



HAL
open science

PERFECT R-NARCISSISTIC NUMBERS IN ANY BASE René-Louis Clerc (january 2024) ((*))

Rene-Louis Clerc

► **To cite this version:**

Rene-Louis Clerc. PERFECT R-NARCISSISTIC NUMBERS IN ANY BASE René-Louis Clerc (january 2024) ((*)). 2024. hal-04376934v2

HAL Id: hal-04376934

<https://hal.science/hal-04376934v2>

Preprint submitted on 31 Jan 2024

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

PERFECT R-NARCISSISTIC NUMBERS IN ANY BASE
René-Louis Clerc (january 2024) ([\(*\)](#))

- ABSTRACT - We defined in [3] the perfect r-narcissistic numbers, or rppdi, which are a natural extension of the classic ppdi or Armstrong number of the first kind ([1]). We have shown that the set of these rppdi is finite and we have given the list of the first 15 rppdi greater than 1 in decimal base ([4], [OEIS2]).

Let us now consider the ppdi and the rppdi in any base b; we will thus define two distinct families (ppdib, ppdi0) on the one hand and (rppdib, rppdi0) on the other hand, associated with any base b other than the decimal base.

We will treat here the cases of bases from 3 to 9 (base 2 only has the trivial solution 1 and base 10 corresponds to the classic ppdi and rppdi) without taking into account the common trivial solution 1.

For a k-digit number n, the function $S_p(n)$ will express the sum of the powers $p \geq 1$ of all the digits of n: $n = \sum_{i=1}^{i=k} a_i 10^{k-i}$, $a_i > 0$, $S_p(n) = \sum_{i=1}^{i=k} a_i^p$, $p \geq 1$.

To move on to rppdi, we apply S_p , not to n (as for ppdi), but to a power r (> 1) of n.

Recall that a perfect r-narcissistic number or rppdi ([3], [4]) is an integer n (> 1) with p digits whose power $r > 1$ ($r = 1$ corresponding to the classic ppdi) is such that the sum of the powers p of all its digits $S_p(n^r)$ is equal to the number n.

While a p-digit ppdi is a solution of $n = S_p(n)$, a p-digit rppdi is a solution of $n = S_p(n^r)$ for $r > 1$.

An rppdi is therefore a p-digit fixed point of the transformation S_p^r

$$(1) \quad n \rightarrow S_p^r(n) = S_p(n^r), \quad r > 1,$$

a ppdi being a fixed point with p digits of (1) but with $r = 1$.

In any base b ($2 < b < 10$), n becomes n_b and we apply the transformation S_p^r to n_b ; we then look for fixed points of: $n = S_p^r(n_b)$, for $r = 1$ (ppdi) or $r > 1$ (rppdi).

With $p =$ number of digits of n_b , then denoted p_b , we obtain the ppdi in base b or ppdib, and the rppdi in base b or rppdib; in this case it is the number n_b which has p digits.

With $p =$ number of digits of n, then denoted p_0 , we obtain fixed points of S_p^r which we will respectively call ppdi0 ($r = 1$) and rppdi0 ($r > 1$); in this case it is the number n which has p digits.

Let us add that for ppdi, only ppdib are considered by the authors ([2], [OEIS1]).

The pleasant character of the transformations S_p^r ([3], [4]) leads to the finiteness of the sets of integers, {ppdib}, {ppdi0} of a on the other hand, {rppdib}, {rppdi0} on the other hand; the set, also finite ([4]), of all the fixed points of the pleasant transformation S_p^r , contains the first two for $r = 1$ and the last two for $r > 1$.

For finiteness, we can also draw inspiration from the general demonstration of [5], and frame such numbers n with k digits in base b by (since $n^r < 10^{kr}$):

$$(2) \quad b^{k-1} < n < rk(b-1)^k, \quad \text{with } r = 1 \text{ for ppdi and } r > 1 \text{ for rppdi,}$$

the lower limit being the smallest k-digit number, the upper limit being the image by S_k^r of the largest.

We can then easily show that for all b, and all r, there exists a $k^*(b, r)$ such that, in (2), the lower limit is strictly greater than the upper limit; there will therefore be no solution for $k \geq k^*(b, r)$, which ensures the finite nature of all of these numbers for any base (and any finite r).

For example, for rppdib, $k^*(10,1) = 61$, $k^*(10,2) = 69$, ..., $k^*(10,6) = 81$, $k^*(9,1) = 53$, $k^*(9,2) = 60$, $k^*(9,3) = 64$, ..., $k^*(8,2) = 51$, $k^*(8,4) = 57$, ...

All ppdi as well as all rppdi are finite in number in all bases and the corresponding definition intervals are $[1, 10^{k^*(b,r)-1}[$, $b > 2$, $r \geq 1$.

The integers common to the two sets {ppdib}, {ppdi0} respectively {rppdib}, {rppdi0}, are n which have exactly the same number of digits in decimal base and in the base b concerned.

To be more readable we will give all our results by expressing them in decimal base.

1 - PPDIs

The ppdi in decimal base ([2]) and in any base in the form of ppdib ([2], [OEIS1]) are well listed in the literature, but not in the version of ppdi0.

Naturally, in decimal base, the ppdib and the ppdi0 are identified with the well-known ppdi: we know ([1]) that there are 88 including: 1, ..., 9, 153, 370, 371, ...

115132219018763992565095597973971522401.

To simplify the results concerning ppdi_b, ppdi₀ in base b, we do not cite or count trivial solutions 1, 2, ..., b-1.

1 - 1 - The PPDIB

Here $p = p_b$.

This choice is the one adopted in [2] as in [OEIS1].

In $[2, 10^9]$, the results, written in base 10, provide 96 solutions.

b = 3 has 3 solutions: 5, 8, 17.

b = 4 has 8 solutions: 28, 29, 35, 43, 55, 62, 83, 243.

b = 5 has 12 solutions: 13, 18, 28, 118, 289, 353, 419, 4890, 4891, 9113, 1874374, 338749352.

b = 6 has 12 solutions: 99, 190, 2292, 2293, 2324, 3432, 3433, 6197, 36140, 269458, 391907, 10067135.

b = 7 has 26 solutions: 10, 25, 32, 45, 133, 134, 152, 250, 3190, 3222, 3612, 3613, 4183, 9286, 35411, 191334, 193393, 376889, 535069, 79 4376, 8094840, 10883814, 16219922, 20496270, 32469576, 34403018.

b = 8 has 21 solutions: 20, 52, 92, 133, 307, 432, 433, 16819, 17864, 17865, 24583, 25639, 212419, 906298, 906426, 938811, 1122179, 20876 46, 3821955, 13606405, 40695508.

b = 9 has 14 solutions: 41, 50, 126, 127, 468, 469, 1824, 8052, 8295, 9857, 1198372, 3357009, 3357010, 6287267.

Example: in base 6, we have $99 = 243_6$, $p = 3$, $S_3(243) = 2^3 + 4^3 + 3^3 = 99$.

1 - 2 - The PPDIO

Here $p = p_0$.

This choice is, to our knowledge, not used, but it is nonetheless also natural since it is associated with the number of digits of the initial n (and not of the transformed n_b). We obtain in our search interval, 34 solutions such that n and n_b have the same length, and therefore common to the two sets {ppdi_b}, {ppdi₀}.

In $[2, 10^9]$, the results, written in base 10, provide 43 solutions.

b = 3 has 0 solution.

b = 4 has 0 solution.

b = 5 has 3 solutions: 13, 18, 118.

b = 6 has 2 solutions: 190, 251.

b = 7 has 8 solutions: 10, 25, 32, 45, 133, 134, 152, 250.

b = 8 has 15 solutions: 20, 52, 133, 307, 432, 433, 16819, 17864, 17865, 24583, 25639, 212419, 1122179, 2087646, 13606405.

b = 9 has 15 solutions: 41, 50, 126, 127, 468, 469, 1824, 65538, 65539, 1198372, 3357009, 3357010, 5300099, 156608073, 156608074.

Example: in base 6, we have $251 = 1055_6$, $p = 3$, $S_3(1055) = 1^3 + 5^3 + 5^3 = 251$.

Let us observe that in $[2, 10^9]$ there are 96 ppdi_b for 43 ppdi₀ ...

Let us state a property common to ppdi_b and ppdi₀ in any base b.

- PROPERTY 1

For any base b, if a fixed point m of $S_p(n_b)$, with $p = p_b$ or p_0 , is a multiple of b, then $m + 1$ is also the fixed point of this application.

Indeed, if $m = 0(b)$, m_b ends with a 0 (in $m_b = \sum_{i=1}^{i=p} a_i b^{p-i}$ we will have $a_p = 0$), and if m is a solution, $m + 1$ is also a solution since a_p will become equal to 1 and will remain so at any power.

Examples: 370, 371 in base 10 for ppdi; 28, 29 in base 4 for ppdi_b; 156608073, 156608074 in base 9 for ppdi₀, ...

2 - RPPDI

In ([3], [4]), the rppdi were defined and the list ([OEI2]) of the first 15 rppdi greater than 1 in decimal base was given (we write 8(3) for solution 8 with $r = 3$):

7(4), 8(3), 9(2);

180(6), 205(2);

38998(2), 45994(2), 89080(2);

726191(2);

5540343(3), 7491889(2), 8690141(3);

167535050(3), 749387107(4);

9945245922(3).

We will determine here the representatives in base b ($2 < b < 10$) for the two choices of p .
 We will observe that in $[2, 10^9]$ there are 17 rppdib for 47 rppdi0 ...

2 - 1 - The RPPDIB

With $p = p_b$, the results, written in base 10, give 17 solutions in $[2, 10^9]$:

(solution 10 for $r = 2$ will be noted $10(2)$)

$b = 3$ has 1 solution: $10(2)$

$b = 4$ has 0 solution

$b = 5$ has 1 solution: $6(2)$

$b = 6$ has 1 solution: $30(3)$

$b = 7$ has 2 solutions: $33(2)$;

$56(3)$

$b = 8$ has 6 solutions: $16(2)$, $41(2)$, $129(2)$, $432(2)$, $9808(2)$;

$7(4)$

$b = 9$ has 6 solutions: $42(2)$, $684(2)$, $52777(2)$;

$8(3)$;

$7(4)$, $468(4)$

Example: $9808 = 23120_8$, $p = 5$, $r = 2$, $23120^2 = 534534400$, $S_5(534534400) = 5^5+3^5+4^5+5^5+3^5+4^5+4^5 = 9808$.

2 - 2 - The RPPDI0

With $p = p_0$, the results, written in base 10, give 47 solutions in $[2, 10^9]$:

$b = 3$ has 5 solutions: $4(2)$, $33(2)$, $95(2)$, $5121(2)$;

$7294(3)$

$b = 4$ has 20 solutions:

$9(2)$, $33(2)$, $129(2)$, $480(2)$, $640(2)$, $736(2)$, $34816(2)$, $69666(2)$, $2129920(2)$, $4259970(2)$,
 $134742016(2)$, $269484546(2)$, $335028856(2)$;

$8(3)$, $20(3)$, $512(3)$, $32768(3)$, $2097152(3)$, $134217728(3)$;

$8(9)$.

$b = 5$ has 1 solution: $81748(4)$

$b = 6$ has 2 solutions: $90646(2)$;

$30(3)$

$b = 7$ has 5 solutions: $9(2)$, $33(2)$, $9667(2)$, $68266(2)$;

$8(3)$

$b = 8$ has 6 solutions: $16(2)$, $41(2)$, $129(2)$, $432(2)$;

$977797(3)$;

$7(4)$

$b = 9$ has 8 solutions: $42(2)$, $684(2)$, $9778(2)$, $52777(2)$, $8767684(2)$;

$8(3)$;

$7(4)$, $468(4)$.

Example: $9667 = 40120_7$, $p = 4$, $r = 2$, $40120^2 = 1609614400$, $S_4(1609614400) = 1^4+6^4+9^4+6^4+1^4+4^4+4^4 = 9667$.

As for the ppdi, the solutions n of the same length as the associated n_b , are common to the two sets $\{rppdib\}$, $\{rppdi0\}$, there are 13 in our search interval.

The results obtained with our two choices allow us to observe some pathologies.

- PROPERTY 2

1) Among the rppdi0, there exists a solution n with two different r for $b = 4$: $8(3)$ and $8(9)$ (this answers a question implicitly asked in [5] concerning the rppdi in decimal base and the uniqueness of the r of a solution).

2) Among the rppdi0, we have 3 times the solution $33(2)$: in bases 3, 4 and 7.

3) For the base $b = 4$, there is no solution of type rppdib but 20 solutions of type rppdi0 in the search interval $[2, 10^9]$.

Base 4 appears to be the most prolific for rppdi0.

4) The solution $n = 8$ appears 4 times in the rppdi0 ($2 < b < 10$) including 3 times with $r = 3$: $8(3)$ and $8(9)$ for $b = 4$; $8(3)$ for $b = 7$; $8(3)$ for $b = 9$.

5) Among the rppdib and rppdi0, the solution $n = 7$ appears twice: with $b = 8$ and $b = 9$ and each time with $r = 4$.

6) The various properties of 8:

8 is equal to the sum of the digits of its cube (512).

8 is equal to the sum of the digits of the cube of its expression in base 4 (which is 8000).

8 is equal to the sum of the digits of the power of 9 of its expression in base 4 (which is $512 \cdot 10^9$).

8 is equal to the sum of the digits of the power of 3 of its expression in base 7 (which is 1331).

- CONCLUSION

After the finite set of {rppdi} ([3], [4], [5], [OEIS2]) which has, in decimal base, 15 representatives in $[2, 10^{10}]$, we have defined three new finite sets of narcissistic numbers in bases 3 to 9, {ppdi0}, {rppdib} and {rppdi0}. In the interval $[2, 10^9]$, they have, respectively, 43, 17 and 47 elements that we have determined.

(*)Honorary professor Paul Sabatier University, Toulouse, France, Webmaster of the site SAYRAC.
This article was published in HAL Open Archive on 08/01/2024 (hal-04376934).

REFERENCES

[1] Armstrong numbers of the first kind or ppdi, [OEIS A005188](https://oeis.org/A005188).

[2] O.Cira, F.Smarandache, Various arithmetic functions and their applications, PONS asbl, Bruxelles, 2016.

[3] R.L.Clerc, Les transformations agréables et une nouvelle classe de nombres narcissiques parfaits, p.1-17, (<https://hal.archives-ouvertes.fr/hal-03619147>), 2022.

[4] R.L.Clerc, The perfect r-narcissistic numbers, p.1-2, (<https://hal.science/hal-04229895>), 2023.

[5] R.L.Clerc, Quelques nombres de Niven-Harshad particuliers, p.1-18, (<https://hal.science/hal-04235744>), 2023.

[OEIS1] Armstrong numbers of the first kind in base 9, [OEIS A010353](https://oeis.org/A010353), N.J.A. Sloane, J.Myers; with references to bases 4 to 16, 2009.

[OEIS2] R-narcissistic numbers (rppdi), [OEIS A364601](https://oeis.org/A364601), R.L.Clerc, 2023.