-tree constituted by an infinite number of finite plays – see Clerbout (2014a,b);
2. implement Perloff and Belnap's (2011, p. 594) proposal to stop the analysis of statements of the form **Will**: α (for complex α), mentioned above, by a formulation of the Socratic Rule specific to the global meaning of the **Will**-operator in the context of games for the vindication of predictions – see next section.

4.2.3 Questions, the Socratic Rule, Statements, and Assertions on Predictions made

Questions and Predictions made

A consequence of focusing only on assertions is that, as Ranta (1994, pp. 137–142) has pointed out, more often than not, other important speech acts such as questions, orders, and instructions (the kind constituting recipes found in cookery books, for instance) are neglected. Relevant to the present study are the so-called *propositional questions* which have the form $A \mid B$ (Ranta 1994, p. 138). Noticing that Ranta's analysis of propositional questions comes very close to the (local) meaning explanation of disjunctions in standard dialogical logic, it can be adapted to the dialogical framework to deal with the specific case of *predictions made*. A *prediction made* is a statement regarding a future contingent which has the name of the agent attached. Thus, when a player has stated a disjunction regarding *predictions made* by agents X and Y at a temporal dialogical context, the interlocutor may challenge with a propositional question of the form $X\text{-left} \mid Y\text{-right}$ at the same temporal dialogical context. For instance, with the sea battle scenario, we have:

$$m_1/(h_1, h_2)$$

? Themistocles-left | Eurybiades-right ■

In order to answer this challenge, the player must state one of the predictions made involved in the challenged disjunction:

! Themistocles **Will**: $A@tmr(m_1)$

Or

! Eurybiades **Will**: $\neg A@tmr(m_1)$

The Socratic Rule

⁸⁵ For writing this kind of thesis with premises see McConaughey (2021).

As mentioned above, the Socratic Rule prescribes which of the Opponent's moves entitle the Proponent to ground his statements based solely on her previous statements; these moves are not further analyzed and are declared to be unchallengeable. In standard formal dialogues, the unchallengeable statements are elementary. In material dialogues, the Socratic Rule might be applied to other kinds of statements, in order to set the *global meaning* of some expression. In fact, one way to look at the rules for global meaning, and in particular at the Socratic Rule, is to consider them as setting the use of the rules for local meaning within the development of a play shaped by the context of the targeted system of argumentation.

This is precisely the case for plays which aim at vindicating or impugning predictions. Despite the fact that the local meaning of **Will** and of negation are set, the global meaning of the negation of an elementary proposition within the scope of a **Will embedded in a prediction made** (on the Proponent's side) and *on the settled past truth of the future contingent* (on the Opponent's side), prescribe those statements as governed by the Socratic Rule, and are therefore handled as non-challengeable. In other words, in the sea battle scenario, A and $\neg A$ are not further analyzed and, once stated by the Opponent **O**, they can be stated by the Proponent **P**. This constitutes the global meaning of predictions.⁸⁶

This results in the following two possibilities, which guarantee the Proponent's successful vindication of the relevant prediction.

- **P** $you(n) \varepsilon A@m_3/h_1$ for the vindication of Themistocles's prediction made !Themistocles **Will:A@tmr(m₁)**;
- **P** $you(n) \varepsilon \neg A@m_4/h_2$ for the vindication of Eurybiades's prediction made !Eurybiades **Will:\neg A@tmr(m₁)**.

These admit the glosses:

You, the Opponent, just conceded at move n that at m_3/h_1 a sea battle has been fought;

You, the Opponent, just conceded at move n that at m_4/h_2 a sea battle has not been fought.

The notation " $a \varepsilon \alpha$ " is a variant of " $a: \alpha$ " introduced in *Immanent Reasoning* whereby " a " stands for a *local reason* for α .⁸⁷ The special case " $you(n) \varepsilon \alpha$ " indicates that the Proponent appeals to the Socratic rule for producing a local reason for α , namely, the Opponent's own statement α at move n .

In order to deal with the more general case of embedded operators, the Socratic rule entitles the Proponent to state **Will: α** , (for complex α), in the context of vindicating a prediction made, iff the Opponent has previously stated the same expression while defending **Sett:Will: α** . Thus, we can speak here of a double contextualization of the local rule for **Will**, namely the context of vindicating a *prediction made* and the context of the sequence of moves that leads the Opponent to state **Will: α** . It is this double contextualization that entitles the Proponent to state **Will: α** and that turns it into a unchallengeable statement.

⁸⁶The importance of this point can be easily seen in the more general case of embedded temporal operators: the prediction that is to be vindicated is marked by the first Will, even if it is a prediction about a prediction. So *in this context*, **Will:A** and **\neg Will:A** will be declared unchallengeable in order to vindicate or impugn the predictions. In this case, the thesis will be: ! **Will:Will:A** \vee **Will:\neg Will:A** [**f:(Sett:Was:Will:Will:A)** \vee (**f:(Sett: Was: Will: \neg Will:A)**)], whereby the disjunction can be further analyzed into the predictions made !_X **Will:Will:A**, !_Y **Will:\neg Will:A**

⁸⁷ See Clerbout & McConaughy (2022, section 3.2). Lorenz already used such a notation in several texts reprinted in Lorenz (2011) in order to express the most basic form of predication. In Lorenz & Mittelstrass (1967, p. 8), this form of predication is called $\delta\nu\omicron\mu\acute{\alpha}\zeta\epsilon\tau\nu$, i.e. naming (what it is).

SOCRATIC RULE FOR “WILL:α” IN THE CONTEXT OF VINDICATING A PREDICTION ON A COMPLEX α □	
Statement	Entitlement
m_i/h_i O ! Will:α (move n_i) Given O ! Sett:Will:α (move $n_j < n_i$)	m_j/h_i P you(n_i) ε Will:α (move k_i)

The structural rules for developing plays involving temporal operators can be found in the appendix.

4.3 The formal dialogues vindicating the sea battle predictions

As mentioned above, the dialogical approach to vindicating the predictions in the sea battle scenario yields two plays: the Opponent’s choice of disjunct she defends calls for a second play in which she chooses the second disjunct; the Proponent wins both plays if he chooses to vindicate the appropriate prediction made.

Opponent		Proponent	
		$m_1/(h_1, h_2)$! Will:A@t_{mr}(m_1) ∨ Will:¬A@t_{mr}(m_1) [f:(Sett:Was:Will:A@t_{mr}(m_1)) ∨ f:(Sett:Was:Will:¬A@t_{mr}(m_1))]	
0.1	$\rho = 1$	$\rho' = 2$	0.
1	$m_1/(h_1, h_2)$! f:(Sett:Was:Will:A@t_{mr}(m_1)) ∨ f:(Sett:Was:Will:¬A@t_{mr}(m_1))	$m_1/(h_1, h_2)$! Will:A@t_{mr}(m_1) ∨ Will:¬A@t_{mr}(m_1)	2
3	$m_1/(h_1, h_2)$? Themistocles- <i>left</i> Eurybiades- <i>right</i>	$m_1/(h_1, h_2)$! Themistocles Will:A@t_{mr}(m_1)	1
5	m_3/h_1 ! Sett:Was:Will:A@t_{mr}(m_1)	$m_1/(h_1, h_2)$? √f	4
7	m_3/h_1 ! Was:Will:A@t_{mr}(m_1)	m_3/h_1 ? Setth_1	6
9	$m_1/(h_1, h_2)$	m_3/h_1	8

	! Will: $A@t_{mr}(m_1)$?Was	
11	<u>$A@m_3/h_1$</u>		$m_1/(h_1, h_2)$	9
13	$m_1/(h_1, h_2)$	12	?will	
	?will		$you(11) \varepsilon A@m_3/h_1$	

Figure 4: Play 1 for the sea battle scenario

Opponent		Proponent	
		$m_1/(h_1, h_2)$	
		! Will: $A@t_{mr}(m_1) \vee$ Will: $\neg A@t_{mr}(m_1)$ [f: (Sett:Was:Will: $A@t_{mr}(m_1)$) \v f: (Sett:Was:Will: $\neg A@t_{mr}(m_1)$)]	
0.1	$\rho = 1$	$\rho' = 2$	0.
1	$m_1/(h_1, h_2)$	$m_1/(h_1, h_2)$	2
	0 ! f: (Sett:Was:Will: $A@t_{mr}(m_1)$) \v f: (Sett:Was:Will: $\neg A@t_{mr}(m_1)$)	! Will: $A@t_{mr}(m_1) \vee$ Will: $\neg A@t_{mr}(m_1)$	
3	$m_1/(h_1, h_2)$	$m_1/(h_1, h_2)$	1
	? Themistocles- <i>left</i> Eurybiades- <i>right</i>	! Eurybiades Will: $\neg A@t_{mr}(m_1)$	
5	m_4/h_2	$m_1/(h_1, h_2)$	4
	! Sett:Was:Will: $\neg A@t_{mr}(m_1)$? \v f	
7	m_4/h_2	m_4/h_2	6
	! Was:Will: $\neg A@t_{mr}(m_1)$?Sett$_{h_2}$	
9	$m_1/(h_1, h_2)$	m_4/h_2	8
	! Will: $\neg A@t_{mr}(m_1)$?Was	
11	<u>$\neg A@m_4/h_2$</u>	$m_1/(h_1, h_2)$	1
	?will		
	$m_1/(h_1, h_2)$		

13	12	$you(11) \varepsilon \neg A@m_4/h_2$
?will		

Figure 5: Play 2 for the sea battle scenario

Notice that these dialogues do not represent the general case with embedded temporal operators. This more general case can however be built either following the indications mentioned in the section on the Socratic Rule for a non “unpacked” solution, or by building a strategy tree constituted by an infinite number of finite plays (see Clerbout 2014a,b).

Note on material dialogues

In the context of material dialogues, each play constitutes a winning strategy; we thus obtain an *assertion made* for the predictions on the future contingents. This differs from a *statement made*, which is the entire sequence of moves constituting each of the plays.

P’s statement made and Themistocles’s Prediction made

The result of vindicating the prediction made by Themistocles amounts to recording all the sequences of pairs of moves that end with a victory for the Proponent in a play in which the Proponent decided to vindicate Themistocles’s prediction as thesis. In the plays above this yields the following sequence in which even numbers correspond to moves by the Proponent and odd numbers to moves by the Opponent. Since the last number is pair, the play is won by the Proponent.

$(\langle 1,2 \rangle, \langle 3,4 \rangle, \langle 5,6 \rangle, \dots, \langle 13,14 \rangle) \varepsilon !_{\text{Themistocles}} \text{Will}:A@t_{mr}(m_1)$ for the vindication of Themistocles’s prediction

If we are provided with a code for substituting the numbers by actual moves we obtain that move 1 corresponds to the Opponent’s challenge to the thesis by stating the premise; move 2 corresponds to the Proponent’s answer stating the conclusion of the thesis; move 6 (say) corresponds to the challenge of the **Was**-operator stated by the Opponent in her move 5 – and so on, we will not present the code here. The codes can thus be read as a sequence of instructions bestowing a *blue print* for reproducing the play.⁸⁸

The important point here linked to the notion of statement made is that by reproducing the sequence following the instructions, we reproduce exactly the moves of the play as developed. Notice that the moves corresponding to the order could have been changed, for example move 4 could have been the Proponent defending his own disjunction rather than challenging the one of the Opponent. The following of the sequences of instructions yields the exact play played by the Proponent while vindicating Themistocles’s prediction. So we distinguish the *prediction made* and the Proponent’s statement made: the latter is the result of vindicating Themistocles’s prediction. Thus, strictly speaking, we should use a double agentive notation

$P ! (\langle 1,2 \rangle, \langle 3,4 \rangle, \langle 5,6 \rangle, \dots, \langle 13,14 \rangle) \varepsilon !_{\text{Themistocles}} \text{Will}:A@t_{mr}(m_1)$ for the vindication of Themistocles’s prediction

The same holds for the Proponent’s vindicating Eurybiades’s prediction.

From assertions made to assertions

If the plays constitute a winning strategy we obtain the following assertion made

$f:(\text{Sett}: \text{Was}: \text{Will}:A@t_{mr}(m_1)) \forall f:(\text{Sett}: \text{Was}: \text{Will}: \neg A@t_{mr}(m_1)) \vdash P ! \text{Will}:A@t_{mr}(m_1) \vee \text{Will}: \neg A@t_{mr}(m_1)$

Letting “ β ” stand for “ $f:(\text{Sett}: \text{Was}: \text{Will}:A@t_{mr}(m_1)) \forall f:(\text{Sett}: \text{Was}: \text{Will}: \neg A@t_{mr}(m_1))$ ”, the assertion can be further analyzed as

⁸⁸This is closely linked to the notion of *state of a dialogue* in Redmond & Fontaine (2011).

$$\beta \vdash_P \langle 1,2 \rangle, \langle 3,4 \rangle, \langle 5,6 \rangle, \dots, \langle 13,14 \rangle \varepsilon !_{\text{Themistocles}} \text{Will}: A @_{\text{tmr}}(m_1)$$

for the vindication of Themistocles's prediction

$$\beta \vdash_P \langle 1,2 \rangle, \langle 3,4 \rangle, \langle 5,6 \rangle, \dots, \langle 13,14 \rangle \varepsilon !_{\text{Eurybiades}} \text{Will}: \neg A @_{\text{tmr}}(m_1)$$

for the vindication of Eurybiades's prediction

In order to abstract away from the *assertion made*, we need to abstract from the concrete player and from the number of moves. In short, (i) we express the assertion without attaching to it any concrete player, (ii) we delete the number of the moves, and (iii) we substitute the codes by general strategic objects – see Rahman et al. (2018, pp. 175–179). Now, this abstraction requires insights regarding the particle and structural rules that set the meaning of each expression involved – the insight in this case requires noticing that substituting the Proponent's move by a defensive one, rather than by a counterattack, has no logical consequences.

5 Conclusion

Research on future contingents has of course a long and venerable history and continues to offer new challenges and perspectives. Perloff & and Belnap's (2011) contribution to the matter combines the idea that speech acts involving future contingents can (and perhaps even should) be included within the family of assertions, and that those assertions obey logical laws.

In relation to the first idea their point is to include speech acts involving future contingents within the realm of assertions. In fact, though in their paper they target predictions, the general approach includes other speech acts such as betting, promising and commanding – see Belnap et al. (2001). Moreover, in order to classify predictions as assertions they put at work the notion of vindication they borrow from Brandom's normative analysis of assertion – explicitly referred to in Belnap & Green (1994, footnote 21). Vindication is then understood as a pragmatist feature that leads to the credit or discredit of the predictor – or, to use Brandom's (2000, chapter 3) terms, to determine if he or she is reliable or unreliable. Their notion of vindication, which they call a pragmatist one, aims at avoiding the syntactic (and semantic) “unpacking” of embedded temporal operators that might occur in the prediction under scrutiny.

However, because predictions are assertions after all, and assertions should obey the laws of logic, they conjugate the pragmatist conception of vindication with a model-theoretic framework for temporal logic expanded with a tree structure in the way mentioned above. The resulting temporal logic follows Carneades's view that the formal truth⁸⁹ of the law of Excluded Middle can be upheld if we are prepared to accept that the disjunction is made true by events happening in the future.

This strategy neglects the fact that, as Brandom (2000, chapter 8) points out, declarations of reliability are the result of an attribution process that takes the form of a language game shaped by games of giving and asking for reasons. This takes us to our own perspective, whereby the process of vindicating is understood as a dialogical interaction.

As discussed, the dialogical perspective does not understand predictions by adding a (model-theoretical) semantics to the pragmatics specific to the speech acts involved; rather, it understands them by associating the process of vindicating to a set of rules that bestow the dialogical meaning explanation of predictions.

More generally, the main philosophical tenet of the dialogical approach rejects the usual reductionist ways to understand the interface between semantics and pragmatics. Kuno

⁸⁹ For a thorough discussion of Carneades's position and his distinction between factual and formal truth see Vuillemin (1984, chapter 11). We owe this reference to Thomas Bénatouïl (Lille U. STL).

Lorenz's general dialogical principle is that the interface between semantics and pragmatics should be understood in the following way:

neither as the result of the semantization of pragmatics—where deontic, epistemic, ontological, and temporal constraints become truth-functional operators; nor as the result of the pragmatization of semantics—where a propositional kernel, when put into use, is complemented by moods yielding assertions, questions, commands and so on. (Lorenz 2011, pp. 519–520)

In short, Lorenz's view (2010, pp. 71–79) is that the differentiation of semantic and pragmatic layers is the result of the articulation within one and the same utterance: each utterance displays in principle both features, as it signifies (semantic layer) and communicates (pragmatic layer).

Per Martin-Löf (2014) also proposed an analysis of Aristotle's Sea Battle challenge which stresses, like Carneades, an analogy between situations where the truth of a past event is indeterminate and situations involving future contingent events.⁹⁰ His reconstruction has been developed within his own Constructive Type Theory (CTT), which, as he pointed out in recent texts, can be linked to a dialogical interpretation of the notions of epistemic assumption and assertoric knowledge.⁹¹ Martin-Löf's analysis highlights the hypothetical nature of this kind of assertion. The proposal amounts to analysing the case of future (and past) contingents as a special case of the truth of empirical propositions, where the notion of proof is extended by including *empirical quantities*. According to such an approach, sentences such as *There was a sea battle* and *There will be a sea battle*, express both an empirical quantity – more precisely, a non-canonical proof-object within the set *Bool*. Still, despite the proof-theoretical insights on empirical propositions such a reconstruction provides, it is unlikely that this corresponds to the views of ancient or medieval philosophy on empirical propositions.

Be that as it may, the CTT framework allows to render the hypothetical structure of future contingents, which when set in the dialogical framework of Immanent Reasoning, opens the door to the study of the speech acts targeted by Belnap et al., which go beyond just predictions. This is still part of an ongoing work in progress, which also displays the intertwining of historic and systematic studies in philosophy of logic, so cherished by Göran Sundholm, and that also shapes the main traits of the research at the Ecole Dialogique de Lille.

Let us finish our essay by paraphrasing a beautiful remark by Steedman (1997, p. 898) in his chapter on temporality:

The large and ramifying literature on the matter in very diverse fields will sometimes require to focus in some of them and there might be a danger of briefly losing sight of others. However, the different perspectives will meet up again as the enquiry proceeds, for historians of philosophy, philosophers of science, logicians, and linguists are linked together in this collective endeavour towards understanding the manifold ways temporality affects assertion.

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⁹⁰ Martin-Löf (2014) discusses here Michael Dummett's (1976, p. 114) example "There was either an even or an odd number of geese which cackled on the Capitol", which refers to the date of the 18th of July in 390 BCE, the day the Gauls attacked Rome. This example resembles those discussed by Carneades, who, as mentioned above, pointed out that in such kinds of situation, which can be placed not only in the past but also in the future, we cannot in the present determine the truth of the event involved since the (causal) traces are either lost or not yet available.

⁹¹ Cf. Martin-Löf's (2017, 2019) – for a detailed development of this interpretation see Klev (2023).

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Appendix

SR0 (starting rule): A play starts with a player stating a proposition called the thesis; that player becomes the Proponent (**P**) and the move is labelled move 0. The other player—the Opponent (**O**)—chooses a repetition rank determining how many times she is allowed to challenge or defend any move in a play. It is usually enough for **O** to choose a repetition rank of 1 ($m := 1$), it is move 1. **P** then chooses a repetition rank: 2 is usually enough ($n := 2$); it is move 2.

SR1i (intuitionistic game-playing rule): After the repetition ranks have been chosen, each move is a challenge or a defence in reaction to a previous move, in accordance with the rules for **local** dialogical meaning explanations. Each player can challenge the same previous move at most n times, where n is the player's repetition rank, or defend against the adversary's *last unanswered* challenge.

SR2 (formal Rule for plays involving Belnap's temporal operators)

1. **P** may state an elementary proposition A at the temporal context m_i/h_j , only if **O** stated first A at m_i/h_j or at m_i/h_k provided histories h_j and h_k cross the same moment m_i .
2. **P** may choose any moment m_k/h_j , such that $m_i < m_k/h_j$ for the future or $m_k < m_i/h_j$ for the past, whereby m_i is the utterance moment at which the choice has been taken (transitivity of the temporal order with regard to the future and the past).
3. In the case of material dialogues some specific moments and histories might have been stated by **O** from the outset: SR2.1 and SR.2.2 apply to these moments and histories.

SR3 (winning rule): The play ends when it is a player's turn to make a move but (s)he has no available move left. That player loses and the other player wins.

In the dialogical approach validity is defined via the notion of *winning strategy*, where winning strategy for **X** means that for any choice of moves by **Y**, **X** has at least one possible move at his disposal such that (s)he (**X**) wins:

- **Validity (definition):** A proposition is valid in a certain dialogical system iff **P** has a winning strategy for this proposition set as thesis of the dialogue

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II. 2 Dialógica, Argumentación y Modalidades Déonticas

II. 2. 1 The Fruitfulness of the Neglected Islamic Theory of Argumentation

The domain of Islamic thought and intellectual history boasts an important body of studies relevant to the Arabic philosophy of language, as well as a growing interest in Islamic argumentation theory and practice.⁹² There remains, however, a dearth of volumes which pool research from both areas and examine them together.

Filling this gap is more critical than ever. Indeed, in our time, significant work is being conducted in argumentation theory, but little of it draws from, or relates to, the rich intellectual traditions of Islam (exceedingly few argumentation specialists have heard, for example, of Islamic traditions of dialectical theory [*jadāl, munāẓara, ādāb al-baḥṭh*], much less benefited from the millennium of discussions and solutions they contain).⁹³

With this in mind, the main objective of this volume of *Methodos* is to provide a venue for studies of hermeneutics, linguistic analysis, and deductive reasoning in the theory / practice of argumentation relevant to the Arabic philosophy of language, including contributions on:

- (1) theories which are geared towards argument (e.g., dialectical justifications and objections, linguistic fallacies, strategies and protocols for engagement on particular issues, etc.);
- (2) the dynamic of argument in shaping concepts and theories (e.g., explaining how systematic argument fashioned certain scholars' solutions); and
- (3) theories "in action" (e.g., argument analyses of the historical, scholarly dialectic on a certain problem).

The papers that constitute our volume offer new insights into the intertwining of these three aspects - including a critical edition of an important text on the issue – that will make of this volume of *Methodos* a reference in the field.

Although the overall discussion intends to be fairly intelligible in its own right – in each individual contribution, as well as collectively—it may nonetheless be helpful to step back a bit from the objectives pursued in each individual paper and to situate them in the larger context of historical and theoretical issues, within which they take shape.

⁹² See Miller (1984, Rep. 2020), Hasan (1986), Hallaq (1997, 2009), Young (2017, 2021, 2022), Rahman, Iqbal & Soufi (2019).

⁹³ See for example the book on the *Dialogical Roots of Deduction*, by Duthil Novaes (2021, p. 140-143) where it is suggested that because of the predominance of Avicenna's approach to logic the dialogical conception of deduction in the Arabic tradition lost the importance it had during the times of the Arabic commentators of Aristotle – it seems that the author relies on Miller's (1984, Rep. 2020) view that Islamicate dialectical theory was directly dependent upon the Aristotelian sources. Unfortunately, this account, that dedicates only three pages to the wealth of authors and hundreds of texts on dialectical theory within the Islamicate tradition, does not consider the criticisms to Miller's view by Hallaq (1997) and Young (2017, 2021). More importantly, Duthil Novaes's suggestion overlooks the later developments of dialectical theory, whereby argumentation was shaped by post-Avicennian logic, and as discussed below, lead to the formulation of the universal "protocols for dialectical enquiry and disputation" (*ādāb al-baḥṭh wa-l-munāẓara*). However, Duthil Novaes's text at least acknowledges the existence of Islamicate dialectical theory, whereas most of the contemporary standard literature on argumentation and logic does not. Just to mention one salient reference book, see the otherwise *excellent Handbook on Argumentation* by van Eemeren et al. (2014), where in chapter 2, which yields an historical overview on ancient and medieval theories of argumentation, the Islamicate tradition is totally absent.

We will start by situating Arabic Argumentation Theory within the Islamicate main division of sciences, then we will proceed by some very brief chronological remarks, and finally we will point out at some general methodological features.

1 Islamicate Dialectical Theory within Rational and Transmitted Sciences

Medieval Muslim scholars writing in Arabic distinguished between purely “rational sciences” (*‘Ulūm ‘Aqliyya*) – that is, sciences whose primary wellspring was the human intellect (such as philosophy, logic, and astronomy) – on the one hand and “transmitted sciences” (*‘Ulūm Naqliyya*) – that is, sciences which, though eminently rational, were ultimately anchored in scripture (such as theology, law, and legal theory) – on the other.⁹⁴

As far as argumentation theory is concerned, the *‘Ulūm ‘Aqliyya* included peripatetic logic (*manṭiq*), dialectic (*jadāl*), rhetoric (*khiṭāba*) and sophistry (*safsāṭa*), *‘Ulūm Naqliyya* included legal theory (*uṣūl al-fiqh*), juristic dialectic and grammatical dialectic (*jadāl / khilāf*), and, at a later date, the “protocols of dialectical inquiry and disputation” (*ādāb al-baḥṭh wa-l-munāzara*). Both “rational” and “transmitted sciences” developed sophisticated theories of argumentation and both branches devoted a great deal of attention to the logical and semantic structure of arguments.

Whereas the study of argumentation in the purely “rational sciences” mostly focused on its logical architecture (within the framework of peripatetic logic), the “transmitted sciences” were concerned first and foremost with moves that were deemed to be legitimate or illegitimate from the point of view of the constitution of meaning.

It is within the transmitted sciences that Islamicate thinkers developed their most original contributions to argumentation theory far beyond the Aristotelian origins. As pointed out in Young (2021, section 2.1):

“In fact, despite contemporaneous translations and commentaries on Aristotle’s *Topics* and *Sophistical elenchi*, and continuing philosophical inquiry culminating in expansions on the Aristotelian tradition (including dialectic) by al-Fārābī and Ibn Sīnā, there was surprisingly little discernible impact on the dialectic of jurists and theologians. The influence of Aristotle – if at all traceable and despite the prominence of syllogistic reasoning in later dialectical disciplines (including the juridical) – is difficult to detect outside of philosophical dialectic. Although similar in some broader features and objectives, the dialectic of religious scholars (especially jurists and theologians) was not the dialectic of Aristotle but something at once indigenous and distinctly Islamic.”

The development of dialectical theory within the “transmitted sciences” involved not only identifying logical moves, but arguments and sub-arguments having to do with the dynamics of meaning: namely, how different situations and different contexts shape the meaning of an expression. Furthermore, as we will briefly discuss below, the intertwining of logical and semantic concepts within *‘Ulūm Naqliyya* is to be understood as being constituted by interaction rules within a general argumentative dynamic framework.

⁹⁴. The division is not mutually exclusive. For instance, the post-classical « *ādāb al-baḥṭh wa-l-munāzara* » (“protocols of dialectical inquiry and disputation”) was, by design, a universal discipline (equally applicable to, e.g., philosophy as to theology and law). That being said, it stands to reason to regroup it with the « transmitted sciences » because – though drawing much from peripatetic logic – it is derivative of the juristic dialectical tradition, and far more similar to it than to Aristotle’s dialectic.

Indeed, one of the most distinctive features of argumentation in the « transmitted sciences », and Islamic Legal Theory in particular, is that they were conceived by their elaborators as dynamic systems of dialectical rules whose main purpose was to provide, by means of rational arguments and on rational grounds, solutions to new and perennially contended problems based on the specialists' understanding of the primary (revelatory) sources. The degree of complexity and sophistication Islamic juridical dialectic reached in this pursuit is nothing short of astounding.

As a matter of fact, certain 11th century manuals of juristic dialectic would come to incorporate collections exceeding three hundred argumentative patterns based on either destructive or cooperative moves, a virtual treasure trove of dialectical justifications, objections, and responses – see Young (2017), (2021) and (2022). Their study not only provides new perspectives on the understanding of illegitimate moves, but solutions for rectifying arguments prone to possible errors and pitfalls. Such works may be said to comprise a double innovation: on the one hand, rather than propounding two dialectics, one destructive and one constructive, both moments are internalized in a single debate wherein contesting and collaborative moves constitute an inquisitive progression towards establishing the truth; on the other hand, they offer a plethora of contesting moves which also engender multiple forms of classification with respect to possible responses. Note that, different to other kinds of debate, constructive responses may in fact be prompted by the challenger, who might in this way collaborate in the search for solutions.

2 Some chronological remarks⁹⁵

The dialectical practice and theory of Law is, amongst Muslim jurists, perhaps almost as old as Islamic Law itself. Thus, the evolution of the theory and practice of Islamic dialectical theory over some 1,400 years is complex, manifold and diversified. Moreover, only a handful of scholars in the past five decades or so have begun to explore, analytically, its vast literature.⁹⁶

At the present stage of the research in the field it is very difficult to pinpoint the chronological emergence of schools and periods with precision. However, the following provisional periodization and listing of main authors on dialectical theory proposed in Young (2021, section 2) might be useful.⁹⁷

(1) From proto-system to full-system disputation theories, first/seventh through fourth/tenth centuries.

Right from the very beginnings of Islamic thought, throughout the second/eighth century, several religio-political, theological, and juristic approaches emerged in contexts of regular and public disputation which eventually configured different schools. By the second/eighth century A *proto-system* displaying common disputation patterns is clearly recognizable,

⁹⁵ The main content of this section stems from Young (2021, section 2).

⁹⁶ See: Miller (1984, Rep. 2020), Hallaq (1990, 1997), Belhaj (2010), Karabela (2010), Dziri (2015), El-Rouayheb (2015); Young (2017, 2021, 2022).

⁹⁷ A caveat is due: we will only mention briefly some few of the leading thinkers and schools in order to place the authors discussed in the contributions of the present volume in the historical context of the development of the Islamicate Dialectical Theories – the diligent reader can find extensive discussions in Young (2021) which provides the main text from which these brief remarks have been extracted from and the references listed in precedent footnote.

particularly in debates involving Law, Theology, and Grammar. Here we should mention the dialect protocols practiced by such master jurists as Abū Ḥanīfa (d. 150/767) and his companions, the sixth Twelver Shī‘ī Imām Ja‘far al-Šādiq (d. 148/765), and the pioneering legal theorist al-Shāfi‘ī (d. 204/820). Refutational treatises and assessments of juristic disagreement (*ikhtilāf*) multiplied.

Full-system dialectical theories emerged in the third/ninth century, throughout the fourth/tenth century. Later developments include the work of pre-eminent theologians such as the Ash‘arī Abū Bakr al-Bāqillānī (d. 403/1013) and the Mu‘tazilī theorist Abū l-Ḥusayn al-Baṣrī (d. 436/1044). Shī‘ī rhetorician Ibn Wahb al-Kātib (fl. c.335/947) formulated a belletristic *jadāl* theory linked with rhetoric and the arts of eloquence, and, in his encyclopaedic history, the *Kitāb al-bad’ wa-l-ta’rīkh* (“Book of the beginning and history”), Abū Naṣr al-Muṭahhar al-Maqdisī (fl. c.355/966) included a preface on *naẓar* (rational enquiry) and *jadāl*.

(2) Dialectical theories and the logical turn: From the further development of dialectal theories (within Law, Grammar and Theology) to the emergence of a universal debate theory, fifth/eleventh through seventh/thirteenth centuries

This is the period where juristic *jadāl* achieved a mature stage from both, the dialectical and the logical point of view. In relation the latter, there were two movements, namely one that incorporated into the dialectical framework Aristotelian logic and the other post-Avicennian logic.

Among the main representatives of the development of legal dialectic salient are the figures of the Shāfi‘ī theorist Abū Ishāq al-Shīrāzī (d. 476/1083) and two of his students, the polymaths Abū l-Walīd al-Bājī and Ibn ‘Aqīl (d. 513/1119) of the Mālikī and Ḥanbalī schools, respectively; and the Shāfi‘ī jurist-theologian al-Juwaynī (d. 478/1085). Al-Juwaynī’s most influential student was certainly the polymath al-Ghazālī (d. 505/1111), who was credited of integrating Aristotelian logic into the dialectical framework

At the same time that the Aristotelian logical turn was taking place a second wave coming from the east fused dialectic and post-Avicennan logic. This fusion is clearly present in the work *Mukhtaṣar* (abridgment by the Mālikī grammarian and legal theorist Ibn al-Ḥājib’s (d. 646/1249), which was the abridgement of his own book *Muntahā l-su’l wa-l-amal fī ‘ilmay al-uṣūl wa-l-jadal* (“The utmost wish and expectation concerning the sciences of legal theory and dialectic”). Al-Ḥājib’s work, attracted in the following centuries, more than ninety commentaries and forty supercommentaries and glosses.

Other significant contributions in juristic *jadāl* were made by the Shāfi‘ī jurist-theologian Sayf al-Dīn al-Āmidī (d. 631/1233) and the Ḥanbalī theorist Najm al-Dīn al-Ṭūfī (d. 716/1316).

Important studies on *‘ilm al-khilāf* (the science focused on the art of examining and resolving the disputes between schools) were written towards the defence of one’s school (*madhhab*) also in this period, including a popular didactic poem by the Ḥanafī polymath Abū Ḥafṣ ‘Umar al-Nasafī (d.537/1142), and, among others, the works of the Ḥanafī dialecticians Raḍī al-Dīn al-Nīsābūrī (d. 544/1149) and Rukn al-Dīn al-‘Amīdī (d. 615/1218), and the influential Shāfi‘ī polymath and disputationist Fakhr al-Dīn al-Rāzī (d. 606/1210) – let us recall that

The teachings of al-Nīsābūrī and of his student al-‘Amīdī, which implemented the infusion of post-Avicennan and post-Rāzian logic into *jadāl/khilāf*; influenced the dialectical theory of the Ḥanafī dialectician Burhān al-Dīn al-Nasafī (d. 687/1288). Al-Nasafī’s *al-Fuṣūl fī l-jadal* (“Chapters on dialectic”) was profusely commented, including the thorough criticism by the famous Ḥanbalī theologian-jurist Ibn Taymiyya (d. 728/1328). The logical turn on dialectics that shaped the

works on dialectic of al-Nīsābūrī and al-‘Amīdī, lead to the formulation of the universal “protocols for dialectical enquiry and disputation” (*ādāb al-baḥth wa-l-munāzara*) by the polymath Shams al-Dīn al-Samarqandī (d. 722/1322), who also commented Nasafī’s *al-Fuṣūl fī l-jadal*.

Al-Samarqandī’s universalist *al-Risāla fī ādāb al-baḥth* (“Treatise on the protocols of dialectical inquiry”) launched the study and implementation of a new epistemological reasoning pattern, that attracted more than fifty commentaries, supercommentaries, glosses, and super-glosses (right through to modern times), and prompted numerous, often eminent, scholars to write and comment upon additional *ādāb al-baḥth* compositions, including the ones by Naṣīr al-Dīn al-Ṭūsī (d. 672/1274) and al-‘Allāma al-Ḥillī (d. 726/1325).

Jadal, assessments of disagreement (*ikhtilāf*) proliferated in grammar, law and theology and experienced extensions to astronomy, in a work by the great polymath al-Bīrūnī (d.440/1048), while the juristic *khilāf* method was extended to medical theory in a work by the physician Najm al-Dīn Ibn al-Lubūdī (d. c.663/1265).

As we will discuss in the methodological section of our introduction the epistemological and logical methods that shaped disputation theory after the logical turn included *talāzum/mulāzama* (entailment), *tanāfin/munāfāt* (mutual exclusivity), and *dawarān* (concomitance), which concerns the identification of a cause).

(3) The spread and predominance of ādāb al-baḥth wa-l-munāzara, eighth/fourteenth century to the present

As already mentioned al-Samarqandī’s *ādāb al-baḥth* launched an extensive and deep research on this form of dialectical inquiry which prompted in the centuries to come a large number of commentaries and glosses and supercommentaries and superglosses by among others, the theologian-jurist ‘Aḍud al-Dīn al-Ījī (d. 756/1355), Mas‘ūd al-Shirwānī al-Rūmī (fl. c.840/1437), Muḥammad al-Tabrīzī al-Ḥanafī (d. 900/1494), ‘Abd al-Rashīd al-Jawnpūrī’s (d.1083/1672), whose *al-Rashīdiyya*, is still taught today in Pakistan and India, and the polymath al-Sayyid al-Sharīf al-Jurjānī.

Works on *ādāb al-baḥth* by the Mamlūk polymath Zakariyyā’ al-Anṣārī (d.926/1520), and by the Ottoman scholars Taşköprüzade Ahmed Efendi (d. 968/1561) and Mehmed Efendi Birgivi (d. 981/1573, had appeared by the close of the tenth/sixteenth century.

There was also intense creative work around Ibn al-Ḥājjib’s *Mukhtaṣar*. Commentaries and glosses were often written by key scholars such as al-‘Allāma al-Ḥillī, the philosopher Quṭb al-Dīn al-Shīrāzī (d. 710/1311); and the distinguished polymath Sa‘d al-Dīn al-Taftāzānī (d. 792/1390).

Juristic *jadal* material, the *khilāf* tradition (and likewise the genre of *ikhtilāf*) also continued to appear in works by, amongst others, Tāj al-Dīn al-Subkī (d.771/1370), Badr al-Dīn al-Zarkashī (d. 794/1392), and Ibn Khaldūn (d. 808/1406).

Commenting on older works continued into the twentieth century, which also witnessed seven new *ādāb al-baḥth* treatises; that of Mauritanian scholar Muḥammad al-Amīn al-Shinqīṭī (d.1973), for example, is popular to this day.

During the second of the periods mentioned above, when dialectical theories evolved in their mature forms, *qiyās* systems emerged as one of among the most salient forms of inference that were elaborated and continuously refined by the mainstream Sunni jurists.

In fact, it is out of the tradition of *qiyās* together with other forms of Islamicate dialectics that al-Samarqandī's *ādāb al-baḥṭh wa-l-munāzara* arose. Moreover, the logical and epistemological shaping of dialectical theories deepened and extended the methods for identifying causes developed within *qiyās*.

Most of the contributions of the present volume relate directly or indirectly to *qiyās* and also to the logical shaping of dialectical debates that extended and developed further its legal origins.

Thus, let us first introduce briefly to this form of dialectical inference and then discuss some systematic points on the logical and epistemological notions involved in the logical turn, which involved discussions on (necessary) implication and several elimination rules (such as *modus ponens*, *modus tollens* and disjunctive syllogism).

• *Qiyās*

The many debates and elaborations on *qiyās*, which might be translated “correlational inference” (more often, if less accurately, “analogy”),⁹⁸ together constitute one of the finest outcomes of the argumentative approach to legal reasoning within Islamic Law. A particularly lucid example is the systematization of the respected Shāfi‘ī theoretician Abū Ishāq al-Shīrāzī (1003-1083 CE) mentioned above, upon which the following is based.⁹⁹

The aim of *qiyās* is to provide a rational ground for the application of a juridical ruling to a given case which has not been directly and unequivocally pronounced upon in the primary juridical sources (i.e., the Qur’ān, Sunna [Prophet’s example], and *Ijmā‘* [consensus]). It combines heuristic (and/or hermeneutic) moves with logical inferences; and the archetype of *qiyās* adheres to the following pattern:

In order to establish whether or not a given juridical ruling (*ḥukm*) applies to a novel or contended case, called the branch-case (*far‘*), we look for a relevant, authoritatively determined root-case (*asl*) bearing that ruling in the primary sources of law (Qur’ān, Sunna, and *Ijmā‘*). We next attempt to determine the property (*wasf*) or set of properties in the root-case which constitutes the causal condition, or occasioning conditions, or *ratio legis* (*‘illa*) giving rise to its ruling. If it “probable” (*ẓann*) that this property occasions the ruling, and it is shared by the branch-case, we may infer that it is equally productive of that ruling in the branch-case. The novel or contended branch-case thus falls under that juridical ruling, and the range of its application is extended.

When the legal occasioning factor (*‘illa*) is made explicit by the sources, or is capable of being rationally inferred by adequately identifying the relevant property or set of properties, we may proceed via a “correlational inference of the cause” (*qiyās al-‘illa*). The crucial step underlying this form of argumentation is thus the identification of the legal causal condition (*‘illa*) that gives rise to prohibition in the authoritative root-case.

⁹⁸ See Young 2017, p. 10; the author’s choice of “correlational inference” renders a narrower sense more consonant with the Shāfi‘ī approach.

⁹⁹ A landmark on the subject of *qiyās* is Hasan (1986). Young (2017) provides a summary of al-Shīrāzī’s systematization of *qiyās*; and on this basis Rahman, Iqbal, & Soufi (2019) develop a logical analysis – see too Iqbal (2022).

When the legal occasioning factor (*illa*) is neither made explicit by the sources, nor capable of being rationally inferred, however, we might next resort to “correlational inference of indication” (*qiyās al-dalāla*), which, in lieu of the *illa*, is based on pinpointing specific, relevant parallelisms between sets of rulings (and thus inferring that whatever the *illa* may be, it is shared by such cases). Should even this prove infeasible, we might finally resort to (the highly contentious) “correlational inference of resemblance” (*qiyās al-shabah*), which is based merely on the presence of shared, but either non-causal or indeterminable, properties.

Thus, *qiyās al-dalāla* and *qiyās al-shabah*—which, far more than *qiyās al-illa*, merit the label “arguments by analogy” (or, better yet, “arguments *a pari*”)—are put into action when the *illa* grounding the application of a given ruling is not known. The plausibility of a conclusion attained by parallelism between rulings (*qiyās al-dalāla*) is considered to be of a higher epistemic degree than a conclusion obtained by resemblance in respect to some set of (relevant) properties (*qiyās al-shabah*). And conclusions by either have a lower epistemic standing than conclusions inferred via a known, pinpointed, and share legal occasioning factor (*qiyās al-illa*).

It is important to keep in mind that the whole of *qiyās* procedure involves two main dialectical steps: one concerns the epistemological and hermeneutical task for identifying the properties claimed to occasion the ruling, the second one, the logical one, that assumes that the first has been set. The first involves sub-arguments and counterarguments that are not formal, but contentual or material, the second involves the dialectical rules for logical reasoning. Contesting some legal argument, might involve contesting a logical mistake, but this concerns rather fallacies in the sense of the traditional theory of syllogism and they constitute a group by their own known as *mughāliṭ* or *mughālaṭa*. In the context of legal reasoning, the most relevant form of objection concerns the first step whereby the semantic link between the property and the ruling is claimed to hold. The sub-arguments and counterarguments concerning this step are not formal, they are rather probable than certain and thus always prone to further revision. Agreement can be settled at some point, but the whole process can start again – cf. Young (2017).

A cardinal feature of al-Shīrāzī’s take on *qiyās al-illa* is his particular notion of efficiency (*ta’tḥīr*), which tests whether the property \mathcal{P} purported to be efficient in occasioning the juridical ruling at stake is indeed so. For al-Shīrāzī, *ta’tḥīr* consists of two complementary procedures:

- co-presence (*ṭard*): whenever the property is present, the ruling is also present
and
- co-absence (*aks*): whenever the property is absent, the ruling is also absent.

While co-presence examines whether ruling \mathcal{K} follows from verifying the presence of property \mathcal{P} , co-absence examines whether exemption from ruling \mathcal{K} follows from verifying the absence of \mathcal{P} .

This test of a property’s causal efficiency is elsewhere and more commonly called “co-presence and co-absence” (*al-ṭard wa’l-aks*) or “concomitance” (*dawarān*), and listed among the “modes of causal justification” (*masālik al-ta’līl*) – see Young (2019) and Hasan (1986), p. 315-330.

As for “efficiency” (*ta’tḥīr*), some authors such as al-Ghazālī deemed it to be a direct designation of the cause (*illa*) by either univocal source-text (*naṣṣ*) or consensus (*ijmā’*), while others held different notions – see Hasan (1986), p. 272-3, 284).

The logical turn focused on the semantic and logical links between property and

effect (rather than restricted to legal effect) and more generally on the logical characterization of concomitance.

Remarks on the logical turn and their dialogical underpinnings

As mentioned above the logical shaping of the dialectical theory focused on the notions of

talāzum/mulāzama (entailment),
tanāfin/munāfāt (mutual exclusivity), and
dawarān(concomitance), which concerns the identification of a cause).¹⁰⁰

The first notion has been quite often also translated as implication or conditional, however, it is important to observe that there are different intertwined levels of analysis of the links between antecedent and consequent, namely a semantic link and one that can be rendered as some kind of logical connective.

The semantic level concerns the links between concepts, such as the semantic link between *Living Being* and *Knowing Being*, to use a frequent example present in the relevant literature. This level does not directly render a proposition but rather the semantics conditions out of which a proposition obtains: *Knowing Being entails Living Being*:

For any instance x of the concept *Knowing Being* the proposition *Living Being* can be obtained by a semantic process that renders instances of the latter out of instances of the former.

In other words, at this level of analysis *Living Being* can be conceived as a propositional function defined over *Knowing Beings*.¹⁰¹

At a further level of analysis this either constitutes a propositional implication such as *if it is knowing then it is living*, or some universal proposition such as *Every knowing being is a living being*.

Now, notice that the connective level, presupposes the semantic level: only when we know how one concept is dependent upon another one, can we render the corresponding logical connective or quantifier.

If we place both levels of analysis, as we should, in a dialogical setting, the point is the following:

if someone asserts that something is knowing, then the contender can ask him to further assert that it is a living.

moreover, the assertion of the implication *If it is knowing, it is living*, presupposes that the semantic link between *knowing* and *living* has been previously endorsed!. If such an endorsement takes place, it also the outcome of a previous dialogical interaction. We might call this, more generally the *dialogical meaning explanation* of the logical constant (including connectives and quantifiers) – cf. Clerbout&McConaughy (2022).

¹⁰⁰ See Young's contribution in the present volume.

¹⁰¹ In contemporary notation : *Living Being* \in prop (x : *Knowing Being*).

once the logical constant has been constituted out of its dialogical meaning explanation, those moves can be identified, that a higher level, namely the strategic level, can be rendered as elimination rules (modus ponens for the case of implication).

From this perspective the semantic and logical analysis is a consequence of the dialogical interaction underlying a debate on meaning and its possible extensions. Many contemporary reconstructions, that appeal to relevant, modal or other forms of so called non-classical logics, seem, at least from one of the authors of the present introduction, to oversee that it is the dialogical constitution of meaning, that leads the way in the logical shaping of dialectics. As dialogicians often put it: the dialogical conception of logic is not logic plus dialogical interaction, but it is dialogical interaction that shapes both meaning and logical reasoning.¹⁰²

Similar applies to the notion of *tanāfin/munāfāt* (mutual exclusivity), and *dawarān* (concomitance).

In relation to *tanāfin/munāfāt* the dialogical meaning explanation requires the endorsement of the incompatibility of two concepts such as *Whiteness* and *Blackness*, *Existence Non-Existence*, out of which the dialogical meaning explanation of the exclusive disjunction results, namely if the Proponent asserts *It is white or black*, and the interlocutor asks which of both is the case, the Proponent can choose one of both, but only exactly one.

Furthermore, if one of the interlocutors also asserted that one of the sides is not the case, say it is not white, then the antagonist can force him to assert that it is black. This is what the disjunctive syllogism (i.e. one instance of elimination rule for disjunction) expresses when its dialogical roots are abstracted away.

The semantic level at stake in the case of *dawarān*(concomitance) is provide a dialogical meaning explanation that makes it plausible to conceptually link the effect to its cause, and this with different degrees of certitude. For instance, once it has established with some degree of certitude that drinking scamonny leads to purgation, different inferential patterns can be further applied.

More generally, the main insight, so we claim, is that different to present in mainstream contemporary Islamicate logical analysis did not seem to practice the syntax first assumption - whereby uninterpreted well-formed lists of signs are given a semantics by means of a metalinguistic device (the interpretation function) that links these lists with a universe of discourse: rather the rules that determine well-formed syntactical expressions are constituted at the same time as conceptual links that give rise to meaningful expressions, and these rules amount to dialogical meaning explanations.

If this is right, the logical turn within the dialogical framework should not be seen as importing a deductive system with its own, say truth-functional, semantics, but rather as the result of a new epistemological process by the means of which the dialogical meaning explanations are generalized beyond the particular conceptual framework that gave rise to them.

4 The contributions

¹⁰² For an overview of the dialogical stance see Clerbout & McConaughy (2022). The classical papers on dialogical logic have been published in Lorenzen & Lorenz (1978). For some recent developments see Rahman et al. (2018), and for applications to the logical analysis of *qiyās* see Rahman, Iqbal & Soufi (2019).

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Ibn Ḥazm on Heteronomous Imperatives. A Landmark in the History of the Logical Analysis of Norms.

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Introduction.¹⁰³

The passionate and staunch defence of logic of the controversial thinker Ibn Ḥazm, Abū Muḥammad ‘Alī b. Aḥmad b. Sa‘īd of Córdoba (994-1064), had lasting consequences in the Islamic world.¹⁰⁴ Indeed his book *Facilitating the Understanding of the Rules of Logic and Introduction Thereto, with Common Expressions and Juristic Examples (Kitāb al-Taqrīb li-ḥadd al-manṭiq wa-l-mudkhal ilayhi bi-l-alfāz al-‘āmmiyya wa-l-amthila al-fiqhiyya)*, composed in 1025-1029, was well known and discussed during and after his time; and it paved the way for the studies of his compatriots Ibn Bājja (d. 1138), Ibn Ṭufayl (d. 1185), and Ibn Rushd (d. 1198), who each gave demonstrative reasoning a privileged place within the methods of attaining knowledge.¹⁰⁵

Unfortunately, as too often in the history of science, Ibn Ḥazm’s innovative perspectives and contributions in logic have been overlooked or considered with an attitude of contempt. On the one hand, his work has been seen, at best, as promulgating the benefits of studying Aristotle’s logic, so that his contribution is assessed as more didactical than conceptual. And on the other hand, those who do examine his innovations often consider them to be mistaken.¹⁰⁶

However, a reassessment of his work on logic has since begun, by delving into the ways the thinker of Córdoba studied the links between deontic and modal qualifications of propositions.¹⁰⁷ In this context an important contribution is Lameer’s (2013) paper on the logical sources of Ibn Ḥazm. He observes (p. 417, n. 1) that—although, strictly speaking, it was al-Fārābī who first drew the parallelism between deontic and modal concepts—it was Ibn Ḥazm who developed it and worked it out in a more precise manner.

In fact, as Lameer elsewhere points out (1994, p. 240), though Al-Fārābī, while paraphrasing the *de Interpretatione*, speaks of the possibility for rephrasing a command in terms of necessity, the convertibility strategy he follows does not achieve the reduction he is after. Indeed, his example: converting “Zayd come over here” into “Zayd must come over here,” does not seem to paraphrase away the deontic component of the command encoded by the expression “must.”

Independently of the success of these attempts, it seems to be the case that these passages by al-Fārābī ground Lameer’s remarks (1994, pp. 240-241, and 2013, p. 417) that both al-Fārābī’s and Ibn Ḥazm’s perspectives appear to be the earliest testimony on record of a transference from deontic to modal concepts.¹⁰⁸ Lameer’s observations can be seen as targeting Knuuttila’s (1981) well-known overview of the history of deontic logic, situating its origins around the 14th century, and Von Wright’s (1981b, p. 3) assertion that deontic logic was born in Leibniz’s *Elementa Juris Naturalis* of 1671. Von Wright’s assertion is based on the fact that Leibniz explicitly states in that work that the

¹⁰³ The present paper shares the formal analysis of deontic modalities with that of Rahman, Granström, Farjami (2019), though it develops further the notion of necessity—both historically and formally.

¹⁰⁴ For a recent comprehensive volume on his work, see Adang, Fierro, and Schmidtke (2013).

¹⁰⁵ For an overview of the reception and reshaping of the Aristotelian Peripatetic work on logic, see Hasnawi and Hodges (2016).

¹⁰⁶ As indicated by Chejne (1984, p. 2), contempt towards the logical work of Ibn Ḥazm was also present in its reception by Eastern philosophers who accused him of deviating from Aristotelian logic and of dabbling in things beyond his capability.

¹⁰⁷ See also Guerrero (1997, 2013), and Lameer (2013).

¹⁰⁸ Lameer acknowledges Gutas (1988, p. 270; 2014, p. 306) for the reference to Ibn Ḥazm.

transference between deontic and modal concepts can be carried out in the following way:

Modal	Deontic
possible, it is intelligible.	(<i>licitum</i>) permissible
necessary, its negation is not intelligible.	(<i>debitum</i>) obligatory
possibly not, its negation is intelligible.	(<i>indebitum</i>) omissible
impossible, it is not intelligible.	(<i>illicitum</i>) forbidden

It is **known** that Leibniz's work on legal reasoning was inspired by his studies in Roman Law and Stoic Logic.¹⁰⁹ Indeed it was the Stoics who, from the background of a dynamic ontology constituted by events and actions, proposed to extend or perhaps even substitute Aristotle's relational approach to necessity with a propositional one,¹¹⁰ whereby connectives and inference rules played the role of the Aristotelian term-relation governed by the metaphysics of essences and the logic of the syllogism.¹¹¹

The propositional perspective on causal necessity allowed the Roman Jurists, and Cicero in particular, to transfer different forms of natural causality into the realm of legal reasoning. This contributed to the inception of the notion of *ratio legis*, the cause grounding a juridical decision. Perhaps one could understand the Stoic theory of signs, not as some early kind of formalism, but as a way to gather a general notion of cause-effect applying to both norms and events.¹¹² Now, however, two main problems arose.

1. While the predicative approach of Aristotle assured contentual relevance, the propositional construction made it difficult to tie cause and effect with bare truth-functional means. Recall the well-known disputes on how to define an implication that expresses causality.
2. The question was raised of the gap between norms as prescriptions (and their actualization), and norms as propositions understood as bearers of truth—particularly in the context of legal reasoning.

These gaps evoke the broader epistemological problem of how to link theory and experience or theory and praxis: **does it make sense to speak of *practical syllogism*?** The Arabic tradition, particularly sensitive to issues concerning *praxis*, developed the insight that the interface theory-praxis should be studied from the perspective of the dyad *prescription-actualization*, precisely in the contexts most cherished by the Stoics; namely, ethics and jurisprudence. This new insight of the Arabic tradition led to the following bold steps:

¹⁰⁹ Cf. Armgardt (2008), Magnier (2013), Rahman (2015).

¹¹⁰ For a thorough discussion on Aristotle's relational view on modalities see Malink (2013), who also proposes a formal reconstruction based on what he calls a *mereological pre-order semantics*.

¹¹¹ Bénatouïl (2017) recalls that quite before Łukasiewicz's (1934) famous paper on the history of propositional logic, Brochard (1892, 1912) and Hamelin (1901) not only acknowledged the *propositional turn* promoted by the Stoics but they also discussed if the Stoics proposal amounted to a replacement—or rather an extension—of the Aristotelian metaphysical framework of essences with one rooted in physics and events. **In fact, according to chapter 9 of the *Peri Hermeneias* the notion of modality as applied to events, individualized by some time structure, amounts to a predicative relation between cause and effect. Moreover, in such a context Aristotle seems to think the relation from the event to the cause, rather than the other way round: *if there is rain, there is necessarily a cause (clouds)*, but rain is not necessary! – for a lucid and thorough study on the subject see Crubellier (2010). The Stoics, who arguably preserved Aristotle's time dimension, undertook the task of constituting the Cause-Effect link by means of a **propositional connective rather than by developing some special predicative relation**. The Leibnizian project (at least in the contemporary **reconstruction** promulgated by Kripke and Hintikka), substituted the time structure with the abstract and meta-logical structure of possible worlds. However, in this move causes are, in a manner of speaking, absorbed by the conditions defining a possible world. More precisely, in such an approach modality is not attached to causality *per se*. **Actually there is a growing consensus within the community of experts in ancient philosophy that traditional modalities cannot be understood as the operators studied in contemporary modal logic – see Bobzien (1993), Malink (2013).****

¹¹² Bréhier (1928) proposes a *semiotic* reading of the logic of the Stoics that moves away from the naturalistic interpretations of Brochard (1892) and Hamelin (1901).

- Prescriptions are understood as prescriptions to **do** rather than prescriptions that take us from one state of affairs to another: *Tun Sollen* rather than *Sein Sollen*.
- Not only events but (performances of) actions are first-class denizens of the universe of discourse. Actions and prescriptions display a contentual link that yields a classification of types of actions. Deontic reasoning is reasoning with content.
- Prescriptions to do are embedded in a system of hypothetical judgements involving implications where actions—the actualizations of the prescriptions—are subjects of predication: actions are bearers of qualifications such as law-abiding or law-breaking.¹¹³ Similarly, events are qualified as necessarily happening, or possibly happening, or not happening at all.
- Norms presuppose freedom of choice: a prescription to do presupposes the possibility of choosing between carrying out, or not carrying out, the action prescribed by the norm.
- The principle *all actions are permissible unless proscribed by Law* encouraged the development and application (among the majority of Sunni jurists) of correlational modes of argumentation, subsumed under the rubric of *qiyās*; these sought to regulate into gradually expanding corpora of substantive law the dynamic integration of explicit “updated” deontic qualifications (possibly different to *permissible*) for novel and unresolved problem-cases.¹¹⁴

To be certain, analogies between deontic, temporal, and modal concepts have a long and rich history before their resurgence in contemporary deontic logic.¹¹⁵ Important lacunae nevertheless exist in the literature on its historic development, even in the most recent overviews, and particularly so in relation to contributions developed within Islamic jurisprudence.¹¹⁶ This remains the case despite the fact that there has been work on the influence of Stoicism on Arabic thinkers in general, and on the moral classification of acts as *obligatory*, *forbidden*, *recommended*, *reprehended*, and *neutral*, including studies by van Ess (1964) and Jadaane (1968).¹¹⁷ In fact, Gutas (1994) shows that the conditions for a grounded assessment of the influence of Stoicism on Islamic thinkers are not yet available. Indeed, he makes it very clear that studies such as the ones just mentioned are not backed

¹¹³ The notion of *conditional assertions* provided the ground for further sophisticated developments within the Islamic tradition of implications (including bi-implications), or *sharṭiyya muttasila*, and disjunctives, or *sharṭiyya munfasila*. For a recent, thorough study of the notion of *sharṭiyya* see Hasnawi and Hodges (2016, section 2.4.3, pp. 63-65).

¹¹⁴ For an overview of the *qiyās*-theory of the Shāfiʿī legal theoretician Abū Ishāq al-Shīrāzī (d. 1083), see Young (2017). The archetypal form of *qiyās* is that which is based on a shared ‘*illa*, or “occasioning factor,” which is known or inferred to have triggered the juridical decision (such as *legally valid*) or deontic qualification (such as *forbidden* or *obligatory*) of a known, authoritative case, and thus allows it to be transferred to the new case which shares that same occasioning factor. The “occasioning factor,” or ‘*illa*, is in certain ways analogous to the concept of *ratio legis* mentioned above.

¹¹⁵ In fact, Knuuttila (1993, p. 182) observes that Peter Abelard (1079-1144) and other early medieval philosophers often endorsed an inverted form of Leibniz’s reduction by defining modal concepts by means of deontic concepts. According to this characterization, necessity is taken to be what nature demands, possibility is identified with what nature allows, and impossibility with what nature forbids.

¹¹⁶ See, for example, Knuuttila (1981), and the otherwise excellent essay by Hilpinen and McNamara (2013, p. 14). Though these works discuss the occurrence of deontic concepts in classical Islamic jurisprudence, they do not mention early testimonies to the parallelism between deontic and modal concepts in that tradition.

¹¹⁷ Jadaane (1968, pp. 184-189) discusses and convincingly relativizes Van den Bergh’s strong assertion (1954, reprinted 1987, vol. II, p. 117 of the notes) that the *obligatory*, *recommended*, *reprehended*, and *forbidden* notions of Islamic jurisprudence correspond (respectively) to the Stoic notions of *recte factum*, *commodum*, *incommodum*, and *peccatum*. Van den Bergh also points out (1954, vol. II, p. 118 of the notes) that Islamic theologians coupled the deontic notion of *permissible* with the modality *not logically impossible*. He does not develop the issue any further, however. Notably, Gutas (1994) develops a thorough critical analysis of the hasty assessments made by Van den Bergh and Jadaane.

by evidence stemming from the sources.¹¹⁸

Be that as it may, it seems that it is precisely in the context of Islamic jurisprudence that the contribution of the Arabic tradition to modality and its logic should be studied and pondered. Avicenna, who was not particularly interested in the logic of jurisprudence, might have influenced contemporary studies which focused on the developments of metaphysical rather than of deontic necessity. In fact, a primary aim of this paper is to stress the role of the work of Ibn Ḥazm in developing a notion of deontic necessity deeply rooted in legal normativity. According to our view, the basic units of Islamic deontic logic are what we might call, indulging in terminological anachronism, *heteronomous imperatives*.¹¹⁹ As it turns out, the heteronomy of imperatives within Islamic legal systems is closely related with those of the moral realm. In premodern Islam, as Hallaq (2009) has shown us, there was no real division between the moral and the legal.¹²⁰ However, in the present paper we will focus on the heteronomous imperatives of legal systems rather than on the imperatives when applied to purely moral contexts. In this context the work of Ibn Ḥazm extends the parallelism, stressed by his predecessors, between the necessity of events and that of human actions. Roughly:

- The relation between *Cause* and *Effect* within natural events is paralleled with the relation between *Action* and *Legal Consequence* (e.g., *Reward* or *Sanction*) **within the Law**.¹²¹

According to our understanding, Ibn Ḥazm's parallelism can be rendered **explicit formally by means of a conditional (or hypothetical) structure shared by both deontic and modal propositions. Thus,**

while in the domain of alethic modalities, given some causal conditions, it makes sense to introduce the categories (causally) necessary, impossible and contingent events in order to distinguish between an event that is more likely to happen than another;

in the domain of deontic modalities, the categories obligatory, forbidden and permissible (optional) actions, can be deployed in order to signalize how the performance of different kind of actions lead to different ways of distributing *Reward* and *Sanction* (in the sense of *penalty*).

While developing our point we will ourselves delve into the logical structure of the heteronomous imperatives. This distinguishes our contribution from the existing literature, such as the papers of Chejne (1984), Lameer (2013), and Guerrero (1997, 2010, 2013), which do not provide logical analyses of the deontic concepts put to work by Ibn Ḥazm.¹²² The true antecedent to the present paper is the work of Farid Zidani (2007, 2015),

¹¹⁸ This situation, so far as we know, has not changed substantially since Gutas' remark.

¹¹⁹ We refer to Kant's distinction between an autonomous imperative, that prescribes to act virtuously for the sake of the virtue itself, and a heteronomous, or hypothetical, imperative, that prescribes to act virtuously in view of attaining some kind of benefit or reward. Von Wright (1981b, p. 34) has already suggested that "traditional deontic logic is not a genuine 'logic of norms' but a logic of structures resembling what Kant called hypothetical imperatives." We agree with the last part of the quote and would add that this shows that the traditional logical analysis of norms is indeed quite different from current deontic logic.

¹²⁰ This lack of division may even be observed in the work of the young Leibniz (1678), who was adamantly disposed to run together moral and legal concepts. Recall for example his proposal to define obligatory as "what is necessary for a good person to do."

¹²¹ As pointed out by Zysow (2013) and Young (2019), there was a historically much-debated parallelism between the natural, or "intellective" cause (*'illa 'aqliyya*), and the legal cause (*'illa shar'iyya*). Some argued that the latter could not be treated like the former, and others that it could.

¹²² When completing the final draft of the current article, the authors learned of Joep Lameer's then-forthcoming article "Deontic Modalities in Ibn Ḥazm" (2019). Though complementary to our study, and contributing important philosophical observations and historical remarks, we should note that Lameer's approach is quite different, employing modality as an operator that builds propositions from propositions in order to express deontic modality. Our approach, on the other hand, employs hypothetical judgements incorporating choice, reward, and sanction to performances of actions. This is, we believe, requisite for understanding Ibn Ḥazm's parallelism.

who, so far as we know, was the first to undertake such a task.

I Ibn Ḥazm's Logic of Heteronomous Imperatives.

I.1 The Main Definitions.

Muslim jurists identified five deontic qualifications for an action. Ibn Ḥazm defines them as follows:¹²³

- 1 **wājib, farḍ, lāzim.** Obligatory action is the one which: If we do it we are rewarded. If we do not do it we are sanctioned.
- 2 **ḥarām, maḥzūr.** Forbidden action is the one which: If we do it we are sanctioned. If we do not do it we are rewarded.
- 3 **mubāḥ mustaḥabb.** Recommended permissible action is the one which: If we do it we are rewarded. If we do not do it we are neither sanctioned nor rewarded.
- 4 **mubāḥ makrūh.** Reprehended permissible action is the one which: If we do not do it we are rewarded. If we do it we are neither sanctioned nor rewarded.
- 5 **mubāḥ mustawin.** Evenly permissible action is the one which: If we do it we are neither sanctioned nor rewarded. If we do not do it we are neither sanctioned nor rewarded.

Note that the classification assumes that *reward* and *sanction* are incompatible but not contradictory. Some actions can be neither rewarded nor sanctioned.

Interestingly, Ibn Ḥazm's classification of actions varies in relation to others, such as that of the prominent Mu'tazilite Qāḍī 'Abd al-Jabbār (935-1025), in his *Mughnī* (vols. 11-14), and in his *al-Uṣūl al-Khamṣa* (pp. 79-96).¹²⁴ 'Abd al-Jabbār's classification distinguishes "evil" (*qabīḥ*) actions (the doer deserves blame) from "good" (*ḥaṣan*) actions. Good actions are classified into (i) acts of grace (*tafaḍḍul*), or the recommended (*nadb*), if and only if the doer deserves praise, and the ommitter does not deserve blame; (ii) the merely permissible [or optional] (*mubāḥ*), if and only if neither the doer nor the ommitter deserves blame or praise; and (iii) the obligatory (*wājib*), if and only if the ommitter deserves blame. As pointed out by Hilipinen and McNamara (2012, p. 14), who briefly discuss this classification of 'Abd al-Jabbār, the set seems to be lacking the category of *reprehended* or *excusable* (*makrūh*) actions. However, 'Abd al-Jabbār, instead of distinguishing a special category for the *excusable*, introduces the category of *not obligatory* (*ghayr wājib*), characterizing all those actions for which the ommitter is not blamed – including the *evil*, the *permissible* and the *recommended*.¹²⁵

Be that as it may, 'Abd al-Jabbār's formulation has, in relation to Ibn Ḥazm's, certain desirable properties, but also one which is less felicitous.

- 'Abd al-Jabbār's category of actions that can be omitted has the logical advantage that it allows logical inferences from categories of *the evil*, *the permissible*, and *the recommended* to *the not obligatory*. Ibn Ḥazm's definitions, given above, are more static.

¹²³ Ibn Ḥazm (1926-1930, vol. 3, p. 77); idem (1959, p. 86; 2003, pp. 83-4). Note that Ibn Ḥazm's extension of *mubāḥ*-permissibility into the categories of recommended and reprehended is atypical. To be certain, this innovative subdivision facilitates the comparison (see below) with nearly possible, distantly possible, and purely possible; but it also highlights the underlying values. All forms of "permissibility" have a value; that is, in terms of doing the recommended or not doing the reprehended, both surpass the neutral value of the "evenly permitted," while not yet reaching the value of doing the obligatory and not doing the forbidden. At the same time, neither doing the reprehended nor neglecting the recommended descends below the neutral value of the "evenly permitted," which latter, always above the status of doing the forbidden and neglecting the obligatory, remains steadfastly in the middle.

¹²⁴ We owe this citation to Hourani (1985, pp. 99-102), who extracted the following definitions from the cited texts.

¹²⁵ Hilipinen and McNamara (2012, p. 14) indicate the proximity of 'Abd al-Jabbār's formulation with that of Alexius Meinong.

- The use of “deserve” in ‘Abd al-Jabbār’s definitions is deontically charged. Already in his day, ‘Abd al-Jabbār’s formulation was qualified as circular, so he attempted to respond to the objection by introducing the notion of *correspondence*. What we have, therefore, is that such actions as are obligatory are those that, if omitted, *correspond* to evil (cf. Hourani, 1985, p. 102).

Obviously, Ibn Ḥazm’s definitions do not suffer from this form of circularity, mainly because *reward* and *sanction*—rather than *blame* and *praise*—are primitive incompatible notions. Notice too that one way to see the problem of ‘Abd al-Jabbār’s definitions is that they are purported to define what “good” is, from both the moral and theological point of view.

I.2 Freedom and Heteronomy: *Ought* Presupposes *Can*.

The following approach is based on the insight that the most salient characteristics of the deontic imperatives listed above are:

Assumption of freedom of choice, or *takhyīr*: the fact that an action can be chosen to be performed or not.

The heteronomy of imperatives: the fact that the way actions are qualified by reward or sanction depends upon the choices made.

Both conditions are linked to the idea of responsibility that is at the core of Ibn Ḥazm’s understanding of obligation. This point has been stressed by Hourani (1985, p. 175), as follows:

The fact that concerns us in a historical account is that in all ethical contexts [Ibn Ḥazm] regards man as responsible for his own actions and liable to Reward and Punishment accordingly.

In our understanding, responsibility manifests itself in the fact that a legally accountable individual can choose to do or not to do some kind of action. On the other hand, reward and sanction are both dependent on the choices made.

In fact, Islamic Jurisprudence makes explicit the presuppositions for the application of a deontic qualification. Indeed, classifications such as obligatory, forbidden, and permissible, grounding a juridical decision (*ḥukm*) for a particular action (e.g., *it is forbidden to eat pork*), presuppose that a) the person who performs an action is legally accountable (*mukallaf*); b) the action in question is one for which the liberty to choose between carrying it out or not has been given (the provision of this liberty of choice is called *takhyīr*).

Notice that this approach is quite different from current studies in deontic logic that include, as axiom, the implication $OA \supset MA$, where “O” stands for “obligatory” and “M” for “possible,” known as the principle that *Ought Implies Can*, and also dubbed *Kant’s Principle (Sollen-Können-Prinzip)*.¹²⁶ According to our analysis of the Islamic conception, however, we find that:

- Every deontic qualification, and not only the obligatory, presupposes **ethical/legal liability**.¹²⁷

So, **in this sense**, *ought presupposes can*, whereby **liability is understood as the**

¹²⁶ Cf. Prior (1958), von Wright (1963, pp. 108-116, 122-125), Hilpinen (1981a, pp. 14-15), Chellas (1974), al-Hibri (1978, pp. 18-21), Hilpinen and McNamara (2013, p. 38).

¹²⁷ Hintikka’s (1981, p. 86) analysis of Kant’s principle is quite close to our view of the role of *takhyīr*—though he speaks of **non logical consequence or deontic consequence** rather than presupposition.

presupposition of being able to perform (or abort) the action prescribed by the norm.¹²⁸

The logical upshot of all this is that the underlying structure is that of a hypothetical, such that if we accept to make the choice between performing or not performing a certain action, we are rewarded or sanctioned (or neither) in relation to this choice.

This already suggests the main idea behind our analysis of judgements involving deontic and modal concepts. In a nutshell, our point is to analyse such judgements as a conjunction of two implications, such that the truth of the antecedent of each of these implications is dependent upon (i.e. presupposes) a disjunction.¹²⁹ Take the case of the *conditional* expressing an obligation. This conditional is constituted by the following implications:

- If an action x of type A is performed, then it will be rewarded; and if it is omitted, then it will be sanctioned (omitting to perform A has been established by the legal system as triggering a sanction, i.e. the contrary of reward) — provided there exists the choice of performing or not performing an action of the type A , that is, provided the disjunction $A \vee \neg A$.

Similarly, for the case of necessity as applied to events, assuming that the cause is both necessary and sufficient (for the sake of simplicity we consider a conjunction of causes as a unity):

- If event E occurs, then it is the presence of C that causes E to happen. The absence of C precludes E from happening — provided C is contingent, that is, provided the disjunction $C \vee \neg C$.¹³⁰

In the next section, before we develop our logical analysis, we will have a closer look at the historic sources of this parallelism, including a translation of the original text from Ibn Ḥazm's *Taqrīb*.

II On Natural and Deontic Necessity.

There are two distinctive approaches to modality in the Arabic tradition; namely: a) an approach closer to the relational view of Aristotle, whereby necessity designates some specific form of predication (including temporality),¹³¹ and b) one that is based on a propositional structure yielding some form of conditional necessity.¹³²

Thinkers in the Arabic tradition proposed innovations to both approaches. Ibn

¹²⁸ However; if *can* is understood as some general form of *permissibility*, then all actions qualified as *mandatory* are also permissible. In such contexts permissible is defined as including all those actions that qualify for reward (i.e., those that are rewarded when carried out and those that are rewarded when not carried out). In fact, in his *al-Iḥkām fī Uṣūl al-Aḥkām* (vol. 8, p. 101), Ibn Ḥazm seems to extend his deontic system with notions of *forbidden to do* and *obligatory not to do*, based only on what is *permissible to do* or *not to do*. Accordingly, *forbidden* is all that is not permissible to do, and *obligatory* is all that is not permissible not to do. Still, there is another sense of “*can*” involved in Kant’s principle—namely, as *ability to fulfil the duty*—that triggers some known puzzles of current deontic logic including Chisholm’s *contrary to the duty obligations*—cf. Chisholm (1963b), Hilpinen and McNamara (2013, section 8). In a recent paper Rahman/Granström/Farjami (2019) discussed Chisholm’s puzzle in the context of Islamic deontic categories

¹²⁹ Cf. Rescher (1963, pp. 76-78), and Jadaane (1968, pp. 117-21). For a recent, thorough study of the notion of *sharṭiyya*-conditionality, see Hasnawi and Hodges (2016, section 2.4.3, pp. 63-65).

¹³⁰ We will come to this point below.

¹³¹ For an overview of work on the first approach, see the sections on modal logic in Hasnawi and Hodges (2016), Strobino and Thom (2016).

¹³² To this point our suggestion is only that, a suggestion. A thorough examination of the implications of the notion of conditional modality for Arabic studies on causality has yet to be undertaken.

Ḥazm’s parallelism, on the other hand, seems to combine both. For while deontic necessity is characterized by the distribution of *Reward* and *Sanction*, both of them defined as predicates over performances of actions, natural necessity is characterized by means of the predicates *Causes* and *Precludes*, both of them defined over the presence or absence of some specific conditions. This provided roots for transference from the realm of actions to the realm of causally (**determined**) events.¹³³ In other words, the mirroring of deontic and modal concepts is a result of:

- 1) understanding modalities as affecting both actions and events;
- 2) mirroring the freedom of choice assumed by the legal system of sanctions and rewards within the contingency of natural events (or actions), given some natural conditions;
- 3) the idea that **both modal and deontic qualifications admit levels that can be put in correspondence**

Witness to the first point is that when Ibn Ḥazm, like other Muslim thinkers before him, speaks about modalities, he refers to them as qualifications of all “things”, *ashyā’* (s. *shay’*), including, here, actions and events. All these “things” can be classified as necessary, possible, or impossible.¹³⁴ Let us have a look at the original text.

Ibn Ḥazm, *al-Taqrīb li-Ḥadd al-Manṭiq wa-l-Mudkhal ilayhi bi-l-Alfāz al-‘Āmmiyya wa-l-Amthila al-Fiqhiyya*.¹³⁵

Extract Translated by Walter Edward Young.

Chapter on Elements (‘ <i>anāṣir</i> ’)	باب العناصر
Know that the elements (‘ <i>anāṣir</i> ’) of all things (<i>ashyā’</i>)—that is, their classes with regard to making assertions (<i>ikhbār</i>) about them—are of three classes, there being no fourth.	اعلم أنّ عناصر الأشياء كلّها أيّ أقسامها في الإخبار عنها ثلاثة أقسام لا رابع لها
[They are] either necessary (<i>wājib</i>), being such as are necessary and manifest, or from among such as must be, like the rising of the sun each morning, and the like of that, this being called in God’s laws the ‘obligatory’ (<i>farḍ</i>) and the ‘binding’ (<i>lāzim</i>);	إمّا واجب وهو الذي قد وجب وظهر أو ما يكون مما لا بدّ من كونه كطلوع الشمس كلّ صباح وما أشبه ذلك وهذا يسمّى في الشرائع الفرض واللازم
or possible (<i>mumkin</i>), being such as might be and might not be, like our anticipation that it will rain tomorrow, and the like of that, this being called in God’s law the ‘lawful’ (<i>ḥalāl</i>) and the ‘permitted’ (<i>mubāḥ</i>);	وإمّا ممكن وهو الذي قد يكون وقد لا يكون وذلك مثل توقّعتنا أن تمطر غداً وما أشبه ذلك وهذا يسمّى في الشرع الحلال والمباح
or impossible (<i>mumtani’</i>) being such as to which there is no path, like a human’s remaining under water for an entire day, or his living a month without food, or his walking in the air without some cunning artifice, and the like of that. And this is the type of thing that, if we saw it manifest in a human, we would know he is a prophet; and this class is called in God’s laws the ‘forbidden’ (<i>ḥarām</i>) and the ‘prohibited’ (<i>maḥzūr</i>).	وإمّا ممتنع وهو الذي لا سبيل إليه كبقاء الإنسان تحت الماء يوماً كاملاً أو عيشه شهراً بلا أكل أو مشيه في الهواء بلا حيلة وما أشبه ذلك وهذه التي إذا ظهرت من إنسان علمنا أنه نبي وهذا القسم يسمّى في الشرائع الحرام والمحظور
Furthermore, the possible (<i>mumkin</i>) is divided into three classes, there being no fourth:	ثم الممكن ينقسم أقساماً ثلاثة لا رابع لها
the nearly possible (<i>mumkin qarīb</i>), like the possibility of occurrence of rain upon a condensing of clouds in the	ممكن قريب كإمكان وقوع المطر عند تكاثف الغيم في شهري كانون وغلبة العدد الكبير من الشجعان العدد

¹³³ In fact, alethic modalities also affect actions, however in this case it seems that those actions are considered as determined by natural causality

¹³⁴ As we shall see, a general *causal possibility* amounts to *contingency* and *permissibility* (deontic possibility) amounts in principle to *optional*. In other words, in principle causal possibility excludes necessity and permissibility obligation.

¹³⁵ Ed. Aḥmad b. Farīd b. Aḥmad al-Mazīdī, (Beirut: Manshūrāt Muḥammad ‘Alī Bayḍūn, Dār al-Kutub al-‘Ilmiyya, 2003), pp. 83-84.

two months of <i>Kānūn</i> , ¹³⁶ or the victory of a large number of the courageous over a small number of the cowardly;	اليسير من الجبناء
and the distantly possible (<i>mumkin ba ʿīd</i>), which is like the defeat of a large number of the courageous at the hands of a small number of the cowardly, and like a cupper (<i>hajjām</i>) [i.e., a practitioner of cupping] taking charge of the Caliphate, and the like of that;	وممكن بعيد وهو كاهزام العدد الكثير من الشجعان عند عدد يسير من جبناء وكحجّام يلي الخلافة وما أشبه ذلك
and the purely possible (<i>mumkin maḥḍ</i>), whose two extremes are equal, this being like one standing—either he will walk or he will sit—and the like of that.	وممكن محض وهو يستوي طرفاه وهو كالمرء الواقف إمّا يمشي وإمّا يقعد وما أشبه ذلك
And likewise we find that this middle class [i.e., the <i>mumkin</i> , corresponding to the <i>mubāḥ</i>] is, in God’s laws, divided into three classes: recommended-permitted (<i>mubāḥ mustaḥabb</i>); reprehended-permitted (<i>mubāḥ makrūh</i>); and evenly permitted (<i>mubāḥ mustawīn</i>) having no tendency towards one of the two sides.	وكذلك نجد هذا القسم المتوسط في الشرائع ينقسم أقساماً ثلاثة فمباح مستحبّ ومباح مكروه ومباح مستوي لا ميل له إلى أحد الجهتين
As for recommended-permitted (<i>mubāḥ mustaḥabb</i>), it is such that when you do it you are rewarded (<i>ujirta</i>), but if you neglect it you do not sin (<i>lam ta ʿtham</i>) and you are not rewarded; like praying two supererogatory prayer-cycles, voluntarily.	فأما المباح المستحب فهو الذي إذا فعلته أجزت وإذا تركته لم تأثم ولم تؤجر مثل صلاة ركعتين نافلة تطوعاً
And as for reprehended-permitted (<i>mubāḥ makrūh</i>), it is such that when you do it you do not sin and you are not rewarded, but if you neglect it you are rewarded; and that is like eating while reclining, and the like.	وأما المباح المكروه فهو الذي إذا فعلته لم تأثم ولم تؤجر وإذا تركته أجزت وذلك مثل الأكل متكئاً ونحوه
And as for evenly permitted (<i>al-mubāḥ al-mustawī</i>), it is such that when you do it or you neglect it you do not sin and you are not rewarded; and that is like dyeing your garment whichever colour you please, and like your riding whichever beast of burden you wish, and the like.	وأما المباح المستوي فهو الذي [إذا] فعلته أو تركته لم تأثم ولم تؤجر وذلك مثل صبغك ثوبك أي لون شئت وكركوبك أي حمولة شئت ونحوه

As pointed out by such scholars as Chejne (1984), Guerrero (1997, 2013), Lameer (2013) and Puerta Vílchez (2013), Ibn Ḥazm’s parallelism between deontic and modal notions emerged from the task of bridging the modal logic of Aristotelians and Post-Aristotelians with Islamic Jurisprudence.

The source of distinguishing between different levels of alethic modality within the natural realm seems to be Aristotle’s distinction, in the *Peri Hermeneias* (chapter 9), between different cases of contingent events:¹³⁷

*Thus it is clear that not everything is or takes place of necessity. Cases there are of contingency ; no truer is then the affirmative, no falser, than the negative statement. Some cases, moreover, we find that, at least, for the most part and commonly, tend in a certain direction, and yet they may issue at times in the other or rarer direction.*¹³⁸

Importantly, Ammonius, in treating this passage in his famous commentary on Aristotle, is strikingly close to Ibn Ḥazm’s above-quoted passage on the possible:

The contingent is divided into three: one is called 'for the most part' (hōs epi to polu), for example that a man is born with five fingers or becomes gray with age (for things behaving otherwise are rare); another is 'for the lesser part' (hōs ep' elatton), for example that one digging comes upon a hoard; and the last is 'equally <often>' (ep' isēs), for example to bathe or not to bathe and to walk or not to walk.

[...]. Concerning the contingent <which occurs> 'equally<often>' there is only choice, for example to go out or not to go out, to converse or not. Only this species of the contingent is called 'however it chances', because its existence is no more or less <frequent> than its non-existence,

¹³⁶ I.e., December and January.

¹³⁷ We owe this reference to Carlo Natali (Università Ca' Foscari Venezia). The authors of the present paper enjoyed further assistance on this topic from Tony Street (University of Cambridge), and Robert Wisnovsky (McGill University).

¹³⁸ *Peri Hermeneias*, chapter 9, 19a18-22, in Aristotle (1962), p139.

*but whichever part of the contradiction it chances can equally occur.*¹³⁹

Now, despite the structural similarities, there is no evidence that Ibn Ḥazm's parallelism is rooted in Ammonius's commentary. There is no evidence either what version of Aristotle's text was accessible to our author.¹⁴⁰

III Deontic Imperatives and the CTT-Analysis of Hypotheticals.

Per Martin-Löf's (1984) Constructive Type Theory (CTT) provides a thorough formal framework whereby categorical and hypothetical judgements can be explicitly distinguished at the object-language level without conflating judgements with the propositions that constitute them.¹⁴¹

Since these distinctions are crucial for the formal reconstruction of traditional logic in general—and of the Arabic tradition in particular—we have chosen to employ the language of CTT for our logical study on the origins of deontic concepts. More precisely, the CTT-framework allows one to distinguish, at the language level, both the *taṣawwur* of a judgement, i.e., its *conceptualization* or (roughly) proposition, and its *taṣdīq*, or *assent* or *believing to be true*, i.e., the act of judgment itself, or, sometimes, the linguistic expression of that act.¹⁴²

III.1 Quantifying over Actions

It seems that the translation of the text suggests the following logical analysis of obligatory actions—with "g" for agent, "O" for obligation:

OgA iff if A is carried out by g, g is rewarded & if A is not carried by g, g is sanctioned & g is able to carry out A and s is able to not carry out A.

In fact, in principle, the overall logical structure of the logical analysis comes quite close to this conjunction. Nonetheless, we would like

1) to express at the object language level that what is the primary subject of sanction or reward is the performance of *some specific type of action*. Sanction and reward are predicates (propositional functions) defined over performance of actions, not just propositional variables. This requires performance of actions be denizens of the domain of quantification. Thus, under this perspective, sanction and reward apply indirectly to the agent: the agent is sanctioned and rewarded for his actions. Perhaps it would be more accurate to display a double qualification: the performance of the action is law-breaking and the agent is therefore sanctioned. However, in order to keep our analysis as simple as possible, we will just focus

¹³⁹ Ammonius (1998) 142.1, pp. 104-105.

¹⁴⁰ Another plausible source is Paul the Persian's summary of Ammonius's commentary (i.e., the translation from Persian into Syriac by Sebokht). Paul the Persian stresses the role of human choice in the context of moral issues in his "l'Élucidation du Peri Hermeneias," section 26. Translated from the Syriac by Henri Hugonnard-Roche (2013, p. 63). Robert Wisnovsky (McGill University) has pointed out to the authors of the present paper that Yaḥyā ibn 'Adī's work on the nature of the possible might also constitute a possible source – see Ehrig-Eggert (1990).

¹⁴¹ See Martin-Löf (1984, pp. 9-10). For a short introductory survey, see Rahman, McConaughy, and Klev (2018, chapter II).

¹⁴² Translating *taṣdīq* by *assent* follows the choice of Hasnawi and Hodges (2016, pp. 56-57), though the authors warn that such a choice might be controversial. In fact, Lameer (2014, p. 403, n. 16) strongly contests this translation and suggests *believing to be true*, such as *what we do when we believe in first principles* or *what we do not do when we accept (or adopt) a false proposition for the sake of an argument* (such as in a proof *per impossible*). Perhaps *endorsement* would also be a good choice, meaning such acts by which we not only accept that a proposition is true but we employ it for our own inferences.

on the performances of the agent rather than on the agent.

2) to render the ability of carrying out A and not carrying out A as a presupposition: the legal presupposition of liability mentioned above. If the presupposition is not fulfilled the norm does not apply.

Let us see work out both of these points

To the first point: It is well known since at least the times Aristotle that examples such as *Some cobblers are good (as cobblers)*, cannot be analyzed as a conjunction—in Aristotle's framework the distinction Subject and Predicate does the job: *Good is said of cobblers*. The CTT has a very distinctive way to approach to such kind of "complex predicates", namely dependent types.

The idea is that the expression $Good(x)$, is a proposition (is of the type proposition), provided (under the hypothesis), that x is an element of the domain (set) of cobblers. For short, $Good(x)$ is propositional function defined over the domain *Cobbler*. Thus, within CTT the familiar notion of propositional function will be rendered as

$Good(x) : prop (x : Cobbler)$, given *Cobbler*: *set*

whereby; "*prop*" stands for the dependent type proposition and "*Cobblers*" for the type *set*, and the colon admits the glosses, "is an instance of", "is an element of". Strictly speaking, because of the Curry-Howard isomorphism between types, sets and propositions, $Good(x)$ and *Cobblers* are both of the same type, *prop* or *set*.¹⁴³ However, the former is a dependent type upon the latter: no cobblers no good cobblers. We use the denominations *set* and *prop* in order to stress the dependence of the second on the former.

Now, the existential (in its general form called a *Sigma-type*) *Some cobblers are good (as cobblers)*, receive the notation

$(\exists x: Cobbler) Good(x)$ *true*

(which comes quite close to the subject predicate structure of ancient logic)

what makes this true is a pair (its *proof-object*), such that, a , the first element of the pair is a cobbler (an element of the set of cobblers) and the second element b , is a function that takes the first element (the chosen cobbler) and makes true *Good cobbler* by rendering an instance of a good cobbler:

$(a,b) : (\exists x: Cobbler) Good(x)$

whereby,

$Good(x) : prop (x : Cobbler)$, given *Cobbler*: *set*

$a : Cobbler$ (a is a cobbler/ a is an element of the set cobblers)

$b : Good(a)$ (b verifies that cobbler a is a good cobbler)

Otherwise (from the point of view of the elimination rules)

¹⁴³ More generally, within TCT a proposition is interpreted as a set the elements of which represent the proofs of the proposition, the solution to a problem, or the fulfilment of an expectation. It is also possible to view a set as a problem description in a way similar to Kolmogorov's explanation of the intuitionistic propositional calculus. In particular, a set can be seen as a specification of a programming problem, the elements of the set are then the programs that satisfy the specification (see Martin-Löf 1984, p.7). Furthermore, in TCT sets are also understood as types, so that propositions can be seen as data- (or proof-) types.

Suppose $(\exists x: \text{Cobbler})\text{Good}(x)$. Then there is some (complex) proof-object making the existential true, say, c

$c: (\exists x: \text{Cobbler})\text{Good}(x)$ – given the same assumptions as before

We know that " c " must be a pair. The functions, $p(c)$ and $q(c)$, called the left and right projections, render the first and the second element of the pair, namely

$p(c): \text{Cobbler}$ and $q(c): \text{Good}(p(c))$

that evaluate as follows

$p(a,b)=a: \text{Cobbler}$ and $q(a,b)=b: \text{Good}(p(c))$

If we bind the free variable with a universal quantifier we obtain the following assertion which in its general form is said to constitute a \forall -type;

$(\forall x: \text{Cobbler})\text{Good}(x)$ *true*

what makes it true is a function, that expresses that there is a method that associates any chosen cobbler with the predicate $\text{Good}(x)$, given $\text{Good}(x): \text{prop } (x: \text{Cobbler})$, given $\text{Cobbler}: \text{set}$.

With regard to our subject, our point is that the specificity of the sanction or law-breaking as qualifying performances of, say the type of action A , is to be understood as a dependent type. After all sanctioning a wanton killing is quite different of sanctioning an act of not paying the alms-giving tax (زكاة, *zakāt*) prescribed in Islam. In other words the formation rules for sanction are

$\text{Sanction}(x): \text{prop } (x: A)$, given $A: \text{set}$

Thus, a first rendering of $\text{Og}A$, yields

$(\forall y: A) R(y) \wedge (\forall z: \neg A) S(z)$

Every performance of A is rewarded and every performance of $\neg A$ is sanctioned

Notice that in our framework, omitting to perform an action that instantiates the action-proposition A (i.e., not doing it) is conceived of as frustrating the performance of an action of that type, e.g., *stopping (or inhibiting) eating or drinking when a day of fasting in Ramadan begins*. This interpretation is close to the notion of aborting a process that grounds the inferential notion of negation (see Martin-Löf 1984, p. 36).

So far so good, however, we would like to express not only that being able to do A and $\neg A$ is a constitutive part of the distribution of reward and sanction prescribed by the norm, but also that performances of A are sanctioned (rewarded) because $\neg A$ could have been chosen. This takes us to the second point.

To the second point: Let us look once more at dependent proof-object and hypothetical judgements. The only means standard frameworks can deal with hypothetical judgments are implications. In CTT the distinction between categorical judgments and hypothetical judgments is rendered in a quite straightforward manner: a categorical judgment is true if there is an independent proof object for the proposition involved C *true* expresses a categorical if some proof-object c can be found that makes it true:

$c : C,$

A hypothetical judgment is true if true under the proviso of some hypothesis (or hypotheses)

$B(x) \text{ true } (x : C)$ whereby $C : \text{set}$ and $B(x) : \text{prop } (x : C)$
 (The bracketed expression to the right is the hypothesis conditioning the truth of the main proposition)

The proof-object of which is a dependent object, that is a function

$b(x) : B(x) \text{ true } (x : C)$

Clearly, an assertion involving a hypothetical judgement does not express that the condition C has been verified, but it asserts that if the condition is verified, then $B(x)$ is true, if a method can be found that transforms this verification into a verification of the main proposition.

Thus, if we have $b(x) : R(x) (x : A)$ as a premise, and we have as a second premise the fact that indeed there is a performance a of the action-proposition A (i.e., if we have as premise $a : A$), then we can infer that performance a will be rewarded (i.e., $b(a) : R(a)$). In plain words, from the premises

- (i) Any performance x of an action will be rewarded, provided it is the performance of an action of the type A ,
 - (ii) a is such a performance ($a : A$),
- we can infer:
- (iii) Performance a is rewarded ($b(a) : R(a)$).

$$\frac{a : A \quad b(x) : R(x) (x : A)}{b(a) : R(a)}$$

The same holds for sanctions for performing forbidden actions, or for omitting to perform obligatory actions.

Relevant to our aims is that the CTT analysis of hypothetical judgments also provides the means for analyzing presuppositions such as the classical example *Sam stopped smoking*, presupposes that Sam smoked:

$\text{Sam stopped smoking}(x) \text{ true } (x : \text{Sam smoked})$
 $b(x) : \text{Sam stopped smoking}(x) (x : \text{Sam smoked})$

In point of fact, with regard to our case, the presupposition of the predicates *Sanction* and *Reward* is that the actions affected by these predications can be carried or not be carried out. In other words, *Sanction* and *Reward* presuppose $A \vee \neg A$.

What we now need¹⁴⁴ is to express the dependence of the rewarding or sanctioning

¹⁴⁴ **Caveat:** In the context of Islamic Law, omitting to perform an obligatory action, or performing a forbidden one, is sanctioned **if the omission z is not excused**; i.e., provided that $\neg E(z)$ applies (the prescribed fasting during Ramadan is not obligatory, for example, while travelling)—this kind of proviso being also very important in contemporary European Civil Law. The proviso can be integrated into the hypothetical as follows:

• $(\forall y : A) [\text{left}^{\vee(y)=\{H\}} x \supset R(y)] \wedge [(\forall z : \neg A) (\neg E(z) \wedge \text{right}^{\vee(z)=\{H\}} x \supset S(z))] \text{ true } (x : A \vee \neg A)$.
 In the present paper, however, we will leave out $E(x)$, since this relates to defeasibility, an issue that is linked

upon the presupposition that a choice has been made. More precisely, if we are describing an *obligatory action*, what we need to express is the following:

Obligatory action: (i) *If the individual g made the choice to perform an action of type A (i.e., if there is a proof-object that makes true the **left side** of the disjunction) then this performance is rewarded.* (ii) *If the individual g made the choice of omitting to perform an action of type A (i.e., if there is a proof-object that makes true the **right side** of the disjunction) then this act of omission is sanctioned.*

If we pull all this together and employ the abbreviation $\{H\}$ for $x : A \vee \neg A$ we obtain:

$$b(x) : [(\forall y : A) \mathbf{left}^{\vee(y)=\{H\}x} \supset R(y)] \wedge [(\forall z : \neg A) \mathbf{right}^{\vee(z)=\{H\}x} \supset S(z)] (x : A \vee \neg A)$$

Where: the expressions “ $\mathbf{left}^{\vee(y)}$ ” and “ $\mathbf{right}^{\vee(z)}$ ” stand for the injections that render the disjunction $A \vee \neg A$ *true*,¹⁴⁵ and where “ $\mathbf{left}^{\vee(y)=\{H\}x}$ ” stands for the choice of performing an action of the type (of the action-proposition) A , and “ $\mathbf{right}^{\vee(z)=\{H\}x}$ ” stands for the choice of not performing such a type of action-proposition.¹⁴⁶ The identity expression can be glossed as follows:

Any performance of A (act of omitting to do A) is identical to the proof-object that renders true the disjunction – by rendering true either the left or the right of $A \vee \neg A$.

Thus:

$$(\forall y : A) \mathbf{left}^{\vee(y)=\{H\}x} \supset R(y) \text{ true } (x : A \vee \neg A)$$

reads

Assuming that, given the choice of performing or not performing an action of type A , performing it has been chosen (i.e., if the left side of the disjunction has been chosen to be performed), then, for any performance y within the set A that is identical to this choice (within the set $\{A \vee \neg A\}$), reward (for performing this action) follows.

Similar reading admits

$$(\forall z : \neg A) \mathbf{right}^{\vee(z)=\{H\}x} \supset S(z) \text{ true } (x : A \vee \neg A).^{147}$$

Notice that the hypothesis or presupposition $x : A \vee \neg A$, carries a modal feature: sanction or reward apply only if the presupposition can be fulfilled. Indeed, within CTT, assertions under hypotheses are understood as modalities — cf. Ranta (1994, chapter 7).

Moreover, if, as suggested below, the presupposition is bounded by a universal quantifier, we have all the universal scope that other frameworks capture by introducing an alethic necessity operator, such as Anderson's (1958) reduction **OA** iff $\Box[(A \supset R) \wedge (\neg A$

to the dynamics of argumentation with which we deal elsewhere (Rahman and Iqbal [2018a] discuss defeasibility in the context of legal argumentation in Islamic Law, though they do not combine it with an analysis of the deontic qualifications of actions).

¹⁴⁵ We have slightly changed the notation for injections, which when they occur as proof-objects of a disjunction usually take the notation $i(x)$ and $j(x)$ (see Ranta 1994, pp. 47). The injection $\mathbf{left}^{\vee(a)}$, takes an element of A , namely a (in our case, the performance a of action A) and renders a as proof-object for the left side of the disjunction.

¹⁴⁶ Cf. Ranta (1994, pp. 52-53).

¹⁴⁷ The notation for propositional identity, namely “ $x_{= \{D\}} y$ ”, standing for “ x is identical to y within the set D ,” is more similar to the one employed in first-order logic. In fact, within the TCT-framework, the usual notation is $\mathbf{Id}(D, x, y)$.

$\supset S$].¹⁴⁸

The point of our approach is that we neither quantify over possible worlds nor do we cast action and reward (sanction) as an implication between elementary propositions. As mentioned above, on one hand possible world semantics does not seem to render the notion of modality at stake in ancient logic, on the other the legal/ethical effects of performing an action are to be conceived as acts of predication. For short, quantifying over events and actions yields, so we claim, renders a framework that makes apparent the insights on modalities in contained the textual sources under consideration.¹⁴⁹

Notice too that our formal reconstruction, though grounded in an inferentialist approach to ancient logic,¹⁵⁰ it does not commit to intuitionistic logic. Within the CTT approach third-excluded is added as an explicit assumption, not unlike standard proof-theoretical presentations of classical logic within the framework of natural deduction.¹⁵¹ In our case, the universal binding of the hypothesis implements the classical assumption. Accordingly obligation receives now the notation

$$(\forall x : A_1 \vee \neg A_1) \{ [(\forall y : A_1) \text{left}^\vee(y)_{=\{H1\}x} \supset R_1(y)] \wedge [(\forall z : \neg A_1) \text{right}^\vee(z)_{=\{H1\}x} \supset S_1(z)] \} \text{true}$$

III.2 Logical Analysis of Ibn Ḥazm's Heteronomous Imperatives.

As already mentioned, deontic qualifications of actions presuppose that the performer is legally accountable and has been given the liberty to choose (*takhyīr*) between two alternatives. The CTT-framework for hypotheticals provides the formal means to express (i) that the deontic qualifications assume such a choice, **and (ii) that sanction and reward are predicates apply to performances of the action at stake.** Each of the deontic concepts determines a **subset** of a general set of actions in a straightforward manner:

- **wājib, farḍ, lāzīm:** Doing A_1 is rewarded. Omitting to do A_1 is sanctioned.
 $b_1(x) : [(\forall y : A_1) \text{left}^\vee(y)_{=\{H1\}x} \supset R_1(y)] \wedge [(\forall z : \neg A_1) \text{right}^\vee(z)_{=\{H1\}x} \supset S_1(z)] (x : A_1 \vee \neg A_1).$
- **ḥarām, mahzūr:** Doing A_2 is sanctioned. Omitting to do A_2 is rewarded.
 $b_2(x) : [(\forall y : A_2) \text{left}^\vee(y)_{=\{H2\}x} \supset S_2(y)] \wedge [(\forall z : \neg A_2) \text{right}^\vee(z)_{=\{H2\}x} \supset R_2(z)] (x : A_2 \vee \neg A_2).$
- **mubāḥ mustaḥabb:** Doing A_3 is rewarded. Omitting to do A_3 is neither sanctioned nor rewarded.
 $b_3(x) : [(\forall y : A_3) \text{left}^\vee(y)_{=\{H3\}x} \supset R_3(y)] \wedge [(\forall z : \neg A_3) \text{right}^\vee(z)_{=\{H3\}x} \supset (\neg S_3(z) \wedge \neg R_3(z))] (x : A_3 \vee \neg A_3).$
- **mubāḥ makrūh:** Omitting to do A_4 is rewarded. Doing A_4 is neither sanctioned nor rewarded.
 $b_4(x) : [(\forall y : A_4) \text{left}^\vee(y)_{=\{H4\}x} \supset (\neg S_4(y) \wedge \neg R_4(y))] \wedge [(\forall z : \neg A_4) \text{right}^\vee(z)_{=\{H4\}x} \supset R_4(z)] (x : A_4 \vee \neg A_4).$

¹⁴⁸ Cf. Chisholm (1963a). For an overview see Hilpinen and McNamara (2013, section 6.2).

¹⁴⁹ Our take on integrating within a TCT framework quantification over events and actions is based on Ranta's (1994) discussions. Indeed, Ranta (1994, p. 54) combines TCT with Davidson's (1980, essays 6-10) idea that an individual action makes an action-proposition *true*. Accordingly, the proposition

(that) *al-Fārābī read Aristotle's Analytica Posteriora*

is made *true* by individual readings of al-Fārābī performing actions of that type. Notice that this not only fits nicely with Ibn Ḥazm's original text, where he uses the term *al-ashyā'*, "things," to include actions and events, but, as mentioned in the preface and discussed below, it is a consequence of the insight that deontic and modal concepts qualify both actions and events.

¹⁵⁰ As the ones by Ebbinghaus (1964), Corcoran (1972), Thom (1981), Marion and Rückert (2016) and Crubellier et al. (2019), for syllogistic in general and Malink and Rosen (2013) and Strobino and Thom (2016) for modalities

¹⁵¹ In fact as pointed by Sundholm (2004) the proof-theoretical meaning explanations of TCT are logically neutral.

- **mubāh mustawin:** Doing A_5 is neither sanctioned nor rewarded. Omitting to do A_5 is neither sanctioned nor rewarded..

$$b_5(x) : [(\forall y : A_5) \mathbf{left}^{\vee}(y)=_{\{H_5\}}x \supset (\neg S_5(y) \wedge \neg R_5(y))] \wedge [(\forall z : \neg A_5) \mathbf{right}^{\vee}(z)=_{\{H_5\}}x \supset (\neg S_5(z) \wedge \neg R_5(z))] (x : A_5 \vee \neg A_5).$$

In some contexts, it might be desirable to define deontic qualifications as expressions building propositions. In fact, it is quite straightforward, since a hypothetical is one inference away from a universal:

$$(\forall x : A_1 \vee \neg A_1) \{ [(\forall y : A_1) \mathbf{left}^{\vee}(y)=_{\{H_1\}}x \supset R_1(y)] \wedge [(\forall z : \neg A_1) \mathbf{right}^{\vee}(z)=_{\{H_1\}}x \supset S_1(z)] \} \mathbf{true}$$

Thus, the whole expression can form new propositions in the usual way; for example, as the consequent of some implication, and so on. The like applies to *Forbidden* and *Permissible*.

IV A Landmark in the History of the Logical Analysis of Norms. Natural and Deontic Modalities.

As discussed above, Ibn Ḥazm's examples of modalities and the denominations he employs make apparent one of the most innovative insights of the Arabic perspective on deontic concepts; namely, the correspondence between deontic and alethic modalities. In the following paragraphs we will propose a logical analysis that renders the causal structure of natural modalities and makes the parallelism apparent. This suggests that heteronomous imperatives and natural modalities share a similar hypothetical structure. In the realm of alethic modalities as applied to natural events the relevant hypothesis/presupposition is the contingency of the causal conditions. Thus, the hypothesis $x : C_1 \vee \neg C_1$, indicates that the casual condition may happen or not. If, again we bind with a universal quantifier, we express the classical assumption that necessarily one of both obtain.

The main idea of our reconstruction of alethic modalities is that the different categories result from comparing the cardinality of the occurrence of causal conditions with the occurrence of the correspondent effect. One main motivation animating our approach is that the logical analysis of alethic modalities should be compatible with the several forms of occasionalism developed by Islamic thinkers.

From the point of view of contemporary logic, generalized quantifiers, as *Most*, *As-Many*, etc., are the instruments for rendering proposition involving cardinality. Let us now introduce the logical analysis of the generalized quantifiers *More* and *As-Many*, recalling briefly their notation in the context of CTT —Sundholm (1989) provides a thorough study on the constructive formation of such quantifiers).

$More(D, (x)A, (x)B)$	There are more A in D than B . Here $(x)A$ indicates that the variable x is bound in the predicate A —the same applies to $(x)B$. (The definition amounts to verifying that there is no surjection from B to A .)
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This can easily be generalized for two domains:

$More(D_1, (x)A; D_2, (x)B)$	There are more A in D_1 than there are B in D_2 .
$As-Many(D, (x)A, (x)B)$	There are as many A in D as B .
$As-Many(D_1, (x)A; D_2, (x)B)$	There are as many A in D_1 as there are B in D_2 . (The definition amounts to verifying that there is a bijection.)

We apply these generalized quantifiers to *Cause* and *Effect*. In the present context,

we obtain the propositional function:¹⁵²

$x={}_{(C)}x(x) : prop (x : C)$ If we take an arbitrary x from the set C , then we gather the proposition that this x is identical to itself.

or in a notation uniform to the one employed for generalized quantifiers

$(C, (x)x={}_{(C)}x) : prop$ “ $(x)\varphi$ ” indicates that x is (existentially) bound in C (in our case φ is $x={}_{(C)}x$).

Let us illustrate by way of two examples:

- *As-Many*($C, (v)E; C, (s)s={}_{C1}S$).
***E* happens as often as *C* does.**
More precisely, there are as many cases of presences of *E* as there are instances of *C*.
- *More*($C_3, (v)E_3, (v)\neg E_3$).
***E* is present more often than not (*Most C* are *E*).**

Now we are ready to deal with the natural modalities. For example, the logical structure of *near possibility*, in both the causal and the simpler formulation, is the following:

mumkin qarīb.

If, *E* is present when *C* is; then *E* is present more often than not (*Most C* are *E*).

$\{ (\forall y : C_3) [\text{left}^{\vee}(y) =_{(H)} x \supset (\forall w : E_3 \vee \neg E_3) \{ (\forall u : E_3) [\text{left}^{\vee}(u) =_{(H')} w \supset \text{More}(C_3, (v)E_3, (v)\neg E_3)] \}] \} \}$ **true** ($x : C_3 \vee \neg C_3$).

Notation: Bear in mind the convention that the subscripts “ $\{H\}$ ” and “ $\{H'\}$ ” stand as abbreviation for the set (in our case a hypothesis) upon which the corresponding propositional identity has been defined, such as $\{A_1 \vee \neg A_1\}$, $\{C_1 \vee \neg C_1\}$, $\{E_1 \vee \neg E_1\}$.

The Logical Structure of Natural and Deontic Modalities

wājīb, farq, lāzim. If, given <i>A</i> or <i>not-A</i> , <i>A</i> is performed, then this performance is rewarded; but if <i>not-A</i> is performed, then sanction follows. $[(\forall y : A_1) \text{left}^{\vee}(y) =_{(H)} x \supset R_1(y)] \wedge [(\forall z : \neg A_1) \text{right}^{\vee}(z) =_{(H)} x \supset S_1(z)]$ true ($x : A_1 \vee \neg A_1$).	Deontic
wājīb. If, given <i>C</i> or <i>not-C</i> , <i>E</i> is present when <i>C</i> is, then <i>E</i> happens as often as <i>C</i> does. $\{ (\forall y : C_1) [\text{left}^{\vee}(y) =_{(H)} x \supset (\forall w : E_1 \vee \neg E_1) \{ (\forall u : E_1) [\text{left}^{\vee}(u) =_{(H')} w \supset \text{As-Many}(C_1, (v)E_1; C_1, (s)s={}_{C1}S)] \}] \} \}$ true ($x : C_1 \vee \neg C_1$).	Modal
ḥarām. If, given <i>A</i> or <i>not-A</i> , <i>A</i> is performed, then this performance is sanctioned; but if <i>not-A</i> is performed, then reward follows. $[(\forall y : A_2) \text{left}^{\vee}(y) =_{(H)} x \supset S_2(y)] \wedge [(\forall z : \neg A_2) \text{right}^{\vee}(z) =_{(H)} x \supset R_2(z)]$ true ($x : A_2 \vee \neg A_2$).	Deontic
mumtani'. If, given <i>C</i> or <i>not-C</i> , <i>not-E</i> is present when <i>C</i> is, then, the number of absences of <i>E</i> equals the number of presences of <i>C</i> . $\{ (\forall y : C_2) [\text{left}^{\vee}(y) =_{(H)} x \supset (\forall w : E_2 \vee \neg E_2) \{ (\forall u : \neg E_2) [\text{right}^{\vee}(u) =_{(H')} w \supset \text{As-Many}(C_2, (v)\neg E_2; C_2, (s)s={}_{C2}S)] \}] \} \}$ true ($x : C_2 \vee \neg C_2$).	Modal
mubāḥ mustahabb. If, given <i>A</i> or <i>not-A</i> , <i>A</i> is performed, then this performance is rewarded; but if <i>not-A</i> is performed, then neither reward nor sanction follows. $[(\forall y : A_3) \text{left}^{\vee}(y) =_{(H)} x \supset R_3(y)] \wedge [(\forall z : \neg A_3) \text{right}^{\vee}(z) =_{(H3)} x \supset (\neg S_3(z) \wedge \neg R_3(z))]$ true ($x : A_3 \vee \neg A_3$).	Deontic
mumkin qarīb. If, given <i>C</i> or <i>not-C</i> , <i>E</i> is present when <i>C</i> is, then <i>E</i> is present more often than not (<i>Most C</i> are <i>E</i>). $\{ (\forall y : C_3) [\text{left}^{\vee}(y) =_{(H)} x \supset (\forall w : E_3 \vee \neg E_3) \{ (\forall u : E_3) [\text{left}^{\vee}(u) =_{(H')} w \supset \text{More}(C_3, (v)E_3, (v)\neg E_3)] \}] \} \}$ true ($x : C_3 \vee \neg C_3$).	Modal
mubāḥ makrūh. If, given <i>A</i> or <i>not-A</i> , <i>A</i> is performed, then this performance is neither rewarded nor sanctioned; but if <i>not-A</i> is performed, then reward follows. $[(\forall y : A_4) \text{left}^{\vee}(y) =_{(H)} x \supset (\neg S_4(y) \wedge \neg R_4(y))] \wedge [(\forall z : \neg A_4) \text{right}^{\vee}(z) =_{(H)} x \supset R_4(z)]$ true ($x : A_4 \vee \neg A_4$).	Deontic

¹⁵² Recall that our notation for propositional identity is “ $x={}_{(D)}y$ ”, standing for “ x is identical to y within the set D ” rather than “ $\text{Id}(D, x, y)$ ”.

<p>mumkin ba ʿīd. If, given <i>C</i> or <i>not-C</i>, <i>E</i> is present when <i>C</i> is, then <i>not-E</i> is more often present than not present (<i>Most C are not-E</i>).</p> $\{ (\forall y : C_4) [\text{left}^{\vee}(y) =_{\{H\}} x \supset (\forall w : E_4 \vee \neg E_4) \{ (\forall u : E_4) [\text{left}^{\vee}(u) =_{\{H\}} w \supset \text{More}(C_4, (v)\neg E_4, (v)E_4)] \}] \} \} \text{ true } (x : C_4 \vee \neg C_4).$	<p>Modal</p>
<p>mubāḥ mustawin. If, given <i>A</i> or <i>not-A</i>, <i>A</i> is performed, then this performance is neither rewarded nor sanctioned; the same holds for performances of <i>not-A</i>.</p> $[(\forall y : A_5) \text{left}^{\vee}(y) =_{\{H\}} x \supset (\neg S_5(y) \wedge \neg R_5(y))] \wedge [(\forall z : \neg A_5) \text{right}^{\vee}(z) =_{\{H\}} x \supset (\neg S_5(z) \wedge \neg R_5(z))] \text{ true } (x : A_5 \vee \neg A_5).$	<p>Deontic</p>
<p>mumkin maḥd. If, given <i>C</i> or <i>not-C</i>, <i>E</i> or <i>not-E</i> are present when <i>C</i> is, then, when <i>E</i> is present, this presence equals the number of absences.</p> $\{ (\forall y : C_5) [\text{left}^{\vee}(y) =_{\{H\}} x \supset (\forall w : E_5 \vee \neg E_5) \{ (\forall u : E_5) [\text{left}^{\vee}(u) =_{\{H\}} w \supset \text{As-Many}(C_5, (v)E_5, (v)\neg E_5)] \}] \} \} \text{ true } (x : C_5 \vee \neg C_5).$	<p>Modal</p>

At this point, the reader might wonder why do we need this heavy hypothetical structuring, given that the entire logical operation seems to be delivered by the generalized-quantifier-expressions. In fact, it is true that the generalized quantifiers perform the logical operation properly; however, if we retain only them, we lose the epistemological features of the relevant texts. Take the example of *distant possibility*. The point is not only to indicate that it is more often the case that the event does not happen than that it does, but also to express explicitly that, *given some specific contingent conditions, if the event is present, then it is unlikely so*.

V Ibn Ḥazm’s Parallelism, *Qiyās*, and the Inferential Structure of Imperatives.

It is interesting that Ibn Ḥazm’s parallelism fits almost perfectly with the general structure of inferences via parallel reasoning and analogy subsumed by the term *qiyās*, and developed with skill and sophistication by the *Shāfi’īs* and other schools of Islamic jurisprudence.¹⁵³ It is particularly significant because, despite having adhered to the *Shāfi’ī madhhab* earlier in his career, Ibn Ḥazm eventually converted to the *Zāhiri madhhab*—in fact becoming its most famous proponent—and rejected with vehemence any form of reasoning by parallelism or analogy, a stance in keeping with the radically literalist *Zāhiri* approach to the interpretation of the sacred sources.

Nevertheless, let us recall that his main motivation for the parallelism was to translate into Islamic juridical terms the logic of the Greek tradition. He did not mean to employ this parallelism as a form of reasoning to interpret the sacred texts, but as a way to bring together the approaches to necessity and normativity in the Greek tradition of logic and Islamic jurisprudence. It might also be argued that the parallelism which is the subject of our paper occupies a completely different level than the sort of parallelism (i.e., *qiyās*) which he rejects.

Still, when Ibn Ḥazm defends the study of logic, he defends it for its fruitful application in jurisprudence. Moreover, Ibn Ḥazm elsewhere explicitly defends a crucial principle of pure permissibility; namely: *All actions are permissible unless proscribed by Law*.¹⁵⁴

Such a principle of Islamic Jurisprudence encouraged the development and application (among most Sunni schools) of that set of correlational modes of argumentation called *qiyās*, regulating the integration of “updated” deontic qualifications (possibly different to *permissible*), for novel and still-contended cases, into expanding bodies of substantive law. In fact, in conjunction with other factors, this principle opens a

¹⁵³ For a comprehensive study of the *qiyās* theory of the *Shāfi’ī* Abū Ishāq al-Shīrāzī, see Young (2017). Rahman and Iqbal (2018) developed a formalism based on Young’s study, already employing the structure of hypothetical imperatives.

¹⁵⁴ That is, the principle that “the original state of things is permissibility except for what the Law proscribes” (*al-aṣl fi’l-ashyā’ al-ibāḥa illā mā ḥazarahu al-Shar’*). See Ibn Ḥazm (1928-1933, vol. 1, p. 177); (1988, vol. 1, p. 176); (2010, p. 81); (1926-1930, vol. 3 pp. 76-77), vol. 6, p. 161.

vast arena for juristic contention and disagreement (*ikhtilāf*). This, coupled with—and governed by—systematized if constantly evolving rules for juristic dialectic (*jadāl / munāzara*), made possible a set of systems capable of continual updating and refinement, in a way Young (2017) calls the *dialectical forge*. Without such principles, methods, and disputational dynamics, Islamic legal systems would have remained closed and static.

Moreover, in this context, iterations such as *It ought to be that A is obligatory*, should not be read as the sheer iteration of the deontic operator, but as the call to integrate the obligation of A into the legal system. In other words, *It ought to be that A is obligatory* reads

The norm "A is obligatory" should be integrated into the legal system.

And this again calls for a legal argument justifying the claim. The same holds for *It ought to be that A is reprehended*, and *It ought to be that A is recommended*.

On the other hand, Ibn Ḥazm's literalist approach is developed within a framework where, as already mentioned, language is conceived quite statically, and where epistemological fictionalism—and in particular the use of *conjecture* for attaining knowledge—is rejected.¹⁵⁵ This might seem to disqualify our suggested approach to understanding the parallelism between natural and deontic necessity as achieved by some manner of parallel reasoning. Moreover, the reader might at this point wonder if the hypothetical structure of natural modalities developed in the present paper does not betray Ibn Ḥazm's rejection of conjectures. As mentioned, however, we can substitute hypotheticals with universal quantifications; these latter, logically speaking, are only one inference away from hypotheticals. Still, this might not be of help in understanding the epistemological mechanism at work when grasping the more general parallelism at issue in our paper.

If we seek to locate an understanding of the parallelism within Ibn Ḥazm's own epistemology, we should recall his notion of *rational perception* (*idrāk al-'aql*), an immediate intellectual act of knowledge, which is closer to the concept of *immediate inference* of contemporary constructivists than it is to Cartesian intuition. This immediate act of knowledge produces rational understanding (*fahm*).

Now, despite its rational and logical nature, *rational perception* is not devoid of sense perception. Arnaldez (1956, p. 128) has pointed out that within epistemology there is always reasoning at the level of the senses. Rational perception is, accordingly, a kind of immediate act of abstraction.¹⁵⁶ And in such an epistemological framework, the correspondence between the world of events and the world of actions seems to have a natural place.

VI Beyond Ibn Ḥazm: Conclusions and the Work Ahead.

VI.1 Brief Remarks on Ibn Ḥazm's Heteronomous Imperatives and Deontic Logic.

We share von Wright's (1981, p. 34) qualification of the traditional logical analysis of norms concerning

[...] *structures resembling what Kant called hypothetical imperatives*

although we certainly take exception to the remark that such approaches do not constitute a *genuine deontic logic*—if indeed that remark cannot be reduced to the trivial assertion that the traditional logic of norms is different from the analysis delivered by contemporary

¹⁵⁵ See Puerta Vélchez (2013, pp. 298-300).

¹⁵⁶ See Puerta Vélchez (2013, p. 301).

formal semantics.

Our preliminary exploration, based on Ibn Ḥazm's analysis of legal norms, does not yet deliver a logic of norms; it delivers a logical analysis of deontic notions, where obligation can be defined both as a particular kind of inference (namely a hypothetical judgement), and as an operator (namely as a **universally quantified expression**).¹⁵⁷ The point of the logical analysis of deontic notions in such a context lies in shaping the argumentation in favour of, or against, transferring the juridical decision from an established case to a new one. Thus, according to our approach, the main focus of deontic qualifications is in contributing to both the content and implementation of legal norms, rather than in studying the logical validity of arguments involving deontic qualifications.

To formulate the point a bit differently, our logical analysis of the deontic content of legal norms takes the side of those who prioritize **ought to do** (*Tunsollen*) over the Leibnizian concept of **ought to be** (*Seinsollen*).¹⁵⁸ While it seems natural to endorse the assertion that *it ought to be the case that contradictions are false*, it is not at all clear how this notion of obligation is linked to *what agents ought to do or bring about*.¹⁵⁹

A direct consequence of this perspective is that, in such a context, it does not make sense to include tautologies within the set of obligatory legal norms—a standard problematic issue in standard deontic logic. On the other hand, certain other known paradoxes of current standard deontic logic, such as paradoxes triggered by conditional obligations, do not seem to arise; Rahman/Granström/Farjami (2019) elaborate this point in the context of legal reasoning.

For example, Ross's (1941) paradox becomes harmless in the logic of deontic imperatives. The paradigmatic example of this paradox is based on the fact that we can infer it is obligatory to send a letter, or to burn it, from the premise that it is, say, legally obligatory to send it. But in the framework of heteronomous imperatives this paradox does not arise.

For if action of type *A* (sending the letter) has been chosen to be performed, and we know that this performance is *law-abiding* (and also that omitting to perform it is *law-breaking*), then weakening the antecedent is harmless.¹⁶⁰ And if carrying out some action *A* is law-abiding, adding the performance of an action of type *B* (burning the letter) does not support the inference that performing *B* will also be law-abiding. More precisely $R(y)$ will still apply to the performance of *A*. Notice that weakening the consequent is harmless, too: extending the consequent with a disjunction does not change the fact that the scope of $R(y)$ is still some performance of *A*.

In short, Ross's puzzle does not apply; whatever performance it is that makes *Sending the letter* true, it is of a type different from the one that makes *Burning the letter* true. Something similar applies to its dual, based on conjunction. **The point is that the seemingly oddity of applying the introduction rule for disjunction in Ross's example stems from the fact that standard semantics leaves the proof-object for the disjunction in the metalanguage.**

¹⁵⁷ Contemporary deontic logicians often point out that a deontic sentence p can be interpreted either *prescriptively* as expressing a mandatory norm, or *descriptively* as a statement that it is obligatory that p , according to some unspecified system of norms or law. See von Wright (1963, viii, pp. 104-5); Stenius (1963, pp. 250-1); Alchourrón (1969, pp. 243-5); Alchourrón and Bulygin (1971, p. 121). Our approach might appear closer to the descriptive interpretation, though perhaps our distinction between the type of action and its performance presents a middle path.

¹⁵⁸ Cf. Castañeda (1970). For a recent discussion on the issue see Hilpinen and McNamara (2013, pp. 59-69, 97-110).

¹⁵⁹ Rahman/Granström/Farjami (2019) discuss Chisholm's (1963b) puzzle on *contrary to duty imperatives* in the context of Islamic deontic categories.

¹⁶⁰ NB: In the context of contemporary law we interpret the expressions $R(x)$, $S(x)$, as the qualifications *Law-abiding*, and *Law-breaking*, rather than *Reward* and *Sanction*.

VI.2 On Normativity the Other Way Round and The Internalization of Nature.

It is the aim of the present study to suggest that an alternative perspective to the logic of norms has been achieved. As already mentioned, our proposal has not yet been developed into a logical system; we will now briefly mention the philosophical concepts grounding that project.

What at least the first author of the present paper is ready to claim is that the logic of norms should be built on a conception where logic itself is understood as normatively constituted. This is the reason why traditional logic involving deontic concepts is so close to inference itself. The point is that traditional logic—from Plato and Aristotle, through to the Stoics and the Islamic tradition, to Leibniz, Kant, Bolzano and even Frege—is about inference; and inference is understood as being normative. Let us briefly elaborate on this point that should certainly, we believe, be the focus of future research.

Logical approaches to meaning subsequent to the work of Frege and Tarski are based on what we might call, in a general manner, the *semantization of pragmatics* (SP): contexts are formalized with the help of some specific indexes, usually in the metalanguage, upon which the truth-value is made dependent. According to this view, a propositional kernel is complemented by “modalities” expressing necessity, commands, temporality and knowledge. These modalities are defined then as truth-functional operators conceived as certain kinds of logical connectives.

As pointed out by Per Martin-Löf (2017, p. 9), however, standard approaches to the layers underlying logic have gotten the order of priority between deontic and epistemic notions the wrong way round. Martin-Löf’s remark is motivated by his fine analysis of the dialogical conception of logic, particularly that which is proposed by Paul Lorenzen (1958), his point being that—different to other standard logical approaches—dialogical logic got the order right.¹⁶¹

[...] *deontic logic deals only with assertions, just like in standard logic, it’s only assertions whose content contains the deontic operations. This puts an opposite order between the deontic notions and the epistemic ones than the one that I have been advocating here, namely that the deontic notions make their appearance in the analysis of what it means to know how to do something; that’s where the deontic notions have their place. So, the deontic notions are at a more basic level than the epistemic notion of knowledge-how [...]. Maybe this is the explanation for the difficulties of deontic logic: [...] maybe it can be explained in this way, that one has got the order of priority between the deontic notions and the epistemic notions the wrong way around.*

Per Martin-Löf (2017, p. 9).

According to the dialogical approach, knowledge and meaning emerge as a fine intertwining of having the *right to ask for reasons* and the *duty to give them*, to put it in the words of Brandom (2000). From this perspective, not only are inferences understood as the duty to give reasons for supporting a conclusion, the interface right-duty is constitutive of the judgements that structure an inference.

Thus, the traditional view on the logic of norms as related to hypothetical imperatives is only a consequence of this overall normative view on knowledge and reasoning. The lesson of the parallelism between natural and deontic necessity is that in order to achieve knowledge both have to be internalized by the same act: they must be *placed in the space of reasons*.¹⁶² This internalization is one way to understand the inverse parallelism that ascribes moral features to nature, and is part and parcel of Ibn Ḥazm’s notion of *rational perception* (*idrāk al-‘aql*) mentioned above.

To know about our world requires to know *why* the events in the world are linked the way they are. But *knowing why* comes always together with *knowing how*: we need to bring to the fore, to the tribunal of public criticism, how those reasons fulfil our obligation to justify our assertions. And this is no different to the way we provide reasons for the guiding of our actions.

¹⁶¹ For a comprehensive presentation of the dialogical conception of logic, see Rahman, McConaughey, Klev, and Clerbout (2018).

¹⁶² The notion of the *space of reasons* stems from Sellars (1991, pp. 129-194).

Still, the underlying logical **building blocks** of Ibn Ḥazm's heteronomous imperatives is hypothetical. Moreover, the parallelism is **based on admitting several levels of knowledge rather than on certainty**. This is a puzzle within Ibn Ḥazm's own epistemology; and our paper does not provide an answer beyond the modest observation that if rational perception is targeted at the parallelism Ibn Ḥazm has bequeathed us, we might grasp in a sole act of consciousness that epistemology requires a gamut of perspectives after all.¹⁶³

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The Good Cobbler, the Writing Man and the Unseen Bride

Some historic and systematic reflections on the family of inferences involving compounds

Abstract: The fallacy of composition – recall Aristotle’s famous example of the *Good Cobbler* –, who is not just *Good* and a *Cobbler*, one of the eldest patterns of illegitimate inferential moves, animated and still animates lively discussions in argumentation and logic which as pointed out by Bar-Hillel (1964) and Woods & Walton (1977), calls for a fresh start for the classification and shaping of the notion of fallacies and illegitimate moves. After motivating an analysis of Aristotle’s within the framework of Ranta’s (1994) Type Theory Grammar; we briefly discuss two, likely independent, formidable developments of Medieval Islamic thought on **a family** of inferences involving composition and division, which not only had long lasting consequences in the history of philosophy, but they offer new perspectives for researches in philosophy of logic and argumentation. Namely, (i) Avicenna’s *divided or substantial* (dāṭī) and *compound or descriptive* (waṣṫī) reading of temporal and modal propositions in predicate logic – e.g. *All (human)writers move their fingers of necessity, all the time while writing*, is true in the descriptonal/compound reading, but false in the divided reading, since writers move necessarily their fingers when those humans are understood as writing-humans; but false in the substantial/divided sense, since writers are not necessarily moving their fingers when we only focus in their substance, namely human beings – interestingly Hodges & Johnston (2017) relate Avicenna’s approach to Buridan’s *de re/de dicto* distinction (ii) The laying down of rules for producing a counterexample (*kasr*-case), based on comparing alternative compounds that preserve the relevant legal meaning of the original compound, within Islamic Legal Argumentation Theory – e.g. the legal invalidity of a transaction involving an unseen-good, cannot be countered by the case of the legal validity of a marriage involving an unseen-bride. In fact, we will also very briefly mention paradoxes on totalities understood as mereological wholes, discussed by philosophers of the eastern Islamic world (Iraq to India) from the thirteenth century to the nineteenth. Let us point out that Bar-Hillel (1964) and Woods & Walton (1977) pointed out that mereological analysis of whole-part relations might also be understood as generalizing some of the features shaping the fallacy of composition. These brief remarks show that the fallacy of composition constitutes a larger family of illegitimate moves, perhaps too large and so it seems that a fresh start on the place of the composition fallacy in the classification of fallacies is due, or even for a rethinking of the general classification of fallacies.

I Introduction

The fallacy of composition – recall Aristotle’s famous example of the *Good Cobbler* –, who is not just *Good* and a *Cobbler*, one of the eldest patterns of illegitimate inferential moves, animated and still animates lively discussions in argumentation and logic which as pointed out by Bar-Hillel (1964) and Woods & Walton (1977), calls for a fresh start for the classification and shaping of the notion of fallacies and illegitimate moves. After motivating an analysis of Aristotle’s within the framework of Ranta’s (1994) Type Theory Grammar; we briefly discuss two, likely independent, formidable developments of Medieval Islamic thought on a whole **family of inferences** on composition and division, which not only had long lasting consequences in the history of philosophy, but they offer new perspectives for researches in philosophy of logic and argumentation; namely:

1. Avicenna’s *divided or substantial* (dāṭī) and *compound or descriptive* (waṣṫī) reading of temporal and modal propositions in predicate logic – e.g. *All (human)writers move their fingers of necessity, all the time while writing*, is true in the descriptonal/compound reading, but false in the divided reading, since writers move necessarily their fingers when those humans are understood as writing-humans; but false in the substantial/divided sense, since writers are not necessarily moving their fingers when we only focus in their substance, namely human beings.

Wilfried Hodges (<http://wilfridhodes.co.uk/arabic44.pdf>) is working out the mathematics behind this insight of Avicenna based on a reconstruction compatible with some form of Kripke-structures. There are other ongoing alternative reconstructions based on Aarne Ranta's (1994, chapter 5) development of temporal reference within Constructive Type Theory, closer to Tony Street's (2002a,b, 2004) interpretation – see Rahman & Zarepour (2021). The latter also motivated new researches on temporal logic inspired by Post-Avicennian thinkers, such as Suhrawardi, see Rahman & Seck and Rahman & Boussad (both forthcoming).

2. The laying down of rules for producing a counterexample (*kasr*-case), based on comparing alternative compounds that preserve the relevant legal meaning of the original compound, within Islamic Legal Argumentation Theory – e.g. the legal invalidity of a transaction involving an unseen-good, cannot be countered by the case of the legal validity of a marriage involving an unseen-bride: a commodity whose attributes are unknown to the buyer at the time of a sales contract is relevantly different from a bride whose attributes are unknown to the groom at the time of a marriage contract, as proven by the buyer's right to rescind and the groom's lack thereof. The accent is here on unsafely inferring the legal validity of a transaction by only focusing on one of the components. More generally the focus is here on illegitimate substitutions within a compound that result from disregarding the inner structure of the compound (conjunctive, disjunctive, or functional) – we might say, borrowing Avicennian terminology, substitutions that disregard the way the compound has been described. Muhammad Iqbal (2020) showed some forms of it can contribute to develop further Paul Bartha's (2010) recent model for parallel reasoning in sciences. Martínez Cazalla et al. (2020) showed such models of argumentation can be used in contemporary Civil Law.

In fact, we will also very briefly mention paradoxes on totalities understood as mereological wholes, which as highlighted by Hassan Rezakhany (2023, forthcoming), was discussed by philosophers of the eastern Islamic world (Iraq to India) from the thirteenth century to the nineteenth. Let us point out that Bar-Hillel (1964) and Woods & Walton (1977) pointed out that mereological analysis of whole-part relations might also be understood as generalizing some of the features shaping the fallacy of composition.

Interestingly, neither of them, at least not the first two cases, are explicitly included in texts or treatises on fallacies, which strongly indicates, that a comprehensive study of fallacies, should comprise those illegitimate forms of inference and argumentation moves that count as fallacy with regard to a norm for correct reasoning given by a context.

II ARISTOTLE ON GOOD COBBLERS

II.1 The Lack of Expressivity of First-Order Logic

Aristotle (*Peri Hermeneias*, chapter XI, 20b35-36 and 21a14-15, *Sophistical Refutations* 20, 177b14-15) pointed out, and rightly so, that from:

Some x is good and a cobbler,

one cannot, without any proviso, infer:

Some x is a good cobbler.

In standard classical contemporary logic, Aristotle’s remark amounts to the fact that from:

$$\exists x (Good(x) \wedge Cobbler(x))$$

one cannot infer:

$$\exists x Good-Cobbler(x).$$

Notably, the latter expression is also unproductive, since it does not support any further inferences upheld by the original *Some cobbler is good (as a cobbler)*.

Clearly, the point is that:

- 1) On the one hand, in:

$$\exists x (Good(x) \wedge Cobbler(x))$$

“*Good*” is left totally unspecified. *Good* at what? *Good* human? *Good* runner? In fact, this logical form is compatible with *x*’s being both a *good* runner (or *good* cook, etc.) and a *cobbler* (possibly a bad cobbler).

- 2) On the other hand, the formalization:

$$\exists x Good-Cobbler(x)$$

does not support the inference that:

Given *some x is a good-cobbler*, it follows that *x is both a cobbler and good as a cobbler*.

In sum, the purported inference:

$$\exists x (Good(x) \wedge Cobbler(x)) \vdash \exists x Good-Cobbler(x)$$

is not only deductively invalid, but:

- The premise and conclusion contravene the patent meaning which is conveyed by the natural language expression, and which allows one to infer from: *some x is a good cobbler*, that *x* is both *a cobbler* and *good as a cobbler*.

Ibn Sīnā’s example,¹⁶⁴ on the other hand, explicitly uses a singular term, which accentuates both the lack of expressivity of standard contemporary first-order logic for analysing the fallacious inference by composition, and the fact that the premisses might lead to a conclusion which is **logically valid but not the targeted one**.

Ibn Sīnā observes that concluding (3) from the conjunction of (1) and (2) yields a fallacious argument.

¹⁶⁴ Proposed in the logic part of his *Ishārāt* (1983, Chapter 10.1, pp. 501-502). We owe this reference to Alexander Lamprakis – cf. Rahman&Zarepour (2021, pp. 411-417).

- (1) Imru' al-Qays is good.
 (2) Imru' al-Qays is a poet.

 (3) Imru' al-Qays is a good poet.

Notice, however, that a naïve first-order translation, with “*a*” standing for Imru' al-Qays, yields a **logically valid conclusion**:

- (1) *Good(a)*
 (2) *Poet(a)*

 (3) *Poet(a) ∧ Good(a)*

In fact, it may even yield the more general conclusion:

There is some *x* called “*a*” (whereby “*a*” stands for *Imru' al-Qays*), who is a poet **and** who is good:

- (1) *Good(a)*
 (2) *Poet(a)*

 (3) $\exists x ((x = a) \wedge Good(x) \wedge Poet(x))$

There is an *x* such that { *x* is called “*a*” } & { *x* is Good (as person, or mathematician, or ...?) } & { *x* is a Poet }

Logically speaking this inference is fine. However, once again the inferred conclusion **does not express the meaning** conveyed by:

Imru' al-Qays is a good poet.

Thus, in such a framework the fallacy results from attempting to infer, from (1) and (2), both

Good-Poet(a)

and/or

$\exists x ((x = a) \wedge Good-Poet(x)).$

This perspective on Aristotle’s example indicates that qualifying whether a step is, or is not, deductively illegitimate—at least in the context of the fallacy of composition—depends upon specifying a targeted conclusion and linking it with a set of premises that can either yield, or not yield, a deductively sound inference.¹⁶⁵

Let us now attempt a formal analysis that renders the meaning of the natural language expression.

II.2 Logic in Reverse (RevLog) and the Fallacy of Composition

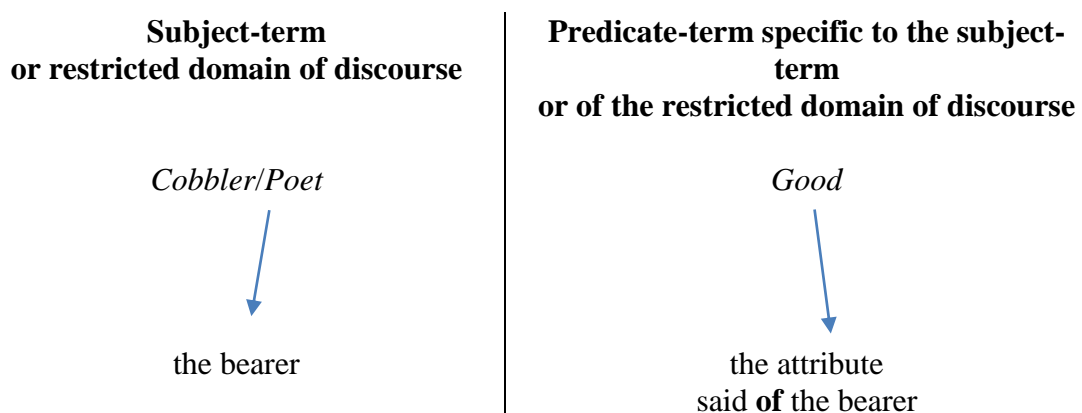
¹⁶⁵ This seems related to Crubellier’s observation (2011, 2014a,b, 2017) that Aristotle’s take on the syllogism often adopts the abductive explanatory perspective rather than the deductive. In other words, Aristotle often examines syllogisms from the conclusion to the premisses, rather than from the premisses to the conclusion. From such a perspective, the syllogism is linked to the task of finding a suitable middle term. Cf. Crubellier (2011), McConaughy (2022), section 2.3.3.

RevLog’s leading idea is to pursue a “reverse” approach to argumentation, seeking to illuminate the principles of sound argumentation through the lens of what goes wrong in fallacious reasoning. So now that we have glimpsed what is wrong in the fallacy of composition, we should determine what the rules allowing sound inferences involving compounds actually look like. In other words, we would like to have our cake and eat it too, i.e., to render the inference rules that allow one to conclude the kind of compounds mentioned above, along with the rules that allow one to infer that *Good Cobblers* are, after all, *good as cobblers*, without falling into the fallacy of composition.

In order to do so we will return to the old Subject-Predicate structure (whereby the former expresses the bearer of attributes and the second what is attributed to the bearer), cast in the language of Per Martin-Löf’s *Constructive Type Theory* (CTT), as applied to natural language by Aarne Ranta (1994). Clearly, other formal languages might offer alternative reconstructions. However, we have chosen this application of CTT because of its expressive-power, which seems very close to the grammatical and logical analyses developed in ancient and medieval logic.¹⁶⁶ After a brief discussion, we will show that this approach renders an analysis of the fallacy of composition which comes very close to the later commentary on Aristotle’s treatment of the good cobbler.

II.2.1 Back to the Good Old Ways: Attributes and Bearers of Attributes

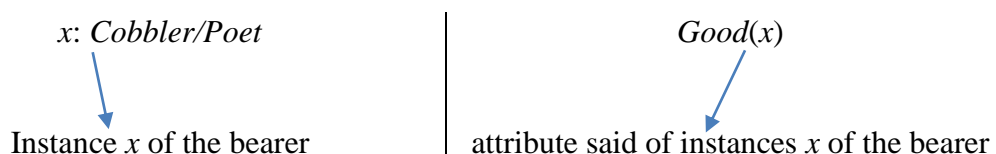
One way to begin analysing the examples above is to explicitly distinguish the bearers of attributes from the attributes predicated of them.



The point here is that **the attribute is restricted** to instances of some specific bearer: **it is one of the things that you can say of some specific bearer**. Thus, it is apparent that:

Good Cobbler/Poet
 means
Good as a cobbler/poet.

In other words, those x of whom it is said that they are good, are instances of the bearer being a cobbler/poet. Thus, $Good(x)$ assumes that x is an instance/token of *Cobbler/Poet*, i.e., x : *Cobbler/Poet*.



¹⁶⁶ A thorough plaidoyer for the fruitfulness of this kind of analysis for Aristotelian logic, when combined with Dialogical Logic, may be found in McConaughey (2021).

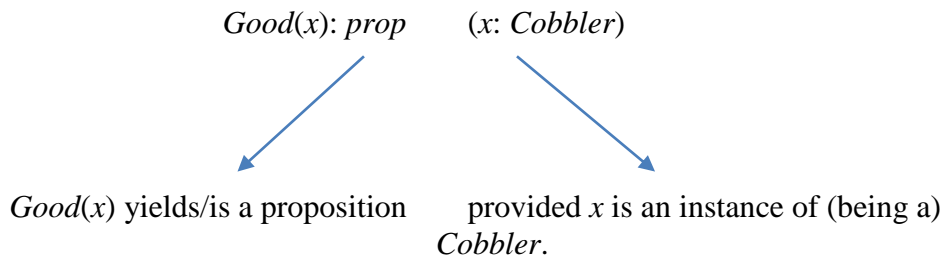
Notice that x : *Cobbler* also admits the following readings (with, of course, similar readings applying to x : *Poet*):

- x **is** an instance of *Cobbler*;
- x **is** an element of the set *Cobbler*;
- x **verifies** that the set/type *Cobbler* is not empty;
- x **is** some instance of the Subject-Term *Cobbler*.

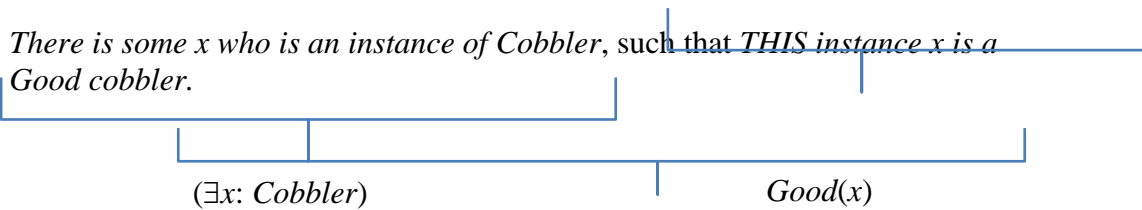
In other words, the logical form of $Good(x)$ is that of a propositional function defined over the restricted domain of *Cobblers*, *Poets*, *Cooks*, and so on. Thus, $Good(x)$ yields a proposition if one substitutes for x instances of a specific bearer to which this attribute of $Good$ is said to apply:

<i>Good Cobbler</i>	$Good(x)$ yields a proposition if x is an instance of (being a) <i>Cobbler</i>
<i>Good Poet</i>	$Good(x)$ yields a proposition if x is an instance of (being a) <i>Poet</i>
...	...
<i>Good B</i>	$Good(x)$ yields a proposition if x is an instance of (being a) <i>B</i>

Formally, we express this with the following well-formation rule for the propositional function $Good(x)$:



Then if we quantify existentially over the domain *Cobbler*, we obtain the following:



$$(\exists x: \text{Cobbler}) \text{Good}(x)$$

Something similar, of course, applies to $(\exists x: \text{Poet}) \text{Good}(x)$.

Notice again that here we do not have the ambiguity of $Good$ mentioned above. $Good$ is **dependent** upon the set of *Cobblers/Poets*; *Cobbler* and $Good$ are **not** two attributes such that the second can be dependent upon another domain, such as $Good$ cooks (or some other subdomain of human being). In short, the composition is **not achieved by the conjunction** of $Cobbler(x)$ and $Good(x)$, but by expressing $Good$ as a compound attribute said of good cobblers/poets.

So far so good, but we would also like to have inference rules which avoid the fallacy of composition and which render both:

1) the synthetic procedure by way of which we obtain, from the premises that a is a cobbler and that a is a good cobbler, the fact that at least one x enjoys the compound attribute of being a *good cobbler*,

and

2) the analytic procedure, by way of which we decompose $(\exists x: \text{Cobbler}) \text{Good}(x)$, into the facts that there is indeed an instance x of *Cobbler*, and that one of those instances enjoys the attribute of being *good as a cobbler*.

II.2.2 Synthetic and Analytic Procedures of Composition

Let us start with the rules that prescribe how to constitute a meaningful expression out of the predicate *Being good (as a cobbler)* and the subject or domain of *Cobblers*. These *well-formation rules* comprise well-formation procedures for the constitution of a proposition which are at the same time syntactic and semantic. According to this approach, an expression is well-formed or meaningful iff it satisfies both semantic and syntactic conditions for the constitution of a proposition (independently of the truth or falsity of the resulting proposition). In other words, and applying the traditional terminology to our main example: The predicate *Being Good (cobbler)* yields a well-formed proposition, such as *Some Cobblers are Good (cobblers)*, iff this proposition has instances of *Cobbler* as its subject.

Well-Formation Rules

From the above discussion we have learned that $(\exists x: \text{Cobbler}) \text{Good}(x)$ is well-formed if *Cobbler* is a set/domain of discourse and *Good(x)* is a propositional function defined over that set/domain. Thus:

Formation Rule	Subject-Predicate Reading	Natural-Language (Contemporary) Gloss
<i>Cobbler: set</i>	→	Instances of <i>Cobbler</i> provide the subject of propositions about cobblers.
<i>Good(x): prop (x: Cobbler)</i>	→	In order to build a proposition out of the predicate <i>Being Good</i> (as a cobbler), the subject of this proposition must be constituted by instances of <i>Cobbler</i> .

$(\exists x: \text{Cobbler}) \text{Good}(x): \text{prop}$	→	The proposition <i>Some cobbler is good (as a cobbler)</i> predicates <i>being Good (as a cobbler)</i> of at least one of the cobblers.

Provided this formation rule—which establishes that the existential has been built by combining a domain and a predicate over this domain, such that the predicate is *restricted* to this domain (in our example, that *Good* is said of good cobblers)—we can say:

a is a *Cobbler*
 this a is *Good (as a cobbler)*

There is at least one instance of *Cobbler* who is a good cobbler

The most delicate step in the inference is the second premise. We need a procedure that combines the first premise with the second one in such a way that it **expresses semantic composition but not conjunction** (which would lead to the fallacy).

The solution appears to be a simple one. We need a *method* that transforms instances of *being a cobbler* into instances of *being good*, in such a way that the composition at stake in the compound is understood as expressing the semantic dependence of the propositional function *Good(x)* upon the set *Cobbler*—i.e., the scope of the meaning of being *Good* is restricted to the set of cobblers. (Linguists would point out that the predicate *Good* **modifies** the predicate *Cobbler*.) Such a method is a function—that is, a function that takes instances of *being a cobbler* and yields instances of *being good* (at *being a cobbler*):

a: Cobbler

b(a): Good(a).

Now, given both premises, we can synthesize them into the compound:

$(\exists x: \text{Cobbler}) \text{Good}(x)$

Remark:

Notice that if we wish to render both *Cobbler* and *Good* as attributes of, say, the species of *Human Beings*, but at the same time stress the fact that *Good* is dependent upon *Cobbler* (i.e. that *Good* modifies *Cobbler*), the structure requires the embedding of two existentials expressing that *Cobbler* is said of *Human Beings*, and *Good* is said of those human beings who are cobblers. This notation is a bit more complex, but in fact amounts to an embedding of two existentials:

$(\exists y: \{(\exists x: \text{Human Being}) \text{Cobbler}(x)\}) \text{Good}(\text{this-}y)$

There is an instance *y* of { being one of those humans *x* who are cobblers }, such that { this instance *y* (of human cobbler) is good (as a cobbler) }.

In order to include the anaphoric “*this-y*” within the formalization we need the notation $(\exists y: \{ x: \text{Human Being} \mid \text{Cobbler}(x) \} \text{Good}(\text{left}(y)))$, which explicitly expresses that the instance of those of which it is said that they are good as cobblers, is of a human – the left component Human-Cobbler –, who is of course a cobbler.

For the sake of simplicity, we will deploy the simpler structure for now.

This yields the searched inference rule, called the “**introduction rule**,” which renders the conditions which must be fulfilled in order to assert $(\exists x: \text{Cobbler}) \text{Good}(x)$:

Synthesis

Introduction Rule

Natural-Language Gloss

a: Cobbler



a is a *Cobbler*

b(a): Good(a)



There is a way to associate instances of *being a cobbler* with instances of

being a good cobbler, by means of the function $b(x)$; and this function can be applied to a , in order to obtain the instance $b(a)$ of a being one of those *good cobblers*.

 $\langle a, b(a) \rangle: (\exists x: \text{Cobbler}) \text{Good}(x)$ \longrightarrow -----
 The assertion *There is at least one cobbler who is good at being a cobbler*, is grounded on the pair $\langle a, b(a) \rangle$, such that a is an instance of *being a cobbler* (given by the first premise)—i.e., some cobbler a has been found—and such that $b(a)$ is a proof (i.e., verification process) witnessing that *this cobbler is a good cobbler* (given by the second premise).

Or, in a horizontal notation, with the turnstile “ \vdash ” standing for the inference bar:

$a: \text{Cobbler}, b(a): \text{Good}(a) \vdash \langle a, b(a) \rangle: (\exists x: \text{Cobbler}) \text{Good}(x)$

Analysis

Let us now assume that we have an instance c of the compound of something *being a good cobbler*—i.e., let us assume c witnesses $(\exists x: \text{Cobbler}) \text{Good}(x)$. Notice that if c witnesses $(\exists x: \text{Cobbler}) \text{Good}(x)$, then c is an element of the set of *all those cobblers who are Good*.¹⁶⁷ Thus, if c is an un-analysed compound c , and we assert that this c is some instance of the set of *all those cobblers who are good (as cobblers)*, i.e.,

$c: (\exists x: \text{Cobbler}) \text{Good}(x)$

then we can infer—by recalling that instances, such as c , of *being a good cobbler*, are built by the compounds prescribed by the synthesis rule—that:

1. The analysis of the left component of c should yield some instance, say a , of *being a cobbler*
2. The analysis of the right component of c should yield an instance $b(a)$, witnessing that cobbler a is *a good cobbler*.

That is, from:

There is an instance c of being a good cobbler,

we can infer:

1. *Some instance of cobbler can be found,*
2. *There is a way to associate the found cobbler with an instance of being a good cobbler.*

¹⁶⁷ Recall that in our remark on notation we indicated that an existential, such as $(\exists x: \text{Cobbler}) \text{Good}(x)$, can be rendered as the set $\{ x: \text{Cobbler} \mid \text{Good}(x) \}$ of all those cobblers who are good as cobblers.

Thus, the analysis procedure yields two decomposition rules: one for the left component of c and one for the right. These rules are called “**elimination rules.**” They prescribe what inferences we commit to when asserting that there is an instance c witnessing the existential *At least one cobbler is good as a cobbler.*

In other words,

- **The first elimination rule** allows us to obtain, from the compound c , its left component—i.e., an instance, let us call it c_1 , of *being a cobbler.*

$c: (\exists x: \text{Cobbler}) \text{Good}(x)$ <hr style="border-top: 1px dashed black;"/> left(c): Cobbler	\longrightarrow	Natural Language Gloss c is one of these cobblers who are good as cobblers <hr style="border-top: 1px dashed black;"/> the <i>left component</i> of the instance c of one of those cobblers who are good, is a cobbler
Whereby: $a: \text{Cobbler}$	\longrightarrow	a is a cobbler
$b(a): \text{Good}(a)$	\longrightarrow	$b(a)$ yields an instance of cobbler a being a good cobbler
<hr style="border-top: 1px dashed black;"/> left(c)=left<a, b(a)>=a: Cobbler	\longrightarrow	<hr style="border-top: 1px dashed black;"/> the <i>left component</i> of the instance c of one of these cobblers who are good is cobbler a

- **The second elimination rule** allows us to obtain, from the compound c , its *right* component—i.e., an instance, $b(a)$, witnessing the left component to be a *good cobbler.*

$c: (\exists x: \text{Cobbler}) \text{Good}(x)$ <hr style="border-top: 1px dashed black;"/> right(c): Good(left(c))	\longrightarrow	Natural Language Gloss c is one of these cobblers who are good as cobblers <hr style="border-top: 1px dashed black;"/> the <i>right component</i> of the instance c (of one of those cobblers who are good) witnesses that its left component (the chosen cobbler) is good (as a cobbler).
Whereby: $a: \text{Cobbler}$	\longrightarrow	a is a cobbler
$b(a): \text{Good}(a)$	\longrightarrow	$b(a)$ yields an instance of a being a good cobbler
<hr style="border-top: 1px dashed black;"/> right(c)=right(a,b(a))=b(a): Good(a)	\longrightarrow	<hr style="border-top: 1px dashed black;"/> the <i>right component</i> of the instance c (of one of those cobblers who are good) namely $b(a)$, witnesses that its left component, i.e. a , is good (as a cobbler).

II.2.3 The Fallacy of Composition Revisited

The upshot of the preceding considerations is the following rendering of the fallacy of composition:

$$\exists x (\text{Good}(x) \wedge \text{Cobbler}(x)) \not\equiv (\exists x: \text{Cobbler}) \text{Good}(x)$$

But in fact, this is not even well-formed, since the expression left of the turn style quantifies implicitly over some universal or restricted domain say *Human*, while the expression right of the turn style quantifies over the domain *Cobbler*:

$$(\exists x: \text{Human}) (\text{Good}(x) \wedge \text{Cobbler}(x)) \quad \not\equiv \quad (\exists x: \text{Cobbler}) \text{Good}(x),$$

Given

$Good(x): prop (x: Human)$

$Good(x): prop (x:$

$Cobbler)$

$Cobbler(x): prop (x: Human)$

This rendering, as pointed out to me by Zoe McConaughey, coincides with Catherine Dalimier’s (2007, p. 343, n. 6) comment on the cobbler passage in her French translation of the *Peri Hermeneias*. In brief, she notes that the fallacy lies in the fact that, when saying: “he is good and he is a cobbler,” “good” is predicated of the man (just as cobbler is predicated of that man); but when saying: “he is a good cobbler,” “good” is predicated of the cobbler.¹⁶⁸

III THE FALLACY OF COMPOSITION AS AN ILLEGITIMATE ARGUMENTATIVE MOVE

In a dialogical setting, fallacious deductive inferences by composition amount to bringing forward illegitimate argumentative moves while justifying an assertion involving *compound attributes*. Recall that, as mentioned above, in an argumentative setting justification proceeds from conclusion to premisses. More precisely, if, when asked to justify their assertion:

Some x is a good cobbler,

a respondent **X** responds:

a is a cobbler and a is Good,

then these will be deemed *illegitimate responses* to the request of justifying the composition at work in the assertion. In fact, they are illegitimate because the questioner can now retort by showing that these responses are compatible with cobbler *a* being good, say, as a musician, but still bad as a cobbler.

III.1 The Good Cobbler as an Illegitimate Argumentative Move

Assertion	Request	Illegitimate Responses	The Antagonist’s Rejoinder
X ! <i>Some x is a good cobbler</i>	Y ?_L _____	X ! <i>Cobbler(a)</i> (X responds: <i>a is a cobbler</i>) _____	} Y ! <i>Cobbler a is a good musician, but still he is a bad cobbler</i>
(X claims: <i>Some x is a good cobbler</i>)	Y ?_R (Questioner Y asks X to justify the left and/or right component of <i>good cobbler</i>)	X ! <i>Good(a)</i> (X responds: <i>a is good</i>)	

Note that canonical rules prescribing the legitimate responses corresponding to our discussion above can also be declined into rules of *composition or synthesis* and rules of *decomposition or analysis*. The synthesis rules prescribe which moves lead to the

¹⁶⁸ Dalimier also provides the reference to the *Sophistical Refutations* 20, 177b14-15.

justification of **X**'s assertion *Some x is a good cobbler*. Moreover, if successful, **X**'s responses should lead to constructing the compound that justifies their initial claim.

SYNTHESIS			
Assertion	Request	Canonical Responses	Final Synthetic Outcome ¹⁶⁹
<p>X ! ($\exists x$: <i>Cobbler</i>) <i>Good(x)</i>¹⁷⁰</p> <p>(X claims: <i>Some x is a good cobbler</i>)</p>	<p>Y ?_L</p> <p>_____</p> <p>Y ?_R</p> <p>(Questioner Y asks X to justify the left and/or right component of <i>good cobbler</i>)</p>	<p>X <i>a</i>: <i>Cobbler</i> (X responds: <i>a is a cobbler</i>)</p> <p>_____</p> <p>X <i>b(a)</i>: <i>Good(a)</i> (X responds: <i>b(a) witness cobbler a being a good cobbler</i>)</p>	<p>X $\langle a, b(a) \rangle$: ($\exists x$: <i>Cobbler</i>) <i>Good(x)</i></p> <p>(X justifies the initial claim by means of the compound $\langle a, b(a) \rangle$—constituted by <i>a</i>, which verifies that a cobbler, namely <i>a</i>, has been found—and the verification process <i>b(a)</i>, witnessing that this <i>a</i> is indeed a <i>good cobbler</i>. The initial justified claim now becomes an assertion.)</p>

The analysis rules, on the other hand, again prescribe how to decompose a non-articulated compound *c*, which is claimed to witness both the fact that there is at least one cobbler, and the fact that this cobbler is good as a cobbler. Moreover, the questioner can continue their interrogation, requiring **X** to bring forward concrete instances of each component, by asking questions such as: *Who is the cobbler that you claim gives witness to there being at least one (cobbler)? How do you verify that this cobbler is a good one?*

ANALYSIS (I)		
Assertion	Request	Canonical Responses
<p>X <i>c</i>: ($\exists x$: <i>Cobbler</i>) <i>Good(x)</i></p> <p>(X claims: the compound <i>c</i> justifies <i>Some x is a good cobbler</i>)</p>	<p>Y ?_L</p> <p>_____</p> <p>Y ?_R</p> <p>(Questioner Y asks X to justify the left and/or right component of <i>good cobbler</i>)</p>	<p>X <i>left(c)</i>: <i>Cobbler</i> (X responds: the left component of <i>c</i> witnesses that there is indeed a cobbler)</p> <p>_____</p> <p>X <i>right(c)</i>: <i>Good(left(c))</i> (X responds: the right component of <i>c</i> witnesses that the left component is a good cobbler)</p>

ANALYSIS (II)		
Assertion	Request	Identifying the Components

¹⁶⁹ The final synthetic outcome is the result of an argumentative process we call “recapitulation.” The term “recapitulation” was inspired by Michel Crubellier, who pointed out that this was in fact one of the earliest senses of the term *sylogismos*, and represents one of most important phases in ancient Greek dialectic (cf. Kapp [1942, pp. 14–16 & 71], Rahman, et al. [2018, section 7.7.2]). The recapitulation process, when achieved, indicates that **X** has a winning strategy for justifying the initial assertion (which corresponds to the proof-theoretical notion of having a demonstration).

¹⁷⁰ Note that the exclamation sign will disappear when the instance of the proposition is made explicit in the responses. The point is that, with **X** ! *A*, the respondent **X** claims that there is some instance verifying his claim *A*, and, with **X** *a*: *A*, the respondent **X** brings forward instance *a*. The initial statement has an exclamation sign because it has no explicit proof object. This must be constructed during the interaction.

X left(c): Cobbler (X claims: the left component of <i>c</i> witnesses that there is indeed a cobbler)	Y ? left(c) (Y asks: who is this left component of <i>c</i> ?)	X left(c)=a: Cobbler (X responds: the left component of <i>c</i> is cobbler <i>a</i>)
<hr/> X right(c): Good(left(c)) (X claims: the right component of <i>c</i> witnesses that the left component is a good cobbler)	<hr/> Y ? right(c) (Y asks: how do you verify that the left component is a good cobbler?)	<hr/> X right(c)=b(a): Good(a) (X responds: the right component of <i>c</i> is the verifier <i>b(a)</i> , which witnesses that cobbler <i>a</i> is a good cobbler)

The point of discussing these composition-governing inferential and argumentative rules is to show that such rules are illegitimate (and fallacious) precisely when they do not correspond to the right way of composing the meaning of the dependent attributes prescribed by the well-formation rules. Rules for the well-formation of the existential within a dialogical framework are quite straightforward. In short, the questioner will require the respondent to make the components of *Some x is a Good-Cobbler* explicit. The respondent will then make explicit how being good is dependent upon being a cobbler.

WELL - FORMATION			
Assertion	Request	Responses	Final Formal Outcome
X Some <i>x</i> is a Good-Cobbler: prop (X claims: <i>Some x is a Good-Cobbler</i> is a well-formed proposition)	Y ?FL <hr/> Y ?FR (Questioner Y asks respondent X to make explicit each of the existential's components)	X Cobbler: set (X responds: <i>Cobbler</i> is a set) <hr/> X Good(x): prop (x: Cobbler) (X responds: <i>Good(x)</i> is a proposition provided <i>x</i> is some element of the set of cobblers)	<div style="display: flex; align-items: center;"> <div style="font-size: 2em; margin-right: 10px;">}</div> <div> X (∃x: Cobbler) Good(x): prop (X concludes that the existential <i>There is at least one Cobbler who is Good as Cobbler</i> is a well-formed proposition) </div> </div>

Avicenna's substantive-descriptive reading has a more complex structure. However, the illegitimate steps can be identified *mutatis mutandis* in a similar way. Indeed, if the assertion is

All writing humans move their fingers (all the time while writing)

the following answers of the responder with regard to any arbitrary *c* chosen by the questioner

*c is human (all the time while being what it is) and
c moves his finger (all the time while being a human),*

are clearly unjustified. Let us have a closer look at Avicenna's approach.

III AVICENNA ON COMPOUND AND DIVIDED SENSE

III. 1 The Writing Human

Abū-ʿAlī al-Ḥusayn ibn-ʿAbdallāh Ibn-Sīnā [hereafter: Avicenna] (980-1037 CE) is, directly or indirectly, the most influential logician in the Arabic tradition. His work is

central to the redefinition of a family of problems and doctrines stemming from ancient and late logic, in particular from Aristotle and the Peripatetic tradition.

If, in general, Avicenna fits squarely into a logical tradition that it is reasonable to qualify as Aristotelian, the treasure of innovations that he introduces makes him a new canonical figure in the field. Every subsequent logician in this tradition discusses his work either as a critic or as a follower or commentator. In fact, with few exceptions, his work rather than the one of Aristotle sets the paradigm that shaped the impressive development of post-classical philosophy in Islam – see Street (). The landmark work of Nicholas Rescher and Tony Street launched a host of researches on Avicenna allowing us to start to grasp the depth and breadth of his innovations – see Rescher (1963), Rescher & vander Nat (1974), Street (2000, 2002a,b, 2004)..

As already mentioned, studies exploring the possibilities offered by Avicenna’s innovations for modal and temporal first-order logic are still work on progress.

Here we will focus on Avicenna’s *divided or substantial* (dātī) and *compound or descriptive* (waṣfi) reading of temporal and modal propositions in predicate logic, and more precisely on only one of the many combinations and variants proposed by Avicenna – cf. Rescher & vander Nat (1974), Street (2002a,b), Hasnawi & Hodges (2016, Chatti (2019a,b), Strobino (2015, 2016a,b, 2018, 2021).

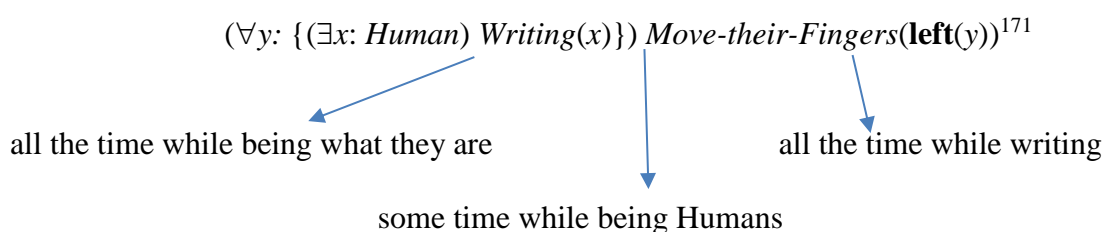
Moreover, following a paper in progress by Rahman & Zidani & Drissi, we will restrict our discussion to an analysis that

one side highlights its links to the fallacy of composition,
and on the other develops a kind of dynamic analysis that spells out the temporal bounds, during the process of justification, rather than burden the surface grammar with a complex logical structure.

In principle the assertion *All writing humans move their fingers (all the time while writing)* can be analysed in a similar way as we developed for the complex version of the assertion *Some (human-)Cobblers are Good*.

$(\forall y: \{(\exists x: \text{Human}) \text{Writing}(x)\}) \text{Move-their-Fingers}(\text{left}(y))$ (*all the time while writing*)

In “principle” since the time restriction requires a finer analysis



Thus, the assertion *All writing humans move their fingers*, can be read

in the compound sense: *All writing (humans) move their fingers (all the time while writing)*.

in the divided sense: *All writing (humans) move their fingers (all the time while being a human)*

¹⁷¹ Notice that though humans have always the capacity/potential of writing or laughing, humans not always actualize this capacity during their existence. This suggests to distinguish those attributes that are actualized by an individual during some period this individual is what it is, from those that are actualized by this individual all the time it is what it is – whereas the former refer to *propria*; the second to essential properties. Of course there are also those attributes that might not be always present not even as a potential, these are the mere accidents. Some post-Avicennian authors such as Suhrawardī’s, live every temporal indication out if it extends to all the time it is what is.

Of course, the second is false and can constitute a fallacy of composition, namely, inferring the second assertion from *All writing humans move their fingers*.

In order to provide a logical analysis, we will think of the assertion as occurring in a dialectical context shaped by the following steps

Give the assertion

All writing humans move their fingers

Questioner will ask Responder if a waṣfi or a dātī is meant.

- 1) **Waṣfi**. If a waṣfi reading is meant, Questioner produces some instance c of humans who are writing and this commits Responder to assert that c is moving her fingers while writing. Responder will ask Questioner to specify a period (within c 's life span) at which this human is writing – i.e. the (writing) period within the span σ_c during which c is what it is, namely human; and then Responder proceeds to assert that c is moving her fingers during that period.¹⁷²
- 2) **Dātī**. If a dātī reading is meant, two analysis are possible.
 - a) The first analysis seems to be closer to Avicenna's own approach. According to this interpretation the divided reading is effectuated by timing the main predicate (in our examples *moving their fingers*) to the the span during which the writing human is what it is, namely the span σ_c . Questioner produces some instance c of a writing human and this commits Responder to assert that human c is moving her fingers at every moment within the span at which c is what she is, namely, human. After Responder fulfilled the commitment Questioner has now the right to choose an arbitrary instant within c 's span σ_c and ask Questioner to show that c is moving her fingers at this instant – clearly, it is expected that Questioner will be able to find an instance of human and a instant within σ_c at which this instance does not move her fingers – compare with *All writing humans are rational*, where such a counterexample cannot be found.
 - b) The second analysis highlights better the links with Aristotle's fallacy of composition. In fact, the only difference to the precedent interpretation is that when Questioner produces an instance of the subject, the resulting instance only picks out the substantive part – i.e. instead of producing a c , who is a *writing human*, c just instantiates just *being a human*. In other words, Questioner breaks the compound into its substantive and descriptonal components, and just focus on the substantive one.

N.B. Hodges has often complained that the terminology *substantive-descriptonal* appeals to Aristotelian categories that, according to his interpretation, Avicenna does not wish to incorporate to his logic – cf. Hasnawi & Hodges (2016), Hodges & Johnston (2017). Indeed, Avicenna does not in principle use explicitly this terminology (letting by side the terminology of *while being what it is*, or *while being its essence*), but this is a terminology used by Avicenna's commentators after all. Moreover, our analysis suggests a generalization that seems to capture most if not all of the different complex embedding used by Avicenna and commentators, namely, the point of the generalization is to distinguish within a subject the **dependent from the independent component – i.e. the component that has been constructed over the other (this also allows more complex embeddings, such as the ones of the iteration of relative clauses). This constitutes one of the main advantages of using Ranta's Type Theoretical Grammar for our reconstruction.**¹⁷³

¹⁷² Johnston & Hodges (2017) and others after them, read Avicenna's formulation *while being what it is*, as referring to an existence predicate which renders formally the life span of the instantiated subject (or of the individual essence). Rahman & Zidani & Drissi (forthcoming), point out that, if we analyse the subject as a (simple or complex) domain explicitly introduced in the object language Avicenna's formulation makes perfect sense, and an existence predicate (with all its notorious connotations) is not needed. Moreover, it seems that individual essences are not that compatible with Avicenna's general views on the knowledge of particulars – cf. Adamson (2005), Strobino (2016a), Strobino (2021), Kaukua (2022).

¹⁷³ Our reconstruction seems to be also compatible with Avicenna's distinction between essence and existence. Take Avicenna's famous example of the bodyless-man, of which we can predicate of necessity that he is rational. Arguably, attributing rationality is dependent upon the bodyless-man being what it is, namely a man: *The bodyless-man is necessarily rational*, however, existence, meaning "real" one, i.e.,

IV.1 Instants, Spans and Periods

IV.1.1 Timing Actualizations

If we have a close look at Avicenna’s and Post-Avicennian texts it seems quite clear that temporal conditions are contextual parameters that can be made explicit in order to *enrich* an assertion that has already content. ¹⁷⁴

They are certainly not values of propositional functions. This way would made of instants bearers of events , i.e. substances, in blatant contravention to very well-known metaphysical views on time in ancient and medieval philosophy.

One way to render formally temporal conditions as contextual parameters, is the following:

if “*T(ime)*” stands for some set of instants, and “@” for a monadic operator that enriches a proposition with elements of *T* we obtain expressions such as $B(d)@t_i$ – whereby “@” can be thought correspond to an adverbial. In fact, $B(d)@t_i$ encodes the outcome of the *timing* function τ which associates presences of $B(d)$ with instants of time – i.e., the function *times* presences of laughing:

Example:

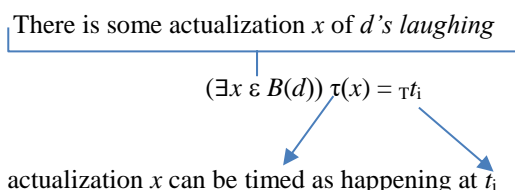
d laughs at t_i

Grammatically, the time indication can be understood as having an adverbial role, expressed by the operator

“@”

d laughs@t_i

The operator @ is in fact understood as the result of associating some actualization *x* of *d*’s laughing, with a instant in time. Thus, $d\ laughs@t_i$ stands as an abbreviation for



Recall that the timing function is on actualizations, so if the property is some capacity that the object always enjoys, such as the capacity of laughing, borrowing from Aristotle’s notion of time, we can say that a period *measures* the instants at which the individual is actually laughing.

- Notice that, though it seems that adverbial structures yield a reconstruction of Avicenna’s use of temporality compatible with his own views on logic and epistemology; interpreting such an adverbial as encoding a process of timing the actualization of an event, might be closer to the *epistemology of presence*, developed by Suhrawardī’s and others post-Avicennian thinkers than to Avicenna’s own framewokr– see Rahman & Seck (2023, forthcoming), Rahman & Zidani & Drissi (forthcoming). However, for our analysis we can remain at the adverbial structure and

outside the form of existence each category has, is, so it seems, determined by other categories let us call them *ontological independent category* and *ontological dependent category*, in our case; mental objects, so we produce *The mental object (or more generally the ontologically dependent object), the bodiless-man, is necessarily rational, so long it is a man*. Of course, we can go deeper and define mental objects as those dependent upon a mind.

¹⁷⁴ We owe the expression *enrichment* to Recanati (2017).

leave its reference to actualizations implicit. Actually “@” operator can be read as an adverbial reformulation of Rescher and vander Nat’s (1974) relational time operator.

IV.1.2 Spans¹⁷⁵

An advantage of the CTT-framework as implemented for time reference is that it provides the opportunity of considering not only time instants, but also time spans and intervals with a beginning and an end. This is particularly important because actions like moving, running, etc. do not happen at just at a time instant. Rather they should be considered as extended events which happen in temporal intervals. It also makes it possible to express how an object can have the same descriptions with different qualifications in different spans of time. For example, an object that is moving in a span of time might be slow-moving in some parts (or sub-spans) of that span and fast-moving in some others. Or even have a description that only applies has a sub-span. In our leading example, writing happens at a subspan of the life-span of the human who is carrying out that actions

So, it seems to be crucial to see how a span of time can be defined in the framework of CTT. The category of the spans of a time scale T can be defined as the Cartesian product of T and the set of natural numbers N . More precisely: $span(T) = T \times N$.

In other words , a span of the time scale T is a pair whose first element refers to the beginning point of that span in T and whose second element refers to the number of temporal units (of the scale T) which must be added to the beginning point to form the span under discussion. Stated differently, the second element determines the length of the span. So if $\sigma = \langle t_0, n \rangle : span(T)$, σ is a span of the time scale T

which begins at t_0 and ends at $t_0 + n$.

For example, if the spans in $span(T)$ are years representing the life-time of an individual,, t_0 , represents, its birthday and n , the length of that individual’s life span.

In order to generalize this, we can instead indicate that the span σ has a beginning $begin(\sigma)$ and end

$end(\sigma) = begin(\sigma) + n: T$, without necessary indicating, which one.

Moreover, useful is the notion of period $\pi(\sigma)$ of a span σ , a function which selects some intervals within σ and yields an interval with it own, specific, beginning and end.

So, if σ_{d_i} is some span, say the life span $\langle begin(\sigma_{d_i}), end(\sigma_{d_i}) \rangle$ of an individual d_i , $\pi(\sigma_{d_i})$, expresses some period during his life, say

$$\langle begin(\pi_{d_i}), \rangle,$$

whereby that period starts at some unities n' after the beginning of d_i ’s life span, that is

$$begin(\pi_{d_i}) = begin(\sigma_{d_i}) + n'$$

and the length of that period is time-unities after the begin of that period. For the sake of keeping things simple we assume that the time unities of a period, say years, are the same as those constituting the span upon which this period has been defined. But we might define finer periods. In fact, in such a framework, instants are periods of length 1.

¹⁷⁵ The following section is an adapted version of the use of spans in Avicenna’s wasfi introduced by Rahman&Zarepour (2021).

If we use the adverbial notation of the previous section that abbreviates the result of a timing function, the expression

$$A(di)@π(σ_{di})$$

indicates that there **is some period** within di 's span $σ_{di}$, during which di is A . So, it contains an implicit existential quantifier over periods. We will leave it implicit, since the dialectical rules below, will bring it to the fore. Indeed, if **X** states $A(di)@π(σ_{di})$, **Y** might ask **X** to specify the period at which di is said to enjoy A – such rules, we may recall, are the dialectical rules involving existentials.

Summing up, for every (contingent) di object of a domain D ,

there is a span $σ_{di}$, with a beginning and an end (perhaps not yet dated), during which di is what it is, namely D – the **independent** component of the subject (in our leading example *Human*); and

there is a period within that span (usually shorter than $σ_{di}$) during which di enjoys some property A – the component of the subject **dependent** upon D (in our leading example *writing* constructed as being dependent upon *Human*),

IV.1.3 The Dialectical Rules

Before we lay down the dialectical rules let us recall that the *dātī* reading admits two analysis:

one which breaks the compound of those D 's that are A , by associating the main predicate to the span during which the instance di of that compound is a D , rather than a D enjoying A ;

the other which breaks the compound by producing an instance of D , that does not convey any information about the component dependent upon D : this instance stands for a *bare* element of D .

In order to refer both, to the span during which di is a D , rather than a D that is an A , and to the span during which it is a *bare* element of D , we deploy Ranta's (1994, pp. 34-36) strategy of distinguishing *separable* compounds, such as the separated subset $\{x: D | A(x)\}$ and the non-separable ones, namely the Σ -type: $\exists x: D A(x)$.

Whereas

an instance of $\{x: D | A(x)\}$ yields a *bare* instance of D – i.e. $di: D$,

in contrast, an instance of $\exists x: D A(x)$, is, as explained above, a compound the left component of which is a D that is an A

By deploying this notation for the *dātī* reading we obtain

$$\mathbf{X} ! (\forall z: \{x: D | A(x)\}) B(\mathbf{left}(z))@σ_{(z)}$$

whereby, **left**(z), picks out a instance that is still a D that is A , but the interval associated to predicating B of that instance is not the period during which z is an A , but the span during which z is D .

for the first analysis and

$$\mathbf{X} ! (\forall z: \{x: D | A(x)\}) B(z)@σ_{(z)}$$

for the second analysis;

Both analyses are different interpretations of the same vernacular expression

All those D 's that are A , are B , while being D

The first analysis suggests that Avicenna’s way to produce the divided sense is quite complex. On one hand the predication of B applies to instances of a compound subject – i.e., to instances of $\{(\exists x: \textit{Human}) \textit{Writing}(x)\}$, on the other the time parameters associate to that predications break apart those instances. This requires to reformulate the implicit timing function in such a way that the timing of $B(\mathbf{left}(z))$, does not time $b(\mathbf{left}(z))$ but $b(\mathbf{fst}(z))$. Since at the end the process breaks apart the compound, we use the separable structure $\{x: D \mid A(x)\}$, but allow the operator $\mathbf{left}(z)$ to construct compound instances of the subject.¹⁷⁶ This suggests that the second analysis is logically speaking simpler.

WAṢFĪ				
Statement	Challenge	Defence		
Informal Dialectical Encoding	Informal Dialectical Encoding	Informal Dialectical Encoding	Informal Dialectical Encoding	Informal Dialectical Encoding
X states: All those D ’s that are A , are B , while being A	Y states: d_i is one of those D ’s that are A during some period π within span σ_{d_i} (during which d_i ’s what it is)	X asks: Which is the period during which d_i is A ?	Y answers: The period during which d_i is A , starts at the instant t_j of d_i ’s span and ends at t_k of that span.	X states: Exactly during this period d_i is also B
X ! (All A_D) B, while being A	Y ! $A(d_i)$ (during some period π within d_i ’s span σ_{d_i})	X Which is $\pi(\sigma_{d_i})$?	Y ! $\langle t_j, t_k \rangle$	X ! $B(d_i)$ during the period $\langle t_j, t_k \rangle$
Formal Dialectical Encoding	Formal Dialectical Encoding	Formal Dialectical Encoding	Formal Dialectical Encoding	Formal Dialectical Encoding
X ! ($\forall z: \{(\exists x: D) A(x)\})$ $B(\mathbf{left}(z)) @ \pi(\sigma_{\mathbf{left}(z)})$	Y ! $A(d_i) @ \pi(\sigma_{d_i})$	X? $\pi(\sigma_{d_i})$	Y ! $A(d_i) @ \langle t_j, t_k \rangle$ Whereby t_j is the begin and t_k the end of the period. ¹⁷⁷	Y ! $B(d_i) @ \langle t_j, t_k \rangle$

DĀTĪ				
Statement	Challenge	Defence	Rejoinder	Defence
Informal Dialectical Encoding	Informal Dialectical Encoding	Informal Dialectical Encoding	Informal Dialectical Encoding	Informal Dialectical Encoding
X states: All those D ’s that are A , are B , while being D	Y states: d_i is one of those D ’s that are A during some period π within span σ_{d_i} (during which d_i ’s what it is)	X states: d_i is B , while being D	Y asks: What about d_i being B at instant t_i within d_i ’s span σ_{d_i} ?	X states: d_i is B at instant t_i within d_i ’s span σ_{d_i}
X ! (All A_D) B, while being D	Y ! $A(d_i)$ (during some period π within d_i ’s span σ_{d_i})	X ! $B(d_i)$ during d_i’s span σ_{d_i}	Y ? $B(d_i) @ t_i: \sigma_{d_i}$	
Formal Dialectical Encoding	Formal Dialectical Encoding	Formal Dialectical Encoding		Formal Dialectical Encoding
First analysis X ! ($\forall z: \{x: D \mid A(x)\})$ $B(\mathbf{left}(z)) @ \sigma_z$	For both analyses Y ! $A(d_i) @ \pi(\sigma_{d_i})$	First analysis X ! $B(d_i) @ \sigma_{d_i}$		For both analyses X ! $B(d_i) @ t_i: \sigma_{d_i}$
Second analysis X ! ($\forall z: \{x: D \mid A(x)\})$		Second analysis X ! $B((d_i) @ \sigma_{d_i})$		

¹⁷⁶ An alternative way to produce the same effect is to use Sundholm’s (1989, pp.9-10) A -injections, which, in an Σ -type (an expression, we might recall, of the form $\langle a, b(a) \rangle : (\exists x: D) A(x)$) only yields bare elements of the first component of the pair. In Rahman&Zarepour (2021) we called such functions, **fst**. Such an analysis yields for the substantive reading $(\forall z: \{x: D \mid A(x)\}) B(\mathbf{left}(z)) @ \sigma_{fst(z)}$. As in the variant we prefer to deploy here, this requires to reformulate the implicit timing function in such a way that the timing of $B(\mathbf{left}(z))$, does not time $b(\mathbf{left}(z))$ but $b(\mathbf{fst}(z))$.

¹⁷⁷ In other words, $\langle begin(\pi_{d_i})=t_j, end(\pi_{d_i}) = t_k, end(\pi_{d_i}) \rangle$.

$B(z)@ \sigma_z$				
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An interesting elegant variant of Avicenna’s divided reading, based on Suhrawardī’s work on modalities, leaves the indication of a temporal indication out if it involves *perpetuity*. This suggest the following analysis, whereby the predicate B carries no temporal parameter, but the dependent component of the subject does. However, since the assertion is that d_i is B , all the time it is a D , the challenger can choose an arbitrary instant (at which d_i is D) and ask the defender if at that instant d_i is indeed is B : if the defender fails to fulfil this commitment, then challenger wins. We only show the first analysis, which is the closest to Avicenna’s dātī-reading.

DĀTĪ IN SUHRAWARDĪ’S FRAMEWORK				
Statement	Challenge	Defence	Rejoinder	Defence
Informal Dialectical Encoding	Informal Dialectical Encoding	Informal Dialectical Encoding	Informal Dialectical Encoding	Informal Dialectical Encoding
X states: All those D ’s that are A , are B , all the time they are D	Y states: d_i is one of those D ’s that are A during some period π of d_i being D	X states: d_i is B , all the time it is a D	Y asks What about d_i being B at instant t_i ?	X states: t_i , is indeed one of those instant at which d_i is B)
X ! (All A_D) B (all the time they are D)	Y ! $A(d_i)$ (during some period π of d_i of d_i being D)	X ! $B(d_i)$ (all the time it is a D)	Y ? $B(d_i)@t_i$ (whereby t_i , a period of length 1, is one of the instants at which d_i is D)	
Formal Dialectical Encoding	Formal Dialectical Encoding	Formal Dialectical Encoding		Formal Dialectical Encoding
X ! ($\forall z: \{x: D \mid A(x)\} B(\text{left}(z))$)	Y ! $A(d_i)@ \pi(\sigma_{d_i})$	X ! $B(d_i)$		X ! $B(d_i)@t_i$

IV.2 Modalities

Ideas

1 de re or de dicto

De re es la misma persona,

Sie es la misma peersona, se puede imaginar que estar no moviendo sus dedos

Si es de dicto quiere deice que en cada hiosy poeude haber otro no moviendo

Y eso puede ser generic

Para periodos fijadoes an una persoan, de re, eso usa bidimensionalidad,

De dicto intereatod como humano ene eneral de dicto

In fact, the dialectical framework to Avicenna’s conceptions above is also inspired by Suhrawardī’s take on modalities. It is a matter of contention if Avicenna’s framework requires both, modal and temporal operators, since Avicenna’s modalities are temporal after all – see Street (2002b), Hasnawi & Hodges (2015), Strobino (2018).

Be that as it may, it is quite straightforward to introduce a necessity (and its dual) operator compatible with a temporality perspective using a branching time structure.

By acknowledging that

some beings such as the chiliagon—a polygon with so many sides that it cannot be distinguished from a circle—exist in the mind but will probably never

exist in the outside world, Avicenna recognizes possibilities that are never actualized in reality. 115 To say that “all animals are humans” is a contingent proposition because we can imagine a time in which there is no animal but man, in spite of the fact that such a time probably never existed in reality. 116

Griffel p 168

The contingency of

the proposition is not verified by the future or past existence of a certain state of affairs in reality but rather through a mental process, namely, whether such a state can be imagined to exist without contradictions. 117 The phrase, “all white things,” may have two different meanings according to the context in which it is uttered. It may refer to all beings that are white at the particular time when the statement is made or to those possible beings that are always described as being white every time they appear in the mind (*al-ḥaqīqat al-ḥaqīqat*).118

F. Griffel (*Al-Ghazālī's philosophical theology*, Oxford University Press, Oxford, 2009, pp. 167–168)

Thus if at a certain time it is assumed, for example, that there is no color except white

..., the statement ‘Every color is white ...’ is then true in an absolute sense by virtue of

the absoluteness of the mode; before that, it was possible. But this possibility is not true

if linked to the predicate. For it is not by proper possibility that every color is white.

Rather, there are colors that are by necessity not white. Similarly, if we assume that at a

certain time there is no animal except the human being, then ‘Every animal is a human

being’ is true at that time in accordance with the absolute sense of the mode

Inati, S.C.: 1984, *Ibn Sīnā, Remarks and Admonitions Part One: logic*, translated from the original

Arabic with an introduction and notes, Pontifical Institute of Medieval Studies, Toronto, p. 291

It is necessary,

for example, that some possible animal is not human; and this would still be necessary if at a certain time it happened that only human animals existed *thom* 372

Indeed, a way to spell this out is to define necessity as what Belnap call *settled-truth*: i.e., true whatever the future brings – cf. Perloff & Belnap (2011) within a branching time structure.

As Perloff & Belnap (2011) point out this is not the notion of necessity of contemporary modal logic, but it just amounts to the idea that the truth of *A* at a moment does not depend on what happens later: If *A* is true at *m*, it will continue to be true at *m* in relation to *every history* that crosses that moment. This might recall Aristotle’s passage on the necessity of the present: “What is, necessarily is when it is”, cf. *De Interpretatione*, 19a23.

According to this analysis, if we interpret *necessity* as involving “for all histories” yields:

Every A is B of necessity

Reads

For each actualization a of A at moment m **for each course of events (or history) h** crossing m (m/h_i), there is an actualization b witnessing that a is B at this very moment m/h_i .

Actually, if we are willing to associate Avicenna’s framework with a branching time structure, a double dimension of time is required, namely, (i) the branching structure just mentioned, where histories pass through a knot, let us call it *moment m* , and (ii) a linear one associated to each moment upon a history, where spans and periods, constituted by instants t , are defined.

De re and de dicto

Furthermore, if we are prepared to distinguish *de re* and *de dicto* positions for the modalities, we might have for the waṣfi reading that

1. Each history passing through a moment m , associates this moment to a **different** period, for a *de dicto* modality ,
 E.g.,
All writing humans move their fingers (all the time while writing) of necessity.
 reads
 For each actualization of *writing human* at moment m for each course of events h crossing m (m/h_i), there is an actualization b witnessing that a is B at this very moment m/h_i .

Since, the truth-conditions of the settled-truth of a proposition C at a moment m , unless it is constituted by embedded temporal operators, amount to evaluating C at that moment, it can be left implicit. However, philosophically it indicates that its truth is settled whatever the future might brings.

According to this analysis, if we interpret as involving “for all histories”

- | | |
|-------|--|
| Reads | <p><i>Every A is B of necessity</i></p> <p>For each actualization a of A at moment m for each course of events (or history) h crossing m (m/h_i), there is an actualization b witnessing that a is B at this very moment m/h_i.</p> |
| Reads | <p>E.g.,
 <i>All writing humans are possibly sitting while writing</i></p> <p>For each actualization a <i>writing human</i> at moment m for each course of events h crossing m (m/h_i), there is an actualization b witnessing that a is B at this very moment m/h_i.</p> |

waṣfi or a dātī

In the next section, through the example of “breaking” (*kasr*), a move developed by medieval Muslim jurist-dialecticians for objecting to incorrectly composed compound causes, we will examine what illegitimate moves can be triggered by reasoning with compounds outside the realm of syllogism or logically valid inferences.

V KASR, OR HOW TO “BREAK” A COMPOUND OCCASIONING FACTOR¹⁷⁸

¹⁷⁸ This section is a succinct presentation of certain parts of Rahman&Young (2022), adding no further

V.1 Composition in the Argumentation Framework of *Qiyās* (Correlational Inference)

In argumentative frameworks that do not focus (primarily, or at all) on logical inference, illegitimate moves involving composition nevertheless assume the kind of well-formation rules described in the previous section. We cannot expect, however, that in argumentative frameworks with a broader scope than logical validity illicit reasoning involving compound attributes reduces to illicitly inferring in precisely the manner described by Aristotle’s discussion of the fallacy of composition.

The framework of our case study is Islamic juridical dialectic (*jadāl / munāẓara*), which shaped arguments targeting the *legal validity* (or lack thereof) of a norm or action, through the justifications of a proponent, the *Respondent* (R), and the critiques of an opponent, the *Questioner* (Q). An important genus of arguments employed by mainstream Sunni jurists in such a framework is called *qiyās*, or *correlational inference*, and takes the form of an inference by parallel reasoning.¹⁷⁹ More precisely, the aim of *qiyās* is to provide a rational ground for the application of a juridical ruling (*ḥukm*)—including such qualifications as *obligatory* (*wājib*, *fard*, *lāzim*), *forbidden* (*ḥarām*, *maḥẓūr*), *permissible* (*mubāḥ*), *valid* (*ṣaḥīḥ*), and *invalid* (*fāsīd*)—to a given case, called the branch-case (*farʿ*), which has not been directly and unequivocally pronounced upon in the primary juridical sources (i.e., the Qurʾān, Sunna [Prophet’s example], and *Ijmāʿ* [consensus]).

The more rigorous, archetypal *qiyās* method—known as *qiyās al-ʿilla*, or “correlational inference of the occasioning factor”—begins with an attempt to determine that property (*wasf*) or set of properties in the root-case (*aṣl*) which constitutes the *ʿilla*, or *occasioning factor* (or *legal cause*, *ratio legis*, etc.), giving rise to its ruling. If it is epistemically certain, or at least probable, that this property occasions the ruling, and it is shared by the branch-case, then we may infer that it is equally productive of that ruling in the branch-case. If, on the other hand, the *ʿilla* cannot be determined, then other (but less epistemically satisfying) forms of *qiyās* may be attempted.¹⁸⁰

With such focus on correctly determining the *ʿilla* in *qiyās* arguments, Muslim jurists elaborated complex discussions around *ʿilla*-oriented justifications and critiques. This included considerations of the “compound occasioning factor” (*ʿilla murakkaba*), which in effect demonstrated different ways in which arguments involving composition can trigger illegitimate moves when deciding whether or not a given ruling applies to a contended case. Particularly relevant is the dialectical objection called *kasr*, or “breaking,” a move in which Q, by taking apart and disposing of its components, aims to destroy that *compound of properties* which R claimed to be the occasioning factor (*ʿilla*) for a specific juridical ruling.

Generally speaking, *kasr* is when Q objects that one (or some) of the properties constituting R’s claimed compound *ʿilla* exists in another case—the “problem-case of breaking,” or *kasr*-case (*masʿalat al-kasr*)—but with a different and incompatible juridical ruling. As such, *kasr* divides into types and subtypes, for some of which several forms of rejoinders were also considered. Rahman&Young (2022b) studied the two main types, and in the present paper we will make reference to what we identified in that study as

results. The aim, rather, is to provide an example of the analysis of an illegitimate move which involves composition but which does not reduce to the syllogistic form of a fallacy of composition.

¹⁷⁹ *Qiyās* is often translated as *analogy*, though, strictly speaking, most conceptions of analogy correspond only to the weakest type of *qiyās*, “correlational inference of resemblance” (*qiyās al-shabah*), deemed to be of the most limited epistemic value.

¹⁸⁰ See the types of *qiyās* detailed in Hassan (1986), Hallaq (1997), and Young (2017).

Type 1a, along with a rejoinder, as illustrated by al-Shīrāzī.¹⁸¹ Here, the objection consists in Q’s bringing a parallel compound *‘illa* that challenges the causal efficiency of R’s claimed compound *‘illa*. As we will discuss below, Q aims to bring forward a compound counterexample which, though different from R’s, seems (at least at first sight) to preserve the same kind of (legal) meaning (*ma’na*). In other words, it is something more precise than, e.g., merely substituting *good cobbler* with *good runner*, as in the cases above.

V.2 *Kasr* and the Unseen Bride

In this example, bear in mind that the occasioning factor (or legal cause, *ratio legis*, etc.) of R’s *qiyās* is a “compound occasioning factor” (*‘illa murakkaba*) consisting of a pair of properties we have labelled *A* and *B*:

<i>A Disputation on Sale of What the Buyer Has Not Seen</i>	
Questioner (Q)	Respondent (R)
<p>1. [Q’s Demand for R’s Ruling]</p> <p>[Though it does not appear in al-Shīrāzī’s abbreviated examples, his prescribed procedure dictates that Q would first ask for R’s ruling—i.e., solution—to the problem-question (<i>mas’ala</i>) at hand.]</p> <p>3. [Q’s Demand for R’s Proof]</p> <p>[Again, though absent from the example, al-Shīrāzī’s procedure dictates that Q would now ask for R’s proof (<i>dalil</i>).]</p>	<p>2. R’s Ruling</p> <ul style="list-style-type: none"> • “Sale of what the buyer has not seen...” <ul style="list-style-type: none"> ○ [= the contended branch-case (<i>far’</i>)] • “... is not valid.” <ul style="list-style-type: none"> ○ [= the claimed ruling (<i>ḥukm</i>)] <p>4. R’s Proof by <i>Qiyās al-‘Illa</i></p> <ul style="list-style-type: none"> • “Because it [i.e., what the buyer has not seen] is [A] an object of sale (<i>mabī’</i>); and it is [B] unknown of attribute to a contractor at the time of contract...” <ul style="list-style-type: none"> ○ [= the claimed compound legal cause (<i>‘illa</i>)] • “...like when the seller merely says ‘I sell you a garment.’” <ul style="list-style-type: none"> ○ [= the authoritative root-case (<i>aṣl</i>), invalid by consensus (<i>ijmā’</i>)]¹⁸² <p>[R’s “indicant” (<i>dalīl</i>) is thus a “correlational inference of the occasioning factor” (<i>qiyās al-‘illa</i>). He argues: the branch-case (<i>far’</i>) has the same legal cause (<i>‘illa</i>) as the root-case (<i>aṣl</i>)—“what the buyer has not seen” and some unspecified “garment” are both objects of sale unknown of attribute to the buyer at the time of contract. Therefore, both should have the same ruling (<i>ḥukm</i>)—i.e., that same legal cause should occasion the same ruling (“not valid”) in the</p>

¹⁸¹ See *al-Ma’ūna*, 246; *Sharḥ al-Luma’*, 893, 898-9.

¹⁸² See *MF*, s.v. [البيع، المبيع وأحكامه وأحواله، حضور المبيع وغيابه، غياب المبيع], 9.23.

<p>5. Q's Objection by <i>Kasr</i></p> <ul style="list-style-type: none"> • “This is broken (<i>yankasiru</i>) by the [case of the unseen] bride...” <ul style="list-style-type: none"> ○ [= the “problem-case of breaking” (<i>mas’alat al-kasr</i>)] • “...because she is [A*] an object of marriage (<i>mankūha</i>) who is [B] unknown of attribute to a contractor at the time of contract... <ul style="list-style-type: none"> ○ [= the parallel compound legal cause (<i>‘illa</i>), with A* substituted for A] • “...but [the contract] is valid.” <ul style="list-style-type: none"> ○ [= the opposite of R’s claimed ruling (<i>hukm</i>)] <p>[Q’s objection is Type 1a <i>kasr</i>. He attempts to nullify R’s compound legal cause (<i>‘illa</i>) by replacing one of its properties (<i>A</i>, <i>mabī’</i>: <i>object of sale</i>) with a supposed parallel in meaning (<i>A*</i>, <i>mankūha</i>: <i>object of marriage</i>)¹⁸³ and showing that the property of primary concern (<i>B</i>: <i>unknown of attribute to a contractor at the time of contract</i>) is thus co-present with the opposite of R’s claimed ruling (<i>hukm</i>), in a seemingly parallel case (the <i>mas’alat al-kasr</i>: marriage of an unseen bride).</p>	<p>branch-case as it does in the root-case (where it is invalid by consensus).]</p> <p>6. R’s Counter-Objection by <i>Farq</i></p> <ul style="list-style-type: none"> • “Marriage is not like sale with respect to this ruling, because lack of knowledge (<i>jahāla</i>) has an efficiency in the category of sale which it does not have in marriage.” <ul style="list-style-type: none"> ○ [= the counter-objection by disqualifying difference (<i>farq</i>)] • “If he marries an unseen bride and then sees her, he has no option [to rescind] (<i>khiyār</i>); but if he buys some commodity he hasn’t seen and then sees it, he has the option [to rescind]. <ul style="list-style-type: none"> ○ [= demonstration of the disqualifying difference (<i>farq</i>)] • “Thus it is possible that being unseen does not prevent validity of marriage, while it prevents validity of sale.” <ul style="list-style-type: none"> ○ [= the conclusion of R’s rejoinder] <p>[R counter-objects, nullifying Q’s “problem-case of breaking” (<i>mas’alat al-kasr</i>) with a charge of disqualifying difference (<i>farq</i>), and then demonstrating that difference: marriage differs from sale with regard to property <i>B</i> because one has the option to rescind with an object of sale having property <i>B</i>, but not so with an object of marriage having property <i>B</i>.]</p>
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In the above sequence, first notice that Q’s objection by *kasr* (move #5) is based on the idea that both objects of sale and objects of marriage are objects of contractual transaction—i.e., they are in the same meaning (*ma’nā*). Thus, according to Q, they should

¹⁸³ Although the replacing property, (*A**) *mankūha*, is not made explicit in the *Ma’ūna*’s abbreviated text, it is in the *Sharḥ al-Luma’* (893, 898-9).

have the same ruling when they share property *B* (“unknown of attribute to a contractor at the time of contract”), but—and this inconsistency is at the heart of the *kasr* critique—in fact they do not have the same ruling. R’s counter-objection, however, specifically targeting Q’s *kasr*-case of the unseen bride, demonstrates that his substituting object of marriage (*mankūha*) for object of sale (*mabī*) in fact contributes a *kasr*-case marred by a disqualifying difference (*farq*)—i.e., they are not in the same meaning. A commodity whose attributes are unknown to the buyer at the time of a sales contract is relevantly different from a bride whose attributes are unknown to the groom at the time of a marriage contract, as proven by the buyer’s right to rescind and the groom’s lack thereof. Thus, the ruling of proscription applies only to selling the unseen object of sale, and not to marrying the unseen bride.

In other words, the efficiency of property *B* (“unknown of attribute to a contractor at the time of contract”) in occasioning a ruling of proscription is not evident outside of apt substitutions of similar sets of objects—i.e., objects in the same *ma’nā*—such as should allow property *B* to play the same causal role in Q’s opposing *kasr*-case as in R’s original, authoritative root-case. Q’s substitution should preserve the *meaning dependences* of the original case even as it disproves R’s juridical ruling’s dependence upon this parallel compound. However, R’s counter-objection by disqualifying difference (*farq*) shows that it does not preserve these meaning dependences. This constitutes a novel approach to illicit moves concerning composition as we have encountered them in previous sections (i.e., the cobbler and the poet). Not only does Type 1a *Kasr* and its unseen bride example occur in a context of determining (legal) causation, but it concerns primarily the *illegitimate substitution* of one of a compound cause’s relevant components.

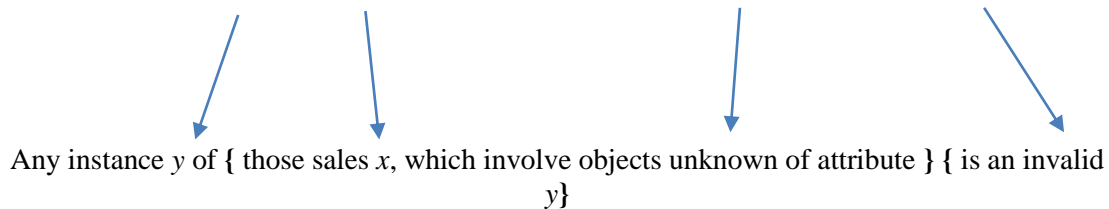
Note the logical structure displays a *double meaning dependence*, namely:

1. In the context of R’s root-case, the ruling *not valid* is linked to a cause composed of two properties: *object of sale* and *attributes unknown to contractor at time of contract* (i.e., *attributes unknown to buyer at time of buying*), and not to any other compound cause.
2. In the context of R’s root-case, the property *attributes unknown to contractor at time of contract* refers only to objects of sale, and not to objects of any other contract (such as marriage).

Thus, we may begin our formalization as follows:

Selling objects of sale, unknown of attribute, is not valid.

$(\forall y: \{ x: \textit{Selling} \mid \textit{Involving objects unknown of attribute} (x) \} \textit{Invalid}(y))$



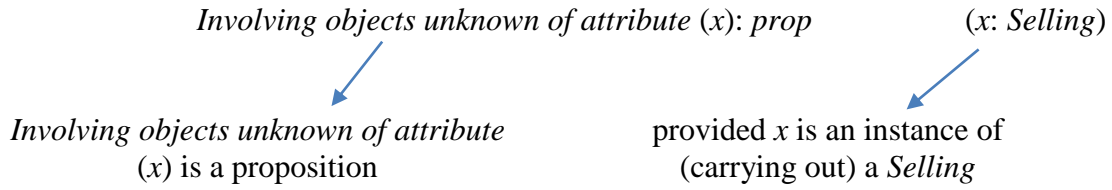
In fact, we can be more precise and indicate that what is invalid is actually the *selling of something* unknown of attribute—that is, what is (contractually) invalid is the *left component* of *y*. Indeed, if:

$\{ x: \textit{Selling} \mid \textit{Involving objects unknown of attribute} (x) \}$

is the set of all those sales of objects unknown of attribute, and *y* is an element of this set:

$y: \{ x: \textit{Selling} \mid \textit{Involving objects unknown of attribute} (x) \}$

then this already assumes the meaning dependence at work:



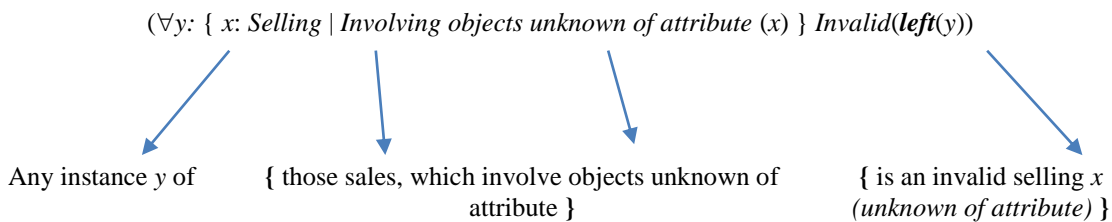
With such being the case, then the left component of y is the *selling* x (which is of those sales involving objects unknown of attribute),¹⁸⁴ that is:

$\text{left}(y)=x$: *selling*,

and the right component of y verifies that this selling x *involves an object unknown of attribute*:

$\text{right}(y)=z(x)$: *Involving objects unknown of attribute* (x).

This leads to the formulation:



The illegitimate use of *kasr* is thus due to the fact that the following substitution does not hold:

$(\forall y: \{ x: \textit{Marrying} \mid \textit{Involving objects unknown of attribute} (x) \} \textit{Invalid}(\textit{left}(y)))$

And why does this not hold? It does not hold due to a disqualifying difference (*farq*), whereby a telling norm holds true for objects of sale but not for objects of marriage—for commodities, but not brides:

Disqualifying difference (*farq*)

$(\forall y: \{ x: \textit{Selling} \mid \textit{Involving objects unknown of attribute} (x) \} \textit{Rescinding-allowed}(\textit{left}(y)))$

$(\forall y: \{ x: \textit{Marrying} \mid \textit{Involving objects unknown of attribute} (x) \} \textit{Rescinding-not-allowed}(\textit{left}(y)))$

V.3 Logic in Reverse (RevLog) and Illegitimate Moves

As mentioned above, the leading idea of “Logic in Reverse” is to study the principles of sound argumentation at work in different traditions, by focussing on what constitutes the criteria for qualifying a move as admissible or not admissible, in the context of certain precise reasoning patterns. In the case of the fallacy of composition, from the

¹⁸⁴ Notice that the set $\{ x: \textit{Selling} \mid \textit{Involving objects unknown of attribute} (x) \}$ is constituted by all those sales which have been carried out while the object is unknown of attribute. Thus, for every x in this set, one can say that it is a sale involving objects unknown of attribute.

inferential point of view, the inference rules should yield the *meaning composition* at work. This also holds if the rules for reasoning with compounds are cast in a dialogical framework, whereby the rules for question and answer are built on top of the rules of syntactic and semantic well-formation. The latter establish that the compounds involved assume the *meaning dependence* of one component upon the other.

Meaning dependences are at the core of Avicenna's *divided or substantial* (dātī) and *compound or descriptive* (waṣfi) reading of temporal and modal propositions in predicate logic. Let us recall that this conception of Avicenna did not only produce formidable developments in Medieval logic – let point out that Hodges & Johnston (2017) relate Avicenna's approach to Buridan's *de re/de dicto* distinction, but it offers a new perspective on the use of modalities and temporality in predicate logic, which animate several contemporary developments.

Within Islamic dialectic, the same governs the criteria of admissible moves with respect to objections against claims involving compound causes. Moves aimed at testing the legal causal efficiency of a compound of attributes cannot simply proceed by replacing that compound with another in which one of the components has been switched out for an arbitrary other. An admissible Type 1a *Kasr* objection must be based on a counterexample (*kasr*-case) that preserves the relevant legal meaning of the original compound.

The compounds of our Islamic case study—namely, marriages involving unseen brides and sales involving unseen commodities—can both trigger inferential fallacies of the kind described by Aristotle's good cobbler and Avicenna's good poet. However, the contribution of Type 1a *Kasr* lies not only in its offering a case wherein we find a complex embedding of components, such that the legal decision is also dependent upon this compound. The discussions around *kasr* also—or perhaps even primarily—concern the development of a reasoning pattern based on comparing alternative compounds in relation to the objectives set by the argumentative framework at work (in our case study, determining the compound's efficiency vis-à-vis the legal invalidity claimed by R for the contended case).

As developed by Hassan Rezakhany in a forthcoming paper, from the thirteenth century to the nineteenth, philosophers of the eastern Islamic world (Iraq to India) studied different forms of the mereological paradox on the totality can take such as the concepts of totality – such as the one of the totality (*majmūʿ*) of all relations (*nisab*). Rezakhany quotes the following example:

Consider the totality of all concepts (mafhumāt), such that none are excluded. The contradictory of this totality is also a concept, and must, therefore, be also included in it. But it follows from this that the totality will have its contradictory as a part, which is absurd. Bihārī and Baḥr al-'Ulūm, *Sharḥ Baḥr al-'Ulūm*, 581.1–3, 581.8–10

Interestingly, Woods and Walton (1977) suggest that a mereological perspective might offer a way to generalize the fallacy of composition and perhaps induce to a new classification. With regard to the first point, indeed, one can apply a mereological reading of *Every -writing-humans move their fingers while writing*, whereby each writing human is taken as an aggregate, and of course, from each part: human, taken as separate we cannot assert that it moves its parts. In fact, mereological readings of Avicenna's logic have been proposed that offer new insights into how the theory of Predicables articulates with his theory of demonstration – see Strobino (2016b).

In this context it is relevant to recall Bar-Hillel (1964) and Woods & Walton (1977) remarks that a mereological contemporary study on the fallacy of composition calls for rethinking the classification of fallacies.

These brief remarks show that the fallacy of composition constitutes a larger family of illegitimate moves, perhaps too large and so it seems that a fresh start on the place of the composition fallacy in the classification of fallacies is due, or even for a rethinking of the very notion of fallacy

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La logique de la présence et les modalités déontiques dans et au-delà de la pensée Islamique

Dans le présent chapitre, basé sur un article en préparation avec le Prof. Shahid Rahman et d'autres, nous explorons les possibilités d'appliquer l'épistémologie de la présence de Suhrawardī en dehors de son propre contexte historique et systématique. En effet, nous sommes convaincus que les idées de Suhrawardī sur la temporalité et la modalité ne sont pas seulement fructueuses pour une analyse du travail de ses prédécesseurs, mais qu'elles offrent également de nouvelles voies pour une compréhension épistémologique de la logique - c'est-à-dire une perspective dans laquelle la logique est conçue comme la théorie et la méthode d'acquisition de la connaissance par la démonstration.

Puisque, pour Suhrawardī, la présence constitue la source première de la connaissance, la démonstration s'enracine dans les témoins réels des propriétés attribuées au terme sujet. En d'autres termes, témoigner de la réalisation effective d'une propriété, y compris potentielle, est le rôle que jouent les présences dans un cadre logique et épistémologique. En fait, ces témoins doivent être conçus comme des instanciations de la propriété attribuée, de telle sorte que témoin et propriété se trouvent dans une relation interne inséparable. Cependant, ce n'est que dans un processus postérieur et abstrait que la présence et la propriété sont articulées en tant que sujet et prédicat.

Dans le cas des attributions de capacités ou de potentialités, l'intuition de Suhrawardī conduit à la perspective que les présences, plutôt que les propositions qu'elles fondent, sont celles qui doivent être temporalisées et que cette temporalisation doit être rendue explicite au cours d'une démonstration dialectique durant laquelle un répondant produit une justification pour les attributions individuelles ou génériques de propriétés. L'idée est que les présences réelles sont des créatures du temps, et non les propositions qu'elles instancient.

Dans un contexte épistémologique, les propositions expriment des schémas abstraits de réponse (ou type de réponse) à une enquête, à laquelle nous pourrions associer à la notion de *maṭlūb* مطلوب (problème, objectif, enquête) d'al-Fārābī, une enquête résolue par une vérification. Comme nous l'avons déjà mentionné, lorsque les présences sont produites pour justifier des affirmations dans un contexte épistémologique, elles ne fournissent pas seulement la source de la connaissance, ou un témoin d'une proposition, ou une instance pour un concept, dans ces contextes les présences vérifient une loi scientifique universelle.

La temporalité est mise en évidence lorsque les présences sont produites afin de fonder une revendication de connaissance en réponse à un examen critique et public. Il s'agit à notre avis de l'une des contributions les plus importantes de Suhrawardī à la logique temporelle lancée par Avicenne - une contribution qui semble avoir été négligée dans la littérature consacrée à son œuvre.

En fait, les reconstructions contemporaines standard ne possèdent pas la puissance expressive nécessaire pour mettre en œuvre cette vision de la logique de la présence. Une reconstruction contemporaine, nécessairement anachronique, requiert le développement d'un nouveau cadre dialectique modal-temporel, dans lequel les présences habitent explicitement le langage de l'objet utilisé au cours d'une démonstration. Dans une telle reconstruction, la justification des propositions et des propriétés (entendues comme des fonctions propositionnelles) est basée sur les présences, intérieurement inséparables des propositions dont elles sont les témoins. Pour une telle reconstruction, nous avons étendu le cadre appelé *Immanent Reasoning*, développé par Rahman et al. (2018), qui combine la logique dialogique avec la Théorie Constructive des Types.

Mais nous aimerions maintenant appliquer aussi le cadre de Suhrawardī pour analyser les catégories déontiques islamiques telles que développées par Ibn Ḥazm, qui, comme le montrent Rahman & Zidani & Young (2018) devrait être compté comme le véritable père de la logique déontique. Dans une telle reconstruction, les catégories déontiques affectent les schémas d'action, réalisés par des performances individuelles réelles. Ainsi, selon cette

approche, les présences constituent en fait des performances qui exécutent un type d'actions.

Si les performances des types d'actions déontiques doivent être temporalisées et en même temps conçues comme satisfaisant ces types déontiques tels que l'obligation, l'interdiction et la recommandation, il semble que nous devions supposer que l'action déontique peut être ou ne pas être exécutée. Cela signifie que, au-delà du cadre de Surhrawādī, la temporalisation des performances doit supposer un temps ramifié dans lequel, à un moment donné, l'action est choisie pour être exécutée ou non.

Rahman & Granström & Farjami (2019) ont montré comment une telle approche offrent une nouvelle approche des modalités déontiques qui évite les paradoxes standard de la logique déontique contemporaine. Dans les articles que nous venons de mentionner, les modalités déontiques d'Ibn Ḥazm ne sont ni explicitement liées au cadre dialectique ni au cadre temporel qui façonnent ces modalités, en particulier dans les contextes juridiques. Cela c'est la tâche que nous visons, dans le présent chapitre ou plus précisément, nous nous contentons de poser les premiers jalons de la résolution d'une telle tâche. Comme mentionné dans la conclusion de ce chapitre, l'approche offre une nouvelle logique déontique qui peut être généralisée pour des contextes éthiques et juridiques au-delà de la pensée islamique.

N.B. Les sections 1.1 à 1.5 ont été extraites de Rahman & Zidani & Young (2018) et Rahman & Granström & Farjami (2019).

- Ma contribution, au-delà du cadre théorique général, qui est le fruit du travail de Rahman, Zidani, Seck, Drissi et Boussad, est d'avoir développé les règles dialogiques et les dialogues, en particulier ceux qui concernent à la fois l'adaptation de la règle Socratique – afin de mettre en évidence les liens conceptuels établis lors de l'interaction d'un dialogue – et les jeux pour les approches temporelles.

1. 1 Brèves remarques sur les modalités déontiques contemporaines et anciennes

Depuis *Meaning and Necessity* de Kripke (1980), il a été rendu public que les premières affirmations concernant la richesse de la sémantique des mondes possibles pour exprimer plusieurs formes de nécessité doivent être nuancées - au moins dans le cas où la logique modale propositionnelle est étendue avec des quantificateurs.

En fait, dès les années soixante, les interprétations de la nécessité déontique fondées sur les mondes possibles, telles qu'elles ont été développées par von Wright (1951, 1963), se sont heurtées à une multitude d'énigmes philosophiques et logiques qui ont menacé le cadre dès le départ, et ce déjà au niveau propositionnel¹⁸⁵. Bien qu'à première vue la notion de monde possible, une situation contrefactuelle, semble offrir un instrument attrayant pour saisir le contenu d'une déclaration normative prescrivant la façon dont le monde *devrait être*, il est désormais évident que la sémantique fonctionnelle de la vérité standard qui sous-tend les modalités de style Kripke n'a aucun moyen direct de traiter la dynamique requise par la logique des actions et des prescriptions¹⁸⁶. Les actions ne sont en principe pas porteuses de vérité, et un appel aux mondes possibles n'explique pas en soi l'incidence dans le monde réel d'une prescription d'action. Après tout, le fait que je grille les feux rouges sera sanctionné dans un état du monde réel après l'infraction, et non dans un monde

¹⁸⁵ Pour des aperçus récents de ces défis, voir Hilpinen et McNamara (2013).

¹⁸⁶ *La logique épistémique dynamique* prend au sérieux le défi dynamique - cf. van Ditmarsh et alii (2007). Cependant, elle partage avec la logique modale *statique* la perspective méta-logique sur la signification. En conséquence, les effets propositionnels des actions, les changements fonctionnels de vérité dans le modèle, sont décrits au niveau méta. Alors que dans le cadre de la logique épistémique dynamique, les expressions pour les annonces publiques (c'est-à-dire les assertions) sont intégrées dans le langage-objet, leur contenu, tel que la *proposition p est vraie*, est établi au niveau méta (voir Baltag, Moss et Solecki 1998). Il ne semble pas que, dans ce type d'approches, les prescriptions soient des éléments de première classe du domaine du monde réel.

virtuel possible. Bien entendu, la nécessité déontique est une sorte de nécessité. C'est simplement que la sémantique standard de la théorie des modèles ne semble pas être l'instrument adéquat pour traiter la dimension temporelle impliquée dans la notion de norme, ou que des raffinements importants sont nécessaires.

On peut dire que la sémantique des mondes possibles en général et la logique déontique standard contemporaine en particulier sont le fruit tardif du *tournant propositionnel* lancé par les stoïciens. En effet, ce sont les stoïciens qui, dans le cadre d'une ontologie dynamique constituée d'événements et d'actions, ont proposé d'étendre ou peut-être même de remplacer l'approche relationnelle de la nécessité d'Aristote par une approche propositionnelle, dans laquelle les connecteurs et les règles d'inférence jouaient le rôle du terme-relation aristotélicien régi par la métaphysique des essences et la logique du syllogisme¹⁸⁷.

La perspective propositionnelle sur la nécessité causale a permis aux juristes romains, et à Cicéron en particulier, de transférer différentes formes de causalité naturelle dans le domaine du raisonnement juridique. Cela a contribué à la création de la notion de *ratio legis*, la cause qui fonde une décision juridique.¹⁸⁸ On pourrait peut-être comprendre la théorie stoïcienne des signes comme un moyen de rassembler une notion générale de cause à effet s'appliquant à la fois aux normes et aux événements.

Deux problèmes principaux se sont alors posés.

3. Alors que l'approche prédicative d'Aristote garantissait la pertinence du contenu, la construction propositionnelle rendait difficile le resserrement de la cause et de l'effet avec des moyens purement vérifonctionnels. Rappelons les disputes très connues sur la façon de définir une implication qui exprime la causalité
4. Elle a soulevé la question de l'écart entre les normes en tant que prescriptions (et leur actualisation) et les propositions entendues comme porteuses de vérité, en particulier dans le contexte du raisonnement juridique.

Ces lacunes évoquent le problème épistémologique plus large de la manière de relier la théorie et l'expérience ou la théorie et la praxis. La tradition arabe, particulièrement sensible aux questions relatives à la *praxis*, a compris que l'interface théorie-praxis devait être étudiée sous l'angle de la dyade *prescription-actualisation*, précisément dans les contextes chers aux stoïciens, à savoir l'éthique et la jurisprudence. La nouvelle vision de la tradition arabe a conduit aux étapes audacieuses suivantes :

- Les prescriptions sont comprises comme des prescriptions de **faire** plutôt que comme des prescriptions qui nous font passer d'un état de choses à un autre: *Tun Sollen* plutôt que *Sein Sollen*.
- Les événements, mais aussi les actions, sont des habitants de premier ordre de l'univers du discours. Les actions et les prescriptions présentent un lien de contenu qui donne lieu à une classification des types d'actions. Le raisonnement déontique est un raisonnement par le contenu.

¹⁸⁷ Pour une discussion approfondie sur le point de vue relationnel d'Aristote sur les modalités, voir Malink (2013), qui propose également une reconstruction formelle basée sur ce qu'il appelle une *sémantique de préordre méréologique*.

¹⁸⁸ Le concept de *droit conditionnel* en droit romain, l'une des formes les plus importantes de normes juridiques en droit civil, est également pertinent pour le développement de la nécessité déontique dans les contextes juridiques : une obligation, telle que l'obligation de payer une somme d'argent fixe, est subordonnée à certaines conditions futures (fixées par le bienfaiteur en faveur d'un bénéficiaire).

- Les prescriptions de faire sont intégrées dans un système de jugements hypothétiques impliquant des implications où les actions, l'actualisation des prescriptions, font l'objet d'une prédication: les actions sont porteuses de qualifications telles que le respect de la loi ou la violation de la loi¹⁸⁹. De même, les événements sont qualifiés comme devant nécessairement se produire, pouvant se produire ou ne se produisant pas du tout.
- Les normes présupposent la liberté de choix : Une prescription de faire présuppose la possibilité de choisir entre effectuer ou non l'action prescrite par la norme.
- La dimension temporelle des notions déontiques condensées dans le principe selon lequel *toutes les actions sont permises à moins d'être interdites par la loi*, a nécessité le déploiement d'un système dialectique d'argumentation appelé *qiyās* qui a régi l'intégration dans le système juridique d'une qualification déontique explicite et actualisée (éventuellement différente de *permise*) pour un nouveau type d'actions.¹⁹⁰

Il est certain que les analogies entre les concepts déontiques, temporels et modaux ont une longue et riche histoire avant leur résurgence dans la logique déontique contemporaine¹⁹¹. Des lacunes importantes sont néanmoins présentes dans la littérature sur ses sources historiques, même dans les synthèses les plus récentes, notamment en ce qui concerne les contributions développées au sein de la jurisprudence islamique¹⁹². C'est encore le cas malgré le fait qu'il existe des travaux sur l'influence du stoïcisme sur les penseurs arabes en général, et sur la classification morale des actes comme étant *obligatoires*, *interdits*, *recommandables*, *répréhensibles* et *neutres*, y compris les études de van Ess (1964) et de Jadaane (1968)¹⁹³. En fait, Gutas (1994) montre que les conditions d'une évaluation fondée de l'influence du stoïcisme sur les penseurs islamiques ne sont pas encore réunies. En effet, Gutas (1994) montre clairement que des études comme celles qui viennent d'être mentionnées ne sont pas étayées par des preuves provenant des sources.

Selon nous, c'est précisément dans le contexte de la jurisprudence islamique que la contribution de la tradition arabe à la modalité et à sa logique doit être étudiée et

¹⁸⁹ La notion d'*assertions conditionnelles* a servi de base aux développements plus sophistiqués, au sein de la tradition islamique, des implications (y compris les bi-implications), ou *sharṭiyya mutṭaṣila*, et des disjonctifs, ou *sharṭiyya munfaṣila*. Pour une récente étude approfondie de la notion de *sharṭiyya*, voir Hasnawi et Hodges (2016, section 2.4.3, p. 63-65).

¹⁹⁰ Pour une étude approfondie de la théorie des *qiyās*, voir Young (2018). La forme canonique du *qiyās* est celle préconisée pour trouver le '*illa*, ou "facteur occasionnel", qui a déclenché la décision juridique (telle que *légalement valide*) ou la qualification déontique (telle que *interdite* ou *obligatoire*) d'un cas connu et la transférer au nouveau cas. Le "facteur occasionnel", '*illa*, est l'analogue islamique du concept de *ratio legis* mentionné ci-dessus.

¹⁹¹ En fait, Knuuttila (1993, p. 182) observe que Pierre Abélard (1079-1144) et d'autres philosophes du haut Moyen Âge ont souvent adopté une forme inversée de la réduction de Leibniz en définissant les concepts modaux au moyen de concepts déontiques. Selon cette caractérisation, la nécessité est considérée comme ce que la nature exige, la possibilité est identifiée à ce que la nature permet, et l'impossibilité à ce que la nature interdit.

¹⁹² Voir, par exemple, Knuuttila (1981) et l'excellent essai de Hilpinen et McNamara (2013, p. 14), qui, bien qu'ils discutent de l'occurrence des concepts déontiques dans la jurisprudence islamique classique, ne mentionnent pas les premiers témoignages du parallélisme entre les concepts déontiques et modaux dans cette tradition.

¹⁹³ Jadaane (1968, p. 184-189) discute et relativise de manière convaincante l'affirmation forte de Van den Bergh (1954, réimprimé en 1987, vol. II, p. 117 des notes) selon laquelle les notions d'*obligatoire*, de *recommandable*, de *répréhensible* et d'*interdit* de la jurisprudence islamique correspondent (respectivement) aux notions stoïciennes de *recte factum*, *commodum*, *incommodum* et *peccatum*. Dans la même note de bas de page, Van den Bergh (1954, vol. II, p. 118 des notes) souligne que les théologiens islamiques ont associé la notion déontique de *permissible* à la modalité *pas logiquement impossible*. Van den Bergh n'approfondit toutefois pas la question. Gutas (1994) développe une analyse critique approfondie des évaluations hâtives de Van den Bergh et de Jadaane.

réfléchie¹⁹⁴.

1.2 Ibn Ḥazm de Cordoue sur la nécessité déontique et naturelle

Leibniz est considéré à juste titre comme l'un des penseurs les plus importants dans l'établissement d'un lien entre la logique et le raisonnement juridique. En particulier en raison de ses premiers travaux sur l'analyse logique du droit conditionnel (1664-1669), qui impliquent de distinguer une forme particulière de jugement hypothétique qu'il appelle l'*implication morale*, et de ses travaux ultérieurs (en 1671) liant la nécessité modale aux obligations juridiques et à la probabilité. Dans ce contexte, on a souvent prétendu que la logique déontique contemporaine était née dans les *Elementa Juris Naturalis* de Leibniz en 1671 - voir Von Wright (1981, p. 3). En fait, il est vrai que Leibniz déclare explicitement dans cet ouvrage que le *transfert* entre les concepts déontiques et modaux peut être effectué de la manière suivante :

Modal	Déontique
possible, elle est intelligible.	(<i>licitum</i>) autorisé
nécessaire, sa négation n'est pas intelligible.	(<i>debitum</i>) obligatoire
Si ce n'est pas le cas, sa négation est intelligible.	(<i>indebitum</i>) inadmissible
impossible, elle n'est pas intelligible.	(<i>illicitum</i>) interdit

L'influence des travaux de Leibniz est indéniable ; cependant la revendication historique sur la naissance de la logique déontique est inexacte. Lameer (1994, pp. 240-241, et 2013, p. 417) souligne cette inexactitude en indiquant que les perspectives d'al-Fārābī et d'Ibn Ḥazm semblent être le premier témoignage enregistré d'un transfert des concepts déontiques vers les concepts modaux¹⁹⁵.

En effet, la défense passionnée et acharnée de la logique dans le raisonnement juridique du penseur controversé Ibn Ḥazm de Cordoue (384-456/994-1064) ('Alī ibn Aḥmad ibn Sa'īd ibn Ḥazm ibn Ghālib ibn Ṣāliḥ ibn Khalaf ibn Ma'dān ibn Sufyān ibn Yazīd al-Fārisī al-Qurṭubī), a eu des conséquences durables dans le domaine du raisonnement juridique. En outre, son livre intitulé "*Faciliter la compréhension des règles de la logique et introduction à celles-ci, avec des expressions courantes et des exemples juridiques*" (*Kitāb al-Taqrīb li-ḥadd al-manṭiq wa-l-mudkhal ilayhi bi-l-alfāz al-'āmmiyya wa-l-amthila al-fiḥhiyya*), composé en 1025-1029, a été bien connu et discuté pendant et après son époque ; et il a ouvert la voie aux études qui ont donné au raisonnement démonstratif une place privilégiée dans les méthodes d'acquisition des connaissances en général et dans la prise de décision juridique en particulier.

En fait, la défense de la logique par Ibn Ḥazm s'est concentrée sur son rôle dans la prise de décision dans des contextes juridiques. Cela l'a conduit à plaider en faveur d'un système logique qui s'opposait aux conceptions "formalistes" de son époque. Dans *al-Taqrīb*, Ibn Ḥazm rejette explicitement l'utilisation de dispositifs syntaxiques pour l'analyse des arguments logiques et tente de développer un langage entièrement interprété sur lequel les arguments sont construits. Si la logique doit jouer un rôle dans la pratique juridique réelle, elle doit être fondée sur l'étude de cas réels paradigmatiques de décisions juridiques. Pour ce faire, il entreprend une étude approfondie des notions déontiques et de leurs contreparties modales, ce qui fait de lui l'un des pères de la logique des normes.

L'extrait suivant de l'ouvrage d'Ibn Ḥazm, *al-Taqrīb li-Ḥadd al-Manṭiq wa-l-Mudkhal ilayhi bi-l-Ālfāz al-'Āmmiyya wa-l-Amthila al-Fiḥhiyya*, éd. Aḥmad b. Farīd b. Aḥmad al-Mazīdī, (Beyrouth : Manshūrāt Muḥammad 'Alī Bayḍūn, Dār al-Kutub al-'Ilmiyya, 2003), pp. 83-84, constitue la principale source historique du parallélisme entre les

¹⁹⁴ Cf. Rahman, Zidani et Young (2018).

¹⁹⁵ Lameer (2013, p. 306) reconnaît Gutas (1988, p. 270) pour la référence à Ibn ḥazm.

modalités déontique et aléthique au sein de la tradition arabe.

Traduit en anglais par Walter Edward Young¹⁹⁶ et nous l'avons par la suite traduit en français.

Chapitre sur les éléments (<i>'anāṣir</i>)
Sachez que les éléments (<i>'anāṣir</i>) de toutes les choses (<i>ashyā'</i>) - c'est-à-dire leurs classes en ce qui concerne les affirmations (<i>ikhbār</i>) à leur sujet - sont de trois classes, il n'y en a pas de quatrième.
[Ils sont soit nécessaires (<i>wājib</i>), étant tels qu'ils sont nécessaires et manifestes, soit parmi ceux qui doivent être, comme le lever du soleil chaque matin, et ce qui est semblable, tels qu'ils sont appelés dans les lois de Dieu "obligatoires" (<i>farḍ</i>) et "contraignants" (<i>lāzim</i>) ;
ou possible (<i>mumkin</i>), c'est-à-dire ce qui pourrait être ou ne pas être, comme notre anticipation qu'il pleuvra demain, et d'autres choses semblables, appelées dans la loi de Dieu "licites" (<i>ḥalāl</i>) et "permises" (<i>mubāḥ</i>) ;
ou impossible (<i>mumtani'</i>) étant tel qu'il n'y a pas manière (de le réaliser), comme le fait pour un humain de rester sous l'eau pendant un jour entier, ou de vivre un mois sans manger, ou de marcher dans les airs sans un artifice astucieux, et d'autres choses de ce genre. Et c'est ce genre de choses qui, si nous les voyions se manifester chez un homme, nous saurions qu'il est un prophète ; et cette catégorie est appelée dans les lois de Dieu "interdit" (<i>ḥarām</i>) et "prohibé" (<i>maḥzūr</i>).
De plus, le possible (<i>mumkin</i>) est divisé en trois classes, il n'y en a pas de quatrième :
et le possible proche (<i>mumkin qarīb</i>), comme la possibilité d'une pluie sur une condensation de nuages dans les deux mois de <i>Kānūn</i> , ¹⁹⁷ ou la victoire d'un grand nombre de courageux sur un petit nombre de lâches ;
et le possible éloigné (<i>mumkin ba'īd</i>), qui comme la défaite d'un grand nombre de courageux face à un petit nombre de lâches, et comme la prise en charge du califat par un cuveur (<i>ḥajjām</i>) [c'est-à-dire un praticien de la ventouse], et ce qui est de cet ordre ;
et le purement possible (<i>mumkin maḥḍ</i>), dont les deux extrêmes sont égaux, comme quelqu'un qui se tient debout - soit il marche, soit il s'assoit - et ce qui lui ressemble.
De même, nous constatons que cette classe moyenne [c'est-à-dire le <i>mumkin</i> , correspondant au <i>mubāḥ</i>] est, dans les lois de Dieu, divisée en trois classes : recommandée-permise (<i>mubāḥ mustaḥabb</i>) ; réprouvée-permise (<i>mubāḥ makrūh</i>) ; et uniformément permise (<i>mubāḥ mustawīn</i>) n'ayant aucune tendance vers l'un ou l'autre des deux côtés.
Quant au recommandé-permis (<i>mubāḥ mustaḥabb</i>), il est tel que lorsqu'on l'accomplit on est récompensé (<i>ujirta</i>), mais si on le néglige on ne commet pas de péché (<i>lam ta'tham</i>) et on n'est pas récompensé ; comme le fait de prier deux cycles de prières surérogatoires, volontairement.
Et pour ce qui est du réprouvé-permis (<i>mubāḥ makrūh</i>), il est tel que lorsqu'on le fait on ne commet pas de péché et on n'est pas récompensé, mais si on le néglige on est récompensé ; et c'est comme le fait de manger en étant allongé, et autres choses de ce genre.
Et pour ce qui est de la permission égale (<i>al-mubāḥ al-mustawī</i>), elle est telle que lorsqu'on la fait ou qu'on la néglige, on ne commet pas de péché et on n'est pas récompensé ; et c'est comme teindre son vêtement de la couleur qu'on veut, et comme monter sur la bête de somme qu'on veut, et ainsi de suite.

Les unités de base de la logique déontique islamique sont ce que nous pourrions appeler, par anachronisme terminologique, des *impératifs hétéronomes*. L'intérêt des *impératifs hétéronomes* est de développer une logique des normes dans laquelle l'analyse du contenu des qualifications déontiques telles que l'*obligatoire*, l'*interdit*, le *permis*, le *facultatif*, est mise en pratique afin de justifier le transfert d'une décision juridique d'un cas connu à un

¹⁹⁶ D'après Rahman, Zidani et Young (2018).

¹⁹⁷ C'est-à-dire décembre et janvier.

cas inconnu. Les notions déontiques islamiques qualifient l'accomplissement d'actions comme méritant d'être *récompensées* (à différents degrés), *sanctionnées* ou *ni l'une ni l'autre*. Dans un cadre plus moderne et plus général, nous pourrions utiliser les qualifications de *respect de la loi*, de *violation de la loi* et de *neutralité juridique* (ni *respect* ni *violation de la loi*) à la place de¹⁹⁸ - sur la base desquelles l'agent peut être sanctionné ou non.¹⁹⁹ D'autres possibilités sont *sanctionnées par la loi*, *non sanctionnées par la loi* et *juridiquement neutres*, ou dans le cadre d'une approche de la valeur de la loi; *juridiquement digne*, *juridiquement indigne*, *juridiquement neutre*²⁰⁰.

1.3 Les impératifs hétéronomes d'Ibn Ḥazm.²⁰¹

Les juristes musulmans ont identifié cinq qualifications déontiques pour une action. Ibn Ḥazm les définit comme suit :²⁰²

- 6 ***wājib, farḍ, lāzim***. L'action obligatoire est celle qui :
Si nous le faisons, nous sommes récompensés.
Si nous ne le faisons pas, nous sommes sanctionnés.
- 7 ***ḥarām, maḥzūr***. L'action interdite est celle qui :
Si nous le faisons, nous sommes sanctionnés.
Si nous ne le faisons pas, nous sommes récompensés.
- 8 ***mubāḥ mustaḥabb***. L'action permise recommandée est celle qui :
Si nous le faisons, nous sommes récompensés.
Si nous ne le faisons pas, nous ne sommes ni sanctionnés ni récompensés.
- 9 ***mubāḥ makrūh***. L'action permise réprouvée est celle qui :
Si nous ne le faisons pas, nous sommes récompensés.
Si nous le faisons, nous ne sommes ni sanctionnés ni récompensés.
- 10 ***mubāḥ mustawin***. L'action uniformément permise est celle qui :
Si nous le faisons, nous ne sommes ni sanctionnés ni récompensés.
Si nous ne le faisons pas, nous ne sommes ni sanctionnés ni récompensés.

Il convient de noter que la classification suppose que la *récompense* et la *sanction* sont incompatibles mais non contradictoires. Certaines actions ne peuvent être ni récompensées ni sanctionnées ; ce dernier point est crucial pour l'introduction des valeurs et des degrés.

Notons également que si la notion de *sanction* correspond au vocabulaire de la jurisprudence européenne contemporaine, la notion de *récompense* à l'œuvre dans la classification des actions semble trouver son origine dans le domaine de la théologie²⁰³. L'idée est simplement que, comme l'a minutieusement développé Hallaq (2009), le Qur'ān constitue le "fondement" inséparable de l'émergence de la morale et du droit islamique. Néanmoins, une interprétation non théologique de la *récompense* dans certains contextes juridiques est possible, comme dans le cas du droit conditionnel, où l'on peut dire qu'un

¹⁹⁸ L'idée de remplacer la *récompense* et la *sanction* par le *respect de la loi* et l'*infraction à la loi* a été suggérée à Rahman par Zoe McConaughy.

¹⁹⁹ Il convient de noter que, selon cette interprétation, bien qu'une prestation ne puisse être ni respectueuse ni contraire à la loi, l'agent de la prestation sera sanctionné si sa prestation enfreint la loi.

²⁰⁰ Les systèmes de valeurs du raisonnement juridique sont souvent considérés comme étant en concurrence avec les systèmes logiques.

²⁰¹ Toute cette section est basée sur Rahman, Zidani et Young (2018).

²⁰² Ibn Ḥazm (1926-1930, vol. 3, p. 77) ; idem (1959, p. 86 ; 2003, p. 83-4).

²⁰³ Cf. Hartmann (1992, pp. 74-75).

bénéficiaire est "récompensé" par un bien, si une certaine condition, spécifiée par le bienfaiteur, a été remplie.

En fait, l'extension par Ibn Ḥazm de la modalité *mubāḥ-permissibilité* dans les catégories du recommandé et du répréhensible est atypique. Toutes les formes de "permissibilité" ont une valeur ; c'est-à-dire qu'en termes de faire ce qui est recommandé ou de ne pas faire ce qui est répréhensible, les deux dépassent la valeur neutre du "même permis", tout en n'atteignant pas encore la valeur de faire ce qui est obligatoire et de ne pas faire ce qui est interdit. En même temps, ni le fait de faire ce qui est répréhensible, ni le fait de négliger ce qui est recommandé ne descendent en dessous de la valeur neutre du "également permis", qui, toujours au-dessus de l'état de faire ce qui est interdit et de négliger ce qui est obligatoire, reste fermement au milieu.

1.4 Contingence déontique : Liberté et hétéronomie : Le *devoir* présuppose le *pouvoir*

L'approche suivante est basée sur l'idée que les caractéristiques les plus marquantes des impératifs déontiques énumérés ci-dessus sont les suivantes :

Hypothèse de liberté de choix, ou *takhyr* : le fait que l'on puisse choisir d'effectuer ou non une action.

L'hétéronomie des impératifs : le fait que la manière dont les actions sont qualifiées par une récompense ou une sanction dépend des choix effectués.²⁰⁴

Ces deux conditions sont liées à l'idée de responsabilité qui est au cœur de la conception de l'obligation d'Ibn Ḥazm. Ce point a été souligné par Hourani (1985, p. 175) comme suit :

Le fait qui nous intéresse dans un récit historique est que, dans tous les contextes éthiques, [Ibn Ḥazm] considère l'homme comme responsable de ses propres actions et susceptible d'être récompensé et puni en conséquence.

La responsabilité se manifeste par le fait qu'un individu juridiquement responsable peut non seulement choisir de faire ou de ne pas faire un certain type d'action, mais il peut également choisir de ne pas choisir du tout ; les actions doivent être subordonnées à nous : nous ne devons pas nécessairement accepter le choix. En effet, la récompense et la sanction dépendent toutes deux des choix effectués.

En fait, la jurisprudence islamique explicite les présupposés de l'application d'une qualification déontique. En effet, les classifications telles que l'obligatoire, l'interdit et le permis, qui fondent une décision juridique (*ḥukm* : حکم) pour une action particulière (par exemple, *il est interdit de manger du porc*), présupposent ce qui suit :

- a) la personne qui accomplit une action est légalement responsable (*mukallaḥ* : مكلف) ;
- b) l'action en question est une action pour laquelle la liberté de choisir de l'accomplir ou non a été accordée (l'octroi de cette liberté de choix est appelé *takhyr* : تغيير).

²⁰⁴ Le terme "hétéronomie" qu'on doit à Kant, est principalement utilisé en philosophie pour décrire des normes, dont le fait de les suivre implique l'obtention d'un objectif tel que la compensation ou l'évitement d'une punition.

Il est à noter que cette approche est assez différente des études actuelles en logique déontique qui incluent, comme axiome, l'implication $OA \supset MA$ - où "O" signifie "obligatoire" et "M" "possible", connu comme le principe selon lequel *Doit implique Peut*, et également appelé *principe de Kant (Sollen-Können-Prinzip)*.²⁰⁵ Or, notre analyse de la conception islamique nous amène à constater ce qui suit :

- Toute qualification déontique, et pas seulement l'obligatoire, **présuppose plutôt qu'elle n'implique que l'action qualifiée peut être choisie.**²⁰⁶

Ainsi, *Doit présuppose Peut*. Cependant, il semble que dans la littérature sur la jurisprudence islamique, il y a trois façons de définir le permis, à savoir :

- 1) Le permis se décline sous les trois formes mentionnées ci-dessus, ce qui revient à définir le permis comme les actions dont l'accomplissement n'est ni obligatoire ni interdit.
- 2) Est permise toute action qui n'est pas sanctionnée lorsqu'elle est accomplie. Cela revient à définir le permis comme les actions dont l'accomplissement n'est pas interdit.
- 3) Est permise toute action qui n'est pas sanctionnée lorsqu'elle n'est pas accomplie. Cela revient à définir la permissivité comme les actions qu'il n'est pas obligatoire d'accomplir.

Nous nous concentrerons sur la première classification, puisque l'analyse logique des autres classifications suit la première.

Notez, que,

d'une part,

la structure logique suggérée par cette classification est celle d'un jugement hypothétique, telle que si face au choix entre exécuter ou ne pas exécuter une certaine action, nous sommes récompensés ou sanctionnés en fonction de ce choix ;

d'autre part,

la structure conceptuelle et éthique suggérée par cette classification est que ces mêmes choix façonnent la contingence déontique des schémas d'action et en même temps elles gouvernent les attributions de responsabilité.

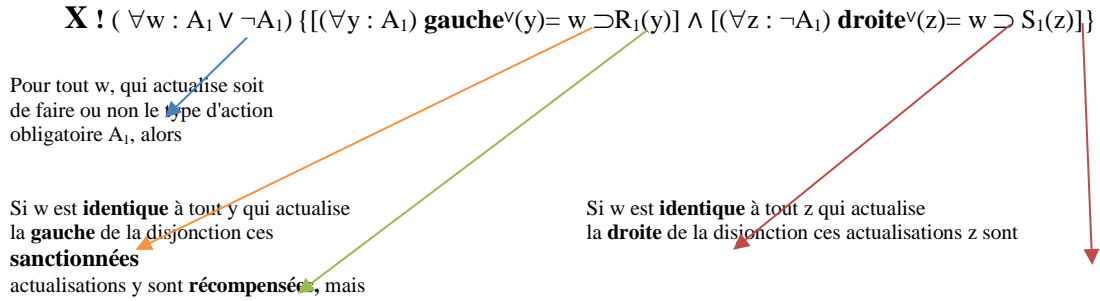
1.5 Les impératifs déontiques et l'analyse des hypothèses.

D'après les analyses de Rahman & Zidani & Young (2018) la forme logique de l'affirmation **W (Obligatoire)**_{A1} est la suivante :

²⁰⁵ Cf. Prior (1958), von Wright (1963, pp. 108-116, 122-125), Hilpinen (1981a, pp. 14-15), Chellas (1974), al-Hibri (1978, p. 18-21), Hilpinen et McNamara (2013, p. 38).

²⁰⁶ Remarquez que l'analyse de Hintikka (1981, p. 86) du principe de Kant est assez proche de notre vision du rôle du *takhyīr* - bien qu'il parle de **conséquence non logique** plutôt que de presupposition.

Notre résultat est en soi très simple, et peut même sembler trivial - une fois qu'il a été établi. (Tous les devoirs devraient être remplis. Il devrait donc être possible de les remplir). Quoi qu'il en soit, la possibilité que le principe "sollen-können" ait été, dès le départ, conçu, même si c'est de manière vague et inarticulée, comme l'expression d'une conséquence déontique plutôt que d'une conséquence logique, confère un intérêt supplémentaire à nos observations. Ce principe a été mis en évidence dans la philosophie morale par Kant. Nous devons donc nous demander comment il l'a conçu. Les explications de Kant ne se distinguent pas par leur lucidité, mais le lien entre le principe "le devoir implique le pouvoir" et le concept de liberté constitue en tout cas un courant de pensée incontestable et récurrent chez Kant (voir par exemple Critique de la liberté). (Voir par exemple Critique de la raison pure A 807, Critique de la raison pratique, 1ère édition, p. 54). Pour Kant, la liberté morale réside dans le fait même qu'un homme peut agir de la manière dont il doit agir.



Cela donne lieu aux analyses suivantes :

wājib, farḍ, lāzim (واجب، فرد، لازم)

Obligatoire : Pour tout w , tel que w revient à faire ou ne pas faire l'action obligatoire A_1 , alors si w est identique à tout y qui actualise la gauche de la disjonction, cette actualisation y est récompensée, et si w est identique à tout z qui actualise la droite de la disjonction, cette actualisation est sanctionnée.

$$(\forall w \in A_1 \vee \neg A_1) \{[(\forall y \in A_1) \text{gauche}^v(y)=x \supset \varepsilon R_1(y)] \wedge [(\forall z \in A_1) \text{droite}^v(z)=x \supset S_1(z)]\}$$

ḥarām, maḥzūr (حرام، مخزور):

Intérendit : Pour tout w , tel que w revient à faire ou ne pas faire l'action interdite A_2 , alors si w est identique à tout y qui actualise la gauche de la disjonction, cette actualisation y est sanctionnée, et si w est identique à tout z qui actualise la droite de la disjonction, cette actualisation est récompensée.

$$(\forall w \in A_2 \vee \neg A_2) \{[(\forall y \in A_2) \text{gauche}^v(y)=x \supset S_2(y)] \wedge [(\forall z \in A_2) \text{droite}^v(z)=x \supset R_2(z)]\}$$

Mubāḥ mustaḥabb (مباح مستحب):

Recommandé : Pour tout w , tel que w revient à faire ou ne pas faire l'action (recommandée) A_3 , alors si w est identique à tout y qui actualise la gauche de la disjonction, cette actualisation y est récompensée, et si w est identique à tout z qu'il actualise la droite de la disjonction, cette actualisation n'est ni sanctionnée ni récompensée.

$$(\forall w \in A_3 \vee \neg A_3) \{[(\forall y \in A_3) \text{gauche}^v(y)=x \supset R_3(y)] \wedge [(\forall z \in A_3) \text{droite}^v(z)=x \supset S_3(z) \wedge R_3(z)]\}$$

Mubāḥ makrūh (مباح مكروه):

Répréhensible : Pour tout w , tel que w revient à faire ou ne pas faire l'action (Répréhensible) A_4 , alors si w est identique à tout y qui actualise la gauche de la disjonction, cette actualisation y n'est ni récompensée et ni sanctionnée, et si w est identique à tout z qui actualise la droite de la disjonction, cette actualisation est récompensée.

$$(\forall w \in A_4 \vee \neg A_4) \{[(\forall y \in A_4) \text{gauche}^v(y)=x \supset S_4(y) \wedge R_4(y)] \wedge [(\forall z \in A_4) \text{droite}^v(z)=x \supset R_4(z)]\}$$

mubāḥ mustawin (مباح مستوین) ε

Neutre ε Pour tout w , tel que w revient à faire ou ne pas faire l'action neutre A_5 , alors si w est identique à tout y qui actualise la gauche de la disjonction, cette actualisation y n'est ni récompensée et ni sanctionnée, et si w est identique à tout z qui actualise la droite de la disjonction, cette actualisation n'est ni récompensée et ni sanctionnée.

$$(\forall w \varepsilon A_5 \vee \neg A_5) \{ [(\forall y \varepsilon A_5) \text{gauche}^v(y)=x \supset S_5(y) \wedge R_5(y)] \wedge [(\forall z \varepsilon A_5) \text{droite}^v(z)=x \supset S_5(z) \wedge R_5(z)] \}$$

2 La Signification dialectique locale des impératifs déontiques

Dans ce tableau le challenger **Y** défie l'universel dans les impératifs déontiques, en choisissant l'exécution ou la performance « a » qui indique que cette exécution ou performance actualise ou bien la **gauche** ou bien la **droite** de la disjonction.

	La Signification	Dialectique	Locale des impératives
Affirmation déontique	Forme logique de l'affirmation déontique	Défi	Défense
X ! WA₁ A₁ est Obligatoire/ Wājib	$X!(\forall w \varepsilon A_1 \vee \neg A_1)[(\forall y \varepsilon A_1) \text{gauche}^v(y) = w \supset R_1(y)] \wedge [(\forall z \varepsilon \neg A_1) \text{droite}^v(z) = w \supset S_1(z)]$	Y ! a $\varepsilon A_1 \vee \neg A_1$ Le challenger Y défie l'universel dans les impératives déontiques, en choisissant l'exécution « a » qui indique que cette exécution actualise ou bien la gauche ou bien la droite de la disjonction.	$[(\forall y \varepsilon A_1) \text{gauche}^v(y) = a \supset R_1(y)] \wedge [(\forall z \varepsilon \neg A_1) \text{droite}^v(z) = a \supset S_1(z)]$ Si l'action a actualise la gauche de la disjonction, cette actualisation est récompensée. ET Si a actualise la droite de la disjonction, cette actualisation est sanctionnée.
X ! HA₂ A₂ est interdit/ Ḥarām	$X!(\forall w \varepsilon A_2 \vee \neg A_2)[(\forall y \varepsilon A_2) \text{gauche}^v(y) = w \supset S_2(y)] \wedge [(\forall z \varepsilon \neg A_2) \text{droite}^v(z) = w \supset R_2(z)]$	Y ! a $\varepsilon A_2 \vee \neg A_2$	$[(\forall y \varepsilon A_2) \text{gauche}^v(y) = a \supset S_2(y)] \wedge [(\forall z \varepsilon \neg A_2) \text{droite}^v(z) = a \supset R_2(z)]$ Si l'action a actualise la gauche de la disjonction, cette actualisation est sanctionnée. ET Si a actualise la droite de la disjonction, cette actualisation est récompensée
X ! MM_{SH}A₃ A₃ est Recommandé/ Mubāh.mustaḥabb	$X!(\forall w \varepsilon A_3 \vee \neg A_3)(\forall y \varepsilon A_3) \text{gauche}^v(y) = w \supset R_3(y) \wedge [(\forall z \varepsilon \neg A_3) \text{droite}^v(z) = w \supset \neg S_3(z) \wedge \neg R_3(z)]$	Y ! a $\varepsilon A_3 \vee \neg A_3$	$[(\forall y \varepsilon A_3) \text{gauche}^v(y) = a \supset R_3(y)] \wedge [(\forall z \varepsilon \neg A_3) \text{droite}^v(z) = a \supset (\neg S_3(z) \wedge \neg R_3(z))]$ Si l'action a actualise la gauche de la disjonction, cette actualisation est récompensée. ET Si a actualise la droite de la disjonction, cette actualisation n'est ni récompensée et ni sanctionnée.
X ! MM_{KR}A₄ A₄ est Réprehensible/ Mubāh makrūh	$X!(\forall w \varepsilon A_4 \vee \neg A_4)[(\forall y \varepsilon A_4) \text{gauche}^v(y) = w \supset [\neg R_4(y) \wedge \neg S_4(y)]] [(\forall z \varepsilon \neg A_4) \text{droite}^v(z) = w \supset R_4(z)]$	Y ! a $\varepsilon A_4 \vee \neg A_4$	$[(\forall y \varepsilon A_4) \text{gauche}^v(y) = a \supset [\neg R_4(y) \wedge \neg S_4(y)]] [(\forall z \varepsilon \neg A_4) \text{droite}^v(z) = a \supset R_4(z)]$ Si a actualise la gauche de la disjonction, cette actualisation n'est ni récompensée et ni sanctionnée. ET Si l'action A ₄ actualise a par la droite de la disjonction, cette actualisation est récompensée.
X ! MM_{SW}A₅ A₅ est (Neutre/ Mubāh mustawin)A₅	$X!(\forall w \varepsilon A_5 \vee \neg A_5)[(\forall y \varepsilon A_5) \text{gauche}^v(y) = w \supset \neg R_5(y) \wedge \neg S_5(y)] [(\forall z \varepsilon \neg A_5) \text{droite}^v(z) = w \supset \neg S_5(z) \wedge \neg R_5(z)]$	Y ! a $\varepsilon A_5 \vee \neg A_5$ (a est choisi par Y) a actualise l'une des composantes de la disjonction v soit en effectuant le type d'action A ₅ , soit en s'abstenant de l'effectuer.	$[(\forall y \varepsilon A_5) \text{gauche}^v(y) = w \supset \neg R_5(y) \wedge \neg S_5(y)] [(\forall z \varepsilon \neg A_5) \text{droite}^v(z) = w \supset \neg S_5(z) \wedge \neg R_5(z)]$ Si a actualise la gauche ou la droite de la disjonction, cette actualisation sera équivalente, c'est-à-dire elle ne sera ni récompensée et ni sanctionnée dans les côtés de la disjonction.

Dans les

dialogues ci-dessous, nous utilisons une variante de la règle socratique. Nous autorisons **O** à contester une affirmation α de **P**, si **O** n'a pas énoncé la même proposition auparavant – même si **P ! α** prend l'une des formes suivantes, **P ! $\neg R_i(a)$** ou **P ! $\neg S_i(a)$** ou **P ! $\neg R_i(a) \wedge \neg S_i(a)$** – dans ce cas le défi prend la forme **O ? α** . La réponse du proposant à une telle contestation prend la forme **P vous_n εA** , ce qui revient à dire que vous (l'adversaire) avez déjà énoncé un même témoin de présence de la proposition avec votre coup n. De plus, par souci de simplicité, nous avons réduit au strict minimum l'utilisation explicite de ces raisons et instructions locales prescrites par les règles dialectiques pour les connexions logiques. Le lecteur peut, s'il le souhaite, les ajouter en consultant l'annexe.

- Le point philosophique de cette adaptation de la **Règle Socratique** est que, contrairement à la règle formelle, le dialogue doit être développé en tenant compte du contenu de l'affirmation en question. Dans ce type de dialogue, il ne s'agit pas seulement de la force logique de l'inférence d'identité, mais plutôt de la **manière dont l'analyse conceptuelle des prémisses** permet de comprendre **pourquoi** le dialogue aboutit à la conclusion exprimée par la thèse. De plus, cette adaptation est liée au concept d'*inanalysable*, utilisé dans les sections sur le syllogisme temporel. Cependant, comme il se doit, cette version met l'accent sur le contenu éthique et juridique d'expressions telles que **ni Sanctionné et ni Récompensé** en tant que **totalité**.

En outre, rappelons que nous utilisons la notation suivante pour classifier les différents types d'actions :

- A₁ indique que A est un type d'actions obligatoires telles que faire les prières, etc. Ainsi, $\neg A_1$ indique que le type d'actions obligatoires A n'est pas accompli.
- A₂ indique que A est un type d'actions interdites telles que tuer une personne, etc. Ainsi, $\neg A_2$ indique que le type d'actions interdites A n'est pas accompli.
- A₃ indique que A est un type d'actions recommandées telles que se brosser les dents avant la prière, etc. Ainsi, $\neg A_3$ indique que le type d'actions recommandées A n'est pas accompli.
- A₄ indique que A est un type d'actions répréhensibles telles que gesticuler exagérément en priant, etc. Ainsi, $\neg A_4$ indique que le type d'actions répréhensibles A n'est pas accompli.
- A₅ indique que A est un type d'actions neutres telles que dîner ensemble, etc. Ainsi, $\neg A_5$ indique que le type d'actions neutres A n'est pas accompli.

3 Dialogues sur modalités déontiques

NB : notation

Modalités déontiques	Obligatoire e/ Wājib	Interdit/Harâm	Recommandé/ Mubâh.mustaḥabb	Répréhensible/ Mubâh makrūh	Neutre/ Mubâh mustawin
Abréviations	WA ₁	HA ₂	MM _{SH} A ₃	MM _{KRA} A ₄	MM _{sw} A ₅

3.1 Dialogues sur quelques types d'actions A₃.

MM_{SH}A₃ Dialogue 1

Coups	Opposante	Proposant	Coups
	Prémisses 1 MMA ₃ $(\forall w \in A_3 \vee \neg A_3) (\forall y \in A_3) \text{gauche}^w(y)=x \supset R_3(y) \wedge$ $(\forall z \in \neg A_3) \text{droite}^w(z)=x \supset S_3(z) \wedge R_3(z)$ Face au choix de faire le type d'action recommandé, si l'individu actualise le type d'action A ₃ , avec la performance « a », cette performance sera récompensée (R ₃ (a)), si « a » actualise $\neg A_3$, cette performance ne sera pas ni récompensée ni et	$! \neg R_3(a) \wedge \neg S_3(a)$ L'exécution ou la performance de l'actualisation « a » du type d'action $\neg A$ n'est ni récompensée ni sanctionnée.	0

	sanctionnée ($\neg R_3(a)$ et $\neg S_3(a)$). Prémisse 2 $a \varepsilon \neg A_3$ Prémisse 3 droite ^v (a) =a (en supposant $a \varepsilon A_1 \vee \neg A_1$) Cette affirmation confirme le choix de la droite de la disjonction.		
1	? $\neg S_3(a) \wedge \neg R_3(a)$	Vous ₁₁ $\varepsilon \neg S_3(a) \wedge \neg R_3(a)$	12
5	$[(\forall y \varepsilon A_3) \text{gauche}^v(y) = a \supset R_3(y)]$ $\wedge [(\forall z \varepsilon A_3) \text{droite}^v(z) = a \supset (\neg S_3(z) \wedge \neg R_3(z))]$	Prémisse 1 $a \varepsilon A_3 \vee \neg A_3$	2
3	? \vee 2	$a \varepsilon \neg A_3$	4
7	$(\forall z \varepsilon A_3) \text{droite}^v(z) = w \supset (\neg S_3(z) \wedge \neg R_3(z))$	5 ? \wedge 2	6
9	droite ^v (a) = a $\supset \neg S_3(a) \wedge \neg R_3(a)$	7 $a \varepsilon \neg A_3$	8
11	$\neg S_3(a) \wedge \neg R_3(a)$	9 droite ^v (a) =a	10
		P gagne	

Bien entendu, il existe d'autres possibilités, à savoir que **P** pourrait affirmer que la performance « a » de l'action recommandé doit être ni récompensée et ni sanctionnée en dépit du fait que A_3 a été exécutée. Dans ce cas, le **P** perdra certainement. Cela montre la signification dialogique de la recommandation. Nous développons également une des autres possibilités, qui donnent lieu au dialogue suivant :

MM_{SH}A₃ Dialogue 2

Coups	Opposante	Proposant	Coups
	Prémisse 1 MMA ₃ $(\forall w \varepsilon A_3 \vee \neg A_3) (\forall y \varepsilon A_3)$ gauche ^v (y) = x $\supset R_3(y) \wedge (\forall z \varepsilon \neg A_3)$ droite ^v (z) = x $\supset S_3(z) \wedge R_3(z)$ Prémisse 2 $a \varepsilon A_3$ Prémisse 3 gauche ^v (a) =a (en supposant $a \varepsilon A_1 \vee \neg A_1$)	! $\neg S_3(a) \wedge \neg R_3(a)$ L'exécution ou la performance de l'actualisation « a » du type d'action A_3 n'est ni récompensée et ni sanctionnée.	0

1	$? \neg S_3(a) \wedge \neg R_3(a)$???	
5	$[(\forall y \in A_3) \text{gauche}^v(y) = a \supset R_3(y)] \wedge [(\forall z \in \neg A_3) \text{droite}^v(z) = a \supset (\neg S_3(z) \wedge \neg R_3(z))]$	Prémisse 1 $a \in A_3 \vee \neg A_3$	2
3	? \vee 2	$a \in A_3$	4
7	$(\forall y \in A_3) \text{gauche}^v(y) = w \supset R_3(y)$	5 ? \wedge 1	6
9	$\text{gauche}^v(a) = a \supset R_3(a)$	7 $a \in A_3$	8
11	$R_3(a)$	9 $\text{gauche}^v(a)$	10
	O gagne		

3.3 Dialogues sur d'autres types d'actions.

3.3.1 Obligatoire/ Wâjib

WA₁

Dialogue 1

Obligatoire/ Wâjib			
Coups	Opposante	Proposant	Coups
	<p>Prémisse 1 ! WA₁</p> <p>$(\forall w \in A_1 \vee \neg A_1) (\forall y \in A_1) \text{gauche}^v(y) = w \supset R_1(y) \wedge (\forall z \in \neg A_1) \text{droite}^v(z) = w \supset S_1(z)$</p> <p>Face au choix entre faire le type d'action obligatoire ou ne pas le faire, si la performance « a » actualise le type d'action A₁, « a » est récompensée (R₁(a)), si la performance « a » actualise $\neg A_1$, « a » est sanctionnée (S₁(a)).</p> <p>Prémisse 2 $a \in A_1$</p> <p>Face au choix entre faire ou ne pas faire A₁, l'individu a choisi de le faire.</p> <p>Dans le contexte de la logique de la présence de Suhrawardī, « a » représente une présence qui témoigne que le type d'action obligatoire A₁ est actualisé par une performance.</p> <p>Prémisse 3 $\text{gauche}^v(a) = a$ (en supposant $a \in A_1 \vee \neg A_1$)</p> <p>Le choix revient à l'affirmation de la choisir la gauche de la disjonction.</p>	<p>! R₁(a)</p> <p>La performance « a » de l'action A₁ est récompensée.</p>	0

1	? $R_1(a)$	$\text{Vous}_{11} \in R_1(a)$	12
5	$[(\forall y \in A_1) \text{gauche}^V(y)=a \supset R_1(y)] \wedge [(\forall z \in \neg A_1) \text{droite}^V(z)=a \supset S_1(z)]$	Prémisse 1 $a \in A_1 \vee \neg A_1$	2
3	? \vee 2	$a \in A_1$	4
7	$[(\forall y \in A_1) \text{gauche}^V(y)=a \supset R_1(y)]$	5 ? $\wedge 1$	6
9	$\text{gauche}^V(a)=a \supset R_1(a)$	7 $a \in A_1$	8
11	$R_1(a)$	9 $\text{gauche}^V(a)=a$	10
		P gagne	

WA₁
Dialogue 2

Obligatoire/ Wâjib			
Coups	Opposante	Proposant	Coups
	Prémisse 1 ! WA₁ $(\forall w \in A_1 \vee \neg A_1) (\forall y \in A_1) \text{gauche}^V(y)=w \supset R_1(y) \wedge (\forall z \in \neg A_1) \text{droite}^V(z)=w \supset S_1(z)$ Prémisse 2 $a \in A_1$ Prémisse 3 $\text{gauche}^V(a)=a$ (en supposant $a \in A_1 \vee \neg A_1$)	$! S_1(a)$ La performance « a » de l'action A_1 est sanctionnée.	0
1	? $S_1(a)$???	
5	$[(\forall y \in A_1) \text{gauche}^V(y)=a \supset R_1(y)] \wedge [(\forall z \in \neg A_1) \text{droite}^V(z)=a \supset S_1(z)]$	Prémisse 1 $a \in A_1 \vee \neg A_1$	2
3	? \vee 2	$a \in A_1$	4
7	$[(\forall y \in A_1) \text{gauche}^V(y)=a \supset R_1(y)]$	5 ? $\wedge 1$	6
9	$\text{gauche}^V(a)=a \supset R_1(a)$	7 $a \in A_1$	8
11	$R_1(a)$	9 $\text{gauche}^V(a)=a$	10
	O gagne		

WA₁
Dialogue 3

Obligatoire/ Wâjib			
Coups	Opposante	Proposant	Coups
	Prémisse 1 ! WA₁ $(\forall w \in A_1 \vee \neg A_1) (\forall y \in A_1) \text{gauche}^V(y)=w \supset R_1(y) \wedge (\forall z \in \neg A_1) \text{droite}^V(z)=w \supset S_1(z)$	$! S_1(a)$ La performance « a » de l'action A_1 est sanctionnée.	0

	Prémisse 2 $a \varepsilon \neg A_1$		
	Prémisse 3 droite ^v (a)= a (en supposant $a \varepsilon A_1 \vee \neg A_1$)		
1	? $S_1(a)$	Vous ₁₁ $\varepsilon S_1(a)$	12
5	$[(\forall y \varepsilon A_1) \text{gauche}^v(y)=a \supset R_1(y)] \wedge [(\forall z \varepsilon \neg A_1) \text{droite}^v(z)=a \supset S_1(z)]$	Prémisse 1 $a \varepsilon A_1 \vee \neg A_1$	2
3	? \vee 2	$a \varepsilon \neg A_1$	4
7	$[(\forall z \varepsilon \neg A_1) \text{droite}^v(y)=a \supset R_1(z)]$	5 ? $\wedge 2$	6
9	droite ^v (a)=a $\supset S_1(a)$	7 $a \varepsilon \neg A_1$	8
11	$S_1(a)$	9 droite ^v (a)= a	10
		P gagne	

3.3.2 Interdit/ Ḥarâm

HA₂ Dialogue 1

Interdit/ Ḥarâm			
Coups	Opposante	Proposant	Coups
	<p>Prémisse 1 ! HA₂</p> <p>$(\forall w \varepsilon A_2 \vee \neg A_2) (\forall y \varepsilon A_2) \text{gauche}^v(y)=w \supset S_2(y)$ $\wedge (\forall z \varepsilon \neg A_2) \text{droite}^v(z)=w \supset R_2(z)$</p> <p>Face au choix entre faire le type d'action interdite ou ne pas le faire, si la performance « a » actualise le type d'action A₂, « a » est sanctionnée (S₁(a)), si la performance « a » actualise $\neg A_1$, « a » est récompensée (R₁(a)).</p> <p>Prémisse 2 $a \varepsilon A_2$</p> <p>Face aux deux choix, l'individu a préféré d'actualiser le type d'action interdit.</p> <p>Prémisse 3 gauche^v(a) =a$\supset S_2 a$ (en supposant $a \varepsilon A_2 \vee \neg A_2$)</p> <p>Le choix revient à l'affirmation</p>	<p>! $S_2(a)$</p> <p>La performance « a » de l'action A₂ est sanctionnée.</p>	0

	de la gauche de la disjonction.		
1	? $S_2(a)$	Vous ₁₁ $\varepsilon S_2(a)$	12
5	$[(\forall y \varepsilon A_2) \text{gauche}^v(a)=a \supset S_2(y) \wedge (\forall z \varepsilon \neg A_2) \text{droite}^v(a) = a \supset R_2(z)]$	Prémisse 1 $a \varepsilon A_2 \vee \neg A_2$	2
3	? \vee 2	$a \varepsilon A_2$	4
7	$[(\forall y \varepsilon A_2) \text{gauche}^v(a)=a \supset S_2(y)$	5 ? $\wedge 1$	6
9	$\text{gauche}^v(a)=a \supset S_2(a)$	7 $a \varepsilon A_2$	8
11	$S_2(a)$	9 $\text{gauche}^v a=a$	10
		P gagne	

HA₂
Dialogue 2

Interdit/ Harâm			
Coups	Opposante	Proposant	Coups
	Prémisse 1 ! HA₂ $(\forall w \varepsilon A_2 \vee \neg A_2) (\forall y \varepsilon A_2) \text{gauche}^v(y)=w \supset S_2(y) \wedge (\forall z \varepsilon \neg A_2) \text{droite}^v(z)=w \supset R_2(z)$ Prémisse 2 $a \varepsilon \neg A_2$ Prémisse 3 $\text{droite}^v(a)$ (en supposant $a \varepsilon A_2 \vee \neg A_2$)	$! R_2(a)$ La performance « a » de l'action A_2 est récompensée.	0
1	? $R_2(a)$ 0	Vous ₁₁ $\varepsilon R_2(a)$	12
5	$[(\forall y \varepsilon A_2) \text{gauche}^v(a)=a \supset S_2(y) \wedge (\forall z \varepsilon A_2) \text{droite}^v(a) = a \supset R_2(z)]$	Prémisse 1 $a \varepsilon A_2 \vee \neg A_2$	2
3	? \vee 2	$a \varepsilon \neg A_2$	4
7	$(\forall z \varepsilon A_2) \text{droite}^v(a) = a \supset R_2(z)$	5 ? $\wedge 2$	6
9	$\text{droite}^v(a) = a \supset R_2(a)$	7 $a \varepsilon \neg A_2$	8
11	$R_2(a)$	9 $\text{droite}^v(a) = a$	10
		P gagne	

3.3.3 Réprehensible/ Mubâh makrûh
MM_{KRA4}
Dialogue 1

Réprehensible/ Mubâh makrûh			
Coups	Opposante	Proposant	Coups
	Prémisse 1 ! MM_{KRA4} $(\forall w \varepsilon A \vee \neg A) (\forall y \varepsilon A) \text{gauche}^v(y)=w \supset [\neg R_4(y) \wedge \neg S_4(y)] \wedge (\forall z \varepsilon \neg A) \text{droite}^v(z)=w \supset R_4(z)$	$! R_4(a)$ L'actualisation de « a » du type d'action A_4 est récompensée.	0

	<p>Face au choix entre faire le type d'action répréhensible ou non, si la performance « a » actualise le type d'action A_4, « a » est ni récompensée et ni sanctionnée.</p> <p>Et si la performance « a » actualise le type d'action $\neg A_4$, « a » est récompensée.</p> <p>Prémisse 2 $a \varepsilon \neg A_4$</p> <p>Prémisse 3 droite^v(a)=a (en supposant $a \varepsilon A_4 \vee \neg A_4$)</p> <p>Le choix revient à l'affirmation de la droite de la disjonction.</p>		
1	? $R_4(a)$	11 Vous ₁₁ $\varepsilon R_4(a)$	12
5	$[(\forall y \varepsilon A_4) \text{gauche}^v(a)=a \supset \neg R_4(y) \wedge \neg S_4(y) \wedge (\forall z \varepsilon A_4) \text{droite}^v(a)=a \supset R_4(z)]$	Prémisse 1 $a \varepsilon A_4 \vee \neg A_4$	2
3	? \vee	$a \varepsilon A_4$	4
7	$(\forall z \varepsilon A_4) \text{droite}^v(a)=a \supset R_4(z)$	5 ? $\wedge 2$	6
9	droite ^v (a)=a $\supset R_4(a)$	7 $a \varepsilon A_4$	8
11	$R_4(a)$	9 droite ^v (a)=a	10
		P gagne	

MM_{KRA4}
Dialogue 2

Répréhensible/ Mubâh makrûh			
Coups	Opposante	Proposant	Coups
	<p>Prémisse 1 ! MM_{KRA4}</p> <p>$(\forall w \varepsilon A \vee \neg A_4) (\forall y \varepsilon A_4) \text{gauche}^v(y)=w \supset [\neg R_4(y) \wedge \neg S_4(y)] [(\forall z \varepsilon \neg A_4) \text{droite}^v(z)=w \supset R_4(z)]$</p> <p>Prémisse 2 $a \varepsilon A_4$</p> <p>Prémisse 3 gauche^v(a) = a (en supposant $a \varepsilon A_4 \vee \neg A_4$)</p> <p>Le choix revient à l'affirmation de la gauche de la disjonction.</p>	<p>! $\neg R_4(a) \wedge \neg S_4(a)$</p> <p>L'actualisation de « a » du type d'action A_4 est récompensée.</p>	0
1	? $\neg R_4(a) \wedge \neg S_4(a)$	Vous ₁₁ $\varepsilon \neg R_4(a) \wedge \neg S_4(a)$	12
5	$[(\forall y \varepsilon A_4) \text{gauche}^v(a)=a \supset \neg R_4(y) \wedge \neg S_4(y) \wedge (\forall z \varepsilon$	Prémisse 1 $a \varepsilon A_4 \vee \neg A_4$	2

	A_4 droite ^v (a)=a \supset R_4 (z)]		
3	? \forall	a : A_4	4
7	$(\forall y \in A_4)$ gauche ^v (a)=a \supset $\neg R_4$ (y) $\wedge \neg S_4$ (y)	5 ? $\wedge 1$	6
9	gauche ^v (a)=a \supset $\neg R_4$ (a) $\wedge S_4$ (a)	7 a $\in A_4$	8
11	$\neg R_4$ (a) $\wedge \neg S_4$ (a)	9 gauche ^v (a)=a	10
		P gagne	

3.3.4 Neutre/ Mubâh mustawin

MMswA₅ Dialogue 1

Neutre/ Mubâh mustawin			
Coups	Opposante	Proposant	Coups
	<p>Prémisse 1 ! MMswA₅</p> <p>$(\forall w \in AV \neg A_5)[(\forall y \in A_5)$ gauche^v(y)=w \supset $\neg R_5$(y) $\wedge \neg S_5$(y)] $\wedge [(\forall z \in \neg A_5)$ droite^v(z)=w \supset $\neg S_5$(z)] $\wedge \neg R_5$(z)]</p> <p>Face au choix entre faire le type d'action neutre ou non, si la performance « a » actualise le type d'action A₅, « a » n'est ni récompensée et ni sanctionnée. Et si la performance « a » actualise le type d'action $\neg A_5$, « a », n'est aussi ni récompensée et ni sanctionnée.</p> <p>Prémisse 2 a $\in A_5$</p> <p>Prémisse 3 gauche^v(a) = a (en supposant a $\in A_5 \vee \neg A_5$) Le choix revient à l'affirmation de la gauche de la disjonction.</p>	<p>! $\neg R_5$(a) $\wedge \neg S_5$(a)</p> <p>L'actualisation « a » du type d'action A₅ n'est ni récompensée et ni sanctionnée.</p>	0
1	? ($\neg R_5 \wedge \neg S_5$ (a))	Vous ₁₁ $\in \neg R_5$ (a) $\wedge \neg S_5$ (a)	12
3	$[(\forall y \in A_5)$ gauche ^v (a) = a \supset ($\neg R_5 \wedge \neg S_5$ (y)) $\wedge (\forall z \in A_5)$ droite ^v (a) = a \supset ($\neg R_5 \wedge \neg S_5$ (a)) (z)]	Prémisse 1 a $\in A_5 \vee \neg A_5$	2
5	? \forall 2	a $\in A_5$	4
7	$(\forall y \in A_5)$ gauche ^v (a) = a \supset $\neg R_5$ (y) $\wedge \neg S_5$ (y)	3 ? $\wedge 1$	6
9	gauche ^v (a) = a \supset ($\neg R_5 \wedge \neg S_5$ (a))	7 a $\in A_5$	8
11	$\neg R_5$ (a) $\wedge \neg S_5$ (a)	9 gauche ^v (a) = a	10
		P gagne	

MMswA₅ Dialogue 2

NB : Dans le deuxième dialogue, le choix d'un type d'action de neutralité, produit le même résultat si l'individu choisit la droite de la disjonction. Mieux ; au cours du dialogue le proposant pourrait demander le conjoint 2 de la prémisse 1 et le résultat demeurera le même.

4 Temporalité et modalités déontiques : actions temporelles

La notion de contingent chez Suhrawardī est guidée par deux grands principes aristotéliens sur le temps qu'il partage avec Ibn Sīnā et la plupart des post- d'Ibn Sīnā, à savoir

1. Le temps est un présupposé logique du contingent - c'est-à-dire qu'étant donné une paire de propositions exprimant deux attributs incompatibles de la même substance, leur vérité doit être relativisée temporellement (si l'on veut éviter la contradiction) ;
2. L'expérience du contingent est une présupposition épistémologique du temps. De plus, ajoute Suhrawardī, le témoignage mental d'une expérience du contingent est une présupposition épistémique de l'existence cf. al-Ishrāq (1999, p. 16).

En d'autres termes, la dimension temporelle est constitutive du sens de la notion d'attributions de contingence chez Suhrawardī. En outre, si l'on examine attentivement les textes de Suhrawardī, il apparaît clairement que les conditions temporelles ne sont pas comprises ici de manière propositionnelle, ni comme des implications, ni comme des indices qui saturent une fonction propositionnelle, ce qui ferait d'ailleurs du temps une substance (en contravention avec le premier principe aristotélien sur le temps mentionné plus haut). Les conditions temporelles sont des paramètres contextuels qui peuvent être explicités afin d'enrichir une assertion qui a déjà un contenu,²⁰⁷ plutôt que de compléter le sens d'une fonction propositionnelle.

Cela suggère que les présences, plutôt que les propositions, sont les premiers porteurs de la temporalité. Ceci est d'une importance majeure dans le domaine des normes éthiques et/ou juridiques.

Une norme, telle que l'obligation de prier, est en quelque sorte atemporelle. Ce qui est temporel, ce sont les réalisations de cette norme, qu'il s'agisse de l'accomplir ou de l'omettre. Par exemple, la prière de Fatima le vendredi 27 octobre 2023 remplit l'obligation de prier, mais Zayd ne la remplit pas. L'accomplissement sera alors récompensé maintenant ou dans le futur et l'omission sanctionnée.

Plus généralement, s'il existe une performance x témoignant que l'obligation de faire le type d'action A_1 , est en train d'être remplie, alors x sera temporalisé. La temporalisation est mise en œuvre par une fonction temporelle τ qui prend ces x et produit un moment t qui est un élément de l'ensemble \mathbf{T} des moments temporels, et ces moments, selon le contexte, peuvent être définis comme des heures, des jours, des mois, etc.

En d'autres termes, si x est l'une de ces performances, l'évaluation de $\tau(x)$ est identique (dans l'ensemble \mathbf{T}) à un certain t_n , qui indique le moment où l'obligation a été remplie,

Il y a une présence x qui témoigne de l'accomplissement du type d'action obligatoire A_1

$$(\exists x \in A_1) \tau(x) =_{\mathbf{T}} t_n$$

et cette présence x peut être chronométrée comme se produisant à t_n

²⁰⁷ Nous devons l'enrichissement de l'expression à Recanati (2017).

Notation adverbiale

Une façon de simplifier la notation est la suivante : si "@" représente un opérateur monadique qui enrichit une proposition avec des moments, $A@t_i$ exprime une construction adverbiale. Dans notre contexte, l'adverbial @ qui apparaît dans $A@t_i$ encode le résultat de la fonction temporelle définie sur les performances (ou actualisations) du type d'action A.

Maintenant, si a est une performance particulière qui remplit le type d'actions obligatoires A_1 , et t_k est le moment précis où a a été effectuée, alors, selon la définition de l'obligation, a sera récompensée. De plus, cette récompense aura lieu à un moment t_k (peu ou longtemps) après que « a » a été exécutée.

Ainsi donné,

$$\begin{aligned} a \in A_1 \\ \tau(a) = t_k \end{aligned}$$

on obtient les notations adverbiales

$$A_1@t_k$$

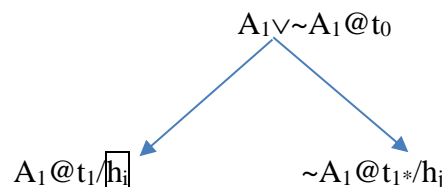
puis,

$$Ra@t_j > t_k$$

Choix et solutions alternatives

Les choix d'un agent déterminent des cours d'événements alternatives après l'instant d'énonciation t_0 , de sorte que, quel que soit l'avenir, une histoire particulière h (une suite linéaire d'instants) se réalisera à un moment donné où le choix se concrétisera ²⁰⁸.

Comme nous le verrons dans la dernière section de notre étude, la ramification des cours d'événements alternatifs produits par un choix façonne le concept de plénitude dans le contexte des actions. Ainsi, si le choix produit l'histoire **réelle** h_1 sur laquelle l'obligation A_1 est remplie à un certain moment $t_1 > t_0$; il existe un cours alternatif (non actualisé) d'événements h_2 sur lequel A_1 n'est pas rempli au moment " **jumeau** " $t_1^* > t_0$. - en supposant que les deux histoires h_i et h_j traversent t_0 , alors $t_{h_0/i}$, $t_{h_0/j}$



Le carré dans lequel s'inscrit h_i indique que h_i est le déroulement effectif des événements, et ' t_0 ' indique le point de référence auquel le choix a été fait.

Cela donne la formulation adverbiale des règles pour la signification locale ci-dessous. Cependant, Remarquez qu'il n'y a pas d'histoire associée au défi $a \in A_1 \vee \sim A_1 @ t_i$ du $\forall w \in A_1 \vee \sim A_1$. Le fait est que, tant qu'on n'a pas encore identifié avec précision quel côté de la disjonction a été choisi, on ne sait pas qui, parmi ces cours actuels d'événements qui traversent t_i , est l'histoire précise,

²⁰⁸ Cela suppose que le flux temporel a une structure ramifiée.

produite par le choix. Notez aussi qu'il est également possible de ne pas inclure toutes les indications temporelles dans les règles, mais de ne les ajouter qu'au cours de l'élaboration d'un jeu. Le raisonnement qui sous-tend cette idée est que la temporalité façonne les mouvements concrets, plutôt que les mouvements abstraits. Cela nécessite d'ajouter des règles structurelles pour la temporalisation des mouvements au cours d'une partie. Nous avons décidé de ne pas le faire pour des raisons de simplicité.

Affirmation déontique	Forme logique de l'affirmation déontique	Défi	Défense
X ! Obligatoire/ Wājib A₁	$X ! (\forall w \varepsilon A_1 \vee \neg A_1) [(\forall y \varepsilon A_1) \text{gauche}^v(y) = w \supset R_1(y)] \wedge [(\forall z \varepsilon \neg A_1) \text{droite}^v(z) = w \supset S_1(z)]$	Y ! a $\varepsilon A_1 \vee \neg A_1 @ t_i$ Dans ce cas, t_i est choisis par Y	$X ! [(\forall y \varepsilon A_1) \text{gauche}^v(y) = a \supset R_1(y) @ t_i > t_i / h_1] \wedge [(\forall z \varepsilon \neg A_1) \text{droite}^v(z) = a \supset S_1(z) @ t_j^* > t_i / h_2]$ t_j/h_1 et t_j^*/h_2 ont été choisis par X . Alors, la défense indique que si « a » actualise la gauche, alors cette actualisation doit être récompensée dans un moment t_j dans l'histoire h_1 future (par rapport à t_i). La défense indique aussi que si « a » actualise la droite, alors cette actualisation doit être sanctionnée dans un moment t_j^* dans l'histoire h_2 future (par rapport à t_i)
X ! Interdit/ Ḥarām A₂	$X ! (\forall w \varepsilon A_2 \vee \neg A_2) [(\forall y \varepsilon A_2) \text{gauche}^v(y) = w \supset S_2(y)] \wedge [(\forall z \varepsilon \neg A_2) \text{droite}^v(z) = w \supset R_2(z)]$	Y ! a $\varepsilon A_2 \vee \neg A_2 @ t_i$	$X ! [(\forall y \varepsilon A_2) \text{gauche}^v(y) = a \supset S_2(y) @ t_j > t_i / h_1] \wedge [(\forall z \varepsilon \neg A_2) \text{droite}^v(z) = a \supset R_2(z) @ t_j^* > t_i / h_2]$
X ! Recommandé/ Mubāh.mustahabb A₃	$X ! (\forall w \varepsilon A_3 \vee \neg A_3) [(\forall y \varepsilon A_3) \text{gauche}^v(y) = w \supset R_3(y)] \wedge [(\forall z \varepsilon \neg A_3) \text{droite}^v(z) = w \supset (\neg S_3 \wedge \neg R_3)(z)]$	Y ! a $\varepsilon A_3 \vee \neg A_3 @ t_i$	$X ! [(\forall y \varepsilon A_3) \text{gauche}^v(y) = a \supset R_3(y) @ t_j > t_i / h_1] \wedge [(\forall z \varepsilon \neg A_3) \text{droite}^v(z) = a \supset \neg S_3(z) \wedge \neg R_3(z) @ t_j^* > t_i / h_2]$
X ! Répréhensible/ Mubāh makrūh A₄	$X ! (\forall w \varepsilon A_4 \vee \neg A_4) [(\forall y \varepsilon A_4) \text{gauche}^v(y) = w \supset (\neg R_4 \wedge \neg S_4)(y)] \wedge [(\forall z \varepsilon \neg A_4) \text{droite}^v(z) = w \supset (R_4)(z)]$	Y ! a $\varepsilon A_4 \vee \neg A_4 @ t_i$	$X ! [(\forall y \varepsilon A_4) \text{gauche}^v(y) = a \supset \neg R_4(y) \wedge \neg S_4(y) @ t_j > t_i / h_1] \wedge [(\forall z \varepsilon \neg A_4) \text{droite}^v(z) = a \supset R_4(z) @ t_j^* > t_i / h_2]$
X ! Neutre/ Mubāh mustawin A₅	$X ! (\forall w \varepsilon A_5 \vee \neg A_5) [(\forall y \varepsilon A_5) \text{gauche}^v(y) = w \supset (\neg R_5 \wedge \neg S_5)(y)] \wedge [(\forall z \varepsilon \neg A_5) \text{droite}^v(z) = w \supset (\neg R_5 \wedge \neg S_5)(z)]$	Y ! a $\varepsilon A_5 \vee \neg A_5 @ t_i$	$X ! [(\forall y \varepsilon A_5) \text{gauche}^v(y) = a \supset \neg R_5(y) \wedge \neg S_5(y) @ t_j > t_i / h_1] \wedge [(\forall z \varepsilon \neg A_5) \text{droite}^v(z) = a \supset (\neg R_5(z) \wedge \neg S_5(z)) @ t_j^* > t_i / h_2]$

Une autre fois, dans les dialogues ci-dessous, nous nous autorisons **O** (et seulement **O**) à contester une affirmation de **P**, si **O** n'a pas énoncé la même proposition (ou la négation d'une proposition) auparavant.

4. Quelques Dialogues temporels

WA₁ Dialogue 1

Coups	Opposante	Proposant	Coups
	Prémisse 1 $X ! (\forall w \varepsilon A_1 \vee \neg A_1) [(\forall y \varepsilon A_1) \text{gauche}^v(y) = w \supset R_1(y)] \wedge [(\forall z \varepsilon \neg A_1)$	$R_1(a) @ t_i > 14h15mn / h_1$ La performance ou l'exécution « a » du type	0

	<p>droite^v(z)= $w \supset S_1(z)$</p> <p>Face aux deux choix de réaliser des types d'actions obligatoires A_1, si la performance « a » de A_1 est faite à l'heure prescrite, elle sera récompensée et si la performance « a » de $\neg A_1$ n'est pas faite à l'heure prescrite, elle sera sanctionnée.</p> <p>Supposons que l'heure prescrite du type d'action A_1 est 14h15mn.</p> <p>Prémisse 2 $a \varepsilon A_1 @ 14h15mn / [h_1]$</p> <p>La performance « a » de A_1 est actualisée à 14H15mn dans l'histoire₁.</p> <p>Prémisse 3 gauche^v(a)= a Le choix correspond à l'affirmation de la gauche de la disjonction.</p>	<p>d'action A_1 est récompensée.</p>	
1	? $R_1(a) @ ti > 14h15mn / [h_1]$ 0	Vous ₁₁ ε $R_1(a) @ ti > 14h15mn / [h_1]$	12
5	$[(\forall y \varepsilon A_1) \text{gauche}^v(y) = a \supset R_1(a) @ ti > 14h15mn / [h_1]] \wedge [(\forall z \varepsilon \neg A_1) \text{droite}^v(z) = a \supset S_1(z) @ ti^* > 14h15mn / [h_2]]$	Prémisse 1 $a \varepsilon A_1$ $V \neg A_1 @ 14h15mn$	2
3	? \vee 2	$a \varepsilon A_1 @ 14h15mn / [h_1]$	4
7	$(\forall y \varepsilon A_1) \text{gauche}^v(y) = a \supset R_1(a) @ ti > 14h15mn / [h_1]$	5 ? $\wedge 1$	6
9	gauche ^v (a)= $a \supset R_1(a) @ ti > 14h15mn / [h_1]$	$a \varepsilon A_1 @ 14H15mn / [h_1]$	8
11	$R_1(a) @ ti > 14h15mn / [h_1]$	9 gauche ^v (a)= a	10
		P gagne	

MMswA5
Dialogue 2

Coups	Opposante	Proposant	Coups
	<p>Prémisse 1</p> <p>X! $(\forall w \varepsilon A_5 \vee \neg A_5)[(\forall y \varepsilon A_5) \text{gauche}^v(y) = w \supset (\neg R_5(y) \wedge \neg S_5(y))] \wedge [(\forall z \varepsilon \neg A_5) \text{droite}^v(z) = w \supset (\neg R_5(z) \wedge \neg S_5(z))]$</p> <p>Supposons l'heure prescrite du type d'action A_5 est 20h.</p> <p>Prémisse 2 $a \varepsilon A_5 @ 20h / [h_1]$ Le fait d'exécuter A_5 dans l'histoire₁ à l'heure indiquée actualise « a ».</p> <p>Prémisse 3 gauche^v(a)= a Ce choix correspond à l'affirmation</p>	<p>$\neg R_5(a) \wedge \neg S_5(a) @ t_j > 20h / [h_1]$</p> <p>La performance ou l'exécution « a » du type d'action A_5 dans l'histoire₁ à l'heure indiquée ne sera ni récompensée et ni sanctionnée.</p>	0

	de la gauche de la disjonction		
1	? $\neg R_5(a) \wedge \neg S_5(a) @ t_j > 20h/h_1$	Vous ₁₁ $\varepsilon \neg R_5(a) \wedge \neg S_5(a) @ t_j > 20h/h_1$ ²⁰⁹	12
5	$[(\forall y \varepsilon A_5) \text{gauche}^v(y)=a \supset \neg R_5(y) \wedge \neg S_5(y)) @ t_j > t_i/h_1] \wedge [(\forall z \varepsilon \neg A_5) \text{droite}^v(z)=a \supset \neg R_5(z) \wedge \neg S_5(z) @ t_j^* > t/h_2]$	Prémisse 1 $a \varepsilon A_5 \vee \neg A_5 @ 20h$	2
3	? \vee 2	$a \varepsilon A_5 @ 20h/h_1$	4
7	$(\forall y \varepsilon A_5) \text{gauche}^v(y)=a \supset \neg R_5(y) \wedge \neg S_5(y) @ t_j > t_i/h_1$	5? $\wedge 1$	6
9	$\text{gauche}^v(a)=a \supset \neg R_5(a) \wedge \neg S_5(a) @ t_j > 20h/h_1$	7 $a \varepsilon A_5 @ 20h/h_1$	8
11	$\neg R_5(a) \wedge \neg S_5(a) @ t_j > 20h/h_1$	9 $\text{gauche}^v(a)=a$	10
		P gagne	

MMswA5
Dialogue 3

Dans le cas de la modalité neutre (Mubâh mustawin), **P** peut perdre dans les cas suivants. En supposant que l'action A_5 a été effectuée alors **P** affirme l'une des thèses :

1. $R(a) \wedge \neg S(a)$
2. $\neg R(a) \wedge S(a)$
3. $R(a) \wedge S(a)$
4. $R(a)$
5. $S(a)$
6. $\neg R(a)$
7. $\neg S(a)$.

Les mêmes dialogues peuvent se produire si A_5 n'a pas été actualisé, c.a.d. si « a » actualise $\neg A_5$. Développons seulement une de ces possibilités.

Coups	Opposante	Proposant	Coups
	Prémisse 1 X! $(\forall w \varepsilon A_5 \vee \neg A_5)[(\forall y \varepsilon A_5) \text{gauche}^v(y)=w \supset (\neg R_5 \wedge \neg S_5)(y)] \wedge [(\forall z \varepsilon \neg A_5) \text{droite}^v(z)=w \supset (\neg R_5 \wedge \neg S_5)(z)]$ Prémisse 2 $A \varepsilon \neg A_5 @ 20h/h_2$ Prémisse 3 $\text{droite}^v(a)=a$	$R_5 @ t_j > 20h/h_2$ La performance ou l'exécution « a » du type d'action A_5 dans l'histoire ₂ à l'heure indiquée sera récompensée.	0
1	? $R_5(a) @ t_j > 20h/h_2$???	

5	$[(\forall y \varepsilon A_5) \text{gauche}^v(y)=a \supset \neg R_5(y) \wedge \neg S_5(y) @t_j > t_i / h_1] \wedge [(\forall z \varepsilon \neg A_5) \text{droite}^v(z)=a \supset \neg R_5(z) \wedge \neg S_5(z) @t_j^* > t / h_2]$	Prémisse 1 $a \varepsilon A_5 \vee \neg A_5 @20h$	2
3	? \vee	2	$a \varepsilon \neg A_5 @20h / h_2$
7	$[(\forall z \varepsilon \neg A_5) \text{droite}^v(z)=a \supset \neg R_5(z) \wedge \neg S_5(z) @t_j^* > t / h_2]$	5? $\wedge 2$	6
9	$\text{droite}^v(a)=a \supset \neg R_5 \wedge \neg S_5(a) @t_j > 20h / h_2$	7 $a \varepsilon \neg A_5 @20h / h_2$	8
11	$\neg R_5(a) \wedge \neg S_5(a) @t_j > 20h / h_2$	9 $\text{droite}^v(a)=a$	10
	O gagne.		

5 Remarques sur la plénitude et le libre-arbitre

Dans la partie précédente on a souligné que la *logique de la présence* de Suhrawardī est le résultat de l'inclusion de l'expérience des présences comme constitutive de la notion de modalités, façonnée par une théorie dialogique de la signification.

Dans la partie B de notre thèse on a souligné que la logique de présence de Suhrawardī est le résultat de l'inclusion de l'expérience des présences comme constitutive de la notion de modalités, façonnée par une théorie dialogique de la signification.

D'une part, Rahman&Boussad (2024, à paraître) ont développé l'idée que le cadre de Suhrawardī propose également une nouvelle perspective épistémologique, qui a d'importantes conséquences philosophiques. En bref, les propositions définitivement nécessaires [al-darūriyya al-batāta], ces propositions que Suhrawardī considère comme les seules conduisant à la certitude, sont régies par la force causale de l'essence du sujet pour la formation des attributions de contingence nécessaire : il est nécessaire que les humains aient la capacité de lire, mais cette capacité (bien qu'elle ne soit pas toujours actualisée), peut être attribuée à chaque humain, uniquement parce qu'il est un être rationnel.

Notez que cette force causale contraste avec la description temporellement conditionnée du sujet qui constitue les waṣfī-propositions d'Ibn Sīnā. Rappelons le célèbre exemple d'Ibn Sīnā : "Celui qui écrit bouge ses doigts tout le temps qu'il écrit. Il est clair que le prédicat "bouge ses doigts" est limité au fait contingent que les individus visés par le quantificateur écrivent : il ne s'applique pas en raison de l'essence de ces individus, à savoir le fait qu'ils sont rationnels - si l'attribution est causée par la propriété essentielle de l'individu, l'attribution, bien sûr, est réputée fautive.

D'autre part, d'un point de vue métaphysique, cette démarche de Suhrawardī revient à faire dépendre l'attribution de la contingence nécessaire *du principe de raison suffisante* (le principe qui établit que chaque événement a une cause pour son existence et son non-existence) et de la plénitude (le principe qui établit que chaque possibilité s'actualise au moins une fois). Le principe de suffisance et la version temporelle du principe de plénitude articulent le lien entre l'épistémologie et la logique de la présence avec les objectifs généraux de la philosophie illuministe de Suhrawardī.

En fait la notion de plénitude générique de Suhrawardī fournit une analyse nuancée de la question du déterminisme : puisque la force causale à l'œuvre dans la plénitude générique

s'applique à une espèce dans son ensemble, elle ne se distribue pas par rapport aux présences individuelles de cette espèce ; alors, la notion de plénitude générique impliquée par le principe de la raison suffisante ne se distribue pas non plus individuellement. Il semble donc que la Plénitude générique ne suppose pas une forme forte de déterminisme.

Ce que la plénitude générique exige est :

(1) ce que Perloff et Belnap (2011) appellent une forme locale d'(in)déterminisme limitée à un moment donné où l'actualisation ou la non-actualisation a été vérifiée - et qui ne s'engage pas à des affirmations impliquant tous les cours possibles d'événements traversant ce moment ;

(2) une position générique en ce qui concerne l'individu qui actualisera ou non la capacité attribuée.

Ainsi, alors que dans le contexte des attributions de capacités, la raison suffisante garantit que toute attribution de contingence nécessaire - c'est-à-dire l'existence d'une relation nécessaire mais contingente entre le sujet et le prédicat - doit être attribuée potentiellement à chaque instance du sujet, dans le contexte de l'attribution de capacités acquises, la plénitude générique garantit qu'elle doit s'actualiser au moins une fois, mais pas nécessairement par chaque individu.

Pour en revenir au cadre des modalités déontiques, l'intérêt de la plénitude est que chaque modalité déontique exige que l'un des deux choix impliqués, à savoir accomplir ou ne pas accomplir le type d'action en jeu, soit un choix concevable. C'est pourquoi une forme faible de plénitude est nécessaire dans contextes étiques et légaux.

En outre, puisque cela implique que chaque être humain a le choix entre effectuer ou non l'action récompensable/recommandée, il semble que nous devions considérer la possibilité de choisir le mal ou de mal agir comme un attribut nécessaire mais contingent des êtres humains.

À ce stade de la discussion, il semble que le fait de faire le mal ou, plus généralement, de mal agir, sous-tende le concept de libre arbitre supposé par les catégories déontiques de la punition et de la récompense et par les attributions de responsabilité.

Nous ne pourrions pas être sanctionnés ou récompensés si les modalités déontiques ne permettaient pas de choisir l'acte répréhensible. C'est en cela que consiste notre contingence en tant qu'êtres moraux.

Conclusion

Les modalités déontiques au-delà de la jurisprudence islamique

L'un des principaux objectifs de cette étude est de proposer une nouvelle approche de la logique déontique qui

1. ne fait pas appel à la sémantique des mondes possibles, comme le fait la logique déontique standard (SDL),
2. est en même temps compatible avec l'idée que les catégories déontiques dans les contextes étiques et juridiques assument la responsabilité des choix que nous faisons,

3. les qualifications déontiques telles que *vertueux* (mérite d'être *récompensé*) ou *blâmable* (mérite d'être *sanctionné*) ne qualifient pas un type d'action, mais l'accomplissement effectif d'un type d'action,
4. la nouvelle approche devrait être appliquée aux contextes éthiques et juridiques en général, c'est-à-dire qu'elle ne devrait pas être limitée à l'éthique ou à la jurisprudence islamique.

Les deux premiers points ont été mis en œuvre par une analyse logique selon laquelle les normes éthiques et juridiques ont la forme d'une hypothétique dont l'antécédent est constitué par le choix d'accomplir ou non l'action prescrite par la norme. En outre, ces choix sont compris comme déterminant un cours d'action précis (ou histoire) dans une structure temporelle ramifiée (préfigurée par les options disponibles). Cette stratégie met l'accent sur la distinction entre la nécessité causale, à l'œuvre dans les approches déterministes de la nature, et la nécessité déontique, qui présuppose le non-déterminisme afin d'attribuer une responsabilité éthique et juridique. Cela permet d'éviter les paradoxes déontiques habituels dans la logique déontique standard qui résultent de l'utilisation de la sémantique des mondes possibles de type Kripke pour la notion de nécessité ontologique ou causale.

Le troisième point applique et généralise la logique de présence de Suhrawardī aux actions. En effet, si dans le contexte de l'épistémologie les présences constituent les vérificateurs effectifs des états de choses, dans le domaine déontique, les présences constituent l'actualisation des types d'actions. Ces actualisations, en fait des performances, accomplissent (ou non) les prescriptions de la norme.

En ce qui concerne le quatrième point, soulignons que si cette nouvelle approche doit être appliquée aux contextes éthiques et juridiques en général, c'est-à-dire sans se limiter à l'éthique ou à la jurisprudence islamique, la généralisation suivante peut être introduite en remplaçant les qualifications déontiques de Récompense ($R(a)$) et de Sanction ($S(a)$) comme suit :

- **dans les contextes éthiques** : $Vir(a)$ - c'est-à-dire "l'exécution a du type d'action A_i est qualifiée de *vertueuse*" et $Blm(a)$ - c'est-à-dire "l'exécution a du type d'action A_i est qualifiée de *blâmable*".

- **dans les contextes juridiques** : $Rsp(a)$ - c'est-à-dire "la performance a du type d'action A_i est qualifiée de *respectueuse de la loi*" et $Vl(a)$ - c'est-à-dire "la performance a du type d'action A_i est qualifié de contraire à la loi ou sa violation".

- Si nous considérons que « Loi » concerne à la fois les lois éthiques et juridiques, la dernière formulation peut être considérée comme exprimant les formes les plus générales de qualifications déontiques pour l'exécution d'actions.

Une caractéristique importante de l'utilisation du cadre dialectique par Suhrawardī est qu'il le déploie pour rendre la signification des connecteurs modaux et logiques. Cela a pour conséquence que le cadre dialectique est celui de dialogues purement antagonistes. Cependant, il est important d'avoir des dialogues coopératifs, en particulier dans le cas de l'argumentation juridique islamique. En effet, l'utilisation des dialogues coopératifs semble être l'un des aspects saillants de l'argumentation juridique islamique :

Ultimately, and most importantly, a truly dialectical exchange – though drawing energy from a sober spirit of competition – must nevertheless be guided by a cooperative ethic wherein truth is paramount and forever trumps the emotional motivations of disputants to “win” the debate. This truth-seeking code demands sincere avoidance of fallacies; it views with abhorrence contrariness and self-

contradiction. This alone distinguishes dialectic from sophistical or eristic argument, and, in conjunction with its dialogical format, from persuasive argument and rhetoric. And to repeat: dialectic is formal – it is an ordered enterprise, with norms and rules, and with a mutually-committed aim of advancing knowledge. Young (2017, p.1)

Les coups coopératifs peuvent être considérés comme des suggestions de l'enseignant pour corriger la thèse ou certaines faiblesses de l'étudiant. Rahman & Iqbal (2019, chapitre 2) ont étudié le dialogue coopératif dans le contexte des qiyās.

Dans un travail en préparation nous étendons les dialogues pour les modalités déontiques islamiques en ajoutant des coups où l'opposant peut suggérer une nouvelle thèse - par exemple, si le proposant affirme à tort que l'actualisation d'une action recommandée donnée ne doit être ni récompensée ni sanctionnée, l'opposant peut suggérer de changer la thèse, en produisant des arguments en faveur d'une révision de la thèse. Le dialogue reprend alors avec une nouvelle thèse. La même chose peut se produire si la thèse est correcte mais que le proposant ne fournit pas les meilleurs coups dialectiques disponibles pour la justifier.

Une telle approche permet de conférer aux dialogues leur rôle fondamental, à savoir celui d'une entreprise collective visant à atteindre signification, savoir et vérité.

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Outside the Logic of Necessity: Deontic Puzzles and “Breaking” Compound Causal Properties in Islamic Legal Theory and Dialectic.

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Abstract:

This paper examines an Islamic dialectical objection called *kasr*, or “breaking,” as treated by the dialecticians and legal theorists Abū Ishāq al-Shīrāzī (d. 1083) and Abū al-Walīd al-Bājī (d. 1081). The fallacious modes of *kasr* which our theorists denounce are confronted with deontic paradoxes and puzzles which fall under *logical extrapolation fallacy*. Our primary claim is that whereas logical extrapolation produces fallacies or paradoxes by unsafely applying inference rules of standard alethic and/or logical necessity to the deontic realm, the fallacies generated by invalid modes of *kasr* in Islamic legal theory (wherein logical rules are expressed dialectically) constitute a genuine source for reflecting on what patterns of reasoning should be endorsed for determining causality in legal and, perhaps, natural epistemological contexts. Ultimately, this is the first step towards a larger study of *kasr*, which will compare it with other dialectical objections and treat relevant critical discourses from later Muslim legal theorists and dialecticians.

Keywords:

Islamic juristic dialectic (*jadal*); correlational inference (*qiyās*); breaking (*kasr*); logical extrapolation fallacy; compound causes; dialogical logic.

Outside the Logic of Necessity
Deontic Puzzles and “Breaking” Compound Causal Properties in Islamic Legal
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1 Introduction

This paper will examine an important group of illegitimate moves involving causal properties as identified by Medieval Muslim jurists in the intertwined domains of legal theory (*uṣūl al-fiqh*) and dialectic (*jadāl*). More precisely, we will focus on the discourse surrounding the dialectical objection called *kasr*, or “breaking,” which deliberates the proper and improper paths to challenging and defending the causal components of a correlational argument (*qiyās*) in which the *ratio legis* (*‘illa*) of the source-case’s ruling (*ḥukm*) is a compound of two or more properties.

The developmental history of this dialectical objection is complicated; long and heated controversies centred on which modes of *kasr* (and responses to *kasr*) were fallacious and which were not. There were even those who rejected *kasr* at the outset, some with arguments paralleling classical and medieval Latinate claims that one cannot refute an argument whose premises have a meaning *in sensu composito* by the blunt separation of its parts.

In the present study, we will therefore restrict our analyses to the relevant discussions of two 11th century CE theorists: the renowned Shāfi‘ī jurist Abū Ishāq al-Shīrāzī (d. 1083) and his equally prominent one-time pupil, the Mālikī jurist Abū al-Walīd al-Bājī (d. 1081), who each elaborated two main pathways to “breaking” an opponent’s compound *‘illa*; namely, (1) by replacing one of its properties, and (2) by removing one of its properties. The rules and responses prescribed in the sources we examined for each type make it clear that they correspond to two primary conjunctive modes for an *‘illa*’s valid composition: either (type 1) the *meaning dependence of one property (or more) upon the other* (with all of them together being claimed to occasion the ruling); or (type 2) *independent conjunctive composition* (from which it follows that if *A, B, ...*, occasion the ruling, then either *A* occasions the ruling or *B* occasions it, or ...) – one or more might be efficient but not the remainders.

Moreover, we will briefly confront the fallacious modes denounced by al-Shāfi‘ī and al-Bājī with forms of certain deontic paradoxes and puzzles we group under the name *logical extrapolation fallacy*. These play a dominant role in contemporary deontic logic, although they have roots in the medieval Latin tradition (or, arguably, are already present in Megarian and Stoic logic). More generally, *logical extrapolation* should in fact include fallacious (or sophistic) reduction of some pattern of argumentation to a pre-existing logical system (be it modal or otherwise).

Our primary claim is that whereas logical extrapolation produces fallacies or paradoxes by unsafely applying inference rules of standard alethic and/or logical necessity to the deontic realm, the fallacies generated by invalid modes of *kasr* in Islamic legal theory (wherein both logical rules and semantic rules for reasoning with deontic modalities are expressed dialectically) constitute a genuine source for reflecting on what patterns of reasoning should be endorsed for determining causality in Law—and, perhaps,

more generally, also for establishing causality in certain natural (as opposed to normative) epistemological contexts.

The current paper is the first step towards a larger study of *kasr*, which will encompass both a comparison between *kasr* and other dialectical objections and analyses of critical discourses from later Muslim legal theorists and dialecticians.

2 The Logical Extrapolation Fallacy

Contemporary approaches to Standard Deontic Logic (SDL) have followed G. H. von Wright's (1951, 1963) idea of applying the developed framework of modal logic to the deontic realm. According to this approach—further extended with the model theoretical semantics of Jaakko Hintikka (1957, 1968) and Saul Kripke (1963)—obligation was conceived as a special reading of the necessity operator of basic modal propositional logic. According to this view deontic reasoning can be reduced to a special kind of modal logic.

Possible-world interpretations of deontic necessity struggled, however, with a wealth of philosophical and logical puzzles that threatened the framework right from the very start, already by the sixties, and at the propositional level.²¹⁰ Of course, deontic necessity is still a kind of necessity. But standard possible-world semantics—without deep modifications—is not the instrument required, *vis-à-vis* the prescriptive dimension on performances of actions, to grasp the meaning of those norms governed by the assumption of liberty—or, more precisely: of legal or ethical *liability*.

P. McNamara and R. Hilpinen (2013) provide, in their contribution to the *Handbook on Deontic logic and Normative Systems*, a systematic, historical overview of such problematic reductionist projects. One of the puzzles they discuss (p. 8) is the following reconstruction of Stephen Langton's (1150-1228) paradox. It results from reducing (Aristotelian) modal necessity to deontic necessity, and constitutes an excellent example for our reflection on the genealogy of fallacious arguments within the deontic realm.²¹¹

Necessarily, if this man visits his sick father, then the father is sick.

But it does not follow that

If this man ought to visit his sick father, then his father ought to be sick.

These and similar counterexamples are rooted in the following form of reductionism for which Leibniz has been praised as much as blamed:

$\mathbf{N}(A \supset B)$ (whereby “N” stands for necessity)

$(\mathbf{O}A \supset \mathbf{O}B)$ (inferred via modus ponens from $\mathbf{N}(A \supset B)$ followed by introduction of implication and substituting \mathbf{N} with “O”, obligatory)

Notice that this example of the sick father is not specific to the modal necessity

²¹⁰ For recent overviews of these challenges see R. Hilpinen and P. McNamara, ‘Deontic Logic: A Historical Survey and Introduction,’ in *Handbook of Deontic Logic and Normative Systems*, eds. D. Gabbay, J. Horty, X. Parent, R. van der Meyden, L. van der Torre (London, 2013), 3-136; Pablo E. Navarro and Jorge L. Rodriguez, *Deontic Logic and Legal Systems* (New York, 2014).

²¹¹ Our rendering of the example is based on S. Knuuttila and O. Hallamaa, “Roger Roseth and Medieval Deontic Logic,” *Logique et Analyse* 38 (1995), 77.

operator but extends to inferences governed by logical necessity as well. This also holds for many other well-known deontic puzzles, including: A. Ross's (1941) *Burning-Letter* paradox; R. M. Chisholm's (1963) puzzle on *Conditional Obligations*; and A. N. Prior's (1958) *The Good Samaritan* paradox, in which latter from *It is obligatory that Jones help Smith who is being mugged*, there follows *It is obligatory that Smith is being mugged* (assuming the premise is rendered as a conjunction).

More generally the problem may be seen as an illicit extrapolation from the modal logic of necessity to the set of rules governing deontic reasoning in the moral and/or legal realms. This signifies a special kind of fallacious argumentation identifying illegitimate moves which consist in extrapolating a set of rules for logical reasoning to a pattern of reasoning within some specific field of knowledge, and drawing a paradoxical or puzzling consequence that patently contravenes the conceptual architecture of that field. As mentioned, we will call this the *fallacy of logical extrapolation*, though this is not new. Ample Megarian and Stoic debates on the fruitfulness of material implication for temporal and modal contexts long ago involved discussions of such puzzles. The garden of logical extrapolation fallacies is quite rich.

This is particularly important in the context of Islamic Dialectical theory, wherein rules for legal reasoning were not constituted by importing logical rules from somewhere else and subjoining them to rules of dialectic, but the dialectical framework itself originated dialectical rules for legal-logical reasoning. In the 14th century general dialectical theory of Shams al-Dīn al-Samarqandī, the *ādāb al-baḥṭh wa-l-munāzara*, which streamlined and universalized a predecessor juristic dialectic, logical rules were (as in the predecessor dialectic) consciously and rigorously formulated as dialogical rules.²¹² This explains why we will not, in principle, find fallacies of extrapolation therein. However, there is no dearth of cases for the identification and study of illegitimate moves in legal and deontic reasoning more broadly.

3 Islamic Deontic Imperatives: Outside Logical Extrapolation

3.1 Ibn Hazm's Deontic Imperatives

In previous joint papers by S. Rahman, W. E. Young, and F. Zidani,²¹³ the authors argue that Ibn Ḥazm of Córdoba's (994-1064) *Facilitating the Understanding of the Rules of Logic and Introduction Thereto, with Common Expressions and Juristic Examples*, where, among other things, he thoroughly investigates deontic notions and their modal counterparts, assures him a place among the fathers of the logic of norms. Moreover, in these papers we show that, in the context of what we called *Islamic heteronomous imperatives*, "puzzles" of extrapolation are not puzzles at all. The point is that this approach does not require one to block the use of any logical rules as standard solutions do, but allows logical inference rules to cohabit with deontic rules—if sufficient care is given to the meaning constitution of deontic assertions generated by the logic that governs

²¹² See L. B. Miller, *Islamic Disputation Theory: The Uses & Rules of Argument in Medieval Islam* (Cham, 2020), 103-123.

²¹³ Most of the current paper's sections 3.1 and 3.2 is reproduced from S. Rahman, J. G. Granström, and A. Farjami, "Legal Reasoning and Some Logic After All. The Lessons of the Elders," in *Natural Arguments. A Tribute to John Woods*, eds. D. Gabbay, L. Magnani, W. Park and A-V. Pietarinen (London 2019), 743-780; S. Rahman, W.E. Young, and F. Zidani, "It Ought to be Forbidden! Islamic Heteronomous Imperatives and the Dialogical Forge," in *Lógica Conocimiento y Abducción. Homenaje a Angel Nepomuceno*, eds. C. Barés-Gómez, F. J. Salguero and F. Soler (London, 2021), 97-114; S. Rahman, F. Zidani, W.E. Young, "Ibn Ḥazm on Heteronomous Imperatives. A Landmark in the History of the Logical Analysis of Legal Norms," in *Agency, Normative Systems, Artifacts, and Beliefs: Essays in Honour of Risto Hilpinen*, eds. P. McNamara, A. Jones, M. Brown (Dordrecht, Forthcoming).

the normative realm. New insights from the Islamic tradition that facilitate our reconstruction are the following:

- Prescriptions are understood as prescriptions to **do** rather than prescriptions that take us from one state of affairs to another: *Tun Sollen* rather than *Sein Sollen*.
- Actualizations (performances) of prescriptions are subjects of predication. In other words, performances of actions are bearers of qualifications such as *law-abiding* (rewarded) or *law-breaking* (penalized). The distribution of reward and penalization yields the classification of deontic modalities into *obligatory*, *forbidden*, *recommended permissible*, *reprehended permissible*, and *evenly permissible*.
- Norms presuppose freedom of choice or moral and legal liability. In other words, each type of prescription (obligatory, forbidden, etc.) assumes as hypothesis that the corresponding type of action can be carried out or (deliberately) not carried out.

3.2 Ibn Hazm's Deontic Imperatives

Muslim jurists identified five deontic qualifications for an action. Ibn Ḥazm defines them as follows:²¹⁴

- 11 **wājib, farḍ, lāzim**. Obligatory action is the one which: If we do it we are rewarded. If we do not do it we are sanctioned.²¹⁵
- 12 **ḥarām, maḥzūr**. Forbidden action is the one which: If we do it we are sanctioned. If we do not do it we are rewarded.
- 13 **mubāḥ mustaḥabb**. Recommended permissible action is the one which: If we do it we are rewarded. If we do not do it we are neither sanctioned nor rewarded.
- 14 **mubāḥ makrūh**. Reprehended permissible action is the one which: If we do not do it we are rewarded. If we do it we are neither sanctioned nor rewarded.
- 15 **mubāḥ mustawin**. Evenly permissible action is the one which: If we do it we are neither sanctioned nor rewarded. If we do not do it we are neither sanctioned nor rewarded.

In the aforementioned papers we proposed a logical analysis based on A. Ranta's (1994) *Type Theoretical Grammar*, based on Per Martin-Löf's (1984) Constructive Type Theory (CTT). This fully interpreted framework allows a logical analysis of imperatives that makes explicit the conceptual links at work here.

More precisely it allows one to express for each deontic qualification both (1) the dependence of rewarding or sanctioning on the presupposition that there was a real choice, and (2) the fact that these performances actualize the type of action specific to the imperative at stake (obligatory, forbidden, etc.) Thus, if we are describing an *obligatory action*, we need to express the following:

Obligatory action:

(i) *If the individual g made the choice to perform an action of type A (i.e., if there is a performance by g that actualizes the **left side** of the disjunction) then some*

²¹⁴ Ibn Ḥazm, *Al-Iḥkām fī Uṣūl al-Aḥkām*, ed. Aḥmad Muḥammad Shākir (Cairo, 1926-1930), 3.77; Ibn Ḥazm, *Kitāb al-Taqrīb li-Hadd al-Manṭiq wa-l-Mudkhal ilayhi bi-l-alfāz al-ʿAmmiyya wa-l-Amthila al-Fiqhiyya*, ed. Iḥsān ʿAbbās (Beirut, 1959), 86; idem, ed. Aḥmad b. Farīd b. Aḥmad al-Mazīdī (Beirut, 2003), 83-4.

²¹⁵ By way of clarification, "sanction" in this study is meant only in its sense of "penalty," and "sanctioned" as "penalized."

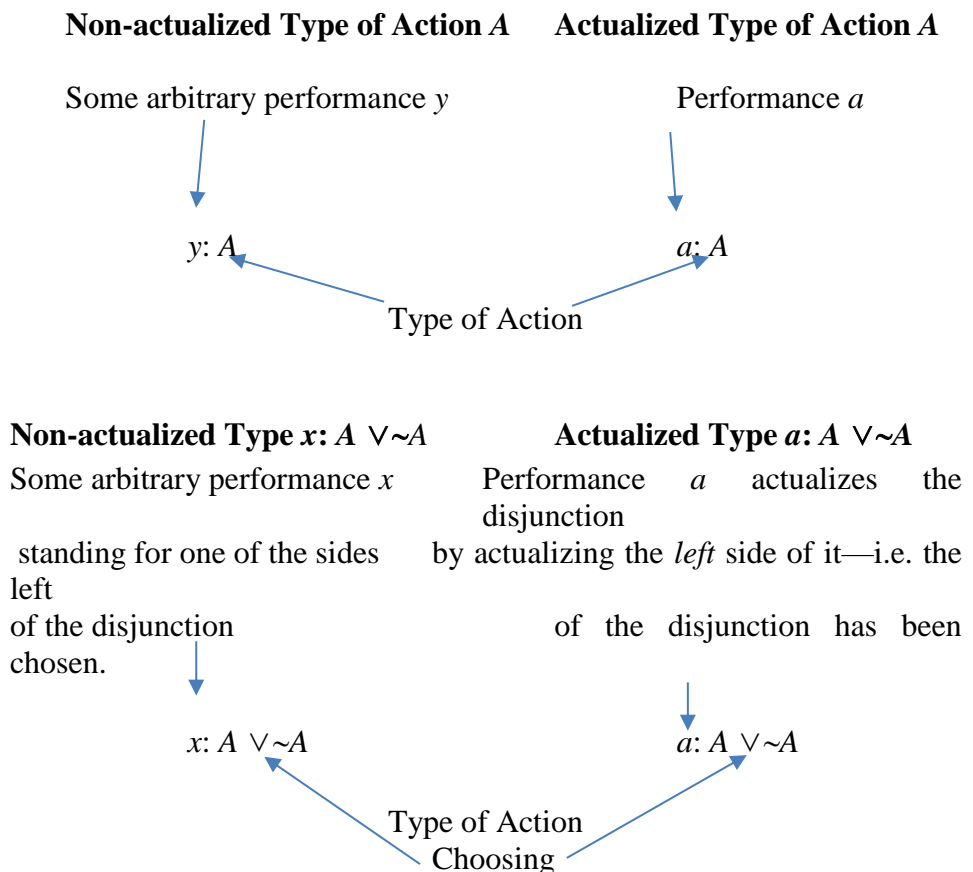
(legal) procedure \mathbf{b} , determined by the Law, attributes a form of **reward** specific to this type of performance.

(ii) If the individual \mathbf{g} made the choice to omit performing an action of type A (i.e., if there is a performance by \mathbf{g} that actualizes the **right side** of the disjunction) then some (legal) procedure \mathbf{b} , determined by the Law, attributes a form of **sanction** specific to this type of omission.

3.3 Rendering the Conceptual Analysis Explicit

Before presenting the notation that makes our analysis explicit, let us provide first the main keys for reading the CTT formal terminology employed here and further on in the paper:

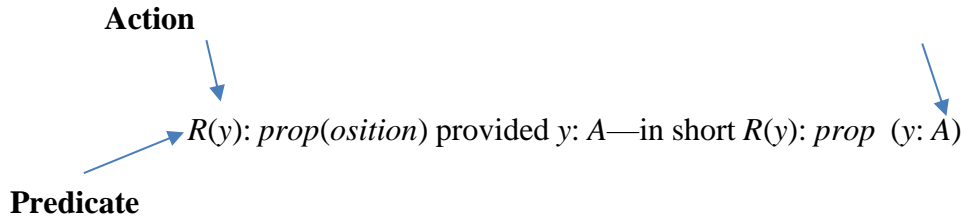
- If “ A ” stands for a type of action and “ y ”, for some not yet actualized performance of an action of type A , we write $y: A$. So, “ x ” in “ $x: A \vee \sim A$ ”, stands for the performance of the type of action of either A or $\sim A$. Thus, more loosely, we can say that “ x ” in “ $x: A \vee \sim A$ ” stands for the choice of carrying out A or (actively) omitting to carry out A :



- If “ y ” stands for some not yet actualized performance of an action of type A , then “ $R(y)$ ” stands for attributing the predicate *Reward* to the performance y of the type of action A . Thus, “ $R(y)$ ” is an expression of the type *prop* (i.e., of the type proposition), whereby y is an element of the set A of performances of action. To put it in a more standard terminology “ $R(y)$ ” stands for a propositional function over the set A of performances. Something similar holds for the propositional function “ $S(z)$ defined over the set of omissions $\sim A$ ”

Performance

Type of



- **Hypothetical judgment or normative prescription**

$R(y) \text{ true } (y: A)$

Any arbitrary performance y of A makes it *true that this performance is rewarded*

$d(y): R(y) (y: A)$

(legal) procedure $d(y)$ attributes *Reward* to any arbitrary performance y of A

- **Categorical judgment or result of carrying out a prescription**

$R(a) \text{ true}$

It is *true* that the performance a is *rewarded*

$d(a): R(a)$

(legal) procedure $d(a)$ attributes *Reward* to performance a of A

- If x indicates the choice between carrying out A or (actively) omitting to carry it out, “ $\mathbf{left}^{\vee}(y) =_{\{H\}} x$ ” indicates that the result of this choice is to perform A (i.e., it is identical to a performance y of A). Something similar holds for “ $\mathbf{right}^{\vee}(z)$ ”.

Accordingly, the first component of the conjunction:

$$(\forall y : A) \mathbf{left}^{\vee}(y) =_{\{H\}} x \supset R(y) \quad (x : A \vee \sim A)$$

reads

Assuming that, given the choice x of performing or not performing an action of type A , performing it has been chosen (i.e., if the left side of the disjunction has been chosen to be performed), then, for any performance y of the type of action A that is identical to the choice x , reward (for performing this action) follows.

A similar reading applies to the second component:

$$(\forall z : \sim A) \mathbf{right}^{\vee}(z) =_{\{H\}} x \supset S(z) \quad (x : A \vee \sim A).^{216}$$

If we pull all this together and employ the abbreviation $\{H\}$ for the hypothesis $x : A \vee \sim A$, expressing the fact that the agent can choose to either actualize (perform) A or actualize an omission of performing A , we obtain:

$$b(x) : [(\forall y : A) \mathbf{left}^{\vee}(y) =_{\{H\}} x \supset R(y)] \wedge [(\forall z : \sim A) \mathbf{right}^{\vee}(z) =_{\{H\}} x \supset S(z)] (x : A \vee \sim A).$$

²¹⁶ The notation for propositional identity, namely “ $x =_{\{D\}} y$ ”, standing for “ x is identical to y within the set D ,” is closer to what is employed in first-order logic.

- Given $x: A \vee \sim A$, “ $b(x)$ ” stands for a procedure (expressed by a function) that distributes **reward** and **sanction** depending upon the actual choice made (by the agent g) between performing A or $\sim A$ and specific to the type of action chosen to be performed. In short, the function relates performances of actions to the kind of sanction or reward specified by the Law. In our context, the procedure is a legal one carried out by the competent legal authority.

The hypothetical can be glossed as:

Obligatory action:

Given the choice x between performing A or (actively) omitting to do so:

*If performing an action of type A has been chosen (i.e., if there is a performance y that actualizes the **left side** of the disjunction)—viz. **left**(y)= x —then (legal) procedure b attributes a form of **reward** specific to this type of performance.*

*If omitting to perform an action of type $\sim A$ has been chosen (i.e., if there is a performance z that actualizes the **right side** of the disjunction)—viz. **right**(z)= x —then (legal) procedure b attributes a form of **sanction** specific to this type of act of omission.*

The above yields:

- **wājib, fard, lāzim:** Doing A_1 is rewarded. Omitting to do A_1 is sanctioned.

$$b_1(x) : [(\forall y : A_1) \mathbf{left}^{\vee}(y)=_{\{HI\}}x \supset R_1(y)] \wedge [(\forall z : \sim A_1) \mathbf{right}^{\vee}(z)=_{\{HI\}}x \supset S_1(z)] (x : A_1 \vee \sim A_1).$$

As mentioned, this approach provides a general framework that prevents the kind of puzzles discussed above. Nonetheless, it works only if the framework is embedded in a general theory on the meaning of legal norms that also accounts for reasoning involving deontic modalities.

It is important to observe that in Islamic jurisprudence there is no theory of deontic syllogisms as such. Rather, within the dialectical theory and practice of *uṣūl al-fiqh*, the distribution and transmission of the deontic modalities described above is governed by argumentation rules on how to contest a claim on the legal validity of a norm, and how to respond to this claim.

In what follows, we will focus on arguments concerning compound premises expressing the causal factors for legal rulings, since enquiries into a ruling’s causal factors were regarded as a critical epistemic endeavour, and rules for challenging arguments with compound causal factors occasioning a legal norm were considered among the more controversial and subtle within juristic dialectic.

4 Beyond Fallacies of Logical Extrapolation

It is interesting to conjecture as to why extrapolation fallacies involving modal necessity drew the attention of premodern scholars in the Latin tradition but not (at last not explicitly) those in the Arabic tradition. From a historical point of view, one possible answer—though perhaps problematic in over-generalizing—might be that in the Latin tradition Aristotle’s modal logic, or at least Aristotle’s modal notions as discussed in the *Perihermeneias* and commented upon by Ammonius, took conceptual priority over

deontic notions, whereas a dominant practice of dialectical legal reasoning in Islam inverted this order of priorities for the Arabic tradition. A possible sign of this might be, in the maturation of dialectical theory following the success of al-Samarqandī's *ādāb al-baḥth*, that discourse on valid and invalid reasoning with regard to both legal and natural causality appears to have blurred older distinctions and embraced a general causality pattern.²¹⁷

Whatever the reason, one might be justified in asking whether fallacies bounded by deontic or legal reasoning can be identified within Islamic disputation theory. The answer is an overwhelming “Yes,” and particularly so in relation to deontic and natural causal necessity, as per Young (2019, Forthcoming A), in the context of that pattern for parallel reasoning called *qiyās*, the basics of which we will now quickly review.

4.1 *Qiyās* Basics

The many debates and elaborations on *qiyās*, which might be translated “correlational inference” (more often, if less accurately, “analogy”),²¹⁸ together constitute one of the finest outcomes of the argumentative approach to legal reasoning within Islamic Law. A particularly lucid example is the systematization of the respected Shāfi‘ī theoretician Abū Ishāq al-Shīrāzī (1003-1083 CE), upon which the following is based.²¹⁹

The aim of *qiyās* is to provide a rational ground for the application of a juridical ruling (*ḥukm*) to a given case, called the branch-case (*far‘*), which has not been directly and unequivocally pronounced upon in the primary juridical sources (i.e., the Qur’ān, Sunna [Prophet’s example], and *Ijmā‘* [consensus]). The method starts by attempting to determine the property (*wasf*) or set of properties in the root-case which constitutes the causal or occasioning factor or factors, or *ratio legis* (*‘illa*) giving rise to its ruling. If it is “probable” (*ẓannī*) that this property occasions the ruling, and it is shared by the branch-case, we may infer that it is equally productive of that ruling in the branch-case.²²⁰

A cardinal feature of al-Shīrāzī’s take on *qiyās al-‘illa* is his particular notion of efficiency (*ta’thīr*), which tests whether the property \mathcal{P} purported to be efficient in occasioning the ruling at stake is indeed so. For al-Shīrāzī, *ta’thīr* consists of two complementary procedures:

co-presence (*tard*): whenever the property is present, the ruling is also present
and

co-absence (*‘aks*): whenever the property is absent, the ruling is also absent.

²¹⁷ See W. E. Young, “Concomitance to Causation: Arguing *Dawarān* in the Proto-*Ādāb al-Baḥth*,” in *Philosophy and Jurisprudence in the Islamic World*, ed. Peter Adamson, (Berlin, 2019), 205-281; W. E. Young, “Islamic Legal Theoretical and Dialectical Approaches to Fallacies of Correlation and Causation (7th-8th/13th-14th centuries),” in [Title TBA], eds. R. Gleave and M. Bedir (Forthcoming).

²¹⁸ See W. E. Young, *The Dialectical Forge: Juridical Disputation and the Evolution of Islamic Law* (Cham, 2017), 10.

²¹⁹ A landmark on the subject of *qiyās* is A. Hasan, *Analogical Reasoning in Islamic Jurisprudence: a Study of the Juridical Principle of Qiyās* (Islamabad, 1986). Young, *Dialectical Forge*, provides a summary of al-Shīrāzī’s systematization of *qiyās*; and on this basis S. Rahman, M. Iqbal, and Y. Soufi develop a logical analysis (*Inferences by Parallel Reasoning in Islamic Jurisprudence. Al-Shīrāzī’s Insights into the Dialectical Constitution of Meaning and Knowledge* [Cham, 2019]). See also M. Iqbal, *Arsyad al-Banjari’s Insights on Parallel Reasoning and Dialectic in Law: The Development of Islamic Argumentation Theory in the 18th Century in Southeast Asia* (Cham, 2022).

²²⁰ Though admitting a range of degrees, the “probability” indicated by the term *ẓannī* does not refer to the statistical view conceived by the time of Leibniz. See W. B. Hallaq, *A History of Islamic Legal Theories: An Introduction to Sunnī Uṣūl al-Fiqh* (Cambridge, 1997), 39. In fact, we might suggest *ẓann* probability is a kind of *qualitative* expression for comparing degrees of acceptability, such as: “there are more indications in favour of accepting that the property **P** occasions the ruling than not”.

While co-presence examines whether ruling \mathcal{K} follows from verifying the presence of property \mathcal{P} , co-absence examines whether exemption from ruling \mathcal{K} follows from verifying the absence of \mathcal{P} .²²¹

Remarks

1. It is important to bear in mind that *qiyās* procedure involves two primary dialectical steps: one is the epistemological and hermeneutical task of identifying the properties claimed to occasion the ruling, the other—that is, the logical step—assumes that the first has been achieved. The first involves sub-arguments and counter-arguments that are not formal, but contentual or material; the second involves dialectical rules for logical reasoning. Contesting a legal argument might involve contesting a logical mistake, but this concerns fallacies more in line with those in the peripatetic syllogistic tradition, which constitute a group of their own known as *mughālaṭāt*. In the context of legal reasoning, the most relevant form of objection concerns the first step whereby a semantic link between the property and the ruling is claimed. The sub-arguments and counterarguments concerning this step are not formal and are therefore always prone to further revision. Agreement can be achieved at some point, but the whole process can also start over again.²²²
2. In section 5 we will distinguish between *occasioning factors* or *properties*, and *occasioning procedure*: the efficient causation that transforms instances of factors or properties into concrete applications of the ruling. Before then, however, we will leave this finer distinction aside.

4.2 *Kasr* or: “How to Break Apart a Compound Occasioning Factor”

An important set of fallacious moves in Islamic argumentation theory relates to challenging claims that a property (*wasf*) or composition of properties constitutes the efficient *‘illa*, i.e., the occasioning (causal) factor upon which the ruling (*ḥukm*) is grounded. What may be subject to fallacy is the form a challenge takes in refuting a claim that some property or properties constitute the *‘illa* for a debated case, or the response to that challenge. In the context of arguments seeking to identify legal or natural occasioning factors, the like of these do not seem to be included among the peripatetic tradition’s *mughālaṭāt* (which is usually limited to the set delimited by *σόφισμα*); rather, we find them in legal theoretical and dialectical works as *invalid* (*fāsid*, *ghayr ṣaḥīḥ*) objections and responses.

Objections and responses that contribute to legitimate refutations and defences, are deemed *valid* (*ṣaḥīḥ*). Bear in mind that in legal contexts, as in Roman Law, *valid* predominantly bears the sense of *legally valid*, as in *valid contract*, but also admits the deontic reading *legally allowed*. However, in Islamic argumentation theory, *valid* (*ṣaḥīḥ*) and *validity* (*ṣiḥḥa*) primarily address arguments, and, by extension, moves that produce valid arguments, understood as in agreement with regard to some set of dialectical rules that establish both the dialectical meaning of the expression involved and the procedural

²²¹ See Rahman, Iqbal, and Soufi, “Inferences by Parallel Reasoning” (preface). NB: this test of a property’s causal efficiency is elsewhere and more commonly called “co-presence and co-absence” (*al-ṭard wa’l-‘aks*) or “concomitance” (*dawarān*), and listed among the “modes of causal justification” (*masālik al-ta’līl*). See Young, “Concomitance to Causation,” and Hasan, “Analogical Reasoning,” 315-330. As for “efficiency” (*ta’thīr*), such as al-Ghazālī deemed it to be a direct designation of the cause (*‘illa*) by either univocal source-text (*naṣṣ*) or consensus (*ijmā’*), while others held different notions (see Hasan “Analogical Reasoning,” 272-3, 284).

²²² See Rahman, Iqbal, and Soufi, “Inferences by Parallel Reasoning,” 46-47.

debate-protocol to be followed.²²³ However, *fāsīd* (invalid) and *fasād* (invalidity) seem to be closer to the notion of winning strategy: a refutation by Q is invalid if, from the outset, whatever Q does after some specific move, R wins without further intervention.²²⁴

Among the most virulent challenges to an *illa*-claim is *naqđ* (inconsistency), whereby Q brings a case wherein R's claimed *illa* is found without R's claimed ruling (*hukm*), thus aiming to destroy R's claimed *illa*. Whereas a fallacious form of *naqđ* is to bring a parallel case that in fact constitutes a particularization or further specification of R's root-case, a legitimate or valid *naqđ* brings a parallel case more general than R's wherein the property claimed to be the occasioning factor is present but R's desired ruling is not.²²⁵

Here we will focus on a special form of *naqđ* called *kasr*, aimed at destroying a *compound of properties* claimed to be the occasioning factor by taking apart and disposing of its components. In short, Q objects that one (or some) of the properties of R's *illa* exists in another case—the “problem-case of breaking,” or *kasr*-case (*mas'alat al-kasr*), but with a different *hukm*.

Formulating the move is delicate and requires some fine distinctions, beyond the pair *sensus composito* and *sensus diviso*. It is therefore not surprising, then, that *kasr* triggered long and heated debates over whether it should be admissible at all.

The notions of *kasr* discussed here are those elaborated by the Shāfi'ī jurist Abū Ishāq al-Shīrāzī (d. 1083) and by his onetime pupil, the Mālikī jurist Abū al-Walīd al-Bājī (d. 1081) who, aware of the pitfalls of breaking a compound apart, developed two main pathways to “breaking” an opponent's compound *illa*:

- (1) by substitution (*ibdāl*): producing a parallel case similar in meaning (intension), *ma'nā*, to R's root- and branch-case, wherein one of R's properties is replaced by another in the same *ma'nā* and that parallel compound is present despite the absence of R's desired ruling (thus, the not-replaced property is not efficient in occasioning the ruling);
- (2) by removal (*isqāt*): producing a parallel case similar in *ma'nā* to R's root- and branch-case, wherein one of R's properties is absent and the remainder present despite the absence of R's desired ruling (thus the remainder property is not efficient in occasioning the ruling).

Each type is further subdivided into moves that do and do not produce valid refutations of R's *illa*. As evident in the above descriptions, a key consideration is that the rules of challenge and defence governing the building of a valid *kasr*-case must be *meaning preserving* in relation to both the form of composition of properties claimed by

²²³ See S. Rahman, Z. McConaughy, A. Klev, N. Clerbout, *Immanent Reasoning. A Plaidoyer for the Play-Level* (Dordrecht, 2018), 57-105, and pp. 279-282 for *refutation*.

²²⁴ The use of valid and invalid in argumentative contexts relates to logical terminology, but an important caveat must be observed. As just mentioned, the notion of *invalidity*—and, by extension, *invalid move*—includes semantically unsound arguments (and refutations) and the notion of *logically* invalid, whereas its counterpart, *validity*, does not refer to logically valid arguments (at least not in its most common use). As we shall see below, however, in Islamic legal reasoning an *invalid move* also may refer to a move where R wins whatever Q attempts to do after this move (since it contravenes the meaning and/or procedural rules established for the thesis involved), but a valid move does not automatically amount to a winning strategy. It refers rather to a legitimate move that can produce the win of one *play* of one of the contenders; but this does not prevent finding another play with a different outcome. More simply put: validity applies to winning just a play, not all relevant possible plays (Rahman, McConaughy, Klev, Clerbout, *Immanent Reasoning*, op cit.) Thus, Islamic debates took place primarily at the play level.

²²⁵ See Young, *Dialectical Forge*, 169-173.

R to constitute the *'illa*, and to the conceptual dependence between the ruling and that compound.

Apparently, this assumes that the framework is thoroughly dialectical. Indeed, the conceptual links are made explicit by what contemporary dialogicians call *dialogical meaning explanations*. That is, the conceptual links expressed by connectives or other terms, are given by rules of challenge and defence. In the sources, these meaning explanations manifest in the context of debates on the legitimacy of a concrete refutation attempt. It is here that the precise mode of composition assumed by each way of constructing a *kasr*-case is determined.

In the *Sharḥ al-Luma'*,²²⁶ al-Shīrāzī appears to divide approaches to building a *kasr*-case into the aforementioned types and subtypes lead by this principle or caveat on meaning preservation.²²⁷ Our texts provide sufficient elements to render a precise formulation of the rules governing valid and invalid moves for each type, though the terse examples, aimed at an audience proficient in Islamic substantive law and its points of derivation, are often difficult to follow. In the following presentation we have attempted to reconstruct arguments regarding the classification of types, with an eye to intrinsic coherence, but this is still to be considered work in progress.

Type 1: *Kasr* by Substitution (*ibdāl*) in Dependent Conjunctive Compounds.

Type 1a: Valid *kasr* by *ibdāl*

- **Valid objection:** Given a compound of properties *AB*, claimed by R to constitute the *'illa*, a valid *kasr*-refutation by *ibdāl* is one in which Q substitutes *A** for *A*, being in the same *ma'nā* as *A*, in a *kasr*-case of the same *ma'nā*, and shows the *naqd* (inconsistency) of the other property *B* (the one not replaced).
- **Valid response (*jawāb*):** Although R might acknowledge that, in principle, the above conditions for a valid refutation have been accomplished by Q, R can nevertheless rejoin if he can produce an argument showing that Q's *kasr*-case involves a significant deviation from the *ma'nā* **due to the substitution of *A** for *A***. R's response takes the form of an objection by *disqualifying difference* (*farq*). The kernel of the rejoinder is that a key difference in *ma'nā* of the *kasr*-case disqualifies it as proof of R's inconsistency. Moreover, that difference in the new case shows that the substitution *A** for *A* also triggers a change in the meaning of *B*—the *ma'nā* of *B* in the context of the new *A* does not contribute to occasioning R's *ḥukm* as did the *ma'nā* of *B* in the context of the old *A*.

Type 1b: Invalid *kasr* by *ibdāl*

- Given a compound of properties *AB*, claimed by R to constitute the *'illa*, an invalid *kasr*-refutation by *ibdāl* is one in which Q substitutes *A** for *A*, but *A** is *not* in the same *ma'nā*, nor in a *kasr*-case of the same *ma'nā*. Moreover, Q's new compound may even prove irrelevant for occasioning

²²⁶ Abū Ishāq al-Shīrāzī, *Sharḥ al-Luma'*, ed. 'Abd al-Majīd Turkī (Beirut, 1988), 892-909, §§1035-1051.

²²⁷ The original sources for our study also include Abū Ishāq al-Shīrāzī, *Al-Ma'ūna fī l-Jadal*, ed. 'Alī b. 'Abd al-'Azīz al-'Umayrīnī (Al-Ṣafāh, Kuwait, 1987); Abū al-Walīd Sulaymān al-Bājī, *Kitāb al-Minhāj fī Tartīb al-Ḥijāj (L'art de la Polémique: Édition Critique d'al-Minhāj de Bāḡī)*, ed. 'Abd al-Majīd Turkī (Paris, 1978); Abū al-Walīd Sulaymān al-Bājī, *Iḥkām al-Fuṣūl fī Aḥkām al-Uṣūl*, ed. 'Abd al-Majīd Turkī (Beirut, 1995). Since al-Bājī's classification mostly follows that of his teacher, and the discussion of *kasr* in the latter's *Sharḥ al-Luma'* is more thorough than the one in his *Ma'ūna*, we will base our presentation mainly on al-Shīrāzī's notion of *kasr* as presented in the *Sharḥ al-Luma'*, sometimes completing the presentation of his *Ma'ūna* with further details found therein. We will also add some of al-Bājī's remarks should they help clarify the subtypes.

the ruling of his own *kasr*-case. At the very least, Q has not properly shown the *naqd* of R's 'illa. R is not bound to do anything except point out that Q's *kasr* is invalid. **The point is that an invalid *kasr*-case does not preserve the meaning dependence between (the new) compound and its ruling as in R's original compound and its claimed ruling.**

In fact, it is the rule for R's rejoinder that signalizes how his compound is to be understood in the first place. In the next section we will provide further details on logical analyses for the form of composition *AB* assumed in *kasr* challenges of types 1 and 2. But for the moment let us say that at the heart of type 1 *kasr* objections and responses is the fact that what is targeted is compounds wherein each of the components *together*, not in isolation, are claimed to occasion the ruling. For instance, as W. B. Hallaq observes (2009, p. 102):

“The ratio may also consist of more than one attribute, all of which must be considered as ‘causing’ a normative rule to arise from them. For instance, the ratio of the theft penalty encompasses five attributes: (1) the taking away of something by stealth; (2) the stolen object must be of a minimum value...; (3) the object must in no way be the property of the thief; (4) it must be taken out of custody (*hirz*); and (5) the thief must have full legal capacity. All of these attributes must obtain for an act to qualify as theft (*sariqa*) punishable by cutting off the hand. Each attribute is necessary; no single one by itself suffices to produce the ratio legis.”

More generally, and although not plainly stated in our sources in these terms, we may infer from the examples that in a successful *kasr*-case the meaning of property *B*, must be dependent upon the meaning of *A* in such a way that the substitution does not change substantially the meaning of *B* when (the original) *A* is present. In short, *B* is a propositional function defined over *A* or its meaning-preserving substitutes. Rejecting a refutation based on *kasr* by *ibdāl* amounts to showing that Q's argument contravenes (partially or totally) the meaning-preservation caveat regarding the dependences inherent to R's compound 'illa.

Examples: Type 1

Type 1a: Al-Shīrāzī provides²²⁸ the following example of a valid attempt at *kasr* and its valid response. The 'illa of R's *qiyās* is a compound 'illa ('illa *murakkaba*) consisting of properties *A* and *B*:

- contended branch-case (*far'*): sale of what buyer has not seen
- claimed legal cause ('illa): it is [A] a saleable object (*mabī'*); and it is [B] unknown of attribute (*majhūl al-ṣifa*) to contractor at the time of contract
- claimed ruling (*ḥukm*): it is not allowed
- endoxon root-case (*aṣl*): when the seller merely says “I sell you a garment”
- Q attempts to invalidate R's 'illa by replacing one of its properties (*A*, *mabī'*: *object of sale*) with another (*A**, *mankūḥa*: *object of marriage*)²²⁹ and showing that *B* is thus co-present with the opposite of R's *ḥukm* (it is valid) in a seemingly parallel case (marriage): the *kasr*-case.

“So, the one disagreeing with him [i.e., Q] says: ‘This is broken (*inkasara*) by [the fact that] if he marries a woman whom he has not

²²⁸ *Sharḥ al-Luma'*, 893, 898-9.

²²⁹ Although this is not made explicit in the example in the *Ma'ūna*, it is in the example in the *Sharḥ al-Luma'* (Turkī ed., pp. 893, 898-9), where (A) *mabī'* is replaced by (*A**) *mankūḥa*.

seen, then she will be [B] **unknown of attribute** to the one contracting at the time of contract—yet it is valid.”²³⁰

- R responds by invalidating Q’s *kasr*-case through an objection of disqualifying difference (*farq*), demonstrating that marriage is not the same as sale **with regard to property B** by reminding Q that the buyer has *khiyār* (i.e., has the option to rescind) in case of sale with property *B*, but not in case of marriage with property *B*.

More importantly, and although the case of marriage produced by Q shares with the contended case (object of sale) the property of a specific form of ignorance, *jahāla*, namely *majhūl al-ṣifa* (unknown of attribute), which “presumes existence [of the object of contract] but involves lack of reasonable knowledge of the thing’s characteristics” (Hallaq, *Sharī‘a*, p. 244)—viz. it lacks knowledge of how the object is—object of sale and marriage take different paths in relation to the contractor’s subsequent recourse to *khiyār*. Thus, the meaning of *B*, unknown of attribute, changes if *A**, marriage, is substituted for *A*, object of sale.

- Q’s objection of *kasr* is thus nullified.

Type 1b: This example begins with the same *qiyās al-‘illa*, but Q’s refutation attempt is invalid due to being clearly *not* in the contended case’s *ma‘nā*.²³¹

- This time, Q attempts to break R’s *‘illa* by bringing the *kasr* case of bequest (i.e., giving something to someone after one dies, in a will) of something whose attribute the legatee (contracted to receive the bequest) does not know, which is [*A**] an object of bequest (*mūṣā bihi*), and [*B*] unknown of attribute (*majhūl al-ṣifa*) to the contracting party [i.e., legatee] when the contract is made, but it is **allowed**, in contrast to the object of sale.

“Such is like when the one drawing indication [R] says: ‘It is [*A*] an object of sale which is [*B*] unknown of attribute to the one contracting at the time of contract, so it resembles when he says: ‘I sell you a garment.’” So he [Q] says to him: ‘This is broken (*yankasiru*) by [*A**] the object of bequest (*mūṣā bihi*), for it is [*B*] unknown of attribute to the one contracting at the time of contract, but the bequeathing of it is valid.’”

Thus, Q has replaced [*A*] with [*A**], then attempted to show the *naqđ* of [*B*] in the *kasr*-case.

- However, Q’s replacing [*A*] *mabī‘*, object of sale, with [*A**] *mūṣā bihi*, object of bequest in the *kasr*-case is not valid, because *A* and *A** are not in the same *ma‘nā*. As al-Shīrāzī explains in the *Sharḥ al-Luma‘*:

“Bequest (*waṣīyya*) is not a parallel case (*naẓīr*) for sale in terms of lack of knowledge (*jahāla*), nor is it in [sale’s] quality / meaning (*ma‘nā*). Don’t you see that no [kind of] lack of knowledge (*jahālāt*) prevents the validity of the bequest? For this reason, if he were to say: ‘I bequeath to you a garment, or something, or wealth / property,’²³² it would be permitted. But

²³⁰ *Ma‘ūna*, 246, §127.

²³¹ *Sharḥ al-Luma‘*, second half of §1041.

²³² Reading [بخير] instead of [بخير].

the like of this is not valid in sale.”

Q’s attempt proposes to replace object of sale with object of bequest, which also has the property of being unknown by the contractor. However, the legatee’s ignorance of the nature of the object or of its characteristics has no efficiency whatever with regard to the contract’s validity. According to our understanding, the case is deceptive precisely because, though it has that property of ignorance which is efficient in R’s case, its irrelevance for determining a bequest’s legal status puts it outside the *ma’nā* of R’s cases. Ignorance in bequest is simply entailed of being an object of bequest, but it does not contribute to the task of identifying relevant occasioning factors for the bequest’s validity or lack thereof.

- Q’s objection of *kasr* is thus nullified from the very start. Different to the *kasr*-case of marriage, property *B*, ignorance, in any degree, does not contribute to the legal status of the contract in bequest, and is thus outside the *ma’nā* of contracting sale of an unknown object.

Conclusions regarding Type 1.

Our texts on *kasr* indicate that the compounds targeted by type 1 involve meaning dependences of the unchanged property (or properties) upon the replaced property. The relation is a conceptual link, whereby the second property is a specification of the first one.

Type 2: *Kasr* by Removal (*isqāṭ*) in Independent Conjunctive Compounds.

In contrast to type 1, a valid type 2 *kasr* works by omitting a non-efficient property of R’s compound ‘*illa* and proving the remainder inconsistent via *naqḍ*. Although at this stage of research we have uncovered no explicit positive statements to this effect, we may also conjecture, by sheer logical analysis of the examples discussed in the sources, that type 2 *kasr* was understood, again in contrast to type 1, to target compounds wherein *each of the components was thought to suffice independently for occasioning the ruling*. For the sake of exploring the full potential of this objection and its responses — without invalidating it from the start, we will carry this assumption forward into our analyses.

Type 2a: Valid *kasr* by *isqāṭ*.

- Given a compound of properties *ABC*, which R claims to be the ‘*illa*, a valid *kasr* by *isqāṭ* is when (1) Q brings a parallel *kasr*-case in which one of R’s properties (e.g., *A*) is absent but the remainder (e.g., *BC*) are present; (2) Q justifies his *kasr*-case by showing that *A* lacks efficiency (a subsidiary objection of ‘*adam al-ta’thīr*), and can thus be removed; (3) Q shows that his *kasr*-case constitutes a *naqḍ* of what remains from R’s ‘*illa* (*BC*)—i.e., that it is present in Q’s *kasr*-case but without R’s *ḥukm*. Note that Q thus contests R’s components in two sets, denying efficiency to one, and destroying the remaining other via *naqḍ*.
- **Valid response (*jawāb*):** R has recourse to either contesting Q’s claim that *A* lacks efficiency (‘*adam al-ta’thīr*), or attempting to disprove the validity of Q’s *kasr*-case in that it does not lead to *naqḍ* of *BC*.

Type 2b: Invalid *kasr* by *isqāṭ*.

- Given a compound of properties *ABC*, which R claims to be the ‘*illa*, an invalid *kasr* by *isqāṭ* is when Q’s *kasr*-case omits *A* as above, while there is clear evidence that *A* is in fact causally efficient. Having thus wrongly removed the causal property *A* of R’s ‘*illa*, his *kasr*-case’s subsequent *naqḍ* of the remaining *BC* does no harm to the truly causative portion of R’s ‘*illa*

(the causally efficient A). A helpful way to think of type 2 *kasr* is to think of Q as accusing R of “stuffing” his *illa* with an extra, non-efficient property (or properties). In type 1 *kasr* by *isqāt*, Q ejects the non-efficient stuffing and asserts the *naqd* of the efficient remainder; in type 2, he wrongly ejects the efficient component, and asserts only the *naqd* of the stuffing.

In the following, we will exemplify only R’s rejoinder to type 2a *kasr*, since it yields a complete illustration of this subtype, whereas type 2b is a special case wherein Q gets it wrong from the start.

Example: Type 2

Type 2a: valid *kasr* and valid response

Before beginning, we must review the relevant substantive legal category of *ibdāl*, or substitution. This is when an original, obligatory act of ritual worship, the *aṣl*, is replaced by another kind of “substitute” act, the *badal*, either because one was not able to do the *aṣl*, or one has neglected / invalidated the *aṣl* and needs to make up for it, or substitute for it some other ritual. For example, a condition (*shart*) of valid prayer is valid *wuḍūʿ*: the minor ritual ablution with water (constituting the *aṣl* in the example below); but should one find oneself in, e.g., a waterless waste, one may instead perform *tayammum*: ritual ablution with sand or clean earth (the *badal* of our example). *Tayammum* is a *badal* for the originally obliged *aṣl* of *wuḍūʿ*.

Now we will break down valid Type 2a *kasr* and response as discussed by al-Bājī.²³³

- R asserts that *niyya*, intent, is not obligatory for doing *wuḍūʿ* because *wuḍūʿ* is [A] a *sabab*, a means of reaching prayer; and [B] not a *badal*; therefore, *niyya* is not obligatory, as in the root-case of removing *najāsa*, filthiness (which also has both properties). As al-Bājī says:

“As for when [Q] omits a non-efficient property, it is a valid *kasr*. This is like when the Ḥanafī [R], for the [issue] of intent (*niyya*) in minor ritual ablution (*wuḍūʿ*), draws indication that [intent] is not obligatory, because [ritual ablution] is [A] a means of access / mediate cause (*sabab*)²³⁴ by means of which one reaches prayer, which is [B] not in the way of a substitution (*badal*),²³⁵ so intent is not obligatory for it—like [in the case of] removal of impure filthiness (*izālat al-najāsa*).”

Thus, R’s *qiyās* has the following components:

- contended branch-case (*farʿ*): *wuḍūʿ*, the minor ritual ablution with water.
- claimed legal cause (*illa*): the compound property of [A] being a *sabab*, and [B] not being a *badal*.
- claimed ruling (*ḥukm*): *niyya*, intent, is not obligatory for it.
- endoxon source-case (*aṣl*): removal of impure filthiness (*izālat al-najāsa*).

In objecting via type 2a *kasr*, Q now “breaks” R’s *illa* in three steps:

²³³ *Minhāj*, §436.

²³⁴ On the *sabab*, see Hasan, *Analogical Reasoning*, 369 ff.

²³⁵ On substitution (*ibdāl*) in general, see *al-Mawsūʿa al-Fiqhiyya* (Kuwait, 1990), 1.140 ff., s.v. [بدال].

First, he brings the *kasr*-case of *tayammum*, the ritual ablution with sand or earth, in which *niyya* is obligatory, and which also contains *A* (being a *sabab*) but not *B* (not being a *badal*)—in fact it *is* a *badal* for *wuḍūʿ*. Property *B* is thus omitted in *Q*'s *kasr*-case (*tayammum* is *A* but not *B*).

Second, *Q* justifies this by claiming that *B* cannot be an efficient property for absence of obligation of *niyya* in the contended case, since when it comes to *niyya* whatever holds for the *badal* holds for the *aṣl* and vice versa—it is never that one requires *niyya* and the other doesn't (as is the consequence of *R*'s claim). *Q* thus attempts to show that *B*, not being a *badal*, has a lack of efficiency (*'adam al-ta'thīr*) in occasioning a *ḥukm* of non-obligation of *niyya*.

Third, once *B* is shown to be non-efficient “stuffing,” *Q*'s *kasr*-case automatically becomes a *naqḍ*-case for the remaining property *A*. In the case of *tayammum*, we find property *A* (being a *sabab*), but not *R*'s ruling (non-obligation of *niyya*). *R*'s *'illa* is present despite the absence of his *ḥukm*—the very definition of *naqḍ*. In al-Bājī's words:

“So the Mālikī [*Q*] says to him: ‘This is broken (*yankasiru*) by ablution with dry earth or sand (*tayammum*), for it is [*A*] a means of access / mediate cause (*sabab*) by means of which one reaches prayer, yet it requires intent.’²³⁶ This is a valid *kasr*, because the property by way of which [*R*] guarded against [inclusion of] ablution with dry earth or sand (*tayammum*) [that is, *B*: it is not a substitution (*badal*)] is not efficient in the falling away of intent (*niyya*).²³⁷ Don't you see that original [acts] (*uṣūl*)²³⁸ and substitutions (*abdāl*)²³⁹ are one with regard to the subject of intent in God's Law?”

- *R*, however, successfully escapes this with a valid response. In short, he revives the possibility that property *B* (not being a *badal*) can have efficiency, by destroying *Q*'s claim of *aṣl-badal* correspondence vis-à-vis *niyya* with counter-examples wherein an *aṣl* does not require *niyya* but its *badal* does. With the possibility of *B*'s efficiency thus restored, *R*'s *qiyās* evades *Q*'s type 2a *kasr* by *isqāṭ*.

Conclusions regarding Type 2.

Our texts on this type of *kasr* make clear the manner in which the compound *AB...*, in order to occasion the ruling, must be constituted by a conjunctive antecedent.

Now, if we link an antecedent of the form $A \wedge B \dots$, and the *ḥukm* with some form implication, and the components of the antecedent do not constitute a chain of dependences as in the first type, then it logically follows that these

²³⁶ Note that *tayammum* is a substitution, done in place of *wuḍūʿ* when there is no water, and so *Q* has omitted property [*B*] “not being in the way of a substitution.”

²³⁷ That is, *R* likely formulated his compound *'illa* with property *B* (not in the way of substitution) in order to exclude the endoxon case of *tayammum*, which has the opposite (intent is obligatory) of the desired *ḥukm*, but that property *B* has no efficiency in the negation of intent in the case at hand. So *Q*'s omission of property *B* in his *kasr*-case of *tayammum* (which is in the way of substitution) is valid, and exposes the inconsistency (*naqḍ*) of the other property *A*: that is, property *A* is present in the *kasr*-case of *tayammum*, but without *R*'s *ḥukm*. Thus, it is a valid *kasr*.

²³⁸ In this case, minor ritual ablution (*wuḍūʿ*).

²³⁹ In this case, ablution with dry earth or sand (*tayammum*).

components distribute **disjunctively** in relation to the *ḥukm*.

This logical fact explains why the challenge Q in 2a requires to do both, denying efficiency to one and destroying the remaining other: challenging a conjunction that distributes over an implication requires challenging all of the resultants of the disjunctive distribution. What the dialectical setting adds to the purely logical analysis is that the challenges are not of the same form (one claims lack of efficiency, the other builds a *naqḍ*-case). The same fact also explains why R can respond by choosing to counter one of these different challenges

Moreover, it is the disjunctive distribution of the conjunction over the implication that explains why in 2b the move that contests the “redundant” components in the antecedent does not destroy the main claim, given that the component known to be efficient has been omitted (and unchallenged)

Thus, the logical analysis suggests that the best way to think about type 2’s target *qiyās* is that it grounds such assertions as:

Performing actions of the types containing *A, B ...* is forbidden... iff performing types containing *A* is forbidden or performing types containing *B* is forbidden or ...

If we do not assume such a logical analysis, the whole argument is wrong from the start on.. For example, we cannot read it as being of type 1. Indeed, consider once more the example of *wuḍū’*. It does not (as R claims) require *niyya*, because it is not a *badal*. If read according to the manner of type 1’s target *qiyās* composition, then the attempted *kasr*-case of *tayammum* would be ruled out from the outset

5 Towards a Dialogical Meaning Explanation of *Kasr*:

Because of space constraints we will focus in developing the logical analysis that will constitute the building blocks for the dialogical framework. However, we will not be able to either describe all the required dialogical steps or build the actual dialogue.

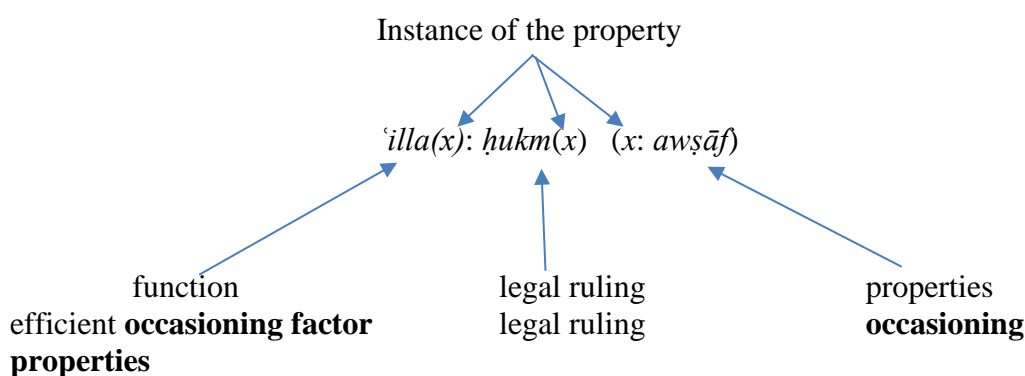
5.1 Functional Analysis of the Occasioning Factor

Rahman and Iqbal have proposed a CTT analysis of the conceptual link between the ruling and the properties constituting the occasioning factor.²⁴⁰ The idea is that a ruling is conceptually dependent upon the properties that occasion that ruling. Certainly, the *Forbidden* occasioned by consuming intoxicating drinks is not the same as the one occasioned by killing someone. The legal system attends to this distinction by the quality and degree of sanctions that penalize such acts.

In the context of a CTT framework, functions are the means to express such dependences; and a function is a method or procedure for relating concepts such that the one is dependent upon the other. In legal contexts, executing this function is a legal procedure by the means of which performances of a type of action occasion the ruling’s application to such performances. In concrete cases, it is the legal authority who carries out the procedure.

²⁴⁰ “Unfolding Parallel Reasoning in Abu Ishaq al-Shīrāzī’s System of Co-Relational Inferences of the Occasioning Factor,” *Arabic Sciences and Philosophy* 28 (2018), 67–132; further developed in Rahman, Iqbal, and Soufi “Inferences by Parallel Reasoning,” 31–40, and Iqbal, *Arsyad al-Banjari’s Approaches to Rationality*.

The upshot of such an analysis is that it allows one to distinguish between the property constituting the *occasioning factor*, and the *actual legal procedure that occasions* the ruling. In other words, according to this view, it is useful to distinguish between the property (Ar. *wasf*, pl. *awṣāf*) and the properly efficient cause / occasioning factor (*illa*), that is, the procedure that transforms instances of occasioning (causal) factors into applications of the ruling to these instances. If we recall our notation in section 3 we have:



- Notice that the CTT framework allows the introduction of potentially infinite types, Each of them intensionally defined—they are not extensional sets. This not only allows one to distinguish between different types of action but between different types of properties as qualities; legal contexts require certain multi-sorted domains. Thus, this multi-sorted language allows one to introduce performances of actions and instances of qualities into the object language. With the term *awṣāf* we may refer to any of these. The distinction between types of actions and types of quality is to be determined in context. And there might be certain arguments supporting the reduction, in legal contexts, of all types to types of actions, and in natural contexts, of all types to types of events.

If the above is granted, we might take a step further and delve into the conceptual links constituting compound occasioning factors.

5.2 Compound Occasioning Factors within *Kasr*-Cases

As discussed in the previous section the *kasr* method of refutation targets compound occasioning factors: the first type targeting compounds wherein the meaning of one property is dependent upon another, the second targeting meaning independent components.

5.2.1 Dependent Composition

The compound *AB* at work in type 1 *kasr*-cases by *substitution* may be understood as *B* constituting a *propositional function* over *A*, in the manner described in section 3; that is, the meaning constitution of *B* is based on *A* (recall “Good” in “Good Cobbler”).

$B(y): prop (y: A)$

When being a *B* is asserted of instances *y* of the type *A* it becomes a proposition

$Unknown\ of\ attribute(y): prop (y: Object\ of\ sale)$

When being *unknown of attribute* by the contractor at the time of the contract, is asserted of instances *y* of the type *object of sale* it becomes a proposition

Now we can constitute the type, or intensional set, or proposition—called *subset separation*—that produces a subset in A by selecting all those elements in A that satisfy B :²⁴¹

All those A that are B

$\{y: A \mid d(y): B(y)\}: prop$

All those objects of sale that are unknown of attribute to the buyer

$\{y: Object\ of\ sale \mid d(y): Unknown\ of\ attribute(y)\}: prop$

What makes this proposition true is in fact a pair, such that the first element of the pair is an instance of A , and the second element is a function from A to B , that relates instances of A with instances of B , in such a way that the latter instances are dependent upon the former:

$(y, d(y): \{y: A \mid d(y): B(y)\})$

Unknown of attribute(y): prop (y: object of sale)

Let us instantiate A , object of sale, with *garment*:

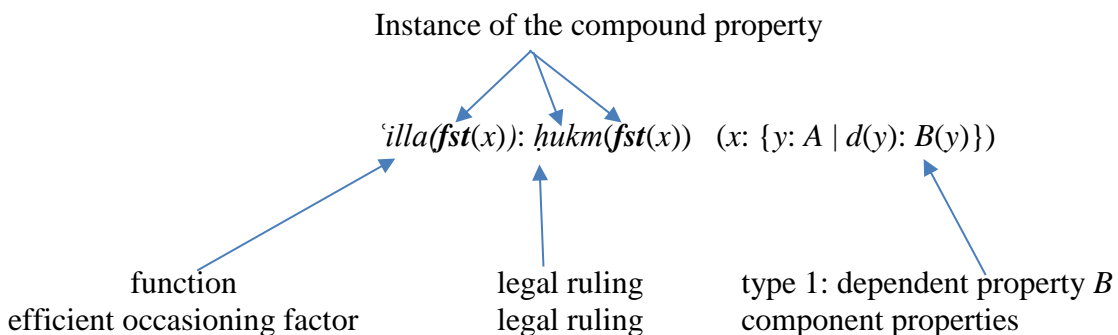
Given

*garment is an object of sale—
garment: Object of sale*

this *object of sale, the garment, is unknown of attribute to the buyer
d(garment): Unknown of attribute*

$(garment, d(garment)): \{y: Object\ of\ sale \mid d(y): Unknown\ of\ attribute(y)\}: prop$

Let us now insert this into the structure wherein this compound expresses the occasioning factor for the ruling:



- Whereby x stands for the pair $(y, d(y))$. Thus $x = (y, d(y)): \{y: A \mid d(y): B(y)\}$, $\textit{fst}(x)$ stands for the first component of x , that is those y 's in A of which B is asserted. Notice that $\textit{fst}(x)$ always evaluates an object that satisfies B

²⁴¹ Notice that this is equivalent to an existential with restricted domain. The sentence *Some Poets are Good* (as poets) expresses the restricted existential quantification $c: (\exists x: Poet) Good(x)$, whereby any c that instantiates the proposition is a good poet.

'illa(fst(x): forbidden(fst(x)) (x: {y: Object of sale | d(y): Unknown of attribute(y)}))

This should clarify why type 1 *kasr* can only be carried out by substitution: what is asserted is that the *ḥukm forbidden* applies only to those *Objects of sale* that are *Unknown of attribute*. The property *Unknown of attribute* does not apply to anything else except apt substitutions of similar set of objects (i.e., objects in the same *ma 'nā*) such that *Unknown of attribute* plays the same role in the new set as in R's original set. Substitution should preserve the meaning dependences of original case and also the dependence of the juridical ruling upon this compound.

5.2.2 Independent Composition

The compound *AB* at work in type 2 *kasr* cases by *removal* may be understood as omitting from the conjunction then non-efficient component and building a proving naqd-case for the remainder. As mentioned above, logically speaking such move is only possible if the meaning of the component that builds the naqd-case is not dependent upon the one removed.²⁴² Let us study the logical structure behind step by step:

$A \wedge B$: *prop*

What makes this proposition true is in fact a pair (or however many components constitute the conjunction) such that the first element of the pair verifies *A*, the second verifies *B*, the third verifies *C*, and so on:

(x_1, x_2) : $A \wedge B$

Whereby x_1 stands for a function that takes elements of the domain *D* and asserts of them that they are *A*. Something similar holds for x_2 and x_3 .

Recall that in our *wuḍū'* example the two members in the conjunction are the properties *being a sabab* (intermediate means) for reaching prayer **and not being a badal** (i.e., a substitute for a ritual component missed or contextually unobtainable).

(x_1, x_2) : $Sabab \wedge Not-Badal$

If we relate the ruling with the compound we might be tempted to express it as:

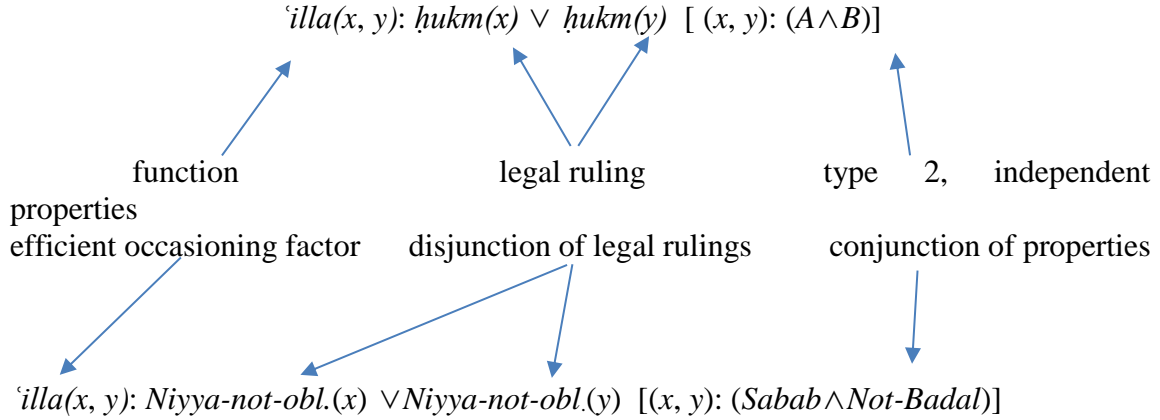
'illa(x,y): ḥukm [(x, y): (A ∧ B)]

However, this does not express that the second might be redundant – as in fact it is in the example under consideration. Thus, what we need is to make apparent the logical consequence of an analysis that contemplates the case that one of the components is not efficient. As mentioned above, this amounts to a disjunctive distribution of the conjunction over the (claimed) occasioned ruling::

²⁴² Each component expresses a property belonging to some domain of objects (or actions)—for the sake of simplicity we leave the domain tacit. In context of TCT grammar, such a process is called *sugaring*: it is the reverse of formalization—see A. Ranta, *Type Theoretical Grammar* (Oxford, 1994), 7-11. Sugaring procedures are those transformations that, step by step, produce a natural language expression. In the case of ambiguity in a natural language expression, one traces bottom up the meaning-constitution sugaring process that yielded this expression.

$$\text{'illa}(x,y): \text{hukm}(\text{fst}(x,y)) \vee \text{hukm}(\text{snd}(x,y)) \quad [(x, y): (A \wedge B)]$$

And since $\text{fst}(x,y)=x: A$, and $\text{snd}(x,y)=y: B$, we may indulge in the following shortcut:



This explains why, if an unskilful Q removes a property known to be efficient, the refutation aborts: verification of one of the components is sufficient. Moreover, it explains why, in order to refute R's claim, Q needs to contest the efficiency of one of the components and to build a *naqd*-case for the other. In the *wuḍū'*-example it amounts to

Contesting the efficiency of

$$\text{'illa}(\text{wuḍū}'_2): \text{Niyya-not-obligatory}(\text{wuḍū}'_2) \quad (\text{wuḍū}'_2: \text{Not-Badal})$$

Building a *naqd*-case against

$$\text{'illa}(\text{wuḍū}'_1): \text{Niyya-not-obligatory}(\text{wuḍū}'_1) \quad (\text{wuḍū}'_1: \text{Sabab})$$

By replacing *wuḍū'* with *tayammum*.

Given:

$$(\text{wuḍū}'_1, \text{wuḍū}'_2): (\text{Sabab} \wedge \text{Not-Badal})$$

$$\text{Whereby} \quad \text{wuḍū}'_1: \text{Sabab} \\ \text{wuḍū}'_2: \text{Not-Badal}$$

5.2.3 Conclusion regarding Dependent and Independent Types

The above CTT analysis reveals that the two different compositional modes of compound factors occasioning a juridical ruling targeted by a *kasr* refutation decline into two different ways of linking the members of such compounds by means of what, in natural language, we call conjunction. In short, this conjunction either (1) might assume that components display some form of conceptual dependence (in type 1 target *qiyās*), or (2) might assume that they do not (in type 2 target *qiyās*).

Note that the dialectic rules established for building a *kasr*-case for both forms of composition require basically the same form of verifiers: namely, a verifier for each of the components (though in the first type [at least], one of the verifiers is a function having as its argument the other verifiers of the compound). This strongly indicates that the two types of compound may be included under a single category we might call “conjunctive composition.”

Here we may appeal to a famous insight of Avicenna's: namely, that an

implication can be reduced to a universally quantified proposition if its components *share content*.

“All conditional and disjunctive propositions, and in particular the conditional in which the antecedent and the consequent share one part, can be reduced to categorical propositions.”²⁴³

This can be generalized as distinguishing, within the category of implications, those for which the content of the antecedent does not occur in the consequent (propositional implications) from those for which it does (universal quantifiers with restricted domains). In CTT and its grammar the category including both forms of implication is known as \forall -Type, also called the *cartesian product of a family of sets*.²⁴⁴

This \forall -Type is the dual, i.e. the logical counterpart, of a category we might call *conjunctive* and which includes conjunctions wherein some of the members *share content with the other* (existential quantifiers with restricted domains or subset separation: those elements in A that are B) and those that do not (propositional conjunctions). In CTT, the general category that includes both forms is called Σ -Type or *disjoint union of a family of sets* Σ -Type.²⁴⁵ The upshot of all this is that whereas our analysis of type 1 amounts to *subset separation* (those elements in A that are B), type 2 corresponds to *propositional conjunction*. Both types of conjunctive composition are at work in the two types of *kasr* we have examined.

Notice that in the case where components of a Σ -Type are dependent, the elimination rule holds, but each eliminated component will encode the information of the dependence. More precisely, from the compound *all those A that are B* ,

$$(y, d(y): \{y: A \mid d(y): B(y)\})$$

The function $\text{fst}(y, d(y)) = y : A$, renders “those A that witness B within the compound,” and $\text{snd}(y, d(y)) = d(y): B(y)$, renders “those B ’s that are A ’s within that same compound.”

And if we again recall our example of the object of sale, $\text{fst}(y, d(y)) = y : A$ renders “an object of sale, unknown of attribute to the buyer,” and $\text{snd}(y, d(y)) = d(y): B(y)$ renders “objects unknown of attribute which are objects of sale.”

Note that, in fact, interdiction in the sale example has, as its scope, $\text{fst}(y, d(y)) = y : A$, i.e., “those objects of sale that are sold unknown of attribute to the buyer.” Thus, if *this garment* is one specific object of sale (unknown of attribute), then $\text{fst}(y, d(y)) = y : A$ can be read as the anaphoric construction:

“If selling this garment is selling one of those objects unknown of attribute to the buyer, then ‘this’ selling (of the garment) constitutes a law-breaking selling.”

5.3 Dialogical Meaning Explanations

²⁴³ Avicenna, *Kitāb al-Shifā’*: *al-Qiyās*, ed. S. Zayed and I. Madkour (Cairo, 1964), 256.11–15. Reference (and translation) is from T. Street, “Arabic Logic,” in *Handbook of the History and Philosophy of Logic*, Vol. I, eds. J. Woods and D. Gabbay (Dordrecht, 2004), 533.

²⁴⁴ See Ranta, *Type Theoretical Grammar*, 45–46. Notice that $(\forall x: A)B$ and $A \supset B$ express the same proposition. However, if B and A “share a part”, and, more precisely, if B is dependent upon A , we have $(\forall x: A)B(x)$. So Avicenna is right: every implication can be subsumed under the general universal $(\forall x: A)B[x]$, where the square brackets indicate that “ x ,” the “part” of A , might occur or not occur in B .

²⁴⁵ Ranta, *Type Theoretical Grammar*, 43–45.

Rahman and Iqbal (2018),²⁴⁶ led by Young's (2017) *Dialectical Forge*, proposed certain first steps towards a dialogical reconstruction of al-Shīrāzī's notion of parallel reasoning, within the dialogical framework called *Immanent Reasoning*.²⁴⁷ This latter is a framework that integrates the fully interpreted language of CTT into Lorenzen & Lorenz's (1978) dialogical logic, in order to achieve a framework which is sensitive both to content and to the perspective of the meaning play level over the strategic and logical validity level.

Being that we cannot here reproduce all the rules for parallel reasoning developed in Rahman, Iqbal, & Soufi (2019), we will therefore restrict ourselves to the case of *kasr*, which was only very schematically and briefly discussed by the authors in 20 lines, moreover we will only mention the crucial steps in context of the examples discussed in the sources:

5.3.1 Dialogical Meaning Explanation of *Kasr* Type 1

Let us recall al-Shīrāzī's example where the 'illa of R's *qiyās* is a compound 'illa ('illa *murakkaba*) consisting of properties *A* being a saleable object (*mabī'*) and *B* being unknown of attribute (*majhūl al-ṣifa*) to contractor at the time of contract, and the claimed ruling (*ḥukm*) is: not allowed. Q attempts to invalidate R's 'illa by replacing one of its properties, namey *A* with another (*A**, *mankūḥa*: *object of marriage*) and showing that *B* is thus co-present with the opposite of R's *ḥukm* (it is valid) in a seemingly parallel case (marriage): the *kasr*-case. However, Q's case is based on a substitution that changes the meaning, since the substitute carries with it the additional meaning of *option to rescind*. Thus, the dialogical point is the following:

Interlocutor **X** observes that if we delve into the meaning of *B*, *unknown of attribute*, it is apparent that it is only one of the kinds of being unknown of attribute, in contexts concerning transactions of goods, the other being *unknown of attribute with option to rescind* (*khiyār*). Both are different specifications of *unknown of attribute*. So, let *U* stand for *unknown of attribute*, with *B₁* for *unknown of attribute without option to rescind* and *B₂* for *unknown of attribution with option to rescind*. Moreover, **X** asserts that *B₁* and *B₂* yield different rulings. Thus, the original proposition and the substitute do not share the same meaning:

X 'illa(*fst*(*x*)): ~allowed(*fst*(*x*)) (*x*: { *z*: {*y*: *A** | *d*(*y*): *U*(*y*)} | *B₁*(*z*) })

X 'illa(*fst*(*x*)): allowed(*fst*(*x*)) (*x*: { *z*: {*y*: *A** | *d*(*y*): *U*(*y*)} | *B₂*(*z*) })

At this point of the debate **Y** might either concede or demand evidence for the latter assertions. Providing such evidence might commit **X** to finding cases in the sources, and thus adding more occasioning factors.

5.3.2 Dialogical Meaning Explanation of *Kasr* Type 2

In relation to our paradigmatic example of removal, let us recall that in the case studied in the sources an original, obligatory act of ritual worship, the *aṣl* of *wuḍū'*, claimed not to require intent (*niyya*) is replaced by another kind of "substitute" act, the

²⁴⁶ Further developed in Rahman, Iqbal, and Soufi, *Inferences by Parallel Reasoning*, pp. 31-40, and in Iqbal, *Arsyad al-Banjari's Approaches to Rationality*.

²⁴⁷ Cf. Rahman, McConaughy, Klev, Clerbout, *Immanent Reasoning*.

badal, such as *tayammum*, which is an *A* does not require intent either – this should prove that being a *badal* or not has no efficiency in relation to the ruling

Valid *Kasr* Type 2

As discussed above it follows from the logical analysis of claims constituted by a compound antecedent, said to occasion a ruling, that it is the disjunctive distribution of ruling over the claimed occasioning factor that makes challenges by *removal* even possible. The explicit claim has the form

$$\text{X } \textit{illa}(x,y): \textit{hukm}(fst(x,y)) \vee \textit{hukm}(snd(x,y)) \ [(x, y): (A \wedge B)]$$

This shapes the dialogical interaction. Q must attempt to show that property *B* (not being a *badal*) is not efficient, despite being an *A* (*sabab*) and can therefore be removed **and** then proceed to build a destroying case against *A*. The disjunctive distribution, requires to carry out both kind of challenges. R, can escape with a valid response, but simply restoring the removed disjunct. In short, he revives the possibility that property *B* (not being a *badal*) can have efficiency.

Overall Conclusions and Work Ahead

As mentioned, the current paper is only a first step towards a larger study of *kasr*, which will encompass both a comparison between *kasr* and other dialectical objections and analyses of critical discourses from later Muslim legal theorists and dialecticians. In fact, such work should contribute to large-scale research on the notion of illegitimate moves in Islamic Dialectic. Here we will advance, very briefly, some remarks—or better yet, working hypotheses—that might motivate or lead such research.

Fallacious—that is, illegitimate—arguments in a specific field of knowledge are arguments that contravene procedural or meaning-setting rules that prescribe what should count as a sound argument in relation to the conceptual framework of that field. From this point of view, the so-called deontic paradoxes or puzzles, that we have referred to as extrapolation fallacies, find a solution—not by adjusting *ad hoc* some inference rule, but by observing the reasoning patterns for linking concepts established in a specific field.

The setting of what is correct and not correct is implemented by a notion that seems to prefigure what we nowadays in proof-theory call “admissibility.” The Aristotelian theory of the syllogism formulates a general form of this notion.

And one way to interpret the Medieval focus on classifying fallacies relates to the task of identifying one or more admissibility criteria for shaping reasoning patterns. In this light, the Islamic tradition seems to have distinguished admissibility criteria at work in argumentation theory from criteria closer to the perspective on syllogistic fallacies.

The key point, so we conjecture, is that

whereas syllogism-based criteria of admissibility focus on the strategic point of view: namely, that what is admissible is what constitutes optimal moves from the point of view of logical validity;

dialectical-based based criteria of admissibility focus on the play-level. In other words, what is admissible is what allows one to carry out dialectical

interactions according to rules that establish the meaning relevant to the concepts at work in a debate. An inadmissible move is what contravenes the rules on how to debate, not on how to win the debate in regard to any possible move of the contender. Note that this leaves room for valid moves, that is admissible moves, which are not always successful.

Islamic texts wisely include discussions on both dialectical illegitimate moves, with a focus on the play-level, and syllogistic-based illegitimate moves (with a focus on the strategy level of logic). Both points of view are complementary: plays are what strategies are made of. What we should not do is impose a purportedly universal, abstract, strategic admissibility criteria on debates wherein concrete interlocutors dispute content constituted and agreed upon during the interaction. And this is, we dare to suggest, what the dialectical stance in Islamic thought is all about.

The Hypothetical Nature of Legal Liability and its Temporal Dimension Towards a Dialogical Approach to Causation in Law

TALK AT THE CONFERENCE ON FORMAL MODELS OF CAUSATION IN LAW, UNIVERSITY OF
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Abstract: Inferential approaches to legal liability (including cases of pre-emption), embedded in a dialogical framework yield, so we suggest, a quite intuitive implementation of Armgardt's proposal to respond to Moore's (2010) study of cases of causal overdetermination in Law, provided the deployment of a fully interpreted language as the one of Martin-Löf's (1984) Constructive Type Theory is casted in a dialogical framework – rather than in a possible-world one –, whereby (1) hypothetical judgements (involving non-actualized tokens) can be explicitly distinguished from categorical conditional judgements, (2) tokens of actions can be introduced as explicit denizens of the object-language, (3) actual harm/tort events are analysed as tokens dependent upon tokens of their (putative) causes – i.e. as (multi) functions over tokens of their causes, (4) tokens of actions can be “enriched” with a timing function. The paper will be divided in two main parts. . The first one follows Armgardt's strategy to start by establishing collective Liability and proceed afterwards with the identification of individual Liability. The second main part, proposes a framework for the attribution of individual responsibility with regard to the determination of individual Causation. This second way is led by the rationale that legally liable is the one who caused the damage, individually or in association with others.

Introduction

In his challenging paper, *Causation in Law, Overdetermination and Normative Ideal Worlds* (2019), Armgardt pleads for a hypothetical and normative approach to legal liability for causing harm in Penal and Tort Law.

Armgardt's paper is a response to Moore's (2010, pp. 410-425) study of overdetermination, as counterexample to the alleged utility of counterfactual reasoning in order to determine legal liability. Moore's study is based on the following cases involving a group of arsonists (varying in number),

1. A group of arsonists whose actions are independent and sufficient to produce the damage – burning the house.
2. A group of arsonists such that the action of one of them, sufficient for the total damage, comes together with one that is not sufficient but when it joins to the first the total damage results, and even might accelerate the effect.
3. A group of arsonists, such that the action of two out of three of them suffice for causing the total damage.
4. A group of arsonists such that the action of one of them comes “too late” since the action of another in the group, already produced the total damage

Armgardt's (2019) main idea; is to reformulate the counterfactual reasoning underlying the use of the rule *Conditio Sine Qua Non* of Roman Jurists, for these and other cases as follows:

[I]n legal cases the counterfactual conditional should have the following structure:
If all involved agents had acted according to their legal duties, would then the harm have not occurred?

In addition to this, we have to respect the *temporal order* of a case. If harm is done before the second tortfeasor starts to act, he is not liable according to private law (and concerning penal law he can only be punished for an attempt). Only if harm develops over time, the second action can lead to liability according to private law. Armgardt (2019, p. 708).

In that paper a possible-worlds framework is suggested and further developed in Andreas, Armgardt and Gunther (2022) – in short AAG.

In the latter paper two main worries of John Woods (2019, pp. 709-714) concerning Armgardt's (2019) proposal have been addressed, namely:

- (1) if a possible-world framework is assumed, it looks as some criteria of similarity between worlds is needed
- (2) how to determine the individual contribution to the caused harm in cases involving collective actors.

In relation to the first of Wood's worries, the authors of the follow-up paper introduce a notion of relevance order.

To address the second problem, iterated counterfactual conditionals are proposed.

For instance, given the independent actions a_1 , a_2 , a_3 and the tort event e (the burning of a house) – all in the actual world –, the following iteration of counterfactuals is applied

- Were the disjunction of the three actions absent (in some counterfactual world), then the event e would not have happened. Moreover, were the disjunction of the three actions absent, then if the presence of one of those actions, say a_3 , validates the iterated counterfactual $a_3 \square \rightarrow \sim e$, then this action is the one of someone at the wrong place and time, not of an arsonist. In other words, if a_3 has no causal impact on the damage e , following holds:

$$\begin{aligned} \sim(a_1 \vee a_2 \vee a_3) \square \rightarrow \sim e, \\ \sim(a_1 \vee a_2 \vee a_3) \square \rightarrow (a_3 \square \rightarrow \sim e). \end{aligned}$$

In contrast, and assuming again that

- Were the disjunction of the three actions absent (in some counterfactual world), then the event e would not have happened. However, were the disjunction of the three actions absent, then, if the presence of independent actions, say, a_1 and a_2 , validate the iterated counterfactuals $a_1 \square \rightarrow e$, $a_2 \square \rightarrow e$, then those actions are said to have causal impact in relation to the tort event e – thus, a_1 and a_2 are indeed the actions of the arsonists:

$$\begin{aligned} \sim(a_1 \vee a_2 \vee a_3) \square \rightarrow \sim e, \\ \sim(a_1 \vee a_2 \vee a_3) \square \rightarrow (a_1 \square \rightarrow e), \\ \sim(a_1 \vee a_2 \vee a_3) \square \rightarrow (a_2 \square \rightarrow e). \end{aligned}$$

An alternative to iteration is to substitute iteration with the following condition:

- If the disjunction constituted by any proper subset of the original set of putative cases were absent then the event would have taken place anyway. In our example, one of those proper subsets is one that only has as elements actions one and two. Clearly, if the negation of the disjunction constituting this proper subset, does not back the consequent, that is, if it is would be false that e would have happened anyway, then, as before, a_3 can be said to be causally irrelevant in relation to e .

If $\neq \sim(a_1 \vee a_2) \square \rightarrow e$, then a_3 did not cause e

Whereby

$$(a_1, a_2) \subset (a_1, a_2, a_3),$$

$$\sim(a_1 \vee a_2 \vee a_3) \square \rightarrow \sim e, \text{ and}$$

the three actions and event e actually happened

Thus, under these assumptions, the set (a_1, a_2, a_3) fails to satisfy the iteration or its alternative proper subset condition

So far so good, however, in principle this seems to make the use of Armgardt's Normative Ideal Worlds (NIW) redundant. However, in sections 5 and 6 AAG reintroduce NIW as third condition in order to tackle cases of omissions and promises or engagements. For instance, if some individual i promised to perform some action a , but omitted to do so

– i.e. if $\sim a$ is the case–, the individual is causally responsible for the event e , since in every NIW i would keep her/his promise of not omitting to perform a , and the two other conditions are also satisfied. Some other actor j , who did not make such a promise, may omit a . Thus, j 's action/omission is causally irrelevant with regard to the harm e .

As pointed out by AAG (2022, section 7) themselves, the logical analysis of legal liability reasoning does not need a possible-worlds framework. Indeed, the most powerful current approach to causation, namely the one of Halpern and Pearl (2005) – based on an inferential conception of causation by Pearl (2000) –, does not deploy possible worlds at all. However, according to AAG, Halpern-Pearl's theory is, from the point of view of computational complexity, quite demanding and perhaps even too difficult to put into practice by lawyers who have to decide on some concrete cases.

The main claim in the present paper, is that inferential approaches to legal liability (including cases of pre-emption), embedded in a dialogical framework – rather than in a possible-world one –, yield a quite intuitive implementation of Armgardt's proposal of reasoning under the hypothesis of ideal agents, provided

The deployment of a fully interpreted language as Ranta's Type Theoretical Grammar (GTS) – based on Martin-Löf's (1984) Constructive Type Theory (CTT) – casted in a dialogical framework (such as in Rahman et al. (2018)), whereby

1. hypothetical judgements (involving non-actualized tokens) can be explicitly distinguished from categorical conditional judgements,
2. tokens of actions can be introduced as explicit denizens of the object-language,
3. actual harm/tort events are analysed as tokens dependent upon tokens of their (putative) causes – i.e. as (multi) functions over tokens of their causes,
4. tokens of actions can be “enriched” with a timing function that can be put into work in order to sort out actions that came “too late” – i.e. every token of a putative causal action must have a timing anterior to the token of the tort event. This also allows the introduction of spans of time.

These points back the following proposal

- Armgardt's (2019) liability counterfactual, is encoded as a hypothetical verified by a multifunction from the tokens of the collective actions of the presumed wrong doers to the token of relevant harm/tort event. We call it *Normative Ideal Hypothetical* (NIH).
- Individual liability of agent g_i can be defined as a relation (of varying arity), which relates agent g_i , with a token of his own action (and possible too with tokens of the actions of other agents), and the damage attributed to that agent. Individual liability presupposes that NIH includes g_i as one of the agents described in the hypothetical.
- actions causally “irrelevant” to the harm at stake, and which extend the antecedent of the hypothetical, are analysed as involving tokens that do not contribute to the constitution of the (multi)function that define the cause of harm at stake – i.e. irrelevant actions introduce tokens by weakening.
- omissions leading to harm/tort events, are understood as tokens dependent on tokens of actions leading to harm – i.e., as composed functions over such tokens,

N.B. The term “enrichment” has been borrowed from Recanati's (2007a,b) pragmatist understanding of those cases of temporalization that are not obtained by studying events as propositional functions over time, but as dynamic adverbials required by the context. Enrichment processes can be given a logical analysis by deploying a dialogical rendering of Ranta's (1994, p. 108) timing function – see Rahman (2021).

Given this framework different formulations are possible. We will here follow a dialogical meaning explanation, which prescribes how causation claims can be challenged and how they can be defended. In this first exploration Armgardt's (2019) main ideas will constitute a kind of guideline for addressing the main problems to be solved.

The framework, attempts to follow closely the legal reasoning that leads to Armgardt's (2019) proposal. Making all this explicit involves some notational bureaucracy. However, so we claim, this brings the legal insights to the fore.

In fact, the paper is divided in two main sections.

- a. The first one follows Armgardt's strategy to start by establishing collective liability and proceed afterwards with the identification of individual liability.
- b. The second main part, proposes a framework for the attribution of individual responsibility with regard to the determination of individual Causation. This second way is led by the rationale that legally liable is the one who caused the damage, individually or in association with others.

The latter, will be grounded on a reconstruction of an ancient test for legal causation developed within medieval juristic dialectics, called "co-presence and co-absence" (*al-ṭard wa'l- 'aks*) or "concomitance" (*dawarān*) – Young (2019)

This second way, which combines the test of causation with a recently developed reconstruction of medieval deontic imperatives, seems to offer a simple way for determining individual liability out of causation – see Rahman&Zidani&Young (2022).

A

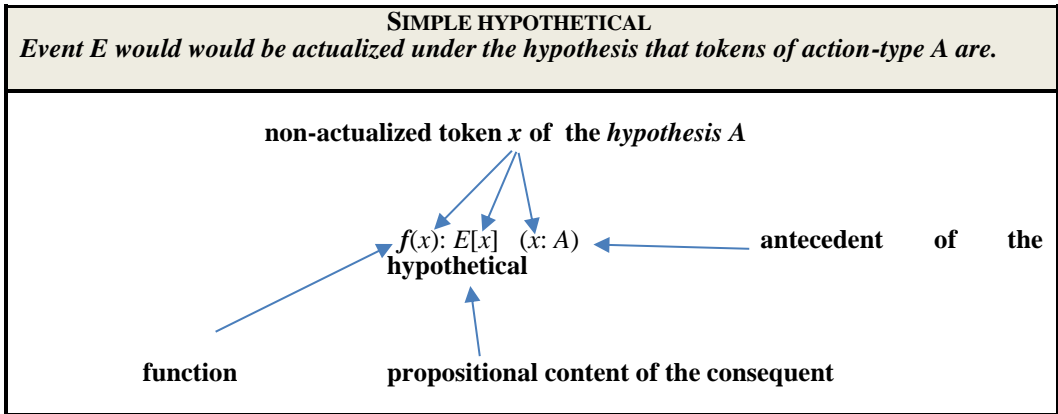
FROM COLLECTIVE TO INDIVIDUAL LIABILITY

A.I Token, Types of Actions and Hypotheticals

Rahman and Iqbal and Rahman, Farid, Young (2022, pp. 152-159),²⁴⁸ deploying the expressivity of the fully interpreted language of Per Martin-Löf’s Constructive Type Theory (CTT) and its further development by Ranta (1994) into a *Type Theoretical Grammar* (TPG), proposed an analysis of imperatives in legal and ethical contexts whereby those imperatives are expressed as hypothetical judgments constituted by action-types and/or action-events.

The verifier of such a hypothetical judgment is a function that takes tokens (the performances of the actions types) of the hypothesis or hypotheses, (i.e the antecedent of the judgment) and yields tokens of the consequent of the hypothetical. In the afore mentioned literature hypotheses were interpreted as constituting the material legal cause, of the legal effect – i.e. of consequent of the hypothetical –, and the function as the legal efficient cause.

A simple hypothetical involving just one type of actions *A* performed by one actor, and the resulting event *E*, as in a CTT-framework the following structure



Now we might wish to make it explicit that the type of action *D* – performing duty *D*, is constituted by tokens carried out by some agent. This yields the set of all those agents how are included in the hypothetical as performing the type of action *D*, for *duty*:

$$\{ y: G \mid D(y) \}$$

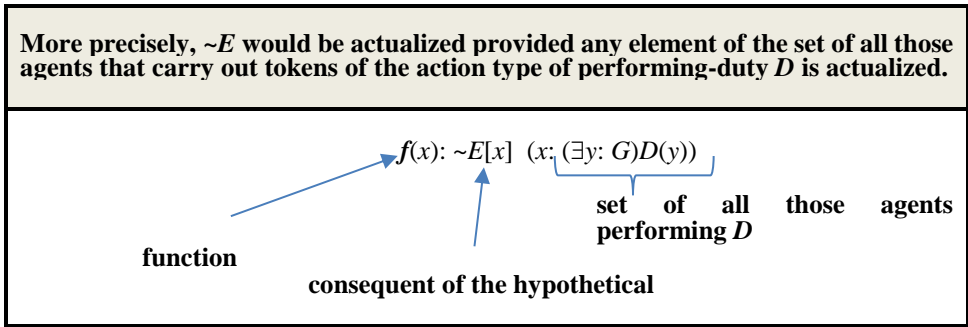
This set is called a Σ -type and it is what an existential amounts to. So, we can write

$$(\exists y: G)D(y)$$

If we interpret the Σ -type as standing for the set of all those agents that carry out tokens of the action type performing-duty-*D*, duty the following **first** approach to Armgardt’s *Normative Ideal Hypothetical* (NIH) results:

NIH-1 <i>~E would be actualized provided any agent g_i involved performs duty <i>D</i></i>

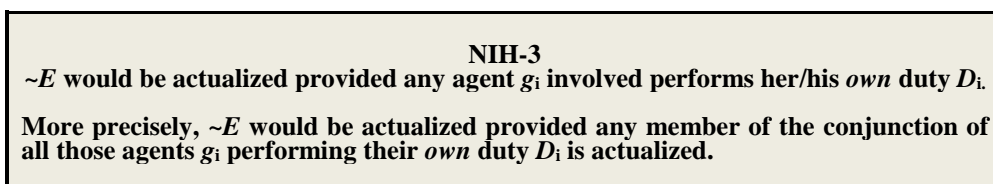
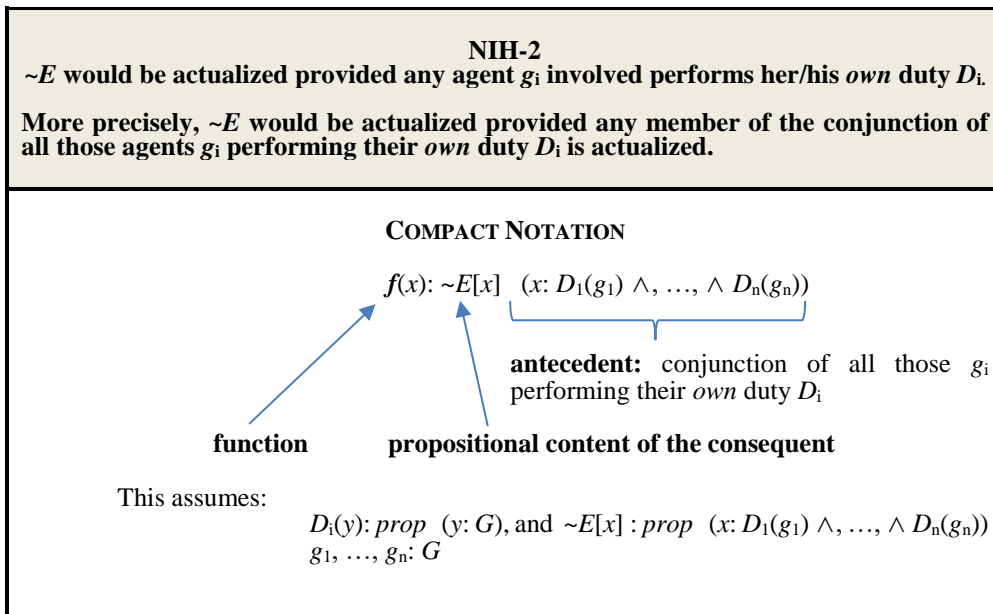
²⁴⁸ See Rahman and Iqbal (2018, 80-84); further developed in Rahman, Iqbal, & Soufi (2019, 31-40), and Iqbal (2022).

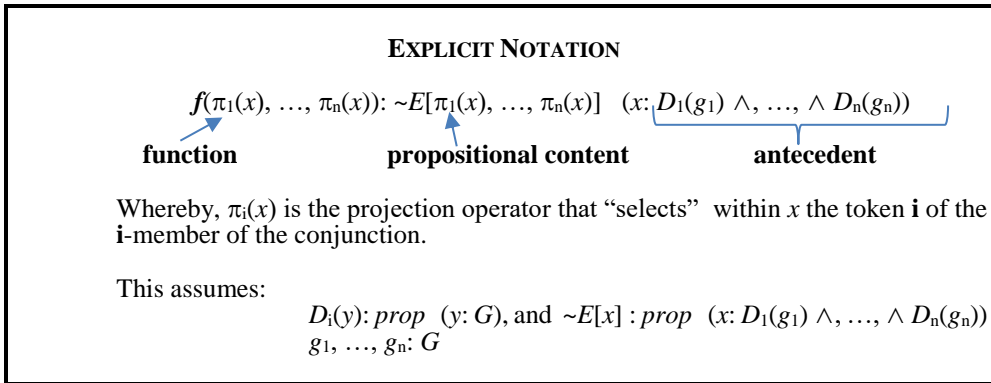


N.B. With *involved*, we mean here those actors performing actions at a place and time that include the location and period where the harm E occurred. In fact, a pragmatic component plays here an important role: the actors included in the hypothetical are those that are *presumed* to be relevant for causing the event E . As pointed out by Armgardt (2019, section 5.5) if a group of agents perform some kinds of actions at the same time, and some harm seems to result, in principle there will be presumption of liability for the whole group. In other words, the constituted of the group will be led by some assumptions on the relevance of their actions for the event at stake. It is now to each actor to defend their case for non-liability.

Notice that we did not use a universal quantifier in order to avoid involving an unrestricted number of agents. Moreover, duty D is some specific duty relevant for the event E not happening.

However, it seems to be desirable to distinguish the duty of each individual agent. For instance, the duty of a fireman in the event of the burning of a house should be distinguished of the duty of some other kind of actor present during that burning. So, assuming that the number of relevant actors can be enumerated we obtain:



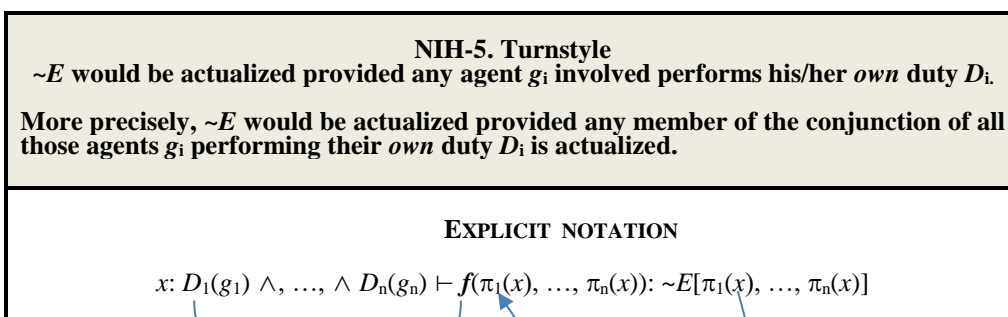
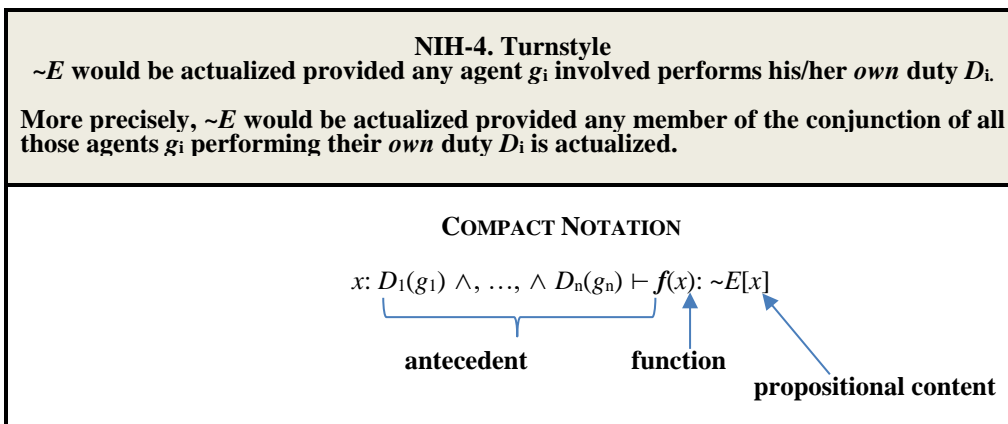


- The multi-function implements Armgardt’s (2019) request that the not-happening of E is the result of the collective performances of the agents involved taken together.

N.B. In the standard CTT-notation, each of the tokens, i.e. the proof-objects, of a conjunction are extracted by means of the left-projection function p , and the right-projection function q . Thus if $c : A \wedge B$, we have $c = \langle p(c), q(c) \rangle : A \wedge B$. If, as in our case the conjunction has more than two members, we need compositions such as $c = \langle p(c), \langle p(q(c), q(q(c))) \rangle \rangle : A \wedge (B \wedge D)$. In order to avoid such bracketings we introduced the projection operators π_i .

We might also wish to specify the time and a locus at which E took place, such that there is both a timing and location function, which determines the constitution of G . In other words, such a specification yields those actors present at some precise time and locus. In order to avoid notational complexity, we will make these parameters explicit dynamically, i.e., they will enrich an assertion when the required by the legal context.

The notation of hypothetical judgments used above, with the hypotheses to the right, follows the original notation of Per Martin-Löf (1984), which focus on the consequent: B would be the case provided the Hypotheses H_1, \dots, H_N . However, sometimes it is useful to use the more familiar back-to-front turnstyle-notation. In the case of formulating NIH we obtain:

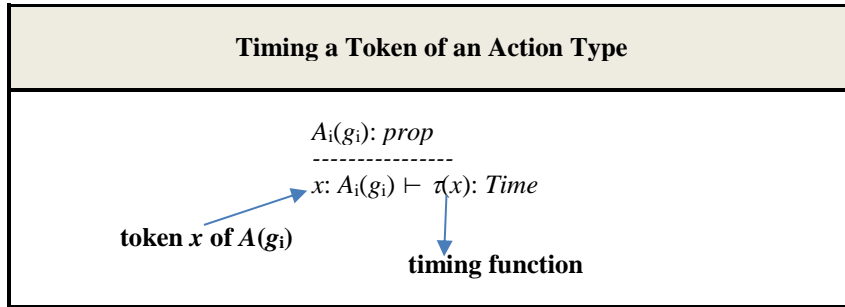


	antecedent	function	propositional content
Whereby			$\pi_i(x)$ is the projection operator that “selects” within x the token i of the i -member of the conjunction
Which assumes			$D_i(y): prop \quad (y: G)$, and $\sim E[x]: prop \quad (x: D_1(g_1) \wedge, \dots, \wedge D_n(g_n))$ $g_1, \dots, g_n: G$

A.II Time Dimension

A. II.1 Timing a Token

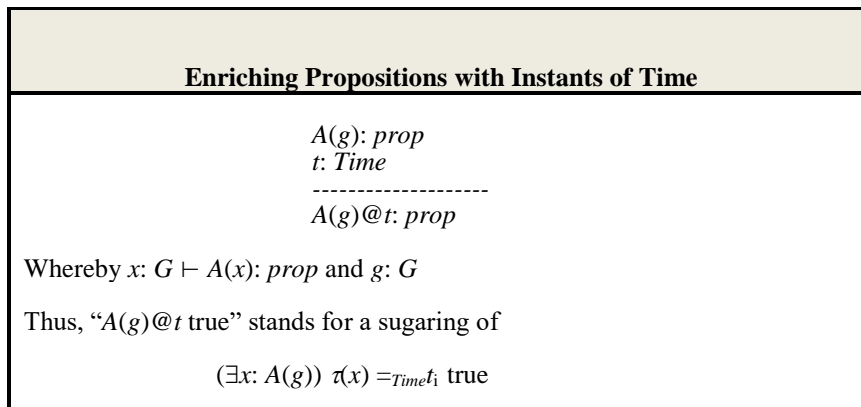
As mentioned in the introduction the idea is to deal with temporal conditions as contextual parameters that can be made explicit in order to *enrich* a proposition that has already content,²⁴⁹ rather than in order to complete the meaning of a propositional function. Thus if “*Time*” stands for some set of instants, “ τ ” for a timing function that takes tokens of some action type and yield instants we obtain:



The assertion g performs an action of the type A at t_i – say, *lighten the fire* –, will be first encoded as

$$(\exists x: A(g)) \tau(x) =_{Time} t_i$$

In this context it is useful to recall Ranta’s (1994, pp. 7-9) notion of *sugaring*, a procedure by the means of which the fully explicit formal encoding undergoes some transformations until the natural language expression results. On our view, sugaring processes have to be built upon some specific contextual background: in this case argumentative interaction, that we call *dynamic encoding*, by the means of which *enriching* will result as an answer to a request for precisizing the time involving some particular token. Enriching has the same effect as adding in natural grammar an adverbial expression, in this case a temporal one.



²⁴⁹ We owe the expression *enrichment* to Recanati (2017).

In other words, “@*t*” occurring in $A(g)@t$ stands for an operator, the formal counterpart of an adverbial, that can be added to a proposition.²⁵⁰

Thus, the sugaring process that yields this enrichment can be (roughly) described as follows:

$$(\exists x: A(g) \ \tau(x) =_T t_i \Rightarrow A(g)@t_i \Rightarrow g \text{ lights the fire at } t_i$$

A.II.2 Spans of Time²⁵¹

An interesting feature of Ranta’s (1994, section 5.1) Type Theoretical Grammar is that it also allows to deal time spans and intervals with a beginning and an end. This is particularly important because actions like moving, running, etc. do not happen in a moment. Rather they should be considered as extended events which happen in temporal intervals. The category of the spans of a time scale T can be defined as the Cartesian product of T and the set of natural numbers \mathbb{N} .²⁵² More precisely:

$$\text{span}(T) = T \times \mathbb{N}$$

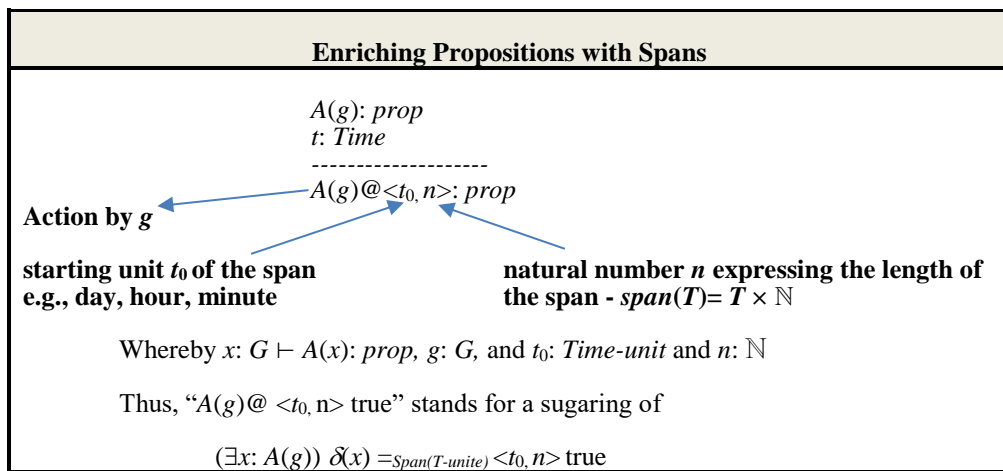
To make it clearer, a span of the time scale T is a pair whose first element refers to the beginning point of that span in T and whose second element refers to the number of temporal units (of the scale T) which must be added to the beginning point to form the span under discussion. Stated differently, the second element determines the length of the span. So if $d = \langle t_0, n \rangle : \text{span}(T)$, d is a span of the time scale T which begins at t_0 and ends at $t_0 + n$. The span d can also be represented as $[t_0, t_0 + n] : \text{span}(T)$. The following functions are also useful:

$$\begin{aligned} \text{left}(d) = \text{begin}(d) &= t_0 : T \\ \text{end}(d) &= t_0 + n : T \\ \text{right}(d) = \text{length}(d) &= n : \mathbb{N} \end{aligned}$$

As an example of the spans of time in the time scale Day , consider the following span:

$$\langle 9 \text{ January } 1991, 365 \rangle : \text{span}(Day)$$

This span of time begins 9 January 1991 and extends for 365 days.²⁵³



²⁵⁰ In our context “@*t*” is not to be read as related to notion of nominal of *Hybrid logic*, since in the latter nominals do not time tokens but rather are to be understood as propositional terms, i.e. they are terms that refer to a proposition such *it is 11 o'clock* – see Blackburn (1993).

²⁵¹ This section is based on Rahman & Zarepour (2021, pp. 421-422).

²⁵² For a detailed technical definition of time spans, see Ranta (1994, p. 115).

²⁵³ It is noteworthy that since 0 is a member of \mathbb{N} , every singular moment of the time scale T can be considered as a span of the length 0. In other word, every t of the time scale T corresponds to $\langle t, 0 \rangle$ which is a member of $\text{span}(T)$. This shows that everything expressible by the terminology of singular moments of time is also expressible by the terminology of time spans, though the other way around does not hold.

A.III The Dialogical Dynamics of Legal Liability

Armgaradt's (2019) main strategy was to determine first liability and then tackle the issue of personal responsibility. In a personal email Armgaradt informed me, that this, so to say two main stages procedure coincides with usual legal practice. That is why, in his first paper Armgaradt starts with a counterfactual having as antecedent everyone one acting according to his duty.

In this context, the claim that the action of an agent is innocent or irrelevant in relation to the harm at stake, but who has been presumed to be liable, is to be proven by the defendant not by the plaintiff.

The dynamic inherent to the dialogical point of view allows quite naturally to implement this two stages procedure. Moreover, the explicit introduction of temporal parameters and so on can be introduced dynamically during the defence: this is what we mean *with dynamic encoding*.

Before describing this let discuss the first of Woods's (2019) worries, namely the problem of similarity-assumption between counterfactual worlds.

A.III.1 Weakening of NIH

The inferential counterpart to the similarity-assumption required by counterfactuals conceived in a possible world-semantics is weakening. Indeed, many of the well-known paradoxes to standard deontic logic come from the fact that weakening to the left, can extend the initial set of premisses to a new set composed with some premise irrelevant to the conclusion – similar can be said to weakening to the right.

In our example, weakening to the left will extend the antecedent with a new conjunct. Now, since our framework the verifiers – the tokens – are explicit, this extension is harmless. Indeed, the new member of the conjunction will have a different verifier to the original one. Let me exemplify this with a simple extension of an obvious hypothetical.

Given the, hypothetical

$$x: A \vdash x: A$$

the following extension (applying weakening and conjunction introduction)

holds

$$x: A \wedge B \vdash f(\pi_1(x)): A$$

What the consequent of the hypothetical expresses is that the hypothetical verification of A , is a method that selects the first member of the hypothetical verifier of the antecedent – that is the verification of the first member of the conjunction $A \wedge B$.²⁵⁴

In other words, the irrelevant extension will be ignored for the justification of the consequent. Similarly, if we have a conjunction of 3 members, say three arsonists

$$x: D_1(g_1) \wedge D_1(g_2) \wedge D_3(g_3) \vdash f(\pi_1(x), \pi_2(x), \pi_3(x)): \sim E[\pi_1(x), \pi_2(x), \pi_3(x)]$$

and extend it to 4 the consequent of the hypothetical can still be verified by the first 3 members

$$x: D_1(g_1) \wedge D_1(g_2) \wedge D_3(g_3) \wedge D_4(g_4) \vdash f(\pi_1(x), \pi_2(x), \pi_3(x)): \sim E[\pi_1(x), \pi_2(x), \pi_3(x)]$$

However, if there is a case for considering the extension as a necessary addition for the verification of the consequent, then the new hypothetical can be asserted

$$x: D_1(g_1) \wedge D_1(g_2) \wedge D_3(g_3) \wedge D_4(g_4) \vdash f(\pi_1(x), \pi_2(x), \pi_3(x), \pi_4(x)): \sim E[\pi_1(x), \pi_2(x), \pi_3(x), \pi_4(x)],$$

²⁵⁴ Certainly, we can also produce the assertion $x: A \wedge B, \vdash right(f(\pi_2(x))): A \vee B$, which selects the hypothetical verifier of the second member of the conjunction and yields a hypothetical verifier for the right of the disjunction. However, notice that the verifier of the disjunction is then then B and A is ignored.

$$\pi_4(x)]$$

Now, perhaps the defendant will claim that the extended hypothetical includes an irrelevant member. This will take the case to plaintiff as discussed further on

A.III.2 Individual Responsibility without Temporal Parameters

As mentioned in the introduction Armgardt’s NI-counterfactual offers in principle a good solution to Moore’s cases and to some others he adds, mainly involving time and one where the presumed liability of one of the actors is contested. However, without refinement the original solution does not distinguish individual responsibility, since it based precisely on the collective liability of all actors.

In the present framework, it looks as if individual liability can be made explicit, by simply singling out the tokens of the action of each agent, and if required enriching them with temporality.

A particular case is omissions the liability of which, should in some cases distinguished from the action “directly” linked to the damage – e.g. the delay of the firemen in the case of the burning house. The idea is to analyse omissions as actions dependent on others.

A.III.2.1 Sufficient and Independent Acts of Harm

Let us start with the simplest case, the one involving independent and sufficient actions.²⁵⁵ This give us too the opportunity to introduce some notational conventions that have been developed in *Immanent Reasoning (IR)* and that will be deployed in our analysis.

$\mathbf{X} ! \dots$, stands for the interlocutor \mathbf{X} claims

Claims commit to further moves, where the claim is backed by some token (in *IR*) such tokens are called *local reasons*), verifying that claim

Thus, if \mathbf{X} claims for example that g_1 performed action A_1 , then the antagonist, might require \mathbf{X} to produce some verification. \mathbf{X} must then produce such a verifier (when the verification has been produced the claim becomes a justified assertion and so the exclamation mark drops out)

Claim	Request	Response
$\mathbf{X} ! A_1(g_1)$	$\mathbf{Y} ?$	$\mathbf{X} b_1: A_1(g_1)$

Coming back to our example. Let us assume that it is agreed by both, Plaintiff and Defendant, that

Three tokens e_1, e_2 and e_3 of the total damage of the type E , have taken place
 Three tokens of independent actions b_1, b_2 and b_3 by three agents g_1, g_2 and g_3 have been performed respectively.

Let us further assume that based on his formulation of the NIH, the Plaintiff claims that each of the three agents are legally collectively liable from the point of view of Penal Law (*PL*), and/or Tort Law (*TL*). Furthermore,

- in order to prove that each agent g_i is individually liable for the total damage produced, the Plaintiff is committed to assert that both, a performance b_i and a

²⁵⁵ Cf. Moore (2010, p. 410).

token e_i , sufficient for the total damage E , can be attributed to the agent g_i , ,
provided the NIH (which involves g_i),

In other words, verifying the assertion on the legal liability of each agent g_i in
relation to E , has the form of a function that associates

the agent g_i ,
the function f : NIH
his performance b_i , of the action $A_i(g_i)$, and
the token e of the event E (sufficient for verifying E)

with the legal liability of that agent in relation to the event E .

We will not include f in the propositional content of the assertion, but only in the
function that verifies the liability. The rationale behind this decision is not simply
notational but it seeks to stress the fact that though verifying an assertion of liability
requires the four components, the **concept of personal liability relates an agent with his
own action and the token of the (total) damage claimed to be of his own
responsibility.**

Assuming $i=3$, the encoding of the verifier of liability has the following form:²⁵⁶

<i>Asserting Agent's g_3 Personal Liability</i>	
Agent Verifier of NIH Token of agent's action Token of the damage E associated to g_3	$g_3: G$ $f: NIH$ $b_3: A(g_3)$ $e_3: E$
$w(\underbrace{g_3, f, b_3, e_3}_{\text{verifier}}; \underbrace{L(g_3, b_3, e_3)}_{\text{relation of personal liability}})$	
Whereby NIH has the form $x: D_1(g_1) \wedge D_1(g_2) \wedge D_3(\mathbf{g_3}) \vdash f(\pi_1(x), \pi_2(x), \pi_3(x)): \sim E[\pi_1(x), \pi_2(x), \boldsymbol{\pi_3}(x)]$	

Such an analysis yields the following general encoding

PERSONAL LIABILITY OF AGENTS OF INDEPENDENT SUFFICIENT ACTIONS OF HARM	
Agreed Assertions	Token of total damage $e: E$ Performances of the Agents $b_1: A_1(g_1)$ $b_2: A_2(g_2)$ $b_3: A_3(g_3)$
NIH	

²⁵⁶ The notation for Personal Liability above is a short-cut, which is closer to the actual dialogical practice, whereby once it is agreed that an assertion is dependent upon an assumption, then the latter is left implicit. The full notation has the form :

<i>Asserting Agent's g_3 Personal Liability</i> <i>Explicit Notation</i>	
$g_i: G, x: D_1(g_1) \wedge D_1(g_2) \wedge D_3(\mathbf{g_3}), f(x): \sim E[x], b_i: A(g_i), z: E \vdash w(\mathbf{g_3}/g_i, f, b_3/g_i, e): L(\mathbf{g_3}/g_i, b_3/b_i, e).$ Alternatively, using the abbreviation NIH for the antecedent: $! NIH \vdash w(\mathbf{g_3}/g_i, f, b_3/g_i, e): L(\mathbf{g_3}/g_i, b_3/b_i, e).$	

Plaintiff's Claims	$\mathbf{X} x: D_1(g_1) \wedge D_1(g_2) \wedge D_3(g_3) \vdash f(\pi_1(x), \pi_2(x), \pi_3(x)): \sim E[\pi_1(x), \pi_2(x), \pi_3(x)]$ Personal Liability $\mathbf{X} ! PL(g_1, b_1, e) \wedge PL(g_2, b_2, e) \wedge PL(g_3, b_3, e)$ $\mathbf{X} ! TL(g_1, b_1, e) \wedge TL(g_2, b_2, e) \wedge TL(g_3, b_3, e)$
Plaintiff's Commitments	$\mathbf{X} ! NIH \vdash w(g_1, f, b_1, e): L(g_1, b_1, e)$ $\mathbf{X} ! NIH \vdash w(g_2, f, b_2, e): L(g_2, b_2, e)$ $\mathbf{X} ! NIH \vdash w(g_3, f, b_3, e): L(g_3, b_3, e)$ Provided an NIH relevant to these agents, tokens of actions and tokens of harm, holds.

If we wish to make explicit a further step that describes how claims of personal liability determine the distribution of the costs of the damage among the agents, a new relation, say the Payment-Duty relation, can be formulated. But we will leave this out for the moment. However, in the following cases such a relation beyond the one of personal liability does not seem to be necessary

A.III.2.2 Collective Acts of Harm

A.III.2.2.1 Mixed Cases Involving Acts of Harm

In such form of mixed collective acts of harm called *asymmetrically overdetermined concurrent-cause cases*, both a sufficient and an insufficient cause come together, e.g. when a sufficient fire merges with another fire that is not individually sufficient to cause total harm.²⁵⁷

Some of the examples for mixed cases brought up by Moore (2011, p. 411) suggest that time considerations can make a difference since the merging of two actions might cause the harm to be produced faster, e.g. two wounds, a small and major one, such that the victim would have died from the major one alone but the merging of the smaller one had an *accelerating* effect on the death. Accelerating effects, will be discussed when we include temporal parameters, though there is an argument to make for the claim that accelerating effects might have an impact when assessing the individual liability of the agent who performed an action insufficient for producing total damage.

Following Armgardt (2019) we start by establishing the collective liability of all the agents involved by means of a suitable formulation of the NIH. The NIH has the same structure as in the case before. What changes is the distribution of Personal Liability that amounts to the conjunction of

1. the liability of the first agent in relation to his (sufficient) performance b_1 and the token e of total damage E
2. the liability of the second agent in relation to his (insufficient) performance b_2 and the token e' of partial damage E ; and
3. the liability of the second agent in relation to his (insufficient) performance b_2 , g_1 's performance b_2 and the token e of total damage E

Such a conjunction determines the commitments of the Plaintiff, which amount to verifying each of the assertions constituting it. Clearly,

- the first member establishes the liability of the first agent in respect to the total damage E ,
- the second the liability of the second agent in relation to the partial damage

²⁵⁷ Cf. Moore (2010, p. 411).

associated to his performance,
and the third of the liability, again of the second agent, but this time in relation to the total damage associated to the merging of his own performance with the one of the first agent.

The latter could be seen as a first approximation to claims of accelerating effect. So, whereas the second assertion associates the second agent with liability to a lesser damage, the latter can be seen as indicating that the merging of both performances makes the second agent liable for accelerating the process that led to total damage. This is displayed in the following table.

PERSONAL LIABILITY MIXING SUFFICIENT AND INSUFFICIENT ACTIONS OF HARM	
Agreed Assertions	Token of total damage $e: E$ Tokens of partial damage $e': E_1$ Performances of the agents $b_1: A_1(g_1)$ $b_2: A_2(g_2)$
Plaintiff's Claims	<p style="text-align: center;">NIH</p> <p style="text-align: center;">$\mathbf{X} x: D_1(g_1) \wedge D_1(g_2) \vdash f(\pi_1(x), \pi_2(x)): \sim E[\pi_1(x), \pi_2(x)]$</p> <p style="text-align: center;">Distribution of Personal Liability</p> <p style="text-align: center;">$\mathbf{X} ! L(g_1, b_1, e) \wedge L(g_2, b_2, e') \wedge L^*(g_2, b_2, b_1, e', e)$</p> <p style="text-align: center;"> ↑ Partial Liability ↑ Accelerating Liability </p>
Plaintiff's Comitments	<p style="text-align: center;">$\mathbf{X} ! \text{NIH} \vdash w(g_1, f, b_1, e): L(g_1, b_1, e)$ $\mathbf{X} ! \text{NIH} \vdash w(g_2, f, b_2, e'): L(g_2, b_2, e')$ $\mathbf{X} ! \text{NIH} \vdash u(g_2, f, b_2, b_1, e): L^*(g_2, b_2, b_1, e', e)$</p> <p style="text-align: center;">Provided an NIH relevant to these agents, tokens of actions and tokens of harm, holds.</p>

A.III.2.2.2 The Merging of the Actions of Subgroups of Agents

In this case, the performance of each individual is insufficient for producing the total damage, however some subgroups build a sufficient cause of harm. For example, three equally sized fires join to burn the plaintiff's house, when any two of them would have been sufficient to produce the total damage – Moore (2010; p. 412)

The NIH has the same structure as in the case before and as pointed out by Armgardt (2019, section 5.2). Indeed, according the NIH here too all of the three tortfeasors are liable, because if all involved agents had acted according to their legal duties, the harm – i.e. the burning of the house –, would have not occurred.

What changes is the distribution of Personal Liability that amounts to the conjunction of

1. the liability of the first agent in relation to his (sufficient) performance b_1 and the token e' of partial damage E ;
2. the liability of the second agent in relation to his (insufficient) performance b_2 and the token e'' of partial damage E ; and
3. the liability of the third agent in relation to his (insufficient) performance b_2 and the token e''' of partial damage E ; and

4. the liability of some agent g_i in relation to his own performance b_i , the performance of b_j of one of the other two agents, and the token e of total damage E .

Thus, each of the agents is Liable with regard to the combination of his/her own action with the action of any other of the agents.

PERSONAL LIABILITY I MERGING SUBGROUPS OF INDIVIDUALLY INSUFFICIENT ACTIONS OF HARM	
Agreed Assertions	<p>Token of total damage $e: E$</p> <p>Partial Damages E_i $E \equiv E_i \wedge E_j$</p> <p>Tokens of partial damage $e': E_1$ $e'': E_2$ $e''': E_3$</p> <p>Performances of the agents $b_1: A_1(g_1)$ $b_2: A_2(g_2)$ $b_3: A_3(g_3)$</p>
Plaintiff's Claims	<p style="text-align: center;">NIH</p> <p style="text-align: center;">$\mathbf{X} x: D_1(g_1) \wedge D_1(g_2) \wedge D_3(g_3) \vdash f(\pi_1(x), \pi_2(x), \pi_3(x)): \sim E[\pi_1(x), \pi_2(x), \pi_3(x)]$</p> <p style="text-align: center;">DISTRIBUTION OF PERSONAL LIABILITY</p> <p style="text-align: center;">$\mathbf{X} ! L^*(g_i, b_i, b_j, e) \quad j \neq i$</p> <p>Whereby in the last member of the conjunction “g_i” stands for one of the three agents, “b_i” for the token of the action of that agent, and “b_j” for the token of the action of one of the two other agents.</p>

Since the actions of any pair of agents out of the three produces the total damage at stake, each of the agents is personally liable, for the total damage with regard to merging of his own action with the one of the others.

PERSONAL LIABILITY II MERGING SUBGROUPS OF INDIVIDUALLY INSUFFICIENT ACTIONS OF HARM	
Plaintiff's Commitments	<p style="text-align: center;">$\mathbf{X} ! \text{NIH} \vdash u(g_i, f, b_i, b_j, e): L^*(g_i, b_i, b_j, e) \quad j \neq i$</p> <p>Provided an NIH relevant to these agents, tokens of actions and tokens of harm, holds.</p>

A.III.2.2.3 The Merging of the Actions of all Agents

This case, discussed in AAG (2022, section 6) describes a scenario where the majority of the members of the executive board of company votes in favour of producing and selling one of their products, despite the fact that each member of the board knows for certain that the product is harmful.

Strictly speaking the collective liability that comes out from the NIH is sufficient, the liability in the context is collective after all. However, we might wish to make the inner structure of the distribution of this collective liability over each agent apparent. In principle this distribution it is quite close to the preceding case of subgroups, however in such a scenario it does not seem to make sense to single out some kind of partial damage, of each of the involved

agents. Rather it each agent is responsible for the total damage caused by the her/his vote. Thus, we can capture personal liability of each agent with one general form for the liability relation, which relates

(for any) g_i his own performance b_i , the performances of b_j, \dots, b_k of **all** the other agents, and the token e_i of total damage E ,

For the sake of simplicity, in the table below, we assume only three agents, each of them, voted in favour of producing the damage. If we wish to add say one or two voters that voted against continuing the production, the case amounts to the one of weakening described above.

COLLECTIVE MERGING OF ACTIONS OF HARM	
Agreed Assertions	<p>Tokens of total damage $e: E$</p> <p>Performances of the agents $b_1: A_1(g_1)$ $b_2: A_2(g_2)$ $b_3: A_3(g_3)$</p>
Plaintiff's Claims	<p>NIH</p> <p>$\mathbf{X} x: D_1(g_1) \wedge D_1(g_2) \wedge D_3(g_3) \vdash f(\pi_1(x), \pi_2(x), \pi_3(x)): \sim E[\pi_1(x), \pi_2(x), \pi_3(x)]$</p> <p>DISTRIBUTION OF PERSONAL LIABILITY</p> <p>$\mathbf{X} ! L^{**}(g_1, b_1, b_2, b_3, e) \wedge L^{**}(g_2, b_2, b_1, b_3, e) \wedge L^{**}(g_3, b_3, b_1, b_2, e)$</p>
Plaintiff's Commitments	<p>$\mathbf{X} ! \text{NIH} \vdash v(g_1, f, b_1, b_2, b_3, e): L^{**}(g_1, b_1, b_2, b_3, e)$</p> <p>$\mathbf{X} ! \text{NIH} \vdash v(g_2, f, b_2, b_1, b_3, e): L^{**}(g_2, b_2, b_1, b_3, e)$</p> <p>$\mathbf{X} ! \text{NIH} \vdash v(g_3, f, b_3, b_1, b_2, e): L^{**}(g_3, b_3, b_1, b_2, e)$</p> <p>Provided an NIH relevant to these agents, tokens of actions and tokens of harm, holds.</p>

A.III.2.2.4 Being at the Wrong Time and Place

The case where some innocent agent has been included in the liability claim takes an important place in AAG (2022). In this context, Armgardt (2019, section 5.5) describes a scenario where there is uncertainty on the legal liability, concerning harm E , of some of the members of a group of agents performing kinds of actions at the same time, presumed to contribute to that harm.

- Let assume that it is agreed by both, Plaintiff and Defendant, that a token e of the harm-type E has taken place and tokens of the agents g_1, g_2 and g_3 performing respective actions at the same time
- Let us further assume that based on his formulation of the NIH, the Plaintiff claims that that three are legally liable. But then the Defendant claims the non-liability of, say, agent three. This commits the Defendant ought to produce some verifier for his claim.

There are two strategies, perhaps even two moments both of them amount to the Defendant's claim of the irrelevance of his duties and actions in relation to the attributed damage. So, the first defence, that underlies Armgardt's focus on the NIH, is the simplest one and is based on the irrelevance of his own duties, on the irrelevance of his (perhaps dangerous actions).

First defence

The defendant claims that his own duties are irrelevant. In other words, the defendant claims that the NIH has the following form, which coincides with a case of weakening – we deploy once more our main example involving three agents (arsonists):

Defendant $x: D_1(g_1) \wedge D_1(g_2) \wedge D_3(g_3) \vdash f(\pi_1(x), \pi_2(x)): \sim E[\pi_1(x), \pi_2(x)]$

Notice that the point is here that defendant is claiming that carrying out his own duties is irrelevant for the absence of damage E . More precisely, the defendant points is that the meaning constitution of the $\sim E$ is NOT dependent upon the meaning of $D_3(g_3)$. Thus, we have the following

Meaning Constitution of Plaintiff's NIH	Meaning Constitution of Defendants's NIH First Defence
$D_1(g_1) \wedge D_1(g_2) \wedge D_1(g_3): prop \vdash \sim E: prop$ Which yields $x: D_1(g_1) \wedge D_1(g_2) \wedge D_3(g_3) \vdash f(\pi_1(x), \pi_2(x), \pi_3(x)): \sim E[\pi_1(x), \pi_2(x), \pi_3(x)]$	$D_1(g_1) \wedge D_1(g_2): prop \vdash \sim E: prop$ Which yields $x: D_1(g_1) \wedge D_1(g_2) \vdash f(\pi_1(x), \pi_2(x)): \sim E[\pi_1(x), \pi_2(x)]$ and by weakening $x: D_1(g_1) \wedge D_1(g_2) \wedge D_1(g_3) \vdash f(\pi_1(x), \pi_2(x)): \sim E[\pi_1(x), \pi_2(x)]$

Second defence

- If the actions under examination are considered to be independent and sufficient actions, the Defendant's claim on the irrelevance of g_3 -actions is to be justified by showing that it is not the case that a token of E is *dependent upon* any token of action $A_3(g_3)$.
- If the actions at stake are considered to collectively cause E , such that only through the joint action of each agent harm E results, then, the Defendant's claim on the non-liability of agent three is to be justified by showing that it is not the case that E is *dependent upon the pairs of actions* $(A_1(g_1), A_3(g_3))$, $(A_2(g_2), A_3(g_3))$ and $(A_1(g_1), A_2(g_2), A_3(g_3))$. This yields the following table
Commitment

Second Defence Irrelevance of g_3 -Performances in the Context of Independent and Sufficient Actions of Harm	Second Defence Irrelevance of g_3 -Performances in the Context of Collective Actions of Harm
Defendant's Commitment $d: \sim(\exists x: A_3(g_3))E(x)$	Defendant's Commitments $d_1: \sim(\exists x: A_3(g_3) \wedge A_1(g_1))E(\pi_1(x), \pi_2(x))$ $d_2: \sim(\exists x: A_3(g_3) \wedge A_2(g_2))E(\pi_1(x), \pi_2(x))$ $d_3: \sim(\exists x: A_3(g_3) \wedge A_1(g_1) \wedge A_2(g_2))E(\pi_1(x), \pi_2(x), \pi_3(x))$

Notice that this can be combined with the first defence.

A.III.2.2.5 Omission as not Preventing

The authors of the AAG paper take the view that omissions are some special kind of process rather than pure negativity. True, many philosophers think of negative acts in

parallel to negative facts and since the latter is usually denied so the former.

However, I think that in the realm of actions, there is conceptually and logically a point to make in favour of considering omissions to constitute some kind of positive actions.

Generally speaking, the agent of an omission is legally liable, since the agent had the choice between preventing some action/event to happen or not doing so. According to this view, there are three main components in an omission, namely

- The object of an omission act is another act or act or event
- The content of an omission act is to not preventing this other act/event from happening
- It is in the agent's *capacity* to prevent that other act from happening

In relation to the third component, the proviso on the agent's capacity to prevent, the point is that the agent had the choice between preventing or not preventing. This is the source of the agent's liability.

In relation to the second point, from a purely deductive point of view, a negation is conceived as a process that leads to the abortion of another process.²⁵⁸ If we focus on actions rather than on processes, the negation of an action is seeing to it that the action does not succeed. Furthermore, according to our view, the agent's preventing an action from being carried out, presupposes the agent, having the choice between seeing to it that this action does or not succeed.

In relation to the first point, and in the context of the other two, an omission amounts to not to prevent another action (the object of the omission act) from succeeding.

The Preventing Relation
<ul style="list-style-type: none"> • If preventing, is preventing the action of <i>someone else</i>, we need to pass from the connective level to a relation. (with connective level, we mean the negation understood as an implication such as $A \supset \perp$, that admits the interpretation A is aborted) • Thus, the meaning of the Prevention relation is shaped on one hand by the <ol style="list-style-type: none"> i. notion of "blocking", which characterizes the deductive meaning of negation; on the other ii. it is shaped by the pragmatist stance of being an action which aims at blocking or aborting another event or action. • Omissions are then defined as instances of not Preventing.

For the sake of simplicity let us consider only two agents,

Our known arsonist g_1 , and the firemen (all collectively) g_2 , who omitted to come (on time), after the arsonist started the fire and before the total damage was produced.

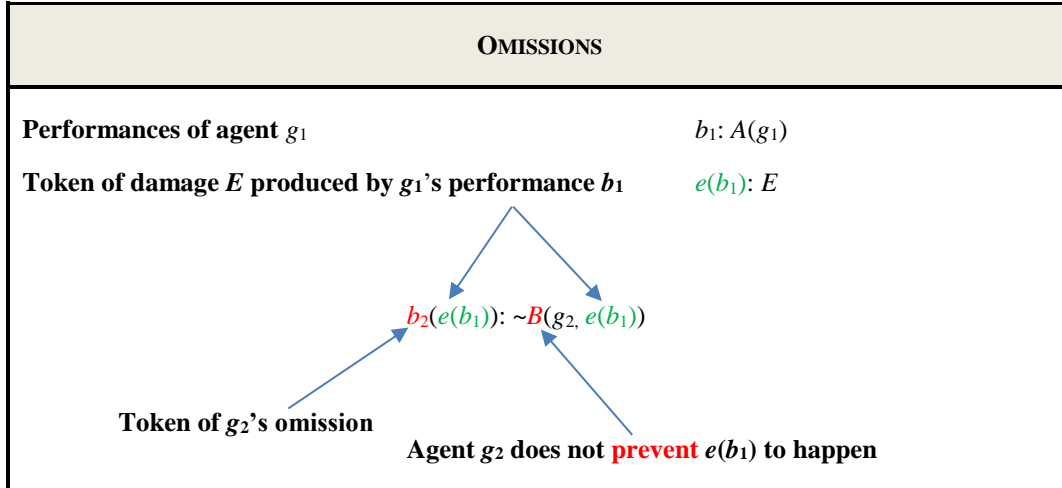
Let us render the token of the arsonist action as

$b: A(g_1)$, and the omission of the firemen as

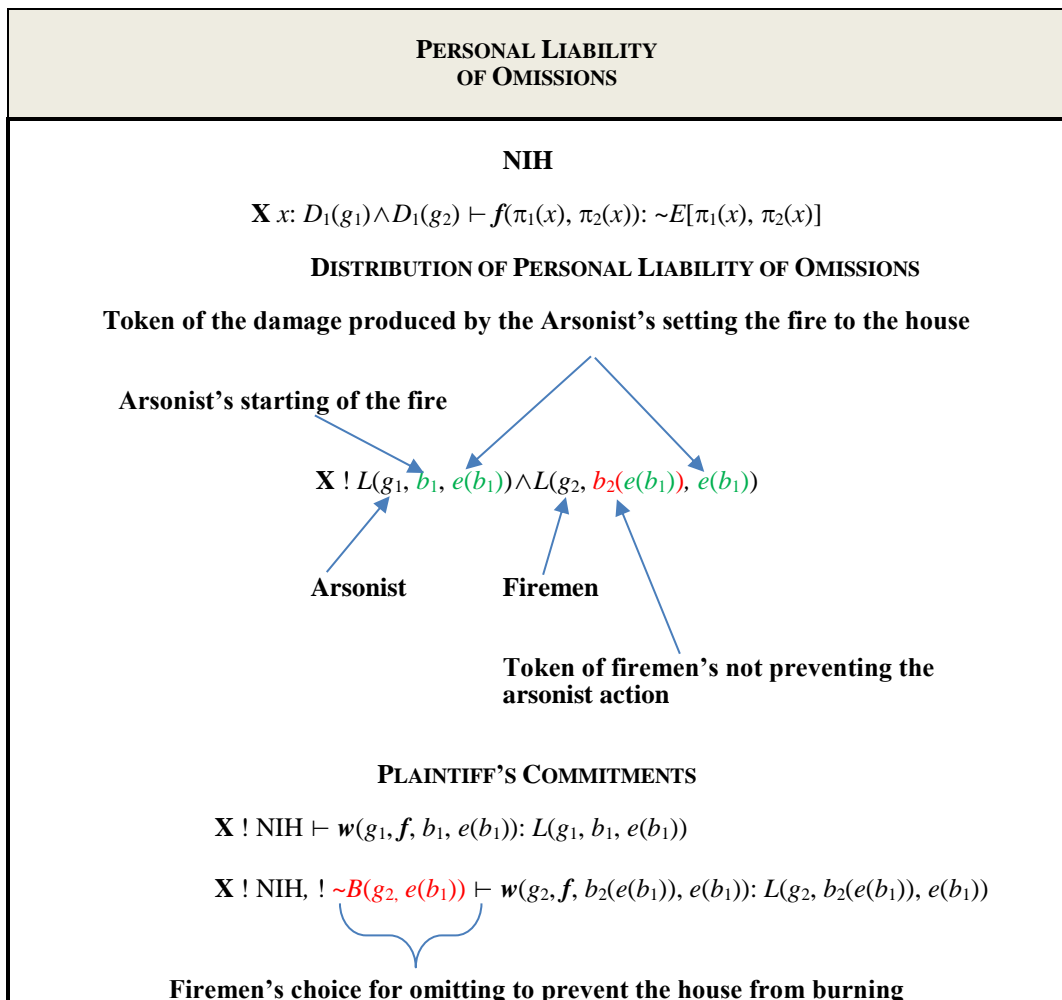
$b_2(e(b_1): \sim B(g_2, e(b_1)))$, whereby " $B(g_2, e(b_1))$ ", stands for " g_2 preventing damage e dependent upon g_2 's performance b ":

²⁵⁸ From the deductive point of view a negation amounts to the implication $A \supset \perp$ true. The proof object of this implication is an operation on a function from A to *Falsum*, such that, when executed, for any proof-object that attempts to proof A a *halt* is produced: $\lambda(x)u(x): A \supset \perp$

Moreover, in order to implement the idea that Preventing is a capacity, we can think of it as the result of a choice, in our example, between preventing or not the house from the burning triggered the arsonist's action. The source of the liability of the firemen is indeed this choice. Since in section B, we will develop a general form for liability that result from choices, I will here simply add as an assumption for the assertion of liability by omission, that, the choice has been already drawn for not Preventing.



Clearly, not preventing the burning (induced by the arsonist to happen) from happening is not the same as actually setting the fire to the house: the firemen's omission is not identical to the arsonist action: the liability of the firemen is dependent upon the arsonist's action after all.



Clearly, an even finer analysis would include time parameters this leads to the next section

A.IV Individual Responsibility and its Temporal Dimension

Strictly speaking, one might think that time parameters are relevant for all the cases discussed above, not only for pre-emptive ones. We might also add locative parameters. But we follow here a dynamic approach according to which temporal and locative parameters are made explicit when required by the context.

A.IV.1 Pre-emption: Liable for Penal Law but Too Late for Private/Tort Law

A pre-emptive case of causation is one where two performances poised to do some damage, each being individually sufficient to do so, are temporally ordered in such a way, that one of both performances, when executed first, pre-empts the capacity of the other one to cause the harm. For instance, two fires independently set, each sufficient to burn the plaintiff's house, do not join. One fire reaches the house first and burns it to the ground, and then the second fire arrives but there is no house to burn – cf. Moore's (2010, p. 412).

As pointed out by Armgardt (2019), both are liable according to *Penal Law* – which does not require the final object of the harmful action to be achieved, but, the second, the *too late* tortfeasor, is not liable according to Private/Tort Law.

In our framework the reasoning is quite straightforward if we decline it in three main steps:

1. it is agreed that according to NIH, would both agents have done their duty, no harm would have resulted
2. it is claimed (we assume, by the defendant), that the second agent is not liable according to Private/Tort Law, since the token of the second agent's actions, arrived after the time acknowledged to correspond to the damage
3. if the second step has been agreed, then it is asserted that the first agent is liable according to both Penal and Private/Tort Law, but the second only according to Penal Law

Shared Assertions	First Main Step NIH	Second Main Step Defendant Y	Third Main Step Concluding Assertions
$e: E@t_j$ $b_1: A_1(g_1)$ $b_2: A_2(g_2)$	$X \quad x: D_1(g_1) \wedge D_1(g_2) \vdash f(\pi_1(x), \pi_2(x)):$ $\sim E[\pi_1(x), \pi_2(x)]$	$Y ! \sim TL(g_2, b_2, e)$ $X ?$ $Y ! b_2: A_2(g_2)@t_{>t_j}$	$X, Y ! NIH \vdash w(g_1, f, b_1, e_1):$ $PL(g_1, b_1, e)$ $X, Y ! NIH \vdash w(g_1, f, b_1, e_1):$ $TL(g_1, b_1, e)$ $X, Y ! NIH \vdash w(g_2, f, b_2, e_2):$ $PL(g_2, b_2, e)$

A. IV.2 Physis Strikes. Liable for Penal Law, but Partially Liable for Private/Tort Law?

Armgardt (2019, section 5.4) discusses an interesting additional case involving temporality, assume that the token of an action and a token of a natural event, say, a lightning bolt, are each being individually sufficient to set the fire to the house. Is the agent liable according to Private/Tort Law?

Armgardt suggests in these paragraphs, that since it seems sensible to grant the insurance company that insured the house, to claim some form of liability of the arsonist (according to Privat/Tort Law), the Duty hypothesis requires some form of reformulation.

In our framework, the reasoning, declined again in three main steps, leads to the conclusion that the arsonist partially liable for the damage according to private Law, provided there is some significant span of time between the agent's setting of the fire to the house and the lightning bolt – if both the natural event and the agent's start of the fire are either simultaneous or quite close, it seems that no liability according to Private/Tort Law can be established. The main steps leading to such a conclusion are the following

- a. it is agreed that according to (the **modified**) NIH, if it is the case that the agent has been done his duty, and it is also the case that **the natural event ($\phi: \Phi$) did not happen**, then no harm would have resulted.
- b. it is claimed (by the insurance company), that the second agent is liable according to Private/Tort Law for the for the partial damage $e': E'$ occasioned during the span of time between his starting of the fire and the instant at which the natural event happened
- c. if the second step has been agreed, then it is asserted that the first agent is liable according to both Penal and Private/Tort Law. Moreover, according to Private/Tort Law the agent is liable for the period established in the previous step.

For the sake of simplicity, we will not explicitly indicate the time when the final damage took place but we assume that it happened at the same time as or immediately after the lightning struck. We further assume that the unity of the span of time are hours.

Shared Assertions	First Main Step NIH	Second Main Step Insurance's Claim Y	Third Main Step Concluding Assertions
Total Damage $e: E$ Partial Damage $e': E'$ Arsonist's Action $b_1: A_1(g_1)$ Natural Event $\phi: \Phi$	$X \ x: D_1(g_1) \wedge \sim \Phi \vdash f(\pi_1(x), \pi_2(x)): \sim E[\pi_1(x), \pi_2(x)]$	$X \ ! TL(g, b, e')_{@< \bar{t}_i, \bar{t}_i+(n=k-1) >}$ <div style="color: red; text-align: center;"> Amount of time passed before the natural event occurred. </div> $Y ?$ $X \ ! \phi: \Phi_{@ \bar{t}_k}$ <div style="color: red; text-align: center;"> Time at which the natural even happened </div> Whereby $x: TL(g, b, e') \vdash \delta(x): Span(hour)$	$X, Y \ ! NIH \vdash w(g_1, f, b_1, e): PL(g_1, b_1, e)$ $X, Y \ ! NIH \vdash w(g_1, f, b_1, e'): TL(g, b, e')_{@< \bar{t}_i, \bar{t}_i+(n=k-1) >}$

A. IV.3 Some More Cases Involving Temporal Order

In fact, Armgardt (2019, section 5.7) records two more cases where temporality is involved.

In *case 1*, A bumps into the car of C and destroys the car's left front light. An hour later, B bumps into the same car and destroys it completely. What about the liability of A according to Private Law?

In *case 2*, A causes an accident by which B loses his eyesight. Since B has a serious disease, he would have lost his eyesight anyway within the next three years. What about the liability, according to Private Law, of A if B is not able to work anymore? Does he or she have to pay a disability pension in favour of B for 3 years only or does he or she have to pay until B dies?

In relation to the first case, Armgardt (2019, *ibidem*) suggests that the analysis must involve the temporal order of both actions. According to our own view, this is indeed the case and the argument runs as follows:

1. If both would have done their duty, no damage would have happened (NIH)
2. The first agent (in Armgardt's example, agent *A*) is liable for the partial Damage that happened *before* the second agent's (in Armgardt's example, agent *B*) bumping into the car. Thus, agent *A* is liable for the car's front light, but not for the total damage of the car.
3. The second agent is liable for the total Damage that happened *before* the second agent's (in Armgardt's example, agent *B*) bumping into the of the car.

Perhaps one could think of a more complicated situation, for instance, if the second agent bumps into the car because it was night and because of the damage produced by the first agent made it impossible to see the car. In such a case, in our framework, the second agent would not be liable. Or perhaps even only by some kind of liability applied to involuntary actions of harm. The logical point is that the second action is dependent upon the first one:

$$x: A_1(g_1)_{@t' < t} \vdash b(x): A_2(g_2)_{@t}$$

In case 2, Armgardt (2019, *ibidem*), concludes that the tortfeasor has to pay only for the three years that "accelerated" the loss of the eye. In our framework, the conclusion coincide with the one of Armgardt, and is lead by an argument similar to the case of the strike of nature. This argument requires also a modification of the NIH. In other words, in our framework, the conclusion follows from the following strategy:

1. If the tortfeasor would have done his/her duty, and **no disease would have affected the victim's eye**, no damage would have happened (NIH)
2. The tortfeasor is liable for the loss of the eye, but this liability is restricted to the span that starts at the time the tortfeasor started the tort and ends, the time at which the disease would have started

The difference to the case of the strike of nature is that in the case of the disease, the damage produced by the disease will not be actualized – assuming the disease is restricted only to the eye. More precisely, g_2 's disease D – whereby g_2 is the victim, would not have started at, say, instant t , if the tortfeasor g_1 would not have carried out action $A_1(g_1)$ before t :

$$x: \sim(A_1(g_1)_{@t' < t}) \vdash c(x): D(g_2)_{@t}$$

N.B. as before, this assumes that all weakening's are irrelevant for the start of the disease

B

FROM CAUSES TO INDIVIDUAL LIABILITY

A PLAIDOYER FOR THE FRUITFULNESS OF HISTORIC STUDIES FOR THE DEVELOPMENT OF CONTEMPORARY LEGAL REASONING

B.I Recovering an Ancient Test for Legal Causation²⁵⁹

In Islamic legal theory a set of methodological tools for legal reasoning were developed, whose elements, valid modes, and proper applications were the focus of continual argument and refinement by Sunnī jurists.²⁶⁰ This centuries-long discourse constitutes a highly developed contribution of the argumentative—more precisely, dialectical—approach to legal reasoning within Islamic Law. A particularly lucid, early output was systematic work of renowned Shāfi‘ī dialectician and legal theorist Abū Ishāq al-Shīrāzī (1003-1083 CE).

A cardinal feature of al-Shīrāzī’s take on legal reasoning is his particular notion of efficiency (*ta’thīr*) which tests whether the property *P* purported to be efficient in occasioning the juridical ruling at stake is indeed so. For al-Shīrāzī, *ta’thīr* consists of two complementary procedures:

- co-presence (*ṭard*): whenever the property is present, the ruling is also present, and
- co-absence (*‘aks*): whenever the property is absent, the ruling is also absent.

While co-presence examines whether ruling *H* is present along with property *P*, co-absence examines whether ruling *H* is absent along with property *P*.²⁶¹

This test of a property’s causal efficiency is elsewhere and more commonly called “co-presence and co-absence” (*al-ṭard wa-l-‘aks*) or “[causal] concomitance” (*dawarān*), and listed among the “modes of causal justification” (*masālik al-ta’līl*) in works of legal theory (*uṣūl al-fiqh*).²⁶² Extensive discussions on this causality test—though it remained a debated technique (especially when considered in isolation from other methods)—evolved both before and after al-Shīrāzī in the legal theoretical literature.²⁶³ Concomitance was a key consideration, and it remained, along with a handful of others—especially the tests of “suitability” (*munāsaba*) and “analytical disjunction and exclusion” (*al-sabr wa-l-taqṣīm*)—among the most commonly (and thoroughly) treated rational modes of causal justification.

In later legal and dialectical theory especially, co-presence and co-absence were expressed as concomitance “in existence” (*wujūdan*) and “in nonexistence” (*‘adaman*), and no operative distinction between natural and normative causality appears to have been maintained.²⁶⁴ In brief, the steps of this test can be generalized as verifying

- (1) concomitance “in existence”, if it amounts to verifying if any token of the occasioning type triggers a token of the effect type, and
- (2) concomitance “in nonexistence” if it amounts to verifying if any evidence for the absence of the occasioning type triggers a token of the negation of the effect type.

When both of the occasioning types are formulated as two members of a disjunction

²⁵⁹ The present summary of the Islamic causation test and its formal encoding below, relies heavily on Rahman, S. & W. Young (2022a,b).

²⁶⁰ That is, what might be called mainstream Sunnīs (excluding, e.g., the Zāhirīs and certain Ḥanbalīs).

²⁶¹ See Rahman, Iqbal, & Soufi (2019, preface).

²⁶² See Young (2019).

²⁶³ See Young (2019, Forthcoming A, B).

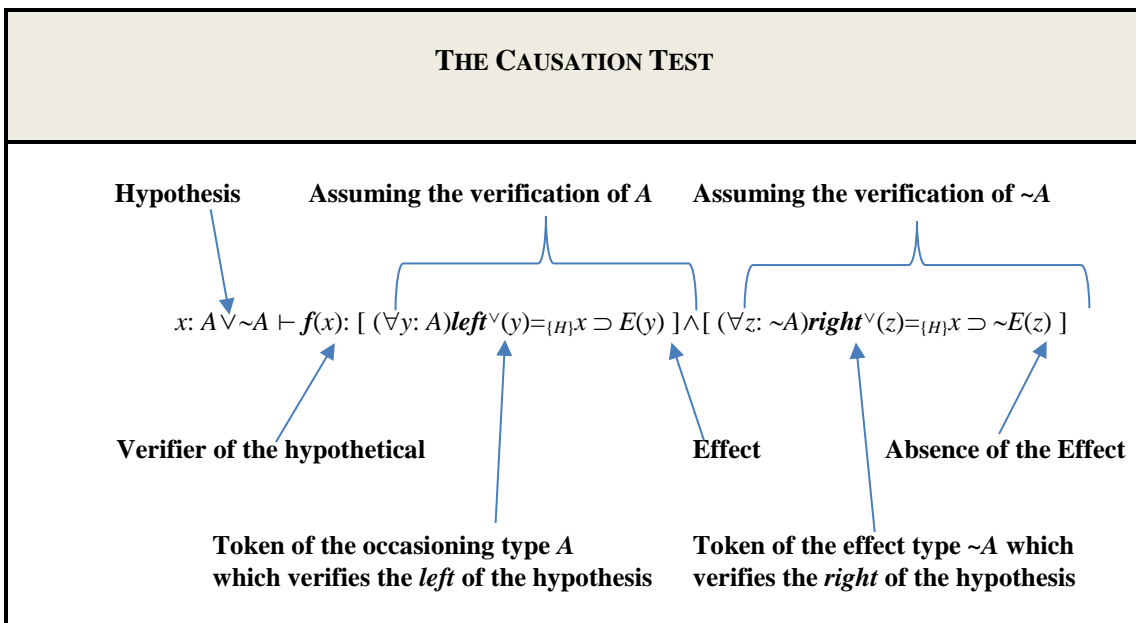
²⁶⁴ See Young (2019, esp. 268 ff.)

that constitutes the antecedent of a hypothetical, this analysis, when developed in the framework of *Immanent Reasoning*, amounts to the following:

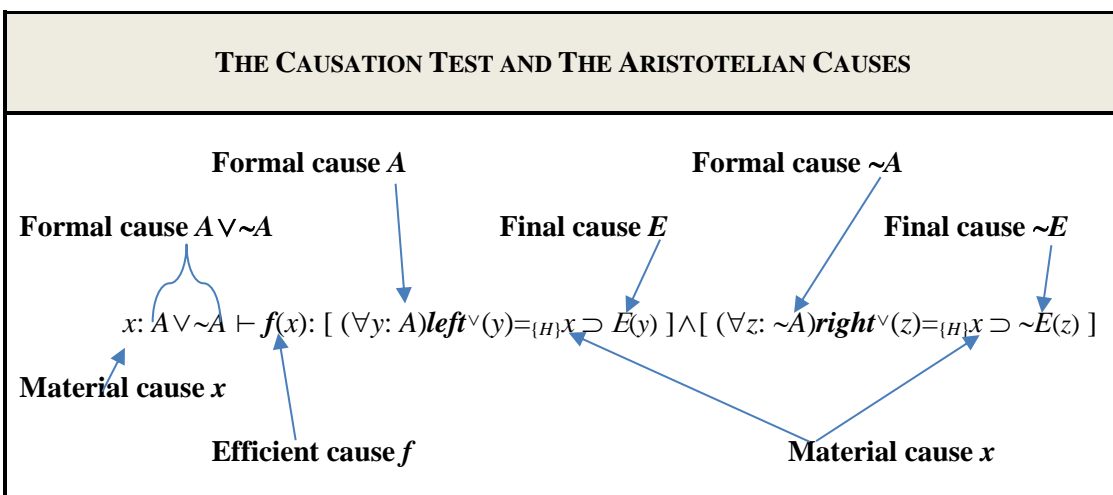
Given the **token x** that witnesses either the presence of A or of $\sim A$:

If any token y of A is identical to a verifier of the **left side** of the disjunction)—viz. **left**(y)= x — then (under such assumption) then token of the effect type E is triggered.

If any token z of $\sim A$ is identical to a verifier of the **right side** of the disjunction)—viz. **right**(z)= x — then (under such assumption) then token of the negation of the effect type E is triggered.



In terms of the Aristotelian theory of causality we obtain:



B.II THE TEST OF CAUSATION AND A MEDIEVAL THEORY OF IMPERATIVES²⁶⁵:

²⁶⁵ The present summary of the Islamic causation test and its formal encoding below, relies heavily on Rahman, S. & W. Young (2022a,b).

The test can be nicely combined with a recently developed theory of Medieval legal Imperatives, by adjusting the hypothesis to the cases under scrutiny – see Rahman&Zidani&Young(2022), see too Rahman&Granström&Farjami (2019)²⁶⁶.

The idea is to apply a two stages procedure that in some way goes the other way around with regard to the procedures discussed in the section A of the present paper.

- 1) The legal procedure starts by testing individual causation. This requires adjusting the disjunction in the hypothesis to the structure of the actions under study.
- 2) Then individual Liability is determined lead by the rationale: an agent if Liable at all, is Liable to the degree of damage his/her action produce.

BII.1 Sufficient and Independent Acts of Harm (SIH)

BII.1.1 The Individual Causation Test (ICT) for SIH

For the sake of simplicity let “A_i” stand for the Action of agent **i**. So, the general form of the test for this case has the form

The General Form of the Causation Test
$x: A(g_i) \vee \sim A(g_i) \vdash f(x): [(\forall y: A(g_i)) \mathbf{left}^{\vee}(y)_{=\{H\}x} \supset E_i[y]] \wedge [(\forall z: \sim A(g_i)) \mathbf{right}^{\vee}(z)_{=\{H\}x} \supset \sim E_i[z]]$

So, in the case of two independent arsonists we have to run the test for each of the arsonists:

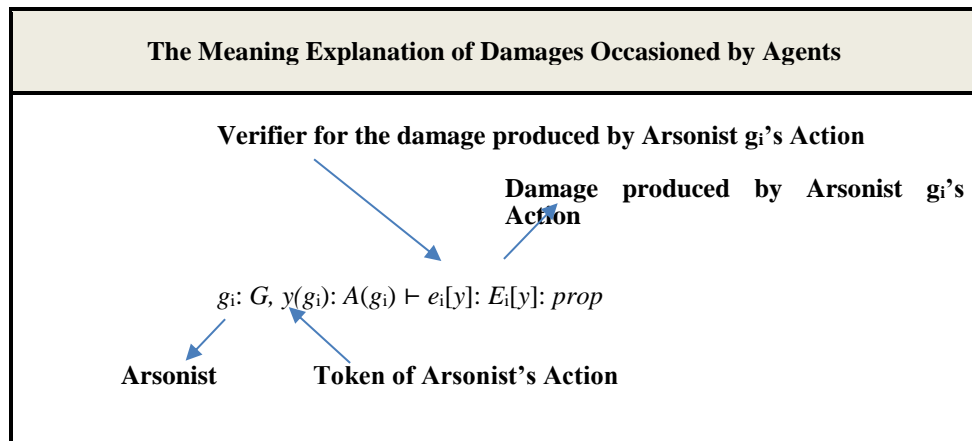
The Causation Test for SIH
$x: A(g_1) \vee \sim A(g_1) \vdash f(x): [(\forall y: A(g_1)) \mathbf{left}^{\vee}(y)_{=\{H\}x} \supset E_1[y]] \wedge [(\forall z: \sim A(g_1)) \mathbf{right}^{\vee}(z)_{=\{H\}x} \supset \sim E_1[z]]$
$x: A(g_2) \vee \sim A(g_2) \vdash f(x): [(\forall y: A(g_2)) \mathbf{left}^{\vee}(y)_{=\{H\}x} \supset E_2[y]] \wedge [(\forall z: \sim A(g_2)) \mathbf{right}^{\vee}(z)_{=\{H\}x} \supset \sim E_2[z]]$
<ul style="list-style-type: none"> • Total damage E is defined as the disjunction of its partial damages $E_1 \vee E_2$. $\sim E$ is defined as $\sim E_1 \wedge \sim E_2$.

BII.1.2 Individual Liability and Absences: NIH revisited?

BII.1.2a Individual Causation of Damage

Notice that, according to the formulation of the test mentioned above, the damage E_i is the product of agent g_i 's action $A(g_i)$ – similar holds for $\sim E_i$:

²⁶⁶ Though Rahman&Granström&Farjami (2019), published in the same volume as Armgardt's (2019) paper, appeared before the paper by Rahman&Zidani&Young (2022), the former relies heavily on the main philological and systematic work developed in the latter. In fact, Rahman&Granström&Farjami (2019) applies Rahman&Zidani&Young (2022) to solve standard paradoxes of contemporary deontic logic. John Woods (2019), was not that impressed by these kind of application, I hope the present one will work as a kind of plaidoyer for studying Islamic dialectic and logic for legal reasoning.



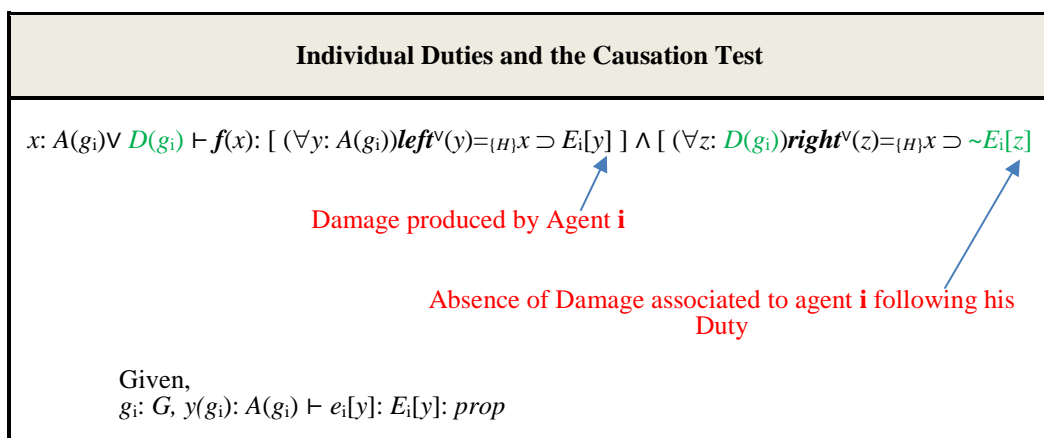
- Thus, the damage produced is the damage produced by **that** agent. Similar for negation. So the absence of the action of one of the agents does not prevent the damage in **general** to happen, but **the damage so far as it has been occasioned by this very agent**

Let us have a closer look at absences in this context.

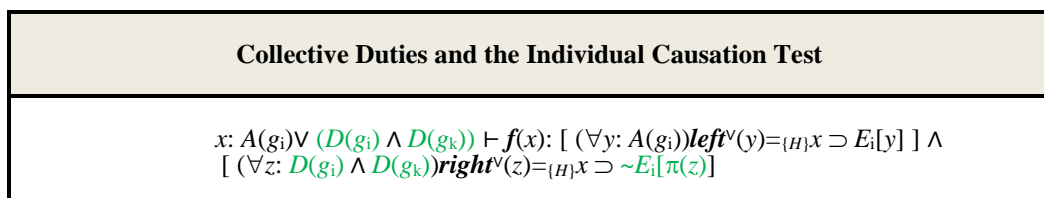
BII.1.2b The Positivity of Absences and Duties: NIH revisited?

In the case of absences, it is the token of an active (self) blocking of an action. One way to understand the positive feature of absences is to understand them as indeed following the duty.

Thus, we might substitute in the hypothesis of the causation test, the negation with **following the duty**. From this point of view, the active part of a negation can be turned into pure positiveness. So, it is the fact that the agent followed his duty, rather than a token of the absence of his action, that occasions a putative token of the absence of Damage:



We might even come closer to Armgardt's (2019) proposal and add the duty-following of all the involved agents, but make the damage dependent upon each individual agent.



Given,
 $g_i: G, y(g_i): A(g_i) \vdash e_i[y]: E_i[y]: prop$

This, in all of its variants, allows to stipulate that if there is a token that verifies E_i , then, agent i is individually liable for E .

BII.1.2c Individual Liability for SIH

Individual Liability for SIH

$x: E_i \vee \sim E_i \vdash f(x): [(\forall y: E_i) \mathit{left}^V(y)_{=(H)x} \supset L_i(g_i)] \wedge [(\forall z: \sim E_i) \mathit{right}^V(z)_{=(H)x} \supset \sim L_i(g_i)]$

Given,
 $g_i: G, y(g_i): A(g_i) \vdash e_i[y]: E_i[y]: prop$

This admits the gloss,

Under the hypothesis that either damage E_i or its absence is verified

If any token y of damage E_i , produced by a token of g_i 's action $A(g_i)$, is identical to a verifier of the (left-side of) hypothesis, then g_i is liable

If any token z of absence of damage E_i , produced by a token of an absence of g_i 's action $A(g_i)$, is identical to a verifier of the (right-side of) hypothesis, then g_i is not liable

BII.1.3 Other Cases without Temporal Parameters

BII.1.3a Mixed Cases

In the case of the merging of one sufficient action by an arsonist with the action of one whose action is insufficient, the formulation of the causation test and of the liability hypothetical is as exactly the same as the one of the precedent case, but

total damage E is defined as the damage E_1 occasioned by the first arsonist and **partial damage** is defined as the damage E_2 occasioned by the second arsonist.

BII.1.3b Damage Occasioned by the Merging of Insufficient Collective Actions

In the case of the collective merging of insufficient actions leading to total tort—recall the case of the executive board of a company that voted in favour of continuing to sell a product, known by the voters to be harmful to health — each E_i is identical to the total damage E . For the sake of simplicity let us assume that each of the votes is necessary and sufficient for the final outcome — either because unanimity is required or because a subset of necessary and sufficient votes has been singled out. Thus, the absence of one suffices to nullify the outcome of the votes.

In such a case, the hypothesis of test of causation requires including the actions of all other agents. Thus, the hypothesis for running the causation test has the form

$$A_1(g_1) \wedge, \dots, \wedge A_k(g_k) \vee \sim (A_1(g_1) \wedge, \dots, \wedge A_k(g_k))$$

The total damage produced has then the form

- $E[\pi_1(x), \dots, \pi_k(x)]$

Which indicates that the total tort (the outcome of the vote), is constituted by the actions of each agent. This all yields:

The Test for Collective Causation
$x: A_1(g_1) \wedge, \dots, \wedge A_k(g_k) \vee \sim(A_1(g_1) \wedge, \dots, \wedge A_k(g_k)) \vdash$ $f(x): [(\forall y: A_1(g_1) \wedge, \dots, \wedge A_k(g_k)) \mathbf{left}^V(y)_{=\{H\}x} \supset E[\pi_1(x), \dots, \pi_k(x)]] \wedge$ $[(\forall z: \sim(A_1(g_1) \wedge, \dots, \wedge A_k(g_k))) \mathbf{right}^V(z)_{=\{H\}x} \supset \sim E[\pi_1(z), \dots, \pi_k(z)]]$

If we are looking for pinpointing at the individual liability of each of the voters, the point is to restrict the liability to the token of the tort of the agent to be individualized:

The Test for Liability in Cases of Collective Causation
$x: E[\pi_1(z), \dots, \pi_k(z)] \vee \sim E[\pi_1(z), \dots, \pi_k(z)] \vdash$ $f(x): [(\forall y: E[\pi_1(z), \dots, \pi_k(z)]) \mathbf{left}^V(y)_{=\{H\}x} \supset L[\pi_i(x)]] \wedge [(\forall z: \sim E[\pi_1(z), \dots, \pi_k(z)]) \mathbf{right}^V(z)_{=\{H\}x} \supset \sim L[\pi_i(x)]]$

BII.1.3c Omissions

Omissions will be based on the idea that if the agent prevented the tort from happening then this agent is not liable and liable otherwise. This requires the hypothesis of the causation test to be formulated as:

$$B(g_i, e(b_j)) \vee \sim B(g_i, e(b_j))$$

Then, individual liability has the same form as the general one: the agent **i** is liable if he did not prevent **j** to produce damage E_j , and not liable if he prevented the tort from happening

Individual Liability for Omissions
$x: B(g_i, e(b_j)) \vee \sim B(g_i, e(b_j)) \vdash f(x): [(\forall y: B(g_i, e(b_j))) \mathbf{left}^V(y)_{=\{H\}x} \supset \sim L_i(g_i)] \wedge [(\forall z: \sim B(g_i, e(b_j))) \mathbf{right}^V(z)_{=\{H\}x} \supset L_i(g_i)]$

BII.2 The Temporal Dimension

Interesting is that in cases of pre-emption and in those of the strike of nature or start of disease, from the purely penal point of view, liability does not require time considerations, beyond those that establish the presence of the action of a presumed tortfeasor.

It is the determination of liability within Private Law that requires a finer temporal analysis

Thus, in the case of pre-emption the test for causation follows the general pattern, without adding temporal parameters. However, in the case of liability according to Private Law, the hypothesis for liability must include some time instant, such that if the damage attributed to the second tortfeasor happens at some time after the one of the first agent, then the second tortfeasor is not liable.

The Test for Tort-Liability in Cases of Pre-emption

$x: E_{2@t_k} \vee E_{2@t \leq t_k} \vdash f(x): [(\forall y: E_{2@t > t_k}) \mathit{left}^V(y)_{=(H)x} \supset \sim TL_1(g_2)] \wedge [(\forall z: E_{2@t \leq t_k}) \mathit{right}^V(z)_{=(H)x} \supset TL_2(g_2)]$

which considers that there are two tortfeasors, that the tort E_2 attributed to the agent g_2 may come either after or before the time t_k at which the tort E_1 attributed to the agent g_1 happened.

With regard to cases of the strike of nature, according to the analysis discussed above the argument is that if the tort attributed to the agent happened before the strike of nature, then the agent is liable for the span of time that starts with his action and ends just before the nature struck.

The Test for Tort-Liability in Cases of Pre-emption

$x: E_{1@<t_i, t_i+(n-k-1)>} \vee E_{1@t \geq t_k} \vdash f(x): [(\forall y: E_{2@<t_i, t_i+(n-k-1)>} t_k) \mathit{left}^V(y)_{=(H)x} \supset TL_1(g_2)_{@<t_i, t_i+(n-k-1)>}] \wedge [(\forall z: E_{1@t \geq t_k}) \mathit{right}^V(z)_{=(H)x} \supset \sim TL_2(g_2)]$

which considers that the tort E_1 attributed to the agent g_1 may have started either before or after the time t_k at which the damage E_ϕ occasioned by some strike of nature Φ happened.

Similar analysis admits the case of the eye and the cars.

Concluding Remarks and Work Ahead

The main idea of the present paper is to focus on tokens of actions and make them explicit at the object language level.

We followed two main strategies, the first closer to the original proposal by Armgardt (2019) and a second one closer to the AAG (2022) paper.

As mentioned above the second strategy, is based on a recent reconstruction of a dialectical pattern of legal reasoning incepted by Islamic jurists and logicians. Now, such dialectical approaches were implemented by means of a whole arsenal of hundred of auxiliary moves that prescribe how to provide reliability to the outcome of a causation test – see Young (2017, 2019, 2021) and Forthcoming A and B)

Moreover, the framework also allows to integrate degrees of liability, dependent upon the strength of the causal link between the so-called occasioning factor, (in our case the actions of the agents) and the damage presumed.

In the context of omissions, a degree of liability can be introduced in the following terms:

if the harmful action to be prevented takes place, then if the agent does not prevent it is liable and not liable if s(h)e prevents it, but it is neither Liable nor not Liable if the harmful action does not take place.

In the context of mixed cases, the following degree of liability seems to be natural: if the action of an agent, sufficient produce the total damage, joins to the one of a second harmful action, that only can produce a smaller damage, the latter is liable, not only for his responsibility for the partial damage, but also for a possible acceleration effect.

But if the first action does not take place at all, then the second agent is only liable to the degree of the partial damage occasioned

- Interesting is that this kind of degrees are degrees of obligation or interdiction, rather than of permissibility – the latter are known in Stoic Logic; Ethics and Law,, such as *commendable*, *abhorrible* and *neutral*. A thorough development should constitute the subject of future interdisciplinary work.

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