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Modeling moderate and extreme urban rainfall at high spatio-temporal resolution

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Abstract

- ▶ High spatial and temporal resolution
- ▶ Univariate modeling of moderate and intense rainfall
- ▶ Analysis of the spatio-temporal extremal dependence
- ▶ Weighted least squares model for dependence modeling

Data [1]



$S = \{17 \text{ rain gauges}\}$

- ▶ **Period:** [2019, 2022]
- ▶ **High temporal resolution:**
Every minute \rightarrow 5-minute aggregation
- ▶ **High spatial resolution:**
Interdistance $\in [77, 1531]$ meters

Spatio-temporal dependence modeling [4]

Let $X = \{X(s, t), (s, t) \in \mathcal{S} \times [0, \infty)\}$ be a strictly stationary isotropic Brown-Resnick process. For a spatial lag $v \geq 0$ and a temporal lag $h \geq 0$, the extremogram of X is given by

$$\chi(v, h) = 2 \left(1 - \phi \left(\sqrt{\frac{1}{2} \delta(v, h)} \right) \right)$$

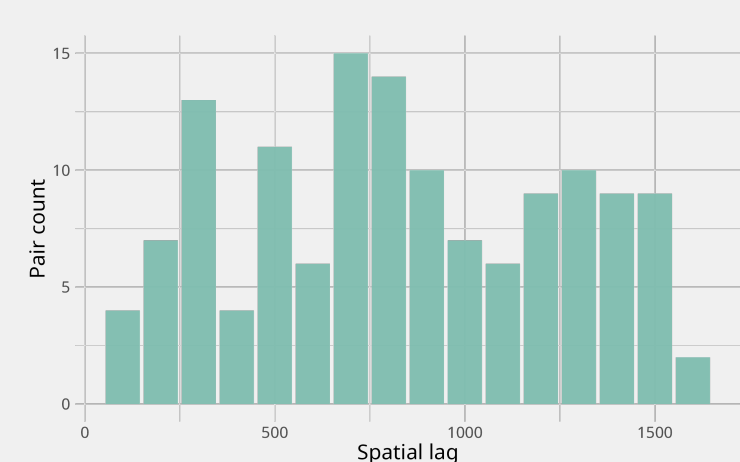
with ϕ the standard normal distribution function and δ a stationary and isotropic variogram.

Assumption of additive separability: $\frac{\delta(v, h)}{2} = \theta_1 v^{\alpha_1} + \theta_2 h^{\alpha_2}$, $0 < \alpha_1, \alpha_2 \leq 2$, $\theta_1, \theta_2 > 0$

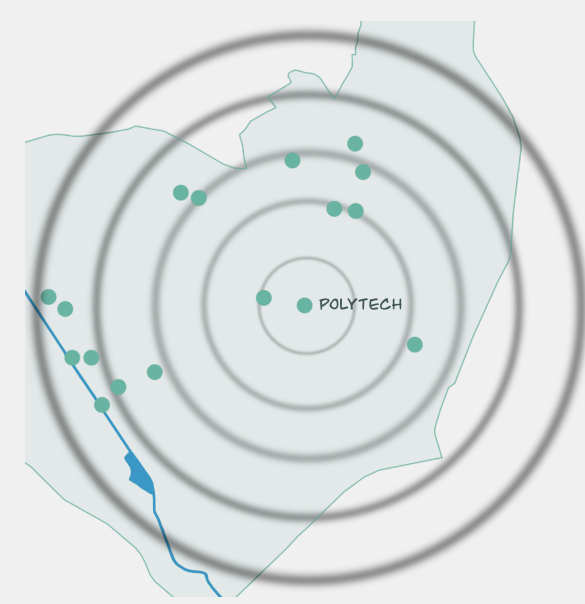
Spatial extremogram

For all spatial lags $v = k \times \Delta v$, $k = 1, 2, \dots$, we define

$$N(v) = \{(s_i, s_j) \mid \|s_i - s_j\| \in [v - \Delta v, v]\}$$



For $\Delta v = 100$ meters



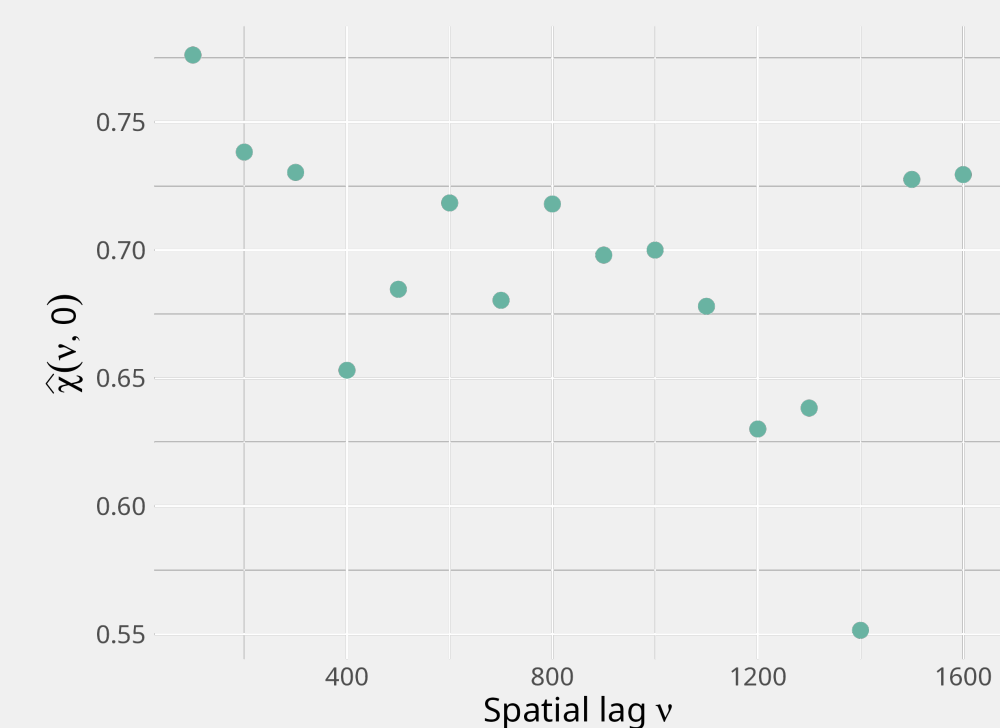
For fixed t and for any $(s_i, s_j) \in N(v)$

$$\chi_{i,j,q}^{(t)}(v, 0) = \frac{\mathbb{P}(X(s_i, t) > q, X(s_j, t) > q)}{\mathbb{P}(X(s_i, t) > q)}$$

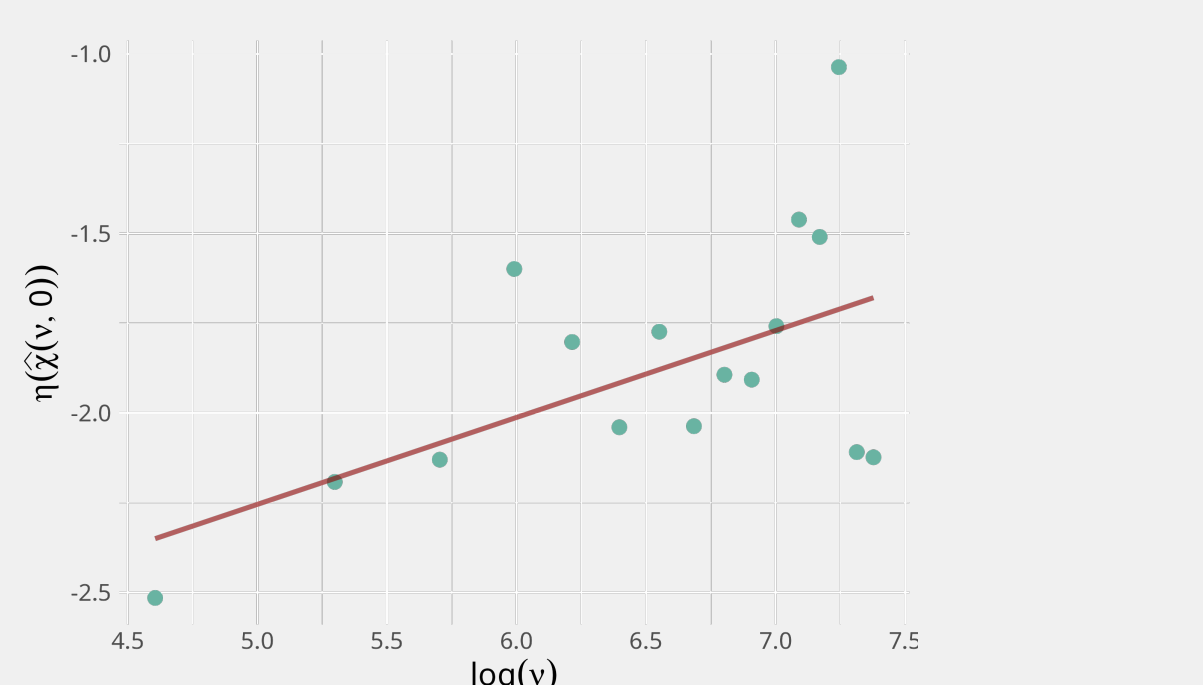
Estimator:

$$\hat{\chi}_q^{(t)}(v, 0) = \frac{\frac{1}{|N(v)|} \sum_{(s_i, s_j) \in N(v)} \mathbb{1}\{X(s_i, t) > q, X(s_j, t) > q\}}{\frac{1}{|S|} \sum_{i=1}^{|S|} \mathbb{1}\{X(s_i, t) > q\}}$$

with q a high quantile (99.8%).



WLSE



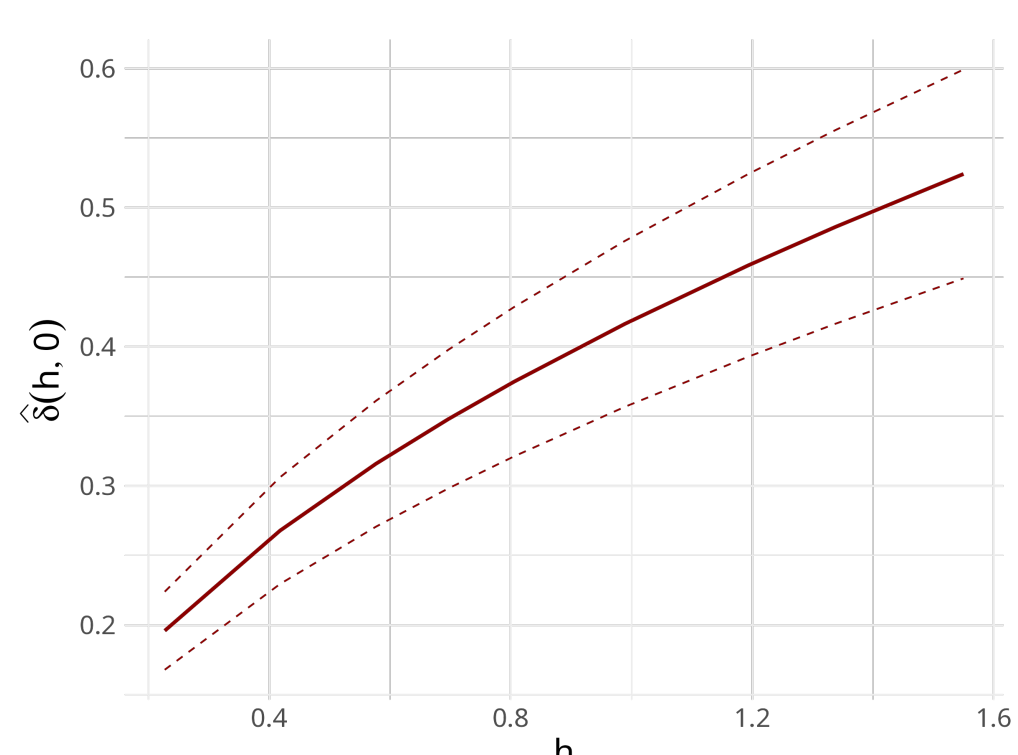
| | Estimate | Std. Error |
|------------------|-----------|------------|
| \hat{c}_1 | -3.465*** | 0.605 |
| $\hat{\alpha}_1$ | 0.242* | 0.093 |

*p-value < 0.05; ***p-value < 0.001

Empirical variogram

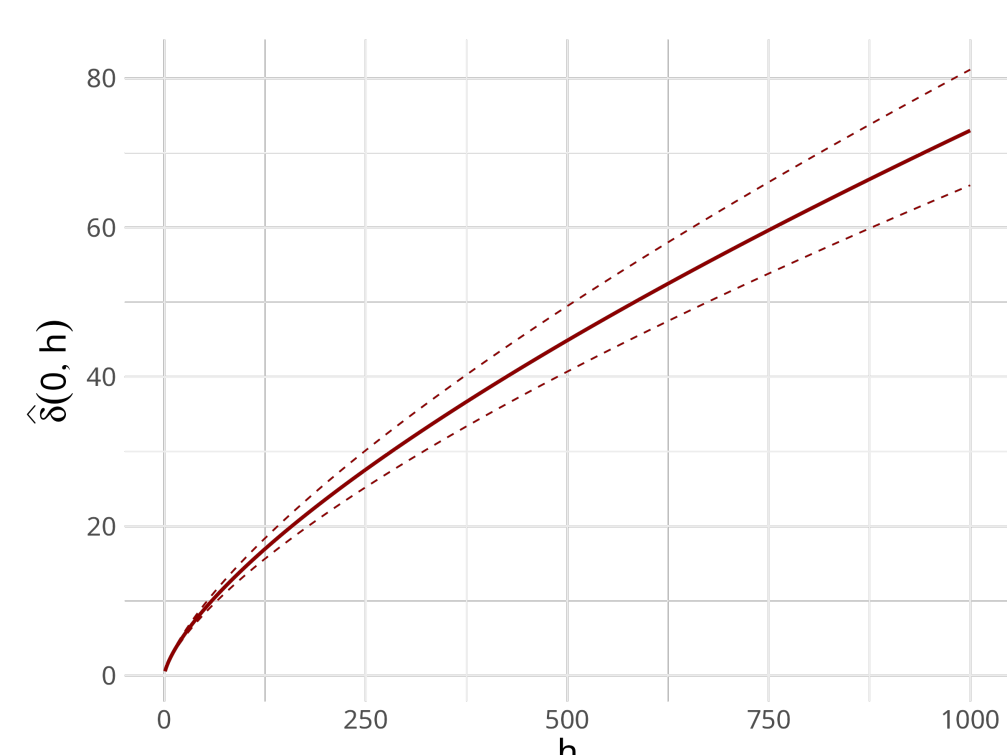
Spatial

$$\hat{\delta}(v, 0) = 2\hat{\theta}_1 v^{\hat{\alpha}_1}$$



Temporal

$$\hat{\delta}(0, h) = 2\hat{\theta}_2 h^{\hat{\alpha}_2}$$



Univariate modeling of the rainfall distribution

Let Y denote the rainfall measurement at a given site.

Generalized Pareto Distribution (GPD) [2] for rainfall excesses above a high threshold u

$$Y - u \mid Y > u \sim H_\xi, \quad \text{where} \quad H_\xi \left(\frac{y}{\sigma} \right) = \begin{cases} 1 - (1 + \xi \frac{y}{\sigma})^{-1/\xi} & \text{si } \xi \neq 0, \\ 1 - e^{-y/\sigma} & \text{si } \xi = 0, \end{cases}$$

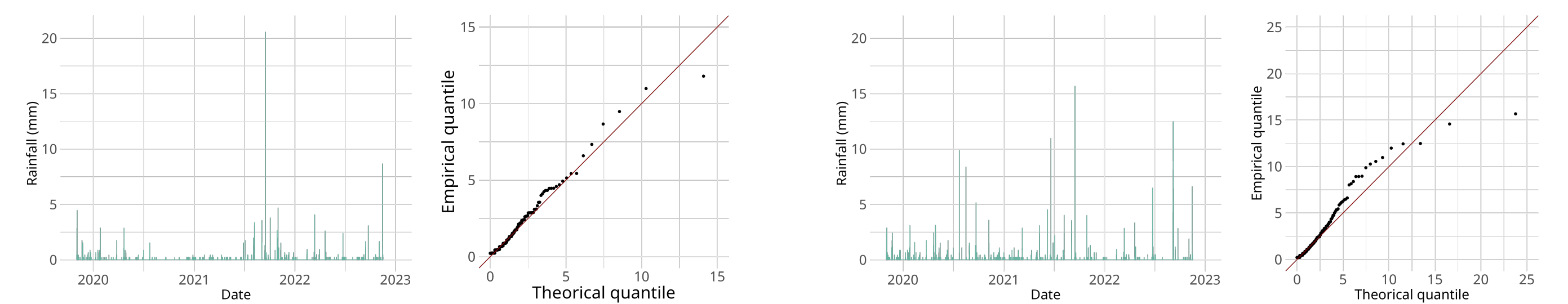
where $a_+ = \max(a, 0)$, $\sigma > 0$, $\xi \in \mathbb{R}$ and $y > 0$.

Extended Generalized Pareto Distribution (EGPD) [3] for the entire distribution

The cumulative distribution function (cdf) of the EGPD is given by

$$F_Y(y) = G \left(H_\xi \left(\frac{y}{\sigma} \right) \right),$$

with $G(x) = x^\kappa$, $\kappa > 0$.



EGPD fit with $\hat{\kappa} = 0.56$, $\hat{\sigma} = 0.26$ and $\hat{\xi} = 0.51$

Spatio-temporal

$$\chi(v, h) = 2 \left(1 - \phi \left(\sqrt{\frac{1}{2} \delta(v, h)} \right) \right)$$

Transformation:

$$\eta(x) = 2 \log \left(\phi^{-1} \left(1 - \frac{1}{2} x \right) \right)$$

Spatial

$$\eta(\chi(v, 0)) = \log \theta_1 + \alpha_1 \log v, \quad v > 0$$

$$:= c_1 + \alpha_1 x_v$$

Temporal

$$\eta(\chi(0, h)) = \log \theta_2 + \alpha_2 \log h, \quad h > 0$$

$$:= c_2 + \alpha_2 x_h$$

Weighted Least Squares Estimation (WLSE)

$$\begin{pmatrix} \hat{c}_i \\ \hat{\alpha}_i \end{pmatrix} = \operatorname{argmin}_{c_i, \alpha_i} \sum_x w_x (\eta(\hat{\chi}) - (c_i + \alpha_i x))^2$$

Temporal extremogram

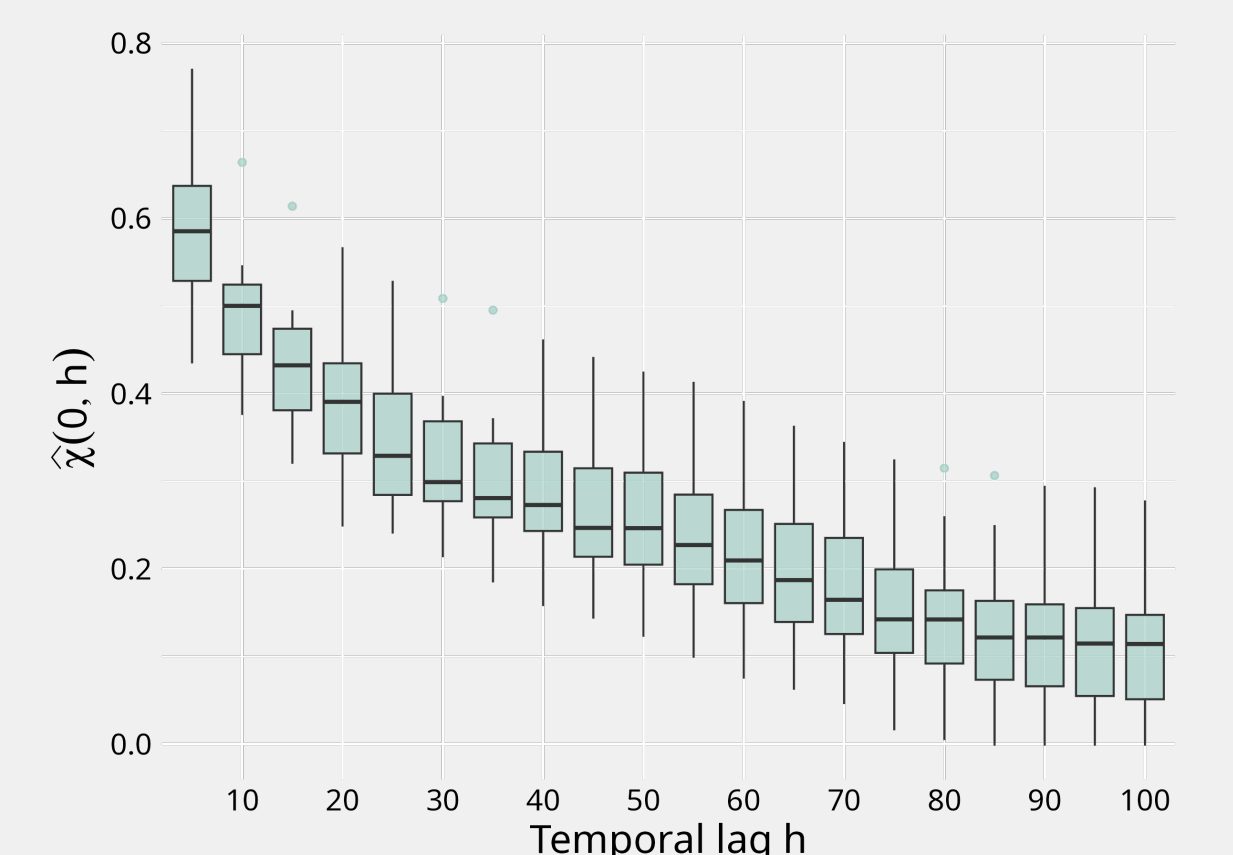
For a site s and for any t

$$\chi_{t,q}^{(s)}(0, h) = \frac{\mathbb{P}(X(s, t) > q, X(s, t+h) > q)}{\mathbb{P}(X(s, t) > q)}$$

Estimator:

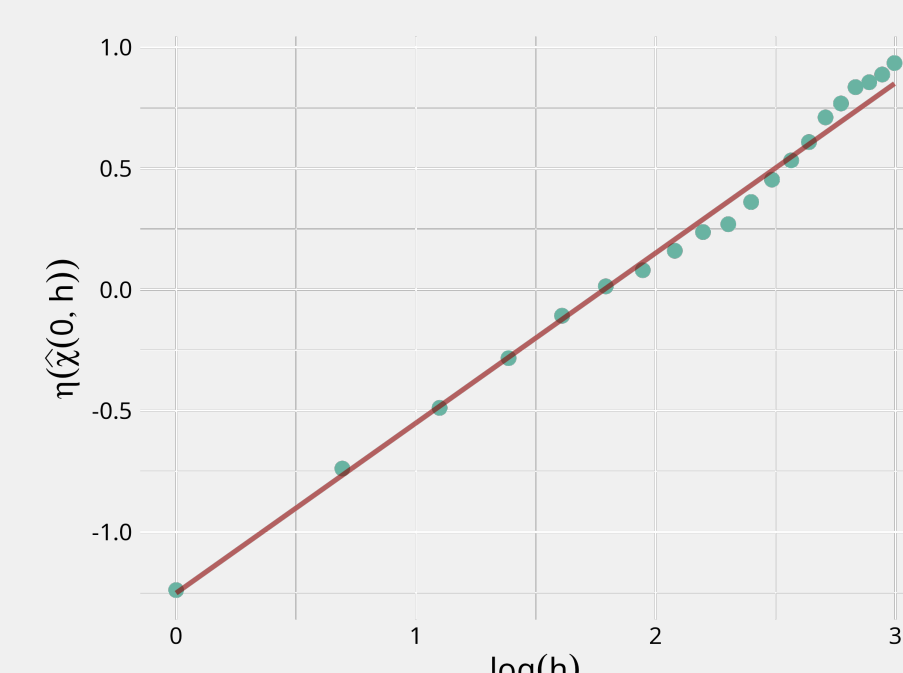
$$\hat{\chi}_q^{(s)}(0, h) = \frac{\frac{1}{T-h} \sum_{k=1}^{T-h} \mathbb{1}\{X(s, t_k) > q, X(s, t_k+h) > q\}}{\frac{1}{T} \sum_{k=1}^T \mathbb{1}\{X(s, t_k) > q\}}$$

with q a high quantile (99.8%) and $t_k \in \{t_1, \dots, t_T\}$.



With $\Delta h = 5$ minutes

WLSE



| | Estimate | Std. Error |
|------------------|-----------|------------|
| \hat{c}_2 | -1.252*** | 0.023 |
| $\hat{\alpha}_2$ | 0.702*** | 0.012 |

***p-value < 0.001

Future works

- ▶ More complex variograms, including space-time non-separability
- ▶ Anisotropic structure and advection
- ▶ Combination of small-scale and larger-scale spatio-temporal modeling using different data sources

References

- [1] Pascal Finaud-Guyot et al. *Rainfall data collected by the HSM urban observatory (OMSEV)*. 2023.
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- [3] Philippe Naveau et al. "Modeling jointly low, moderate, and heavy rainfall intensities without a threshold selection". In: *Water Resources Research* (2016).
- [4] Sven Buhl et al. "Semiparametric estimation for isotropic max-stable space-time processes". In: *Bernoulli* 25.4A (2019), pp. 2508-2537.



Author portfolio website