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Rene-Louis Clerc

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PERFECT R-NARCISSISTIC NUMBERS IN ANY BASE

René-Louis Clerc (january 2024) ([\(*\)](#))

- ABSTRACT - We defined in [3] the perfect r-narcissistic numbers, or rppdi, which are a natural extension of the classic ppdi or Armstrong number of the first kind ([1]). We have shown that the set of these rppdi is finite and we have given the list of the first 15 rppdi greater than 1 in decimal base ([4], [OEIS2]).

Let us now consider the ppdi and the rppdi in any base b; we will thus define two distinct families (ppdib, ppdi0) on the one hand and (rppdib, rppdi0) on the other hand, associated with any base b other than the decimal base.

We will treat here the cases of bases from 3 to 9 (base 2 only has the trivial solution 1 and base 10 corresponds to the classic ppdi and rppdi) without taking into account the common trivial solution 1.

For a k-digit number n, the function $S_p(n)$ will express the sum of the powers $p \geq 1$ of all the digits of n:

$$n = \sum_{i=1}^{i=k} a_i 10^{k-i}, a_i > 0, S_p(n) = \sum_{i=1}^{i=k} a_i^p, p \geq 1.$$

To move on to rppdi, we apply S_p , not to n (as for ppdi), but to a power r (> 1) of n.

Recall that a perfect r-narcissistic number or rppdi ([3], [4]) is an integer n (> 1) with p digits whose power $r > 1$ ($r = 1$ corresponding to the classic ppdi) is such that the sum of the powers p of all its digits $S_p(n^r)$ is equal to the number n.

While a p-digit ppdi is a solution of $n = S_p(n)$, a p-digit rppdi is a solution of $n = S_p(n^r)$ for $r > 1$.

An rppdi is therefore a p-digit fixed point of the transformation S_p^r

$$(1) \quad n \rightarrow S_p^r(n) = S_p(n^r), r > 1,$$

a ppdi being a fixed point with p digits of (1) but with $r = 1$.

In any base b ($2 < b < 10$), n becomes n_b and we apply the transformation S_p^r to n_b ; we then look for fixed points of: $n = S_p^r(n_b)$, for $r = 1$ (ppdi) or $r > 1$ (rppdi).

With p = number of digits of n_b , then denoted p_b , we obtain the ppdi in base b or ppdib, and the rppdi in base b or rppdib; in this case it is the number n_b which has p digits.

With p = number of digits of n, then denoted p_0 , we obtain fixed points of S_p^r which we will respectively call ppdi0 ($r = 1$) and rppdi0 ($r > 1$); in this case it is the number n which has p digits.

Let us add that for ppdi, only ppdib are considered by the authors ([2], [OEIS1]).

The pleasant character of the transformations S_p^r ([3], [4]) leads to the finiteness of the sets of integers, {ppdib}, {ppdi0} of a on the other hand, {rppdib}, {rppdi0} on the other hand; the set, also finite ([4]), of all the fixed points of the pleasant transformation S_p^r , contains the first two for $r = 1$ and the last two for $r > 1$.

For finiteness, we can also draw inspiration from the general demonstration of [5], and frame such numbers n with k digits in base b by (since $n^r < 10^{kr}$):

$$(2) \quad b^{k-1} < n < rk(b-1)^k, \text{ with } r = 1 \text{ for ppdi and } r > 1 \text{ for rppdi,}$$

the lower limit being the smallest k-digit number, the upper limit being the image by S_k^r of the largest.

We can then easily show that for all b, and all r, there exists a $k^*(b, r)$ such that, in (2), the lower limit is strictly greater than the upper limit; there will therefore be no solution for $k \geq k^*(b, r)$, which ensures the finite nature of all of these numbers for any base (and any finite r).

For example, for rppdib, $k^*(10,1) = 61$, $k^*(10,2) = 69$, ..., $k^*(10,6) = 81$, $k^*(9,1) = 53$, $k^*(9,2) = 60$, $k^*(9,3) = 64$, ..., $k^*(8,2) = 51$, $k^*(8,4) = 57$, ...

All ppdi as well as all rppdi are finite in number in all bases and the corresponding definition intervals are $[1, 10^{k^*(b,r)-1}[$, $b > 2$, $r \geq 1$.

The integers common to the two sets {ppdib}, {ppdi0} respectively {rppdib}, {rppdi0}, are n which have exactly the same number of digits in decimal base and in the base b concerned.

To be more readable we will give all our results by expressing them in decimal base.

1 - PPDIs

The ppdi in decimal base ([2]) and in any base in the form of ppdib ([2], [OEIS1]) are well listed in the literature, but not in the version of ppdi0.

Naturally, in decimal base, the ppdib and the ppdi0 are identified with the well-known ppdi: we know ([1]) that there are 88 including: 1, ..., 9, 153, 370, 371, ...

115132219018763992565095597973971522401.

1 - 1 - The PPDIB

Here $p = p_b$.

This choice is the one adopted in [2] as in [OEIS1].

In $[2, 10^9]$, the results, written in base 10, provide 96 solutions.

$b = 3$ has 3 solutions: 5, 8, 17.

$b = 4$ has 8 solutions: 28, 29, 35, 43, 55, 62, 83, 243.

$b = 5$ has 12 solutions: 13, 18, 28, 118, 289, 353, 419, 4890, 4891, 9113, 1874374, 338749352.

$b = 6$ has 12 solutions: 99, 190, 2292, 2293, 2324, 3432, 3433, 6197, 36140, 269458, 391907, 10067135.

$b = 7$ has 26 solutions: 10, 25, 32, 45, 133, 134, 152, 250, 3190, 3222, 3612, 3613, 4183, 9286, 35411, 191334, 193393, 376889, 535069, 79 4376, 8094840, 10883814 , 16219922, 20496270, 32469576, 34403018.

$b = 8$ has 21 solutions: 20, 52, 92, 133, 307, 432, 433, 16819, 17864, 17865, 24583, 25639, 212419, 906298, 906426, 938811, 1122179, 20876 46, 3821955, 13606405, 40695508.

$b = 9$ has 14 solutions: 41, 50, 126, 127, 468, 469, 1824, 8052, 8295, 9857, 1198372, 3357009, 3357010, 6287267.

Example: in base 6, we have $99 = 243_6$, $p = 3$, $S_3(243) = 2^3 + 4^3 + 3^3 = 99$.

1 - 2 - The PPDIO

Here $p = p_0$.

This choice is, to our knowledge, not used, but it is nonetheless also natural since it is associated with the number of digits of the initial n (and not of the transformed n_b). We obtain in our search interval, 34 solutions such that n and n_b have the same length, and therefore common to the two sets $\{\text{ppdib}\}$, $\{\text{ppdi0}\}$.

In $[2, 10^9]$, the results, written in base 10, provide 43 solutions.

$b = 3$ has 0 solution.

$b = 4$ has 0 solution.

$b = 5$ has 3 solutions: 13, 18, 118.

$b = 6$ has 2 solutions: 190, 251.

$b = 7$ has 8 solutions: 10, 25, 32, 45, 133, 134, 152, 250.

$b = 8$ has 15 solutions: 20, 52, 133, 307, 432, 433, 16819, 17864, 17865, 24583, 25639, 212419, 1122179, 2087646, 13606405.

$b = 9$ has 15 solutions: 41, 50, 126, 127, 468, 469, 1824, 65538, 65539, 1198372, 3357009, 3357010, 5300099, 156608073, 156608074.

Example: in base 6, we have $251 = 1055_6$, $p = 3$, $S_3(1055) = 1^3 + 5^3 + 5^3 = 251$.

Let us observe that in $[2, 10^9]$ there are 96 ppdib for 43 ppdi0 ...

Let us state a property common to ppdib and ppdi0 in any base b .

- PROPERTY 1

For any base b , if a fixed point m of $S_p(n_b)$, with $p = p_b$ or p_0 , is a multiple of b , then $m + 1$ is also the fixed point of this application.

Indeed, if $m = 0(b)$, m_b ends with a 0 (in $m_b = \sum_{i=1}^{i=p} a_i b^{p-i}$ we will have $a_p = 0$), and if m is a solution, $m + 1$ is also a solution since a_p will become equal to 1 and will remain so at any power.

Examples: 370, 371 in base 10 for ppdi; 28, 29 in base 4 for ppdib; 156608073, 156608074 in base 9 for ppdi0, ...

2 - RPPDI

In ([3], [4]), the rppdi were defined and the list ([OEI2]) of the first 15 rppdi greater than 1 in decimal base was given (we write $8(3)$ for solution 8 with $r = 3$):

7(4), 8(3), 9(2);

180(6), 205(2);

38998(2), 45994(2), 89080(2);

726191(2);

5540343(3), 7491889(2), 8690141(3);

167535050(3), 749387107(4);

9945245922(3).

We will determine here the representatives in base b ($2 < b < 10$) for the two choices of p .

We will observe that in $[2, 10^9]$ there are 17 rppdib for 47 rppdi0 ...

2 - 1 - The RPPDIB

With $p = p_b$, the results, written in base 10, give 17 solutions in $[2, 10^9]$:

(solution 10 for $r = 2$ will be noted 10(2))

$b = 3$ has 1 solution: 10(2)

$b = 4$ has 0 solution

$b = 5$ has 1 solution: 6(2)

$b = 6$ has 1 solution: 30(3)

$b = 7$ has 2 solutions: 33(2);

56(3)

$b = 8$ has 6 solutions: 16(2), 41(2), 129(2), 432(2), 9808(2);

7(4)

$b = 9$ has 6 solutions: 42(2), 684(2), 52777(2);

8(3);

7(4), 468(4)

Example: $9808 = 23120_8$, $p = 5$, $r = 2$, $23120^2 = 534534400$, $S_5(534534400) = 5^5 + 3^5 + 4^5 + 5^5 + 3^5 + 4^5 + 4^5 = 9808$.

2 - 2 - The RPPDI0

With $p = p_0$, the results, written in base 10, give 47 solutions in $[2, 10^9]$:

$b = 3$ has 5 solutions: 4(2), 33(2), 95(2), 5121(2);

7294(3)

$b = 4$ has 20 solutions:

9(2), 33(2), 129(2), 480(2), 640(2), 736(2), 34816(2), 69666(2), 2129920(2), 4259970(2), 134742016(2), 269484546(2), 335028856(2);

8(3), 20(3), 512(3), 32768(3), 2097152(3), 134217728(3);

8(9).

$b = 5$ has 1 solution: 81748(4)

$b = 6$ has 2 solutions: 90646(2);

30(3)

$b = 7$ has 5 solutions: 9(2), 33(2), 9667(2), 68266(2);

8(3)

$b = 8$ has 6 solutions: 16(2), 41(2), 129(2), 432(2);

977797(3);

7(4)

$b = 9$ has 8 solutions: 42(2), 684(2), 9778(2), 52777(2), 8767684(2);

8(3);

7(4), 468(4).

Example: $9667 = 40120_7$, $p = 4$, $r = 2$, $40120^2 = 1609614400$, $S_4(1609614400) = 1^4 + 6^4 + 9^4 + 6^4 + 1^4 + 4^4 + 4^4 = 9667$.

As for the ppdi, the solutions n of the same length as the associated n_b , are common to the two sets $\{rppdib\}$, $\{rppdi0\}$, there are 13 in our search interval.

The results obtained with our two choices allow us to observe some pathologies.

- PROPERTY 2

1) Among the rppdi0, there exists a solution n with two different r for $b = 4$: 8(3) and 8(9) (this answers a question implicitly asked in [5] concerning the rppdi in decimal base and the uniqueness of the r of a solution).

2) Among the rppdi0, we have 3 times the solution 33(2): in bases 3, 4 and 7.

3) For the base $b = 4$, there is no solution of type rppdib but 20 solutions of type rppdi0 in the search interval $[2, 10^9]$.

Base 4 appears to be the most prolific for rppdi0.

4) The solution $n = 8$ appears 4 times in the rppdi0 ($2 < b < 10$) including 3 times with $r = 3$: 8(3) and 8(9) for $b = 4$; 8(3) for $b = 7$; 8(3) for $b = 9$.

5) Among the rppdib and rppdi0, the solution $n = 7$ appears twice: with $b = 8$ and $b = 9$ and each time with $r = 4$.

6) The various properties of 8:

8 is equal to the sum of the digits of its cube (512).

8 is equal to the sum of the digits of the cube of its expression in base 4 (which is 8000).

8 is equal to the sum of the digits of the power of 9 of its expression in base 4 (which is $512 \cdot 10^9$).

8 is equal to the sum of the digits of the power of 3 of its expression in base 7 (which is 1331).

- CONCLUSION

After the finite set of {rppdi} ([3], [4], [5], [OEIS2]) which has, in decimal base, 15 representatives in $[2, 10^{10}]$, we have defined three new finite sets of narcissistic numbers in bases 3 to 9, {ppdi0}, {rppdib} and {rppdi0}. In the interval $[2, 10^9]$, they have, respectively, 43, 17 and 47 elements that we have determined.

(*)Honorary professor Paul Sabatier University, Toulouse, France, Webmaster of the site SAYRAC.

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