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# Reliable fatigue design of personal vehicle chassis parts from multi-input loads and unsupervised statistical analyses

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## Abstract

A vehicle's chassis plays a critical role in its reliability and durability. To ensure occupant safety and vehicle maneuverability, it is necessary to assess the fatigue strength of chassis components, i.e. their ability to withstand repeated loads during use. This assessment begins at the design stage, with the identification of operating conditions and associated loads. In the case of personal vehicles, various loads must be considered due to diverse road types (e.g., highway, city, ...) and driving styles (aggressive, sporty, ...). In this paper we use a multi-dimensional characterization of the damage caused by external multi-input loads on wheels during vehicle use, including load combinations between the left and right wheels of the front and rear axles. Field measurements are used to calculate pseudo-damages for each load and road type, creating multivariate data with hierarchical structure. Unsupervised statistical analyses are used to explore correlations between pseudo-damages and identify driving profiles, providing a multi-dimensional assessment of severity while avoiding overlearning. A multi-dimensional Gaussian mixture model is then fitted to damage-equivalent constraints. This probabilistic model extrapolates damage computing and simulates driving styles, providing design teams with a stress analysis tool for accurate, realistic fatigue design of chassis components in future vehicles.

**Keywords:** Automotive fatigue design, Multi-input loads, Operational loads, Damage accumulations, Unsupervised statistical analyses, Finite mixture distributions

## 1. Introduction

### 1.1. Fatigue design

The main objective of modern structural design is to reduce production costs while maintaining the reliability and durability of the structure. Particularly in the context of the automotive industry, which must meet the requirements of reducing fuel consumption by minimizing the weight of vehicles, while ensuring

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6 a high level of safety. The chassis plays a key role in safety, supporting the vehicle and ensuring its stability  
7 and handling. As a result, its design is a critical step in the development of a new vehicle model. Material  
8 fatigue is one of the major failure factors for mechanical parts: chassis components are subjected to repeated  
9 stresses due to wheel-transmitted loads caused by road irregularities and driving maneuvers [1, 2, 3, 4].  
10 These stresses can lead to premature localized cracking and failure of parts essential to the stability and  
11 handling of the vehicle.

12 The chassis is comprised of numerous components such as wheel suspension, springs, shock absorbers,  
13 steering mechanism, drive shafts, steering column, etc. These components account for approximately 20%  
14 of the total weight of a personal vehicle [5]. To ensure occupant safety and chassis durability in a future  
15 vehicle, the design office must assess the fatigue strength of the various components, that is, their capability  
16 to endure recurrent loads during service. This involves identifying the vehicle operating conditions and  
17 analyzing the resulting loads. When analyzing the loads for a personal vehicle, there is considerable  
18 variability. This variability comes from the different uses of the vehicle, such as daily commuting, weekly  
19 shopping, leisure and vacation trips, etc., and from the different driving styles of the drivers, who may have  
20 aggressive, sporty, or economical driving styles, etc.

21 Load analysis for a single input signal benefits from extensive documentation and well-established  
22 fatigue damage calculation methods [6, 7, 8, 3, 9]. When the vehicle passes over a bump, the vertical load  
23 applied to the front left wheel generates a tensor of local stresses on zones of the suspension components.  
24 The stress tensor is related to the geometry of the component and the type of loads it is subjected to (tension,  
25 compression, etc.). The components of the stress tensor are then combined using a multi-axial method to  
26 obtain an equivalent stress. This is subsequently used to determine the component ability to withstand this  
27 loading. In fact, the local stress on an zone of a component results from the superposition of forces acting  
28 on the component and, therefore external loads to which the vehicle is subjected. Therefore, it is necessary  
29 to consider the loads applied to the wheel in all three directions (vertical as well as lateral and longitudinal),  
30 applied to all four wheels of the vehicle, as well as the load combinations between the left and right wheels  
31 of the front and rear axles.

32 Accurate evaluation of these various loads is critical to the design office, as it allows them to predict the  
33 fatigue strength of the different chassis components of a vehicle during the design phase. To achieve this,  
34 the design office uses either standard loads corresponding to specific scenarios such as acceleration, braking  
35 or cornering, or loads recorded during field tests with customer-driven vehicles equipped with sensors [10,  
36 11]. Specific scenarios are characterized by their precision in defining load levels and their frequency.  
37 However, they cannot reproduce actual driving conditions due to driver behavior (e.g., aggressive or relaxed  
38 driving, etc.), use (e.g., urban or rural driving, etc.), road imperfections (e.g., potholes, bumps, etc.), traffic  
39 congestion, or weather conditions. This may result in deviations from the actual loads that the vehicle will  
40 be exposed to during its operational life. Customer-measured loads provide authentic data from vehicles on

41 the road, allowing the natural diversity of driving conditions to be taken into account. However, collecting  
42 this data in the field can be costly in terms of time, resources and budget. In addition, it is important to note  
43 that data collected in this way only reflects conditions specific to the location where the vehicles were used,  
44 which may not be an exhaustive representation of all possible conditions.

### 45 1.2. Objectives of the article

46 To ensure the reliable design of chassis components, it is imperative to thoroughly and accurately  
47 consider all potential load configurations that the future vehicle could be subjected to. This article begins by  
48 taking a holistic and realistic approach to modeling the loads applied to the vehicle chassis. This modeling  
49 takes into account the multi-dimensional nature of the load history by simultaneously analyzing the vertical,  
50 horizontal and lateral loads measured at each wheel, as well as the load combinations between the left and  
51 right wheels of the front and rear axles to account for the correlations between these multiple loads. In  
52 addition, we introduce the concept of "local context" to improve our understanding of potential damage that  
53 could occur in different zones of a part. We conducted a field measurement campaign on different types  
54 of roads representative of actual customer use. Using the load histories, we calculated a pseudo-damage  
55 for the local context and road type. Thus, we obtained multi-dimensional data with a hierarchical structure  
56 created by taking into account road types. Therefore, this methodology makes it possible to analyze damage  
57 caused by multi-input loads without having to refer to a specific chassis component to which these loads  
58 are applied. We then applied unsupervised statistical analyses adapted to the hierarchical structure of these  
59 data. These analyses allowed us to identify the main sources of variation in the different damages and to  
60 reduce sampling noise to make our analyses more robust. This allowed us to characterize driving profiles  
61 that are likely to cause damage in different zones of the structure. Secondly, the damage-equivalent stresses  
62 calculated for a reference road consisting of a mixture of road types were examined. Based on these data, a  
63 multi-dimensional Gaussian mixture model is fitted to estimate the fatigue strength of stress concentration  
64 zones for local contexts.

65 In section 2, we first recall the classical procedure for damage calculation from a history for a given  
66 load direction. A multi-dimensional characterization of chassis damage is presented in section 3. In Section  
67 4, we will present the measurement campaign from which we will implement our methodology. Section 5  
68 details the unsupervised multivariate statistical analyses performed on the damage measurements by road  
69 types. Probabilistic stress modeling for fatigue design of chassis components is discussed in Section 6.  
70 Section 7 concludes this paper.

## 71 2. Fatigue design based on load history

72 The analysis of the durability and reliability of a chassis part requires the calculation of the fatigue life  
73 of its components. External loads are forces or combinations of forces that are applied to the chassis and are

74 considered to be "causes" of the fatigue phenomenon in the parts. Local stresses or strains in the parts are  
75 thus the consequences of the external loads and are considered to be "effects" of the fatigue phenomenon.

76 We adopt the framework of Baroux thesis [12], assuming that the parts under study exhibit linear elastic  
77 behavior. For zones of the chassis where these assumptions does not apply, such as the attachment points  
78 where the vehicle suspensions are attached to the chassis, specific analyses are performed to evaluate the  
79 actual stresses [13]. In addition, we assume that the loads considered are representative of normal use,  
80 excluding accidental situations, which implies that the structure is resistant to deformation.

### 81 2.1. From load history to load spectrum

82 Sensors installed on the wheel collect data that provide a load history at regular intervals,  $(t_i; F(t_i))$ .  
83 Mechanical fatigue is the progressive deterioration of a material in response to repeated loads. The first step  
84 is then to identify and counting the load cycles present in the history (see [9] Chapter 3). Rainflow counting  
85 is the most commonly used for this purpose. This provides an accurate representation of repetitive load  
86 cycles  $(k_i, \Delta F_i)$ , where  $k_i$  is the occurrence of the load amplitude  $\Delta F_i$ . Load spectrum model can be fitted to  
87 the cycle counting [10, 14, 15]. The load spectrum represents the distribution of load amplitudes over the  
88 load history. The use of a load spectrum model facilitates storage and, more importantly, allows the data to  
89 be interpolated to obtain load values for periods or cycle amplitudes not directly observed in the raw data.  
90 It is also a probabilistic model that can be used to simulate loads [16, 17].

### 91 2.2. Cumulative Damage

92 Fatigue life is then evaluated by counting the number of load cycles to failure. If the cycles have constant  
93 amplitudes, then it is common to use a Wöhler curve that relates the number of cycles to material failure  
94 (N) to the amplitude of the stress cycles (S). There are numerous fatigue models for this purpose, the most  
95 popular being the Basquin model (see [18] Chapter 7). In practice, however, cycle amplitudes are variable,  
96 in which case a cumulative damage calculation is used [3, 9, 19]. From a cycle counting  $(k_i, \Delta F_i)$ , we then  
97 proceed as follows:

- 98 • Wöhler curve modeling: the Basquin model is used to fit the Wöhler curve, relating  $N_i$ , the number  
99 of cycles required to cause part failure, to  $\Delta F_i$ , the amplitude of the load cycle:

$$N_i \cdot (\Delta F_i)^b = C, \quad (1)$$

100 where the Basquin exponent  $b$  represents the sensitivity of the material to the load amplitude: when  
101 subjected to alternating loads, a material with a Basquin coefficient equal to 8 (e.g. shot-peened zone)  
102 will have a longer life than a material with a Basquin coefficient equal to 4 (e.g. weld zone) [20].  
103 Since the structure has a linear mechanical response, the constant  $C$  depends on the material and  
104 vehicle mass. In addition, the Basquin model generally takes into account the average load over a

complete cycle. In our case, the vehicle mass varies very slightly, so Rainflow cycles averages do not have a significant impact on marginal load induced damage (see [12]). Then, here we ignore the effect of cycle mean or cycle load ratio (see [21, 22, 23] for alternatives).

- Cumulative damage calculation: the damage accumulation hypothesis of Palmgren-Miner [24] is that each cycle with amplitude  $\Delta F_i$ , uses a fraction  $1/N_i$ , of the total life:

$$D_i = \frac{1}{N_i} = \frac{1}{C} \cdot (\Delta F_i)^b.$$

Thus, cumulative damage is given by:

$$D(b, F) = \sum_i k_i \cdot D_i = \frac{1}{C} \sum_i k_i \cdot (\Delta F_i)^b. \quad (2)$$

The lifetime can then be estimated by assuming that the failure occurs when the damage reaches the value of 1.

Note that the Palmgren-Miner rule assumes that each stress cycle is independent of the other cycles.

In other words, the damage caused by one cycle does not affect the response to subsequent cycles.

In order to make the fatigue analysis independent of the vehicle type, we will use the pseudo-damage (see [3] Chapter 3).

Pseudo-damage is a factorization of the damage:

$$d(b, F) = \sum_i k_i \cdot (\Delta F_i)^b. \quad (3)$$

It should be noted that while the linear mechanical response assumptions may be restrictive (and prohibitive for a vibration fatigue study), it is conservative, which means that the pseudo-damages obtained will always be overestimated (see [12]).

If the pseudo-damage does not make it possible to calculate the lifetime, its calculation does not require knowledge of the material constant  $C$  and nevertheless makes it possible to compare the relative impact of different load cycles. This makes it a practical choice for our future analyses.

Often pseudo-damage is difficult to interpret because the unit is not physically comprehensible. It is therefore preferable to use the "damage magnitude"  $DM(F)$ :

$$DM(b, F) = \left( \sum_i k_i \cdot (\Delta F_i)^b \right)^{\frac{1}{b}}. \quad (4)$$

The damage magnitude is proportional to an "equivalent stress" (see [3, 25]). It corresponds to the amplitude of a single load cycle under given one-dimensional loading conditions that would produce the same pseudo-damage as in the local context. This equivalent stress is a "metric of severity" that can be used to compare different load histories or to characterize a population of drivers in terms of the cumulative damage done to the part.

130 It is also important to note that values of  $b$  equal to 4 and 8 introduce a high degree of variability in  
 131 the values of the normalized pseudo-damages, which can affect data interpretation and statistical analyses.  
 132 Using the damage magnitude, the  $b$ -th root of the pseudo-damage, reduces this difficulty.

133 Note that a load history  $F$  depends on the length of the road  $L(F)$ . Therefore, we will use "damage  
 134 intensity", which is the  $b$ -th root of the pseudo-damage per kilometer:

$$DI(b, F) = \left( \frac{1}{L(F)} \sum_i k_i \cdot (\Delta F_i)^b \right)^{\frac{1}{b}}. \quad (5)$$

### 135 2.3. Stress-Strength Interference

136 The main formalism implemented in the industry for the fatigue design of automotive structures is the  
 137 probabilistic "Stress-Strength Interference". This method was first introduced by [26] and is widely used in  
 138 the design of electrical systems and structures subjected to mechanical loads. Stress is a random variable  
 139 that represents the potential amount of damage caused to a part by load applied to the structure (e.g. forces,  
 140 moments, vibrations, etc.) [9]. Stress can thus be used to compare two loads or to characterize a population  
 141 of drivers. Fatigue design then involves determining the strength of parts, controlling the risk of the least  
 142 strength part being installed on a vehicle used by the most severe driver. Strength therefore quantifies  
 143 the highest level of stress that can be applied for a given risk of failure. In the automobile industry, the  
 144 application of the stress-strength method for fatigue design involves the following steps [27, 11, 28, 29]:

- 145 1. Identify the Stress distribution: this stress distribution must be representative of the iterations and load  
 146 levels experienced by the structure. Therefore, based on the loads measured on the wheel, the damage  
 147 magnitudes are an excellent indicator of the stress that a part is subjected to;
- 148 2. Define the reliability requirements: this requirement can be defined for a part or the structure and is  
 149 generally expressed in terms of failure risk;
- 150 3. Calculate the Strength: for a given material, the strength is determined by considering the stress  
 151 distribution and the reliability requirement; the risk of the expected stresses exceeding the strength  
 152 must be less than the defined reliability requirement. To ensure high reliability and durability, the  
 153 fatigue strength of a part is set so that the risk of failure is very low (e.g. less than 0.01%) under  
 154 normal vehicle operating conditions, even when driven by a "severe" driver in term of fatigue. This  
 155 "target customer" is often defined based on the 99th percentile of the stress distribution (see Fig. 1).

156 Local stresses result from the forces or combinations of forces applied to the vehicle by its tires as it as  
 157 it travels along the roadway. Therefore, it is necessary to determine the external loads applied to the chassis  
 158 during vehicle use [2, 4]. These external loads have multiple inputs (longitudinal, lateral and vertical and are  
 159 applied to all four wheels). In the multi-input case, the stress-resistance method is then applied separately  
 160 to each of the loads, resulting in a set of specifications for a target conductor, even under different operating

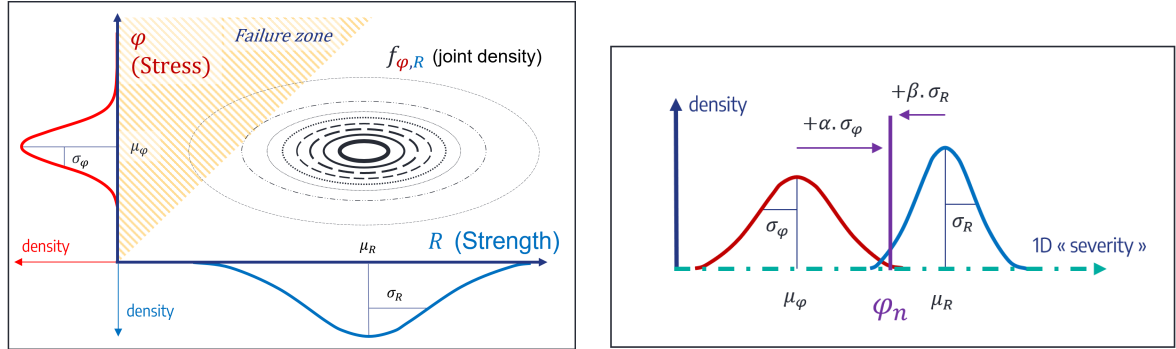


Fig. 1: 1D Stress-Strength interference. Strength is distributed over units/issues of a component/part/system. Load distribution must be representative of the iterations and load levels experienced by the structure. The objective is to determine the fatigue strength of parts, while controlling the risk of the least strength part being installed on a vehicle used by the most severe customer:  $\varphi_n$  is equal to the 99th percentile of the Stress distribution. The risk of failure is set to 0.01%.

161 conditions [29]. Selecting a limited number of "relevant" loads can reduce the time required for the design  
 162 phase. Therefore, the first step is to pre-select the relevant loads to simplify the analysis while trying to  
 163 maintain a high level of completeness.

### 164 3. A Multi-dimensional Fatigue Characterization from Multi-input Load Measurements

165 These external loads applied to the chassis during vehicle use result from various factors, such as road  
 166 conditions (bumps, potholes, etc.), vehicle maneuvers (cornering, braking and acceleration), and aggressive  
 167 driving (sudden acceleration and braking, frequent overtaking, high speed, etc.) (see [3] Chapter 1). Therefore,  
 168 the most exhaustive possible knowledge of the loads applied to the vehicle, during normal use, and a  
 169 thorough understanding of the stresses generated in the chassis parts, are essential from the earliest stages  
 170 of the design process [11, 28, 3].

#### 171 3.1. Reference loads

172 Load histories are given in the three directions, longitudinal, lateral and vertical (X, Y, Z), with different  
 173 phase angles for the left and right wheels of the front and rear axles of the vehicle as shown in Fig. 2.

174 For the three axes  $I \in \{X, Y, Z\}$ , the two axles  $J \in \{\text{Front (f), Rear (r)}\}$  and the phase angles  $\alpha \in$   
 175  $\{0^\circ, 45^\circ, 90^\circ, 135^\circ\}$ , we define the loads:

$$FI\_J\alpha = \cos(\alpha) \cdot F_{I,J,l} + \sin(\alpha) \cdot F_{I,J,r}. \quad (6)$$

176 These different load cases will cause chassis deformation (see [1]). For instance:

$$FX\_f0 = F_{X,f,l},$$



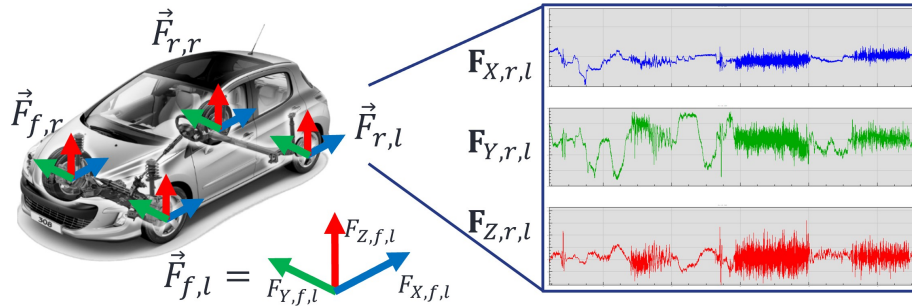


Fig. 2: Multi-axial four-wheel loads and load histories

177 represents the longitudinal load on the left wheel of the front axle and induces bending around the Z-axis of  
 178 the left front suspension parts;

$$F_{Y\_f90} = F_{Y,f,r},$$

179 represents the lateral load on the right wheel of the front axle and induces shearing along the Y-axis of the  
 180 right front suspension parts.

181 We also use linear combinations of two of the measured loads to generate new loads resulting from  
 182 particular situations:

$$F_{Z\_r45} = \cos(45^\circ) \cdot F_{Z,r,l} + \sin(45^\circ) \cdot F_{Z,r,r},$$

183 represents the load on the rear axle when the vehicle passes over a bump and induces bending under in-phase  
 184 loads;

$$F_{Z\_r135} = \cos(135^\circ) \cdot F_{Z,r,l} + \sin(135^\circ) \cdot F_{Z,r,r},$$

185 represents the load on the rear axle as the vehicle passes over a pothole, causing torsion under out-of-phase  
 186 loads. In addition,

$$F_{XY\_fl45} = \cos(45^\circ) \cdot F_{X,f,l} + \sin(45^\circ) \cdot F_{Y,f,l},$$

187 represents coupled deformation of left-hand front suspension arm.

188 In this way, we enrich the family of measured loads on left and right wheels ( $\alpha = 0^\circ, 90^\circ$ ), with linear  
 189 combinations ( $\alpha = 45^\circ, 135^\circ$ ).

190 For durability analysis, this enriched load family will allow us to:

- 191 • model more driving conditions: evaluate the mechanical response of the chassis under more varied  
 192 conditions such as cornering, acceleration, braking, potholes, bumps etc. This provides a better  
 193 understanding of the mechanical response of the chassis to specific roads and driving situations. It  
 194 also increases confidence in the analysis performed during validation testing;
- 195 • optimize design: by analyzing the response of chassis parts to specific loads, it is possible to identify  
 196 zones that may be susceptible to repeated torsional or bending stresses. These observations can then

197 be used to guide the design by reinforcing these zones or opting for stronger materials, especially  
 198 at the level of parts such as suspensions, dampers or springs, to better meet chassis durability and  
 199 performance requirements.

### 200 3.2. Local Contexts

201 It's important to note that different types of load, such as cornering or driving over a bump, will cause  
 202 damage to specific zones of the part. Therefore, we will focus on zones that are potentially susceptible to  
 203 damage from reference loads. These are not only zones that are critical to the operational safety of the part,  
 204 but also zones where weight reduction or geometry changes could be considered by the design office. For  
 205 this reason, we will refer to the "local context" as the given Basquin coefficient  $b$  and load history  $F$ . For  
 each pair  $(b, F)$ , we compute a damage magnitude  $DM(b, F)$ , see Fig. 3.

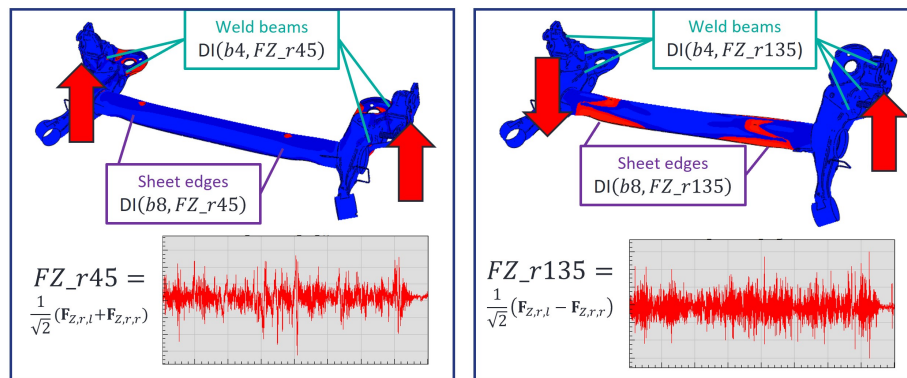


Fig. 3: Loads and local contexts

206

207 Consequently, we have a set of reference loads representing different contexts, see Table 1. During a  
 208 field measurement campaign, several itineraries are studied, each of which is used by several drivers. As a  
 209 result, the research department has a data set of damage intensities calculated for each context, itinerary and  
 210 driver. This allows us to analyze damage caused by multi-input loads without having to refer to a specific  
 211 chassis part to which those loads are applied.

Table 1: The 26 local contexts for the suspension, for different solicitation directions and application points (front, rear, left, right)

Load case	Local context		Events	Zones	Basquin
Front longitudinal	b4_FX_f0 (left) b4_FX_f90 (right) b4_FX_f45 (in-phase) b8_FX_f45	4	Maneuver: braking	wheelpost, triangle	4, 8
Front lateral	b4_FY_f0 (left) b4_FY_f90 b4_FY_f45	3	Maneuver: cornering	gusset, triangle joints	4
Front vertical	b4_FZ_f0, b4_FZ_f90 b4_FZ_f45 (in-phase) b4_FZ_f135 (out-of-phase) b8_FZ_f45, b8_FZ_f135	6	Obstacles: speed bump, ditch, pothole	Cross member edges and joints, wheel posts	4, 8
Rear solicitations	b4_FX_r0, b4_FX_r90 b4_FX_r45, b4_FY_r45 b4_FZ_r0, b4_FZ_r90 b4_FZ_r45, b4_FZ_r135 b8_FZ_r45, b8_FZ_r135	10	Obstacles: speed bump, ditch, pothole	Rear suspension parts	4, 8
Triangle specific	b8_FXY_fl45 b8_FXY_fr45	2	Coupled maneuvers	Front triangle edge	8

#### 212 4. Field Measurement Campaigns

213 Measurement campaigns with sensor-equipped vehicles provide authentic load data. However, we need  
 214 to consider the many factors that explain load variability, focusing on those that are not intrinsically related  
 215 to the structure of the future vehicle, such as its use [3]. While the usage of a vehicle is simply to get  
 216 from point A to point B, its uses are very diverse and depend on the driving environment (road type, traffic,  
 217 weather, etc.), the payload being transported, and the driver's driving style. Therefore, the choice of an  
 218 itinerary for field testing must accurately reflect the actual conditions to which the vehicle may be exposed.  
 219 This means taking into account the driving habits of future users, with particular emphasis on the type of  
 220 roads used.

##### 221 4.0.1. Road types

222 Generally, a distinction is made between:

- 223 • Highways: highways allow driving at high speeds and are characterized by a generally straight  
 224 alignment, the presence of multiple lanes offering overtaking opportunities, and particularly high  
 225 speed limits. They have very low occurrences of pavement imperfections such as potholes or bumps.
- 226 • Urban areas: urban roads require frequent maneuvers such as sharp turns, frequent stops and starts,  
 227 and reduced speeds due to speed limits or general traffic density.
- 228 • Country roads: these roads are characterized by an higher occurrences of pavement defects. Traffic  
 229 density is lower than in urban areas. Speed limits are higher than in urban areas, but still lower than  
 230 on highways.
- 231 • Damaged road: this is a special case of a country road with numerous defects in the pavement.



Fig. 4: Road types in the test campaign

232 In the context of fatigue design, the focus is often on extreme load values, as these can be the most  
 233 critical for the service life of a component or structure. It's worth noting that a vehicle used primarily in  
 234 an urban area will not generate the same extreme multi-axial loads as a vehicle used primarily on country  
 235 roads. For measurement campaigns, it may be useful to use itineraries with portions of all four road types,  
 236 but at specific percentages.

#### 237 4.0.2. Driving styles

238 Loads vary according to driver behavior. For example, loads differ between aggressive driving, characterized  
 239 by hard acceleration and braking and frequent overtaking, and more relaxed driving. However, it is difficult  
 240 to determine the driving style *a priori*. We therefore perform an *a posteriori* evaluation, considering the  
 241 speed and also the correlations between the damage intensities.

#### 242 4.0.3. Traffic and Weather

243 Since traffic density and weather conditions cannot be determined *a priori*, these two factors are not  
 244 studied here. Therefore, any unpredictable event on a route (e.g., traffic jam, slowdown due to heavy rain,  
 245 etc.) is considered and treated as sampling noise.

#### 246 4.0.4. Payload

247 Variations in vehicle speed affect on the distribution of loads measured at the wheels due to various  
248 mechanical factors such as inertia forces, load transfer, etc. In this study, the payload will be identical for  
249 all test vehicles.

#### 250 4.0.5. Specific vehicle characteristics

251 Specific vehicle characteristics, such as weight, engine power, type of suspension and shock absorbers,  
252 etc., influence the loads to which the chassis parts are subjected. We want to design the chassis of future  
253 vehicles whose specific characteristics we don't know *a priori*. That's why we use pseudo-damage, which  
254 doesn't take this information into account.

### 255 5. Analysis of data from a US measurement campaign

256 The measurement campaign took place in the US state of Michigan. A total of 44 drivers were asked  
257 to drive the same SUV, one per day, with the same payload. The reference itinerary is 240 km long and  
258 includes secondary roads (a rural road B1 and a suburban road B2), an urban road (C), a damaged road (D)  
259 and highways (H1 and H2).

260 The procedure for collecting road load data at Stellantis is described in detail in [30]. Prior to this  
261 measurement campaign, the same vehicle was tested on test tracks with accelerometers and wheel force  
262 transducers. In service, the vehicle was not equipped with the load cells. The in-service wheel axle  
263 loads are predicted from the acceleration channels using transfer functions learned from the proving ground  
264 measurements.

#### 265 5.1. Road Types Analysis

266 Our data set consists of 6x26 variables corresponding to damage intensities (Eq. 5) calculated for the 26  
267 different local contexts selected (Eq. 1) and the six road types. Each of these variables represents a unique  
268 combination of factors:

- 269 • a local context;
  - 270 – a Basquin coefficient ( $b=4$  or  $8$ ), which defines the material fatigue behavior of a zone of a part;
  - 271 – an axis of the load measured on the wheel (for example, the longitudinal axis of the left front  
272 wheel), which represents the source of local stress in the zone.
- 273 • a type of road (e.g. city or highway), which characterizes the driving environment.

274 We also have 44 individuals corresponding to the drivers who use the vehicle on these specific road segments.

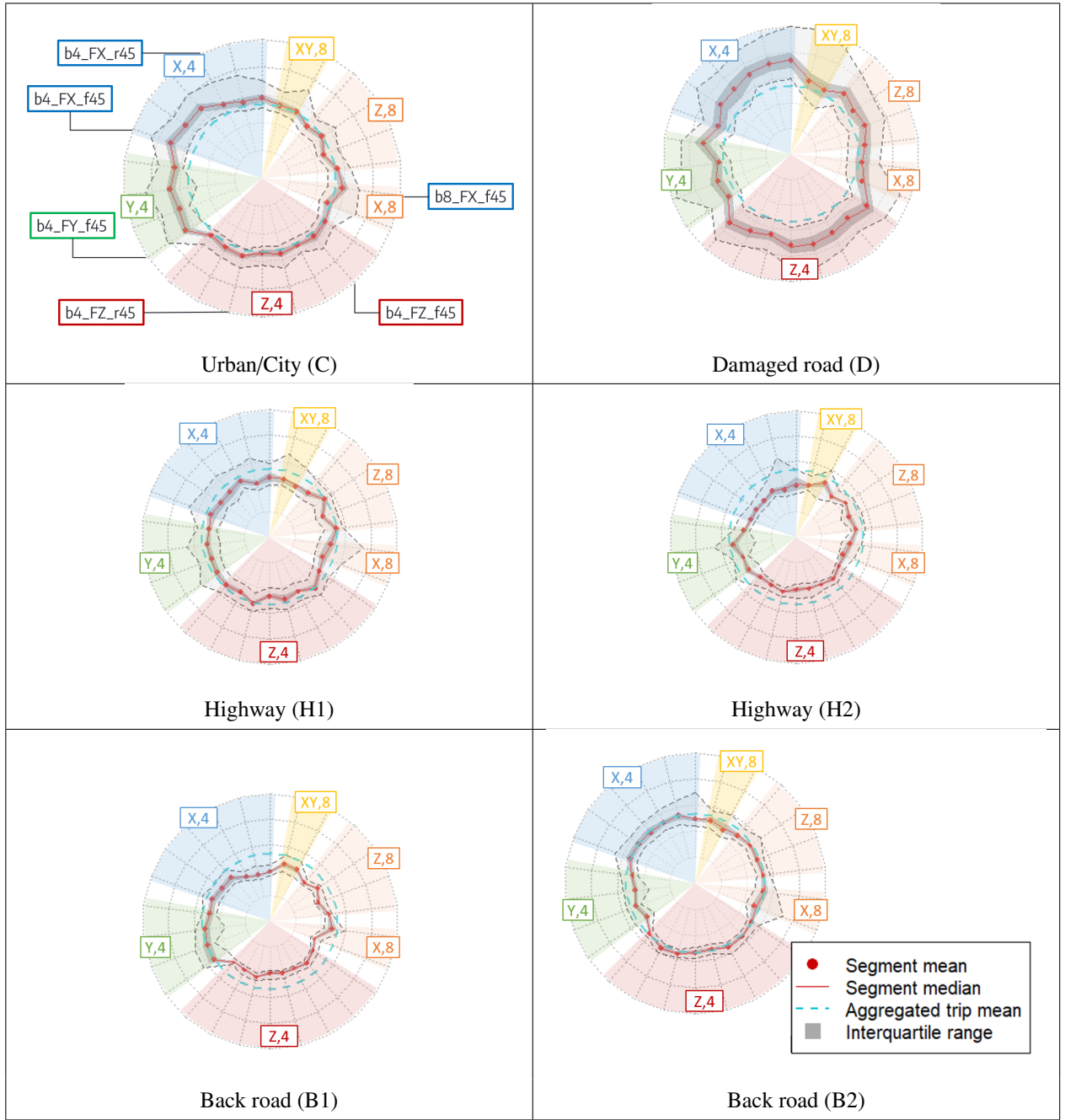


Fig. 5: Normalized damage intensity radar plot for each of the six road types. The light blue dashed circle is the average damage intensity for all drivers across all road segments. It is the basis for normalizing the damage intensity for each of the six road types. The red dots are the average of each damage intensity. The solid red line is their median. The dark gray area with blue borders is the interquartile range. The dotted limits of the light gray zones are the extremes of the road types.

### 276 5.1.1. Multivariate analysis

277 Thus, we have a multivariate data set containing 44 points in the space  $\mathbb{R}^{26 \times 6}$ , where each point represents  
278 a driver and each dimension corresponds to a damage intensity of a specific local context. We are then faced  
279 with a number of issues:

- 280 1. Variable correlation analysis: if two damage intensities are correlated, then the load axes have a  
281 mutual influence on the damage levels;
- 282 2. Drivers similarity analysis: if two drivers are similar, i.e., show proximity in the variables space, this  
283 indicates that they generate similar levels of damage intensities when using the same type of road in  
284 comparable local contexts;
- 285 3. Road types similarity analysis: two road types are similar, if two drivers are similar on one road  
286 segment, they are also similar on the other road segment.

287 But let's start with a descriptive analysis of the data. Fig. 5 shows radar plots of the 26 standardized  
288 intensities for each of the six road types. It can be seen that the C and D segments generally have higher  
289 damage intensities than the other ones, and also have large variations.

### 290 5.1.2. Correlations Structure Analysis

291 Our goal here is to characterize the different road types based on the observed interactions between  
292 damage intensities. The objective is to identify the relevant segments to reproduce fatigue critical load  
293 configurations. We have a data set with a large number of variables (i.e.,  $6 \times 26$  measures of damage  
294 intensity). Our first goal is to understand how these variables are related to each other in order to facilitate  
295 our analyses. Exploratory factor analysis (EFA) is used to analyze the correlation structure of a large number  
296 of variables using a reduced number of underlying factors independently of sampling noise [31]. Factors  
297 corresponding to the main latent sources of variability in the data. Fig. 6 shows the model with the optimal  
298 number of factors to explain the variability in damage intensities for each road type.

299 For C, H1, B1, and B2, we see a similar two-factor structure: the first factor explains the correlations  
300 between longitudinal and vertical intensities, corresponding to driving events such as braking or acceleration.  
301 The second factor explains the relationship between longitudinal and lateral intensities, corresponding to  
302 driving events such as cornering. The correlation between these two factors varies according to the road  
303 type. For segment H2, we also obtain a two-factor structure, one of which explains only the correlation  
304 between the load intensities of the front and rear axles, which could be explained by driving over potholes at  
305 high speed. For segment D, we again obtain a two-factor structure, one of which is related to the correlation  
306 between lateral and longitudinal intensities on the front axle, probably due to the presence of many sharp  
307 bends on this segment.

308 It is interesting to note that here we find results observed by [32] for a rear axle prototype subjected to a  
309 load history recorded during tests on a proving ground.

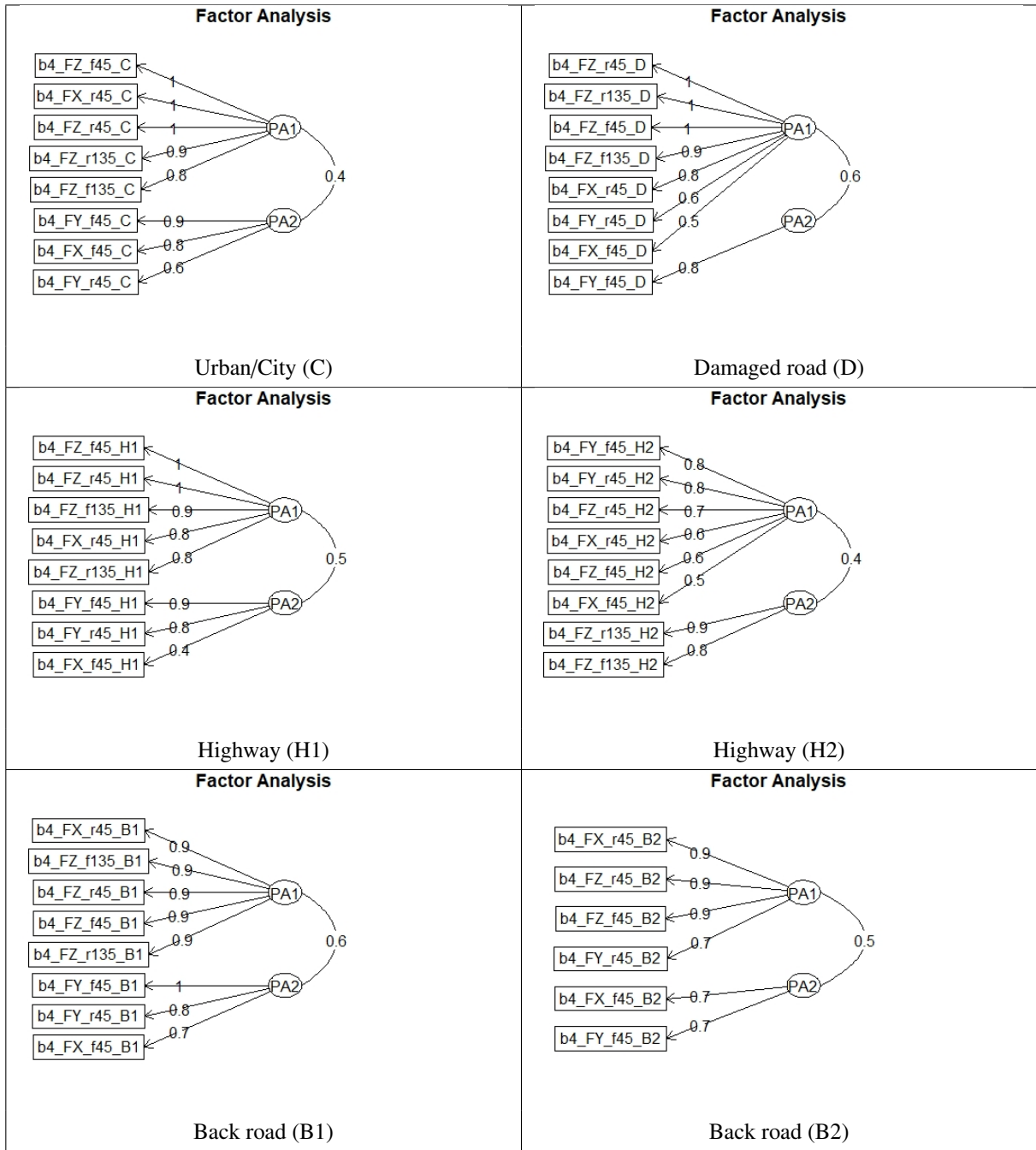


Fig. 6: Exploratory Factorial Analysis (EFA) identifies the main sources of variability in damage intensities for each road type. The weights on the arrows from factors to variables correspond to the percentage of variance in the variable explained by the factor. Only weights greater than 0.3 are displayed. The weights on the arrows between two factors correspond to the correlation between the two factors.



### 310 5.1.3. Driving profils analysis

311 Our goal is to identify which driving profiles are more or less damaging to chassis parts, i.e., to highlight  
 312 a typology of pairs (driver  $\times$  road type) in terms of damage. To do this, we used a clustering technique.  
 313 Unfortunately, with only 44 drivers for  $6 \times 26$  variables, we run the risk of facing the "curse of dimension"  
 314 [33]. This situation, characterized by a large number of variables and a small number of observations,  
 315 could lead to a problem of overlearning, where our analyses could be excessively influenced by the specific  
 316 characteristics of the sample. The data collected may reflect fortuitous driving situations, such as a traffic  
 317 jam when the vehicle is in use. This "unusual" variability in environmental conditions and vehicle use is  
 318 considered to be "sampling noise", as it can distort the detection of truly significant trends. To overcome  
 319 this difficulty and to obtain results that are robust to the random fluctuations induced by the sample, we can  
 320 use Principal Component Analysis (PCA) to reduce the dimensionality of the data (see [31] Chapter 9).

321 *Principal Component Analysis (PCA)*. The general principle of PCA is as follows: let  $\mathbf{Y} = (\mathbf{Y}_1, \dots, \mathbf{Y}_p)$ ,  
 322 be  $p$  variables observed on  $n$  individuals. The data set is a matrix  $Y[n \times p]$  with  $n$  rows and  $p$  columns.  
 323 Without loss of generality, we assume that the variables are centered. PCA consists in creating  $p$  new  
 324 features  $\mathbf{PC} = (\mathbf{PC}_1, \dots, \mathbf{PC}_p)$ , called "principal components". These principal components are obtained  
 325 by searching for linear combinations of the initial variables  $\mathbf{Y}$  that are uncorrelated with each other and  
 326 that maximise the total variance of the data at each stage. In practice, these components are computed by  
 327 diagonalising the variance-covariance matrix of  $Y[n \times p]$ . The eigenvalues  $(\lambda_1 > \lambda_2 > \dots > \lambda_p)$  correspond  
 328 to the proportion of variance explained by each component ( $\mathbf{PC}_i$ ). The result is the factorial model:

$$Y[n \times p] = PC[n \times p]A[p \times p] \quad (7)$$

329 where the  $PC[n \times p]$  matrix corresponds to the coordinates of the  $n$  individuals in the space of the  $p$  principal  
 330 components; the matrix  $A[p \times p]$  is a transition matrix from one space to another.

331 PCA then allows us to do two important things:

- 332 • **Dimension reduction:** it is possible to reduce the dimension of the data, and thus tackle the "curse  
 333 of the dimension", by approximating the original matrix  $Y[n \times p]$  with the matrix  $\tilde{Y}[n \times p]$ :

$$\tilde{Y}[n \times p] = PC[n \times k]A[k \times p]. \quad (8)$$

334 The number of principal components retained  $k$ , is obtained by selecting the first  $k$  principal components  
 335 that explain 95% of the total variance. It is reasonable to assume that the remaining components are  
 336 associated with sampling noise and can therefore be eliminated. Note that the matrix  $\tilde{Y}[n \times p]$ , is the  
 337 best least squares approximation of  $Y[n \times p]$  by a matrix of rank  $k$  (see the Eckart-Young theorem  
 338 [34]). This allows us to reduce the size of the data (if  $k \ll p$ ) while preserving the data structure as  
 339 much as possible.

340 • **Data visualization:** PCA allows us to visualize  $Y[n \times p]$ , a scatter plot of  $n$  points in  $\mathbb{R}^p$ , by projecting  
 341 it onto planes based on two principal components, such as the first plane  $PC[n \times 2]$ .

342 Here, damage intensities are obtained for each of the six road types. In the case of an *a priori* group  
 343 structure of the variables, Multiple Factor Analysis (MFA) is used [35].

344 *Multiple Factor Analysis (MFA).* MFA is used as follows:

- 345 1. Partial PCA is performed for each road type;
- 346 2. For each road type, the damage intensities are then normalized by the largest eigenvalue of the partial  
 347 PCA;
- 348 3. a global PCA is then performed on the overall normalized damage intensities.

349 As seen in the Fig. 5, segments C and D have high damage intensity values and may artificially contribute  
 350 more than other road types during principal components construction. Step 2 aims to mitigate this unwanted  
 351 effect in the analysis.

352 Fig. 7 shows the eigenvalue scree plot of the global PCA. It can be seen that the first 23 principal  
 353 components explain 95% of the total variance of the  $6 \times 26$  variables. The remaining components, representing  
 the remaining 5% of the variance, can be considered as sampling noise.

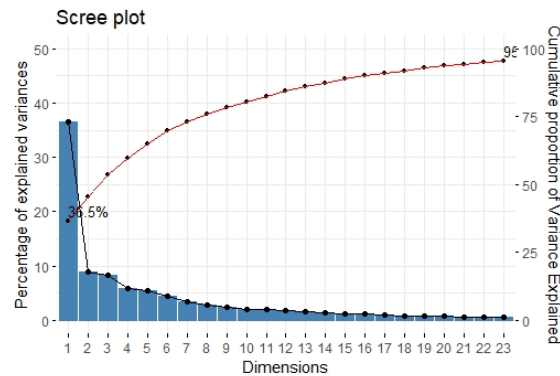


Fig. 7: MFA Scree Plot

354 Speed is an aggravating factor in most driving events. It increases the acceleration of the wheel axles and  
 355 therefore the load on the vehicle when maneuvering and negotiating obstacles. To make the MFA results  
 356 easier to interpret, we have added the speeds (i.e., average value and S90, the 90% percentile) for each  
 357 road type, as well as the damage intensity for the entire itinerary made up of the six road types (i.e., features  
 358 marked with TOT). These intensities allow for a driving style that is unique to the driver. Additional features  
 359 are not used in the principal components computation and are shown in black in the various plots. Fig. 8  
 360 shows the most significant correlations between damage intensities and the first three principal components.  
 361

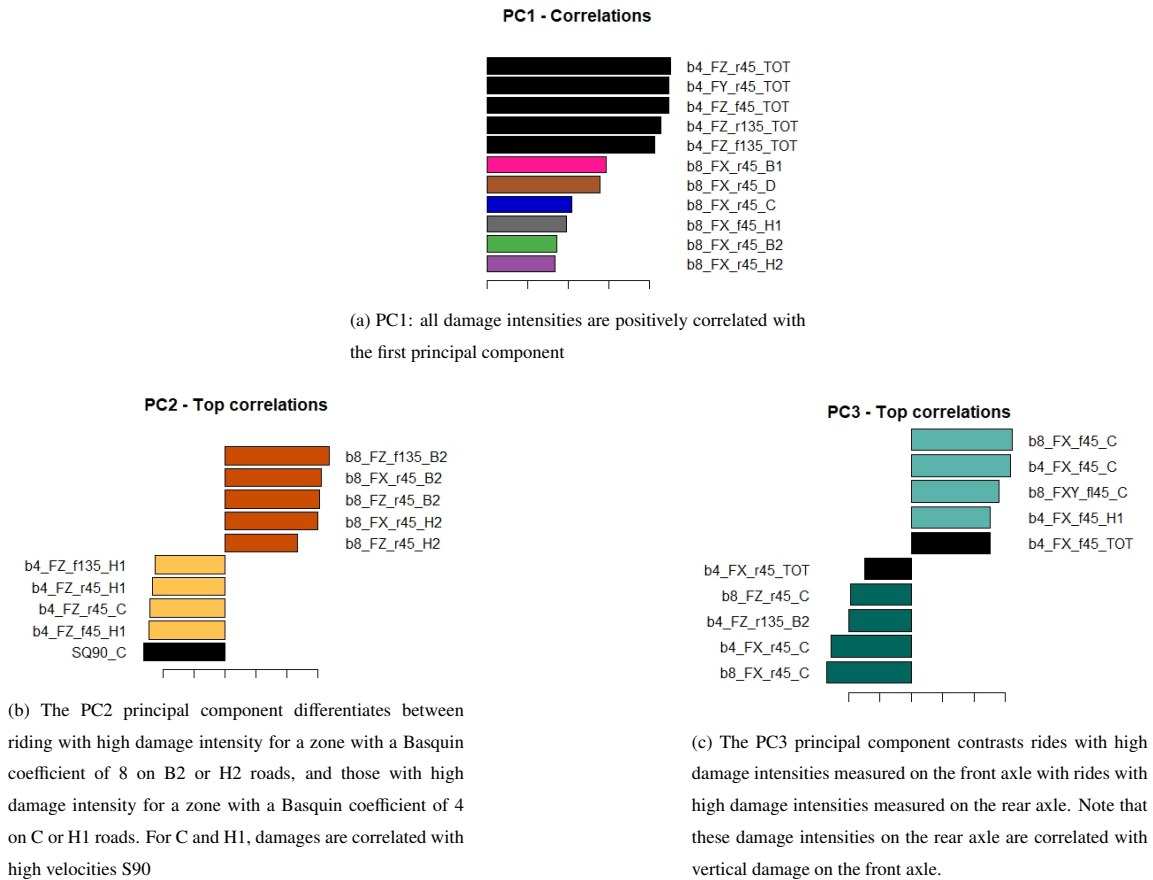


Fig. 8: Correlations with the first three principal components of the AFM

363 **First principal component (PC1):** it can be seen that all damage intensities are positively correlated  
 364 with the first principal component. This component, which is a linear combination of the damage intensities,  
 365 can be considered as a "severity score" to compare drivings in terms of damage caused to a part: a high  
 366 value for this first principal component indicates driving that causes significant damage in all local contexts,  
 367 regardless of road type;

368 **Second principal component PC2):** this principal component allows us to distinguish between driving  
 369 with high damage intensity for a zone with a Basquin coefficient of 8 (e.g. sheet metal edges) when the  
 370 vehicle is on B2 or H2 roads, and those with high damage intensity for a zone with a Basquin coefficient of  
 371 4 (e.g. welds) when the vehicle is on C or H1 roads. For C and H1 roads, damage intensities are correlated  
 372 with high velocities (SQ90, the 90% Speed Percentile). Recall that the Basquin coefficient is a measure that  
 373 is physically related to the way a material responds to fatigue:

- 374 • a zone with a Basquin exponent between 3 and 5 (e.g., a weld bead) will always show initial microcracks  
 375 due to material shrinkage after fabrication (see [36]). Consequently, even loads slightly above the

376 fatigue limit will always result in significant damage;

- 377 • this is not the case for a zone with Basquin exponent between 6 and 20 (e.g., a sheet metal edge),  
378 where much higher loads, compared to the fatigue limit, are required to cause significant damage (see  
379 [20]).

380 Sudden acceleration and deceleration cause significant load transfers, resulting in increased vertical and  
381 longitudinal loads. This results in high stresses on chassis parts. Hard accelerating and braking are characteristic  
382 of aggressive driving. The PC2 component differentiates between damages caused by aggressive behavior  
383 when acceleration and braking, and damages caused by a smoother driving style.

384 **Third principal component (PC3):** this principal component allows us to distinguish between driving  
385 with significant damage measured on the front axle and driving with significant damage measured on the  
386 rear axle. Note that these damage intensities on the rear axle are negatively correlated with vertical damage  
387 intensities on the front axle. These situations are once again characterized by aggressive behavior, but this  
388 time when cornering and overtaking (see [37]):

- 389 • if the driver is too fast when entering the corner, the load on the front axle increases significantly,  
390 which can lead to understeer;
- 391 • if the driver suddenly depresses the accelerator when the vehicle is already in the corner, the load on  
392 the rear axle increases significantly, which can lead to oversteer.

393 Again, oversteer and understeer are characteristic of aggressive driving, especially when cornering and  
394 overtaking. Component PC3 differentiates damage caused by poor speed management when cornering and  
395 overtaking.

396 *Clustering.* Two drivers are considered similar if they show proximity in the fatigue damage space. Clustering  
397 involves constructing a partition of the set of drivers into groups, or clusters, such that drivers within the  
398 same cluster show similarity, while drivers in different clusters do not show similarity to each other. By  
399 characterizing each cluster, we can discover the underlying factors associated with it. In our fatigue design  
400 context, this makes it possible to identify the driving behaviors that, for certain road types, will cause damage  
401 in certain local contexts.

402 The 44 rows of the data set  $Y[44 \times 6 \cdot 26]$  correspond to the drivers and the columns correspond to the 26  
403 damage intensities for each of the 6 road types. To tackle the "curse of dimension" and to avoid overlearning,  
404 we perform a classification based on  $PC[44 \times 23]$ , the projection of the scatterplot  $Y[44 \times 6 \cdot 26]$  into the space  
405 of the first 23 principal components. We then combined two clustering methods. First, we use Hierarchical  
406 Ascending Classification (HAC): this allows us to obtain an optimal number of classes using the Ward jump  
407 criterion [38]. We then use the HAC solution to initialize the k-means algorithm, which allows us to obtain

408 more homogeneous classes in terms of damage intensity (see Fig. 9 (a)). By projecting the data cloud  
 409 onto the first three principal components, we can characterize the three clusters identified (see Fig. 9 (b)).  
 Using PC1, the three clusters can be ranked according to their damage severity, from least severe (Cluster

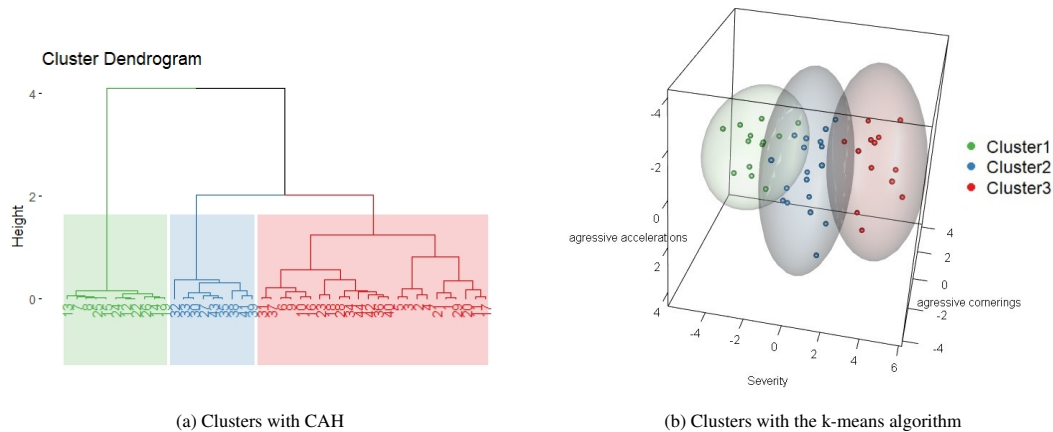


Fig. 9: Fig.(a): HAC was performed on the first 23 principal components of the MFA. Using Ward jump criterion, three clusters are retained. This solution is then improved using the k-means algorithm. Fig.(b): using PC1, the three clusters can be ranked according to their damage severity, from least severe (Cluster 1) to most severe (Cluster 3).

410

411 1) to most severe (Cluster 3). Usually, using the pseudo-damage for a given local context, it is possible  
 412 to compare drivers in terms of damage, taking the "target severe customer" to be at the 99th percentile of  
 413 the distribution. Using PC1, it becomes possible to generalize the notion of damage severity for a set of  
 414 local contexts. Within each cluster there are drivers who are more or less aggressive when accelerating and  
 415 braking, or more or less aggressive when cornering or overtaking, especially in urban traffic.

## 416 6. Reference Itinerary Analysis

417 During the useful lifetime of the vehicle, the conditions of use may vary, as may the owners or drivers.  
 418 Therefore, sizing specifications are defined based on a total distance covered (e.g. 240,000 km) and a  
 419 certain combination of road types. By combining the different road types, it becomes possible to assess the  
 420 contribution of fatigue resulting from each driving condition associated with a specific road type, on the  
 421 total distance covered during the vehicle lifetime. Here we look at the total itinerary created by combining  
 422 all six road types. It is an illustrative example of the reference itineraries that can be used to represent  
 423 typical customer usage. The dataset  $Y[44times26]$  has 44 rows corresponding to the drivers and 26 columns  
 424 corresponding to the calculated damage magnitudes from the load histories along the entire itinerary for the  
 425 26 local contexts (Table 1).

426 *6.1. Correlations Structure Analysis*

427 Using an EFA, we obtain a two-factor model to explain the relationships between the 26 damage values  
 428 (see Fig. 10). The second factor is strongly correlated with longitudinal and lateral loads on the front axle  
 429 and can be associated with maneuvers that primarily involve the front steering of the vehicle. The first  
 430 factor is related to other maneuvers: acceleration/braking, high speed overtaking, etc. The two factors are  
 correlated.

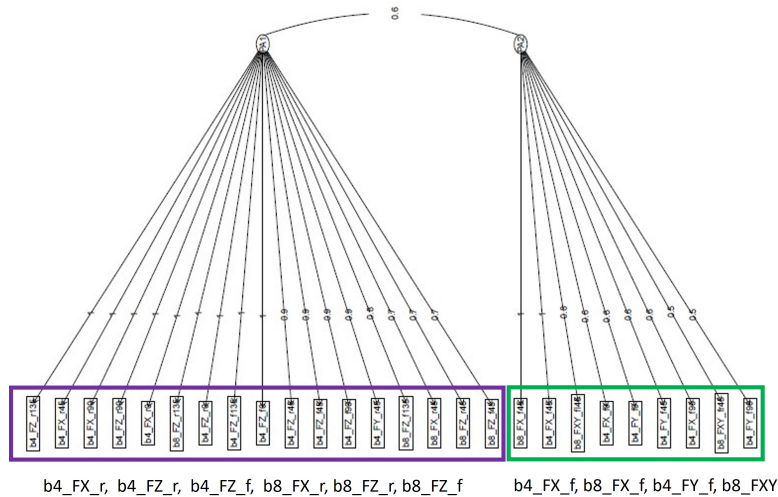


Fig. 10: Total itinerary: Exploratory Factor Analysis of damage magnitudes for the 26 local contexts. The second factor is associated with forward steering maneuvers, while the first factor is associated with various maneuvers (acceleration/braking, high-speed overtaking). The two factors are correlated.

431

432 *6.2. Damage Magnitude Multi-dimensional Distribution*

433 Just as the load spectrum represents the distribution of load amplitudes in load histories, here we are  
 434 looking for the distribution of damage magnitudes in load histories. The damage magnitude is proportional  
 435 to an "equivalent stress" (see [3] Chapter 3) and then can be used to estimate the fatigue strength of a part  
 436 zone for the corresponding local context. We then seek to model the multi-dimensional probability density  
 437 function of the damage magnitudes for the 26 local contexts. First, we perform a PCA to reduce the sampling  
 438 noise of the data set  $Y[44 \times 26]$ .

439 *6.3. Principal Components Analysis*

440 The first seven components of the PCA explain more than 95% of the total variance in the data (see  
 441 Fig. 11).

442 Fig. 12 shows that the first principal component is positively correlated with all damage magnitudes: it  
 443 allows to distinguish globally severe drivings. The second component is positively correlated with lateral  
 444 and longitudinal damage magnitudes: it distinguishes aggressive drivings during cornering.

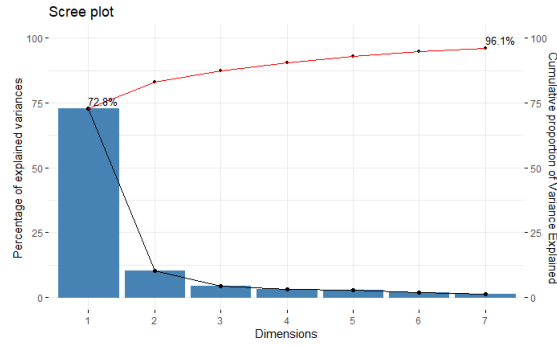


Fig. 11: PCA Scree plot: The first seven PCA components explain over 95% of the total variation in the data, while the remaining components can be assimilated to sampling noise.

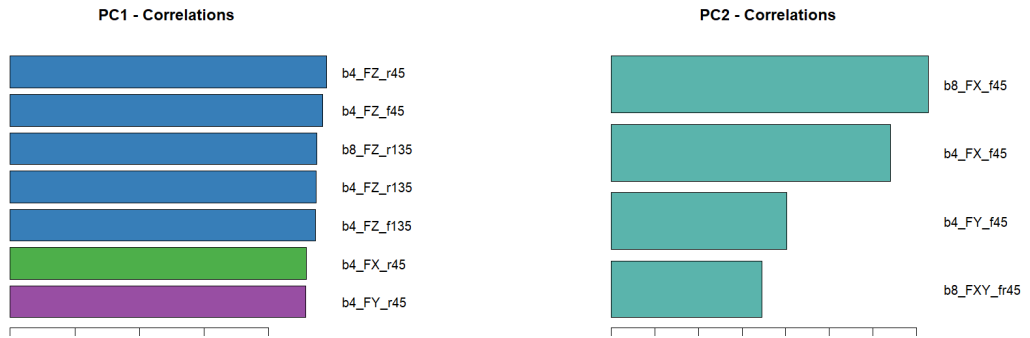


Fig. 12: Correlations with the first two principal components: the first principal component is positively correlated with all damage measures, making it an overall indicator of fatigue severity. The second component is positively correlated with the damage magnitudes of lateral and longitudinal loads: It helps to distinguish aggressive driving when cornering.

445 Using the data collected on the sample of 44 drivers, we obtain an empirical distribution of damage  
 446 magnitudes. Our aim is then to estimate the probability density function so that we can extrapolate beyond  
 447 the observed ranges of values.

#### 448 6.4. Probabilistic PCA

449 As shown earlier, the first seven PCA components account for more than 95% of the total data variance  
 450 (see Fig. 11). We then restrict our analysis to data reconstructed from the first seven principal components:

$$\tilde{Y} = PC[44 \times 7]A[7 \times 26]. \quad (9)$$

451 We propose to fit this data set  $PC[44 \times 7]$  with a probabilistic model.

452 In order to account for the heterogeneity of driving profiles along the entire itinerary, it is common to  
 453 use mixture models [16, 39, 40]. We have chosen here a multi-dimensional Gaussian mixture in which the  
 454 mixture components correspond to the driving behaviors.

455 **6.4.1. Multi-dimensional Gaussian Mixture distributions**

456 A mixture of  $\kappa$  multi-dimensional Gaussian distributions (MMG) is defined as follows [41, 42]: for an  
457 integer  $d \in \mathbb{N}^*$ , and  $\mathbf{x} \in \mathbb{R}^d$ , the probability density function is

$$f_{MMG}(\mathbf{x} | \boldsymbol{\theta}) = \sum_{k=1}^{\kappa} \alpha_k \phi(\mathbf{x} | \boldsymbol{\mu}_k, \boldsymbol{\Sigma}_k), \quad (10)$$

458 where for  $k = 1, 2, \dots, \kappa$ :

- 459 •  $\phi(\mathbf{x} | \boldsymbol{\mu}; \boldsymbol{\Sigma})$  is a multi-dimensional Gaussian probability density function with mean  $\boldsymbol{\mu}$  and covariances  
460 matrix  $\boldsymbol{\Sigma}$ :

$$\phi(\mathbf{x} | \boldsymbol{\mu}, \boldsymbol{\Sigma}) = \frac{1}{(2\pi)^{\frac{d}{2}} |\boldsymbol{\Sigma}|^{\frac{1}{2}}} \exp\left(-\frac{1}{2}(\mathbf{x} - \boldsymbol{\mu})^\top \boldsymbol{\Sigma}^{-1} (\mathbf{x} - \boldsymbol{\mu})\right); \quad (11)$$

461 where  $|\boldsymbol{\Sigma}|$  is the determinant of the matrix  $\boldsymbol{\Sigma}$ .

- 462 •  $0 < \alpha_k < 1$  is the mixing proportion of the  $k$ -th component, with  $\sum_{k=1}^{\kappa} \alpha_k = 1$ ;

463 Let us denote  $\boldsymbol{\theta} = (\kappa, (\boldsymbol{\mu}_k), (\boldsymbol{\Sigma}_k))$ , the set of model parameters. In our problem,  $\kappa$  represents the number of  
464 different driving styles. For a fixed  $k$ , the parameters  $\boldsymbol{\mu}_k$  and  $\boldsymbol{\Sigma}_k$  are generally fitted by maximum likelihood,  
465 using the Expectation-Maximization (EM) algorithm [43].

466 **6.4.2. Expectation-Maximization (EM) algorithm**

467 Let us denote  $\mathbf{z} = (z_1, \dots, z_n)^\top$  the component labels: for  $k = 1, \dots, \kappa$ ,

468  $z_i = k$  if the  $i$ -th driver adopts the  $k$ -th driving style.

469 Unfortunately, the values of  $\mathbf{z}$  are not known *a priori*: it is a latent variable. The EM algorithm is designed  
470 to maximise likelihood despite the presence of a latent variable [44]:

- 471 1. Initialize the means ( $\boldsymbol{\mu}_k^{(0)}$ ), covariance matrix ( $\boldsymbol{\Sigma}_k^{(0)}$ ) and proportion ( $\alpha_k^{(0)}$ ) of each Gaussian component.
- 472 2. For  $\ell \geq 1$ , repeat until convergence:
  - 473 • Expectation-step: Calculate the posterior probabilities:

$$p(z = k | \mathbf{x}_i; \boldsymbol{\mu}_k^{(\ell)}; \boldsymbol{\Sigma}_k^{(\ell)}) = \frac{\alpha_k^{(\ell)} \cdot f(\mathbf{x}_i | \boldsymbol{\mu}_k^{(\ell)}; \boldsymbol{\Sigma}_k^{(\ell)})}{\sum_{j=1}^{\kappa} \alpha_j^{(\ell)} \cdot f(\mathbf{x}_i | \boldsymbol{\mu}_j^{(\ell)}; \boldsymbol{\Sigma}_j^{(\ell)})} = w_{ik}^{(\ell)}.$$

474 The posterior probabilities ( $w_{ik}^{(\ell)}$ ), provide the ability to classify a driver to a cluster (i.e a driving  
475 behaviour), according to its driving characteristics  $\mathbf{x}_i$ . Each driver is assigned to the component  
476 with the highest posterior probability.



- Maximization-step: Update the parameters of each component by maximizing the likelihood on the data assigned to that component. In the Gaussian case, this means calculating

$$\mu_k^{(\ell+1)} = \frac{\sum_{i=1}^n w_{ik}^{(\ell)} \mathbf{x}_i}{\sum_j w_{jk}^{(\ell)}} \quad \Sigma_k^{(\ell+1)} = \frac{\sum_{i=1}^n w_{ik}^{(\ell)} (\mathbf{x}_i - \mu_k^{(\ell+1)}) (\mathbf{x}_i - \mu_k^{(\ell+1)})^t}{\sum_j w_{jk}^{(\ell)}} \quad \alpha_k^{(\ell+1)} = \frac{\sum_{i=1}^n w_{ik}^{(\ell)}}{n}$$

477 3. Convergence criterion: a commonly used indicator for evaluating the convergence of the EM algorithm  
478 is the stability of the log-likelihood between two iterations.

479 It's worth noting that the EM algorithm is an iterative process and its performance depends heavily on the  
480 initial conditions. Sub-optimal initialisation can result in slow convergence or a sub-optimal solution [45].  
481 To initialize the EM algorithm, we use a solution obtained from the hierarchical ascending classification  
482 dendrogram (see Fig. 13a).

#### 483 6.4.3. Model Selection

484 Optimal selection of the number of components to use, is achieved by minimizing a bias-variance  
485 criterion that strikes a balance between modeling bias (i.e., the error introduced by overly simplistic modeling)  
486 and estimator variance (i.e., the error introduced by overly complex modelling). The Akaike Information  
487 Criterion (AIC) is one of the most widely used [46]. The three-components mixture model is thus selected  
(see Fig. 13b).

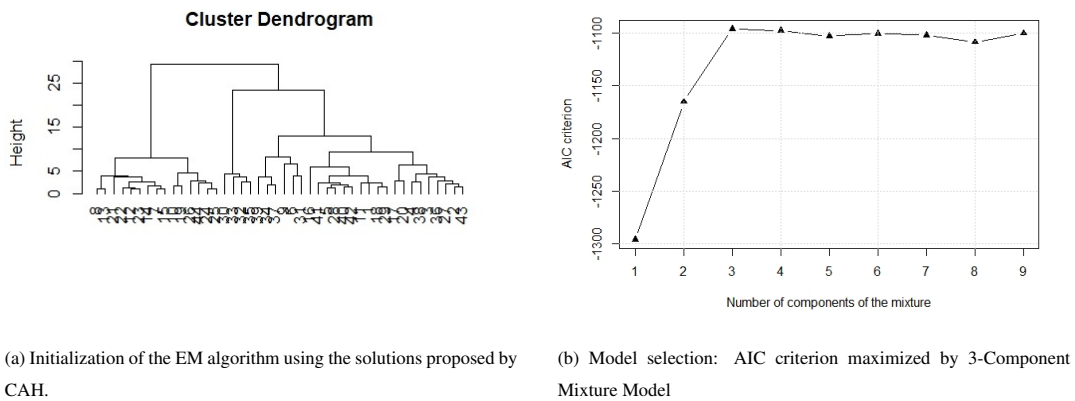


Fig. 13: Multi-dimensional Gaussian Mixture distributions with the EM algorithm

488

489 Fig. 15 show the resulting three-components multi-dimensional Gaussian mixture model. Fig. 14 shows  
490 the projections of the mixture of multi-dimensional Gaussian distributions onto the seven principal components.

491

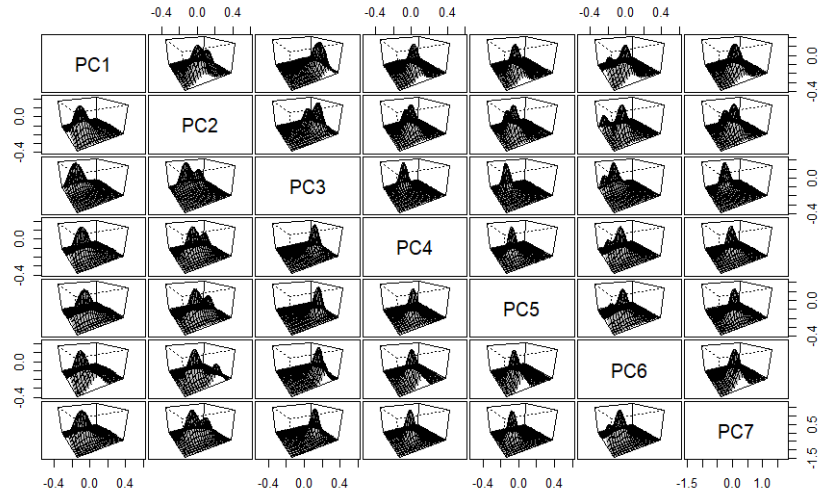


Fig. 14: Visualization of the mixture of three multi-dimensional Gaussian distributions obtained by the EM algorithm on data restricted to the first seven principal components: projections by dimension pairs

492 Fig. 15 shows the projections of the mixture of multi-dimensional Gaussian distributions onto the first  
 493 two principal components, together with the marginal distributions: two components of the mixture that  
 494 have high density values on the large coordinates of PC1, indicating frequent severe behaviours in all local  
 495 contexts (see Fig. 12). These two components are distinguished by more or less high frequencies on the  
 496 second principal component (PC2), i.e., more or less frequent aggressive behaviour when cornering. Finally,  
 497 the last component corresponds to cases of less severe behaviours.

#### 498 6.5. Damage Magnitudes Distribution

499 We fitted a multi-dimensional Gaussian distributions mixture model to  $\mathbf{PC}[7]$ , the vector of the first seven  
 500 principal components. To obtain the damage magnitudes distribution, we use the reconstruction procedure  
 501 with the PCA transition matrix (see Eq.8):

$$\mathbf{Y} = \mathbf{PC}[7] \mathbf{A}[7 \times 26]. \quad (12)$$

502 Thus, the distribution of  $\mathbf{Y}$ , the damage magnitudes, is also a mixture of multi-dimensional Gaussian  
 503 distributions. In this way, we can analyze and reconstruct a damage magnitude distribution for any desired  
 504 local context or driver profile. Fig. 16 shows two reconstruction examples of the joint distribution and  
 505 marginal distributions of damage magnitudes of two local contexts. Correlations between the different  
 506 contexts are clearly visible. For example, when a suspension spring is exposed to both longitudinal and  
 507 lateral loads during a sharp cornering maneuver, it becomes clear that the risk of generating high stress  
 508 concentrations is notable, especially for cluster 3 drivers (represented by red triangles). Similarly, simultaneous  
 509 longitudinal and vertical loads during braking or acceleration can also present similar risks.

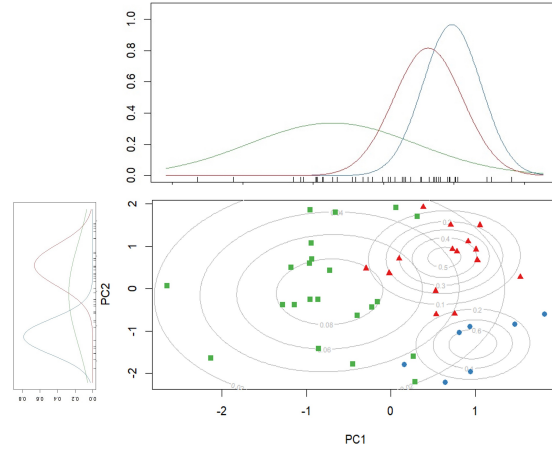


Fig. 15: Projections of the mixture of three Gaussian distributions onto the first two principal components and the marginal distributions. Classification is done with posterior probabilities. Two components of the mixture have high density values on the large PC1 coordinates, indicating severe behavior common to all local contexts. These two components stand out with higher or lower frequencies on the second principal component (PC2), i.e., more frequent aggressive behavior during cornering (cluster 3 with red triangles) and less frequent aggressive behavior (cluster 2 with blue dots). The last component corresponds to less severe behaviors (cluster 1 with green squares).

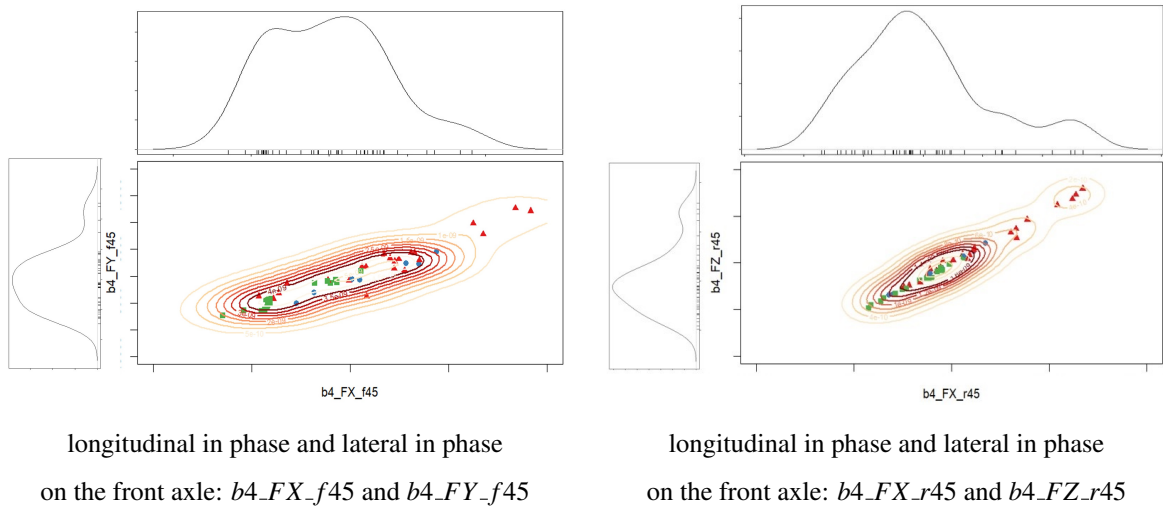


Fig. 16: Joint and marginal distributions of local contexts on the front axle. Data have been centered

510 Note that the magnitude distributions are multi-modal: this can be explained by the different driving  
 511 styles identified by the mixture model. For example, for the  $b4\_FX\_f45$  local context, Fig. 17 shows the 99%  
 512 quantile of the damage magnitude distributions conditional on the clusters. A cluster of severe behaviors is  
 513 clearly identified for this local context.

514 Note that the probabilistic PCA allows us to overcome the challenge of high-dimensional estimation  
 515 while avoiding over-learning [47].

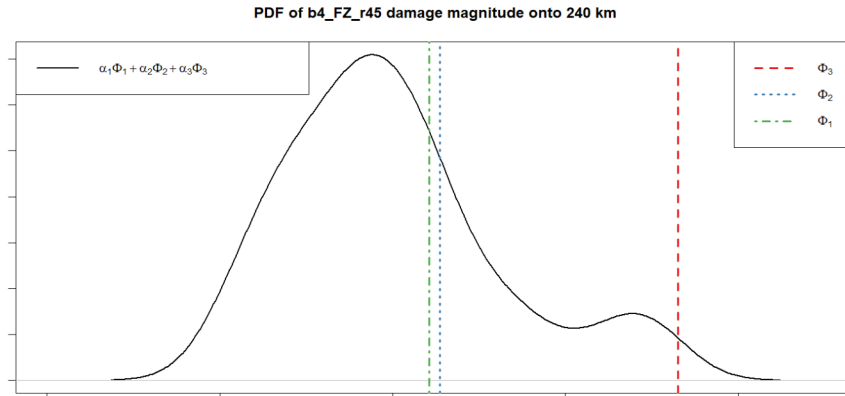


Fig. 17: Local context  $b4\_FZ\_r45$  damage magnitude distribution. Vertical lines represent the 99% quantile of the damage magnitude distributions conditional on the clusters (upper bound of a right-sided 95% confidence interval cf. Eq. 13). Values have been suppressed to protect data confidentiality.

516 It is also important to note that there is a relationship between the load spectra and these damage  
 517 magnitudes distributions: the load spectrum corresponds to the probability density of the load amplitudes  
 518 observed in the load histories of a local context [48, 39]. Here, we obtain a multi-dimensional probability  
 519 density of equivalent damage amplitudes observed in the load histories of all the local contexts studied.

## 520 6.6. Defining Validation Test Conditions

521 This probabilistic model can also be used for the definition of test environments that should have effects  
 522 similar to those observed in real-world conditions [28, 3]:

- 523 • numerical simulation tests on a finite element model of a component: they are used to validate  
 524 resistance values by exposing the component to high levels of severity;
- 525 • rig tests on a prototype of a multi-component part: designed to ensure part reliability and compliance  
 526 with specifications, these tests are performed at a moderate level of severity;
- 527 • proving ground tests with a vehicle: their aim is to validate the performance, safety and handling of  
 528 the vehicle by exposing it to severe driving conditions.

529 To implement these various tests, it is necessary to define load scenarios and load levels. The EFA helps  
 530 to select the relevant loads to define scenarios, and the quantiles of the magnitude distribution are used to  
 531 define load levels.

532 *Proposition:* Conditional to a driving profile, let  $DM_p(F, b)$  the  $p$ -quantile of the damage magnitude of a  
 533 local context  $(F, b)$ . Let  $(\bar{y}, s_y)$  be the average and standard deviation of the sample of the damage magnitude  
 534 in the local context  $(F, b)$ , for  $n$  drivers of the considered driving profile, then the  $100(1 - \alpha)\%$  confidence

535 interval of  $DM_p(F, b)$  is:

$$\left[ \bar{y} - t(1 - \alpha/2; n - 1, -\sqrt{n} z_p) \frac{s_y}{\sqrt{n}}; \bar{y} - t(\alpha/2; n - 1, -\sqrt{n} z_p) \frac{s_y}{\sqrt{n}} \right], \quad (13)$$

536 where  $t(\beta; k, \delta)$  is the  $\beta$ -quantile of the Student distribution of order  $k$  and non-centrality parameter  $\delta$ , and  
537  $z_p$  is the  $p$ -quantile of the gaussian distribution  $N(0; 1)$ .

538 *Proof:* The distribution of  $\mathbf{Y}$ , the damage magnitudes, is a mixture of multi-dimensional Gaussian  
539 distributions (see Eq. 12). For each Gaussian component of the mixture, we can use a pivotal method to  
540 construct confidence intervals for the parameters and, in particular, for the  $p$ -quantiles (see [49] Appendix  
541 E).

## 542 6.7. Extrapolation

543 Durability certification ensures that the vehicle performance and structural integrity will be maintained  
544 for 240,000 km, although the tests are limited to a 240 km itinerary. It is therefore necessary to extrapolate  
545 the results of the 240 km tests to the full 240,000 km distance. Obviously, the cumulative damage over  
546 240,000 km is not simply 1,000 times the cumulative damage over 240 km: a driver will never take a curve  
547 the same way every time, or drive over a pothole the same way. This variability inherent in driving conditions  
548 is used as a basis for introducing probabilistic models into the analysis of stress spectra or to account for  
549 the statistical variability of rainfall count matrices. Our proposal is to extrapolate from damage magnitude  
550 distributions.

551 *Proposition:* Let  $DM$  be the damage magnitude on the reference itinerary for a local context, and  $DM_c$   
552 it's extrapolation to an itinerary  $c$  times as long. If the driving conditions remain stationary, then for large  $c$   
553 (i.e.,  $c \geq 30$ ),  $DM_c$  probability density function can be approximated by:

$$f(x) = \frac{(1/b) x^{1/b-1}}{\sqrt{2\pi} \cdot c\sigma^2} \exp\left(-\frac{(x^{1/b} - c\mu)^2}{2 \cdot c\sigma^2}\right), \quad (14)$$

554 where  $\mu$  and  $\sigma$  are the mean and standard-deviation of  $DM^b$ :  $\mu = \mathbb{E}[DM^b]$  and  $\sigma = \sigma(DM^b)$ .

555 *Proof:* If a driver travels the reference route with his vehicle, the number of occurrences of cycles with  
556 amplitude  $\Delta F_i$  has an intrinsic random nature, we denote it as the random variable  $v_i$  (see [50]). Therefore  
557 Eq. 2 gives us the pseudo-damage, which is also random:

$$DM^b = \sum_i v_i \cdot (\Delta F_i)^b.$$

558 If the driver repeats this operation  $c$  times,  $D_c$ , the total pseudo-damage is equal to the sum of the successive  
559  $n$  random pseudo-damages  $DM_1^b, DM_2^b, \dots, DM_c^b$ :

$$D_c = \sum_{\ell=1}^c DM_{\ell}^b = \sum_{\ell=1}^c \sum_i v_{\ell i} (\Delta F_i)^b.$$

560 The stationarity of driving conditions allows us to have  $(v_{fi})$  independent and identically distributed. By  
 561 invoking the central limit theorem [51],  $D_c$  is then distributed according to a Gaussian distribution of mean  
 562  $c\mu$  and standard-deviation  $\sqrt{c}\sigma$ . This allows us to calculate the density of the damage magnitude  $DM_c =$   
 563  $(D_c)^{1/b}$  □.

564 In the previous proposal, we assume that driving conditions, including driving style, do not change from  
 565 one trip to the next within the same identified cluster. Thus, calculations are performed for each cluster  
 566 identified above. Fig. 18 shows the distribution of pseudo-damage extrapolation for a local context as well  
 567 as the 99% percentile for each cluster. It can be seen that with extrapolation, the distributions conditional on  
 the clusters become well separated.

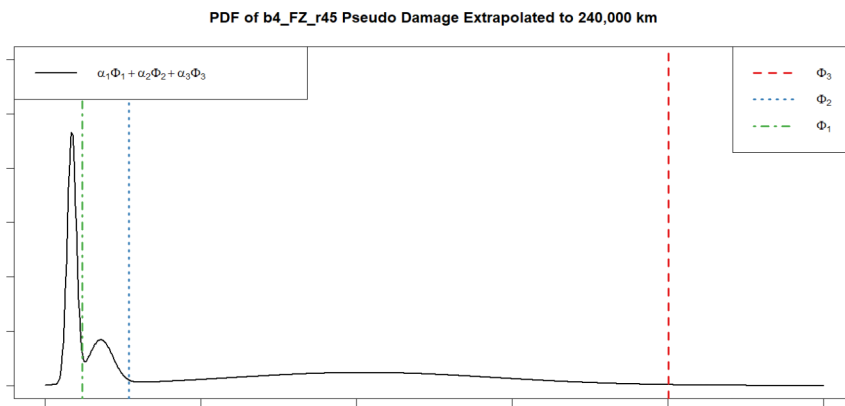


Fig. 18: Local context  $b4\_FZ\_r45$  pseudo-damage extrapolation. Vertical lines represent the 99% quantile of the distributions conditional on the clusters. Values have been suppressed to protect data confidentiality.

568

569 In addition, it is now possible to simulate pseudo-damage on different road mixes and with driver profile  
 570 changes.

## 571 7. Conclusion

572 The vehicle chassis plays a critical role in safety by supporting the weight of the vehicle while providing  
 573 stability, maneuverability and handling. Fatigue analysis of chassis parts is then a critical step in the  
 574 development of any new vehicle. This analysis considers multiple input loads (i.e., loads acting in the  
 575 three longitudinal, lateral, and vertical directions, measured at each wheel and the various combinations  
 576 between the left and right wheels of the front and rear axles). The complexity of the analysis lies in the need  
 577 to also study the correlations between these loads, as their interactions play a decisive role in the generation  
 578 of stress concentrations. What's more, loads must be collected on a variety of road types to accurately reflect  
 579 real-world conditions experienced by customers, resulting in a wide variety of loads to analyze.

580 In this article, we have presented a methodology that meets the challenge of the complexity of multidimen

581 -sional load analysis and provides a more complete and in-depth perspective on the development of a  
582 personal vehicle chassis.

- 583 1. As a first step, we introduce the concept of "local context", which takes into account both the load  
584 direction and the material behavior through the use of a Basquin coefficient.
- 585 2. A field measurement campaign was conducted with a sensor-equipped vehicle: 26 damage magnitudes  
586 were computed for six different road types, involving a total of 44 drivers.
- 587 3. We then use unsupervised statistical analyses on "damage intensities", which correspond  $b$ -th root of  
588 the pseudo-damage per kilometer. An exploratory factor analysis (EFA) of damage intensities allows  
589 us to identify two main sources of variation for each road type. For most road types, the first factor is  
590 related to vehicle steering (lateral and longitudinal loads) and the second to acceleration and braking  
591 (longitudinal and vertical loads). These two factors are correlated with each other. With 44 drivers  
592 and 6x26 variables, we face the risk of overlearning in our statistical analyses. To address this risk,  
593 we perform a multiple factor analysis (MFA) to reduce sampling noise. Then, using a clustering  
594 method, we obtain clusters of pairs (driver ; road type). The first principal component of the MFA  
595 can be seen as a "severity score" for comparing driving in terms of damage to a part: a high value  
596 of this first principal component indicates driving that causes significant damage both longitudinally,  
597 laterally, and vertically, regardless of the road type. The second and third components help us identify  
598 severe driving profiles in terms of damage associated with poor speed management. This may include  
599 acceleration/braking in fast lanes or high-speed cornering in urban areas.
- 600 4. During the life of the vehicle, the conditions of use and the owners or drivers may vary. For this reason,  
601 durability specifications are based on the total mileage traveled by the vehicle (e.g., 240,000 km) and a  
602 specific combination of road types. Therefore we will look at fatigue design using a combination of all  
603 six road types over a distance of 240 km. An EFA allows us to identify two main sources of variations  
604 of the damage magnitudes. The second factor is associated with forward steering maneuvers, while the  
605 first factor is associated with various maneuvers (acceleration/braking, high-speed overtaking). The  
606 two factors are correlated. Just as the load spectrum represents the distribution of load amplitudes  
607 in load histories, here we're looking for the distribution of damage amplitudes in load histories.  
608 To reduce sampling noise and avoid overlearning, we use probabilistic PCA: to account for the  
609 heterogeneity of driving styles, we use a mixture model of multi-dimensional Gaussian distributions  
610 on the first principal components. The parameters are estimated by maximum likelihood using the  
611 EM algorithm. Next, the probability density function of the extent of damage is constructed using the  
612 PCA transition matrix and then extrapolated to cover a distance several times longer.

613 Significant advantages of this approach include:

- 614 • The use of local contexts makes it possible to analyze damage caused by multi-input loads without  
615 having to refer to a specific chassis part to which those loads are applied;
- 616 • Unsupervised statistical analyses allow us to study the latent structure of correlations between damage  
617 intensities and the latent structure of driver behavior:
  - 618 – Exploratory Factor Analysis (EFA) enables the design office to select the local context(s) that  
619 best describe fatigue on the reference route. This then proves invaluable in guiding the design  
620 of chassis parts;
  - 621 – Multiple Factor Analysis (MFA) helps to deal with the complexity of the hierarchical structure  
622 of multivariate data resulting from the consideration of different road types. First, it solves the  
623 problem of overlearning due to the limited number of drivers and the large number of local  
624 contexts to be considered. In addition, the first principal component of the MFA is a linear  
625 combination of the damage intensities of all local contexts, which allows us to compare the  
626 different pairs (driver; road type) in terms of induced damage. In other words, we obtain a  
627 multidimensional measure of severity. We then use it to establish profiles of more or less severe  
628 drivers.
- 629 • The mixing model for damage magnitude probability distribution function, offers the possibility of  
630 a more detailed analysis of the damage distribution according to different driving profiles, whether  
631 more or less severe, and allow extrapolation and simulations for different road mixes and driver profile  
632 changes.

633 All of this is made possible by field data collection, which adds a more realistic dimension to reference loads  
634 for design, simulation and test rig validation.

635 The main objective of this work is to propose a methodology for multidimensional load analysis. Two  
636 axes of perspectives are outlined: on the one hand, the statistical analysis of more complete data sets,  
637 including in particular the consideration of the payload effect, and on the other hand, the applications of this  
638 methodology in the design process, in particular to define validation test levels for chassis components.

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### 642 **Authors contributions**

643 **Emilien Baroux:** performed Conceptualization, Writing - original draft, Writing review. **Patrick**  
644 **Pamphile:** performed Conceptualization, Writing - original draft, Writing review & Editing, Supervision.



645 **Benoit Delattre:** performed Conceptualization, Supervision. **Andrei Constantinescu:** performed Supervision.  
646 **Laurent Rota:** performed Supervision.

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