



**HAL**  
open science

# Modeling moderate and extreme urban rainfall at high spatio-temporal resolution

Chloé Serre-Combe, Nicolas Meyer, Thomas Opitz, Gwladys Toulemonde

► **To cite this version:**

Chloé Serre-Combe, Nicolas Meyer, Thomas Opitz, Gwladys Toulemonde. Modeling moderate and extreme urban rainfall at high spatio-temporal resolution. EVA 2023 - 13th International Conference on Extreme Value Analysis, Jun 2023, Milano, Italy. hal-04374090

**HAL Id: hal-04374090**

**<https://hal.science/hal-04374090v1>**

Submitted on 9 Jan 2024

**HAL** is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

## Abstract

- ▶ High spatial and temporal resolution
- ▶ Univariate modeling of moderate and intense rainfall
- ▶ Analysis of the spatio-temporal extremal dependence
- ▶ Weighted least squares model for dependence modeling

## Data [1]



$S = \{17 \text{ rain gauges}\}$

- ▶ **Period:** [2019, 2022]
- ▶ **High temporal resolution:**  
Every minute  $\rightarrow$  5-minute aggregation
- ▶ **High spatial resolution:**  
Interdistance  $\in [77, 1531]$  meters

## Spatio-temporal dependence modeling [4]

Let  $X = \{X(s, t), (s, t) \in \mathcal{S} \times [0, \infty)\}$  be a strictly stationary isotropic Brown-Resnick process. For a spatial lag  $v \geq 0$  and a temporal lag  $h \geq 0$ , the extremogram of  $X$  is given by

$$\chi(v, h) = 2 \left( 1 - \phi \left( \sqrt{\frac{1}{2} \delta(v, h)} \right) \right)$$

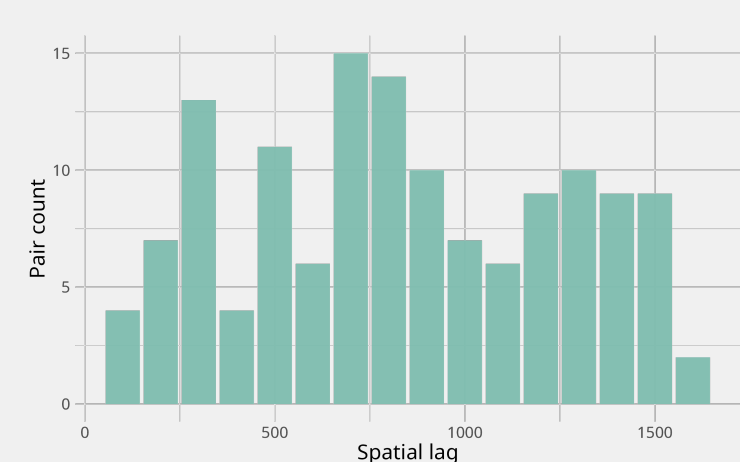
with  $\phi$  the standard normal distribution function and  $\delta$  a stationary and isotropic variogram.

**Assumption of additive separability:**  $\frac{\delta(v, h)}{2} = \theta_1 v^{\alpha_1} + \theta_2 h^{\alpha_2}$ ,  $0 < \alpha_1, \alpha_2 \leq 2$ ,  $\theta_1, \theta_2 > 0$

## Spatial extremogram

For all spatial lags  $v = k \times \Delta v$ ,  $k = 1, 2, \dots$ , we define

$$N(v) = \{(s_i, s_j) \mid \|s_i - s_j\| \in [v - \Delta v, v]\}$$



For  $\Delta v = 100$  meters



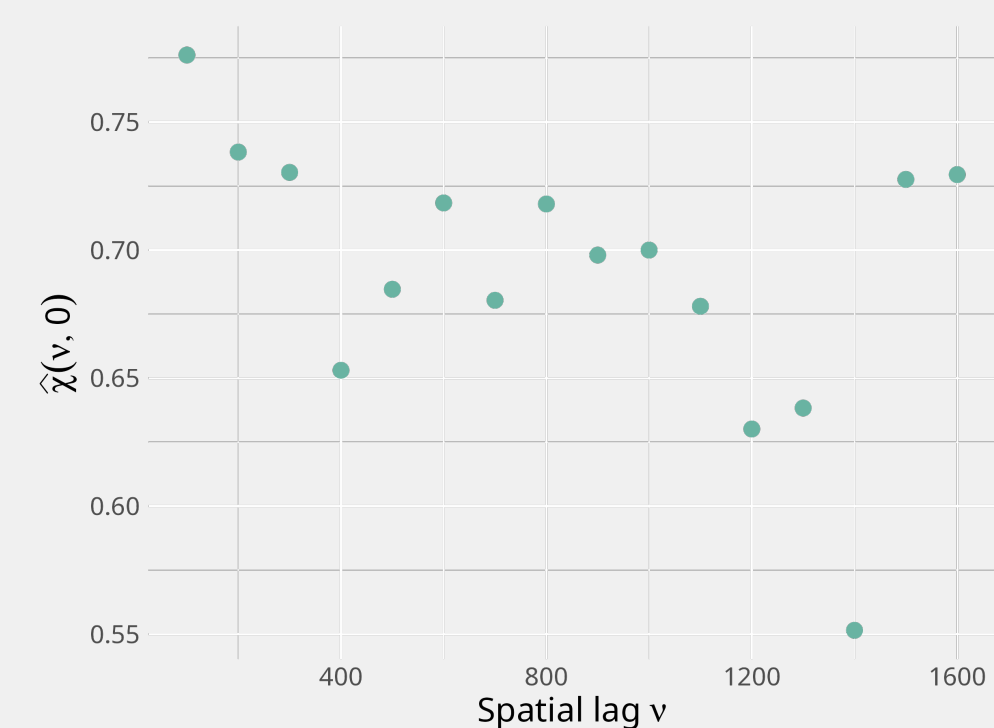
For fixed  $t$  and for any  $(s_i, s_j) \in N(v)$

$$\chi_{i,j,q}^{(t)}(v, 0) = \frac{\mathbb{P}(X(s_i, t) > q, X(s_j, t) > q)}{\mathbb{P}(X(s_i, t) > q)}$$

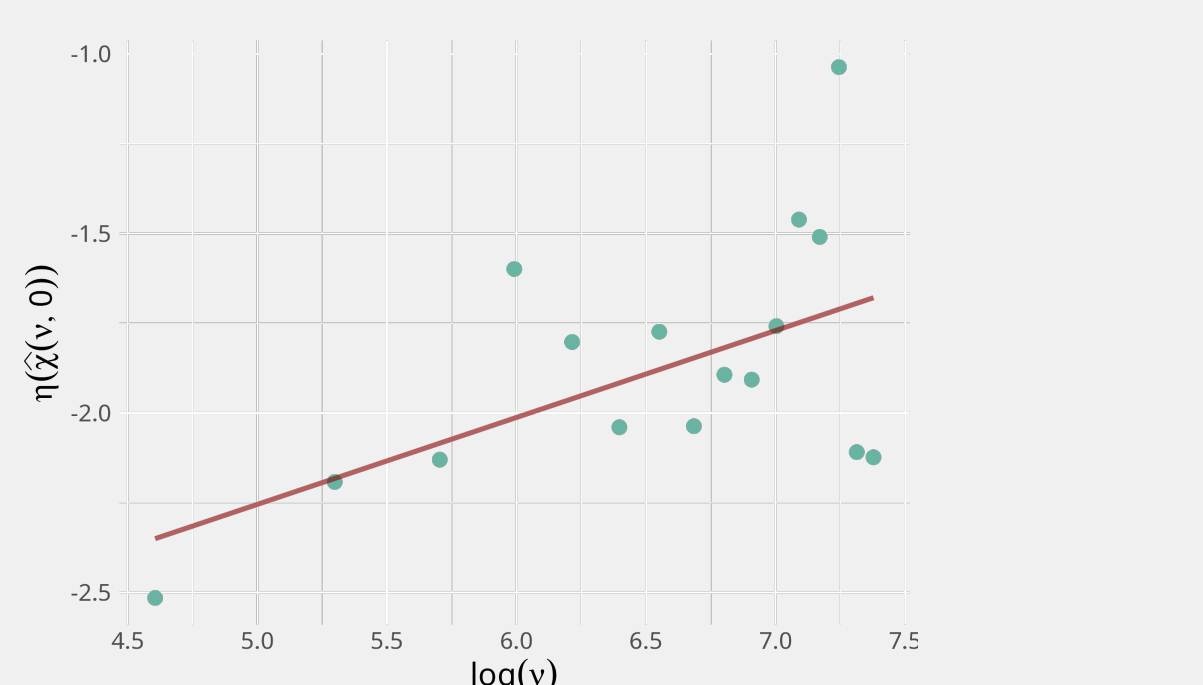
**Estimator:**

$$\hat{\chi}_q^{(t)}(v, 0) = \frac{\frac{1}{|N(v)|} \sum_{i,j \mid (s_i, s_j) \in N(v)} \mathbb{1}\{X(s_i, t) > q, X(s_j, t) > q\}}{\frac{1}{|S|} \sum_{i=1}^{|S|} \mathbb{1}\{X(s_i, t) > q\}}$$

with  $q$  a high quantile (99.8%).



WLSE



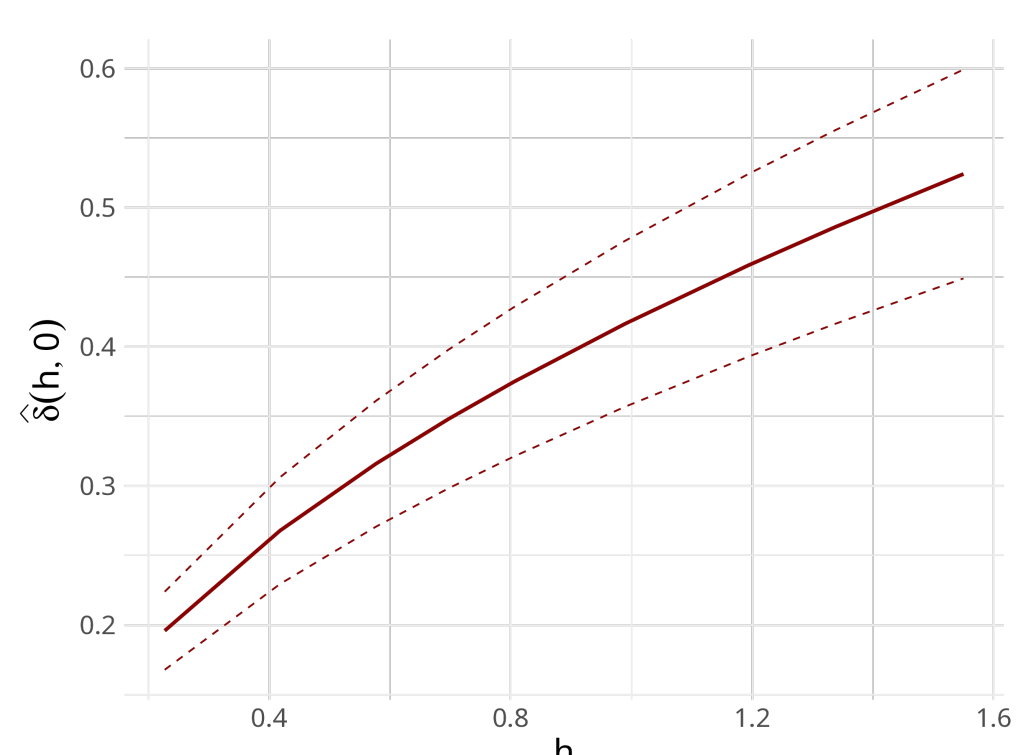
	Estimate	Std. Error
$\hat{c}_1$	-3.465***	0.605
$\hat{\alpha}_1$	0.242*	0.093

\*p-value < 0.05; \*\*\*p-value < 0.001

## Empirical variogram

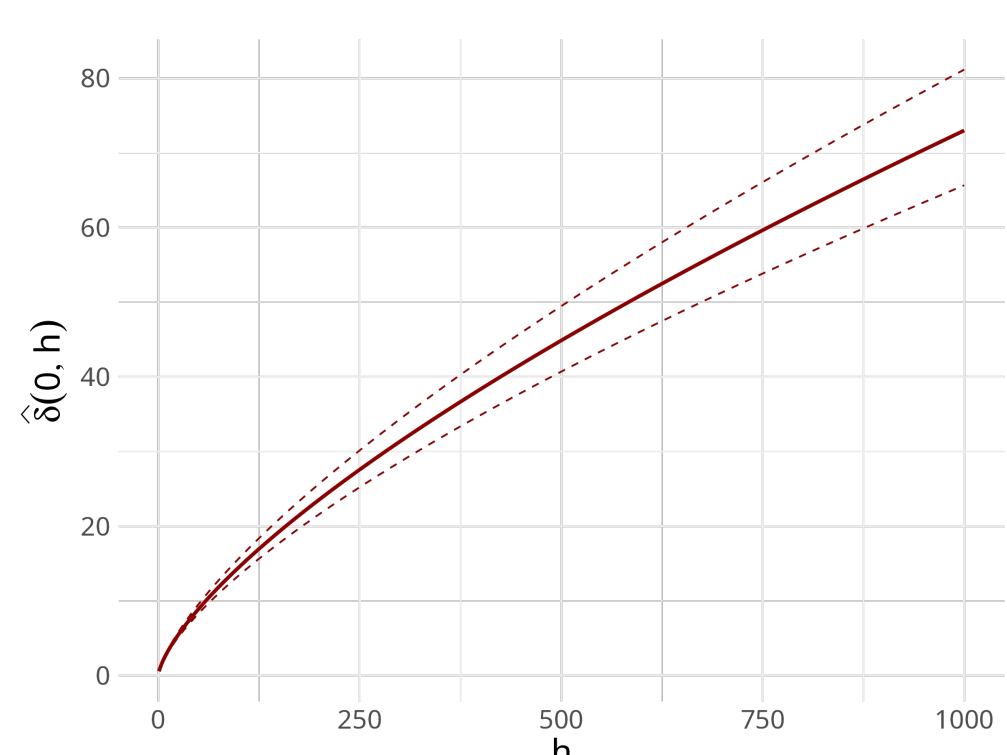
**Spatial**

$$\hat{\delta}(v, 0) = 2\hat{\theta}_1 v^{\hat{\alpha}_1}$$



**Temporal**

$$\hat{\delta}(0, h) = 2\hat{\theta}_2 h^{\hat{\alpha}_2}$$



## Univariate modeling of the rainfall distribution

Let  $Y$  denote the rainfall measurement at a given site.

**Generalized Pareto Distribution (GPD) [2]** for rainfall excesses above a high threshold  $u$

$$Y - u \mid Y > u \sim H_\xi, \quad \text{where} \quad H_\xi \left( \frac{y}{\sigma} \right) = \begin{cases} 1 - (1 + \xi \frac{y}{\sigma})_+^{-1/\xi} & \text{si } \xi \neq 0, \\ 1 - e^{-y/\sigma} & \text{si } \xi = 0, \end{cases}$$

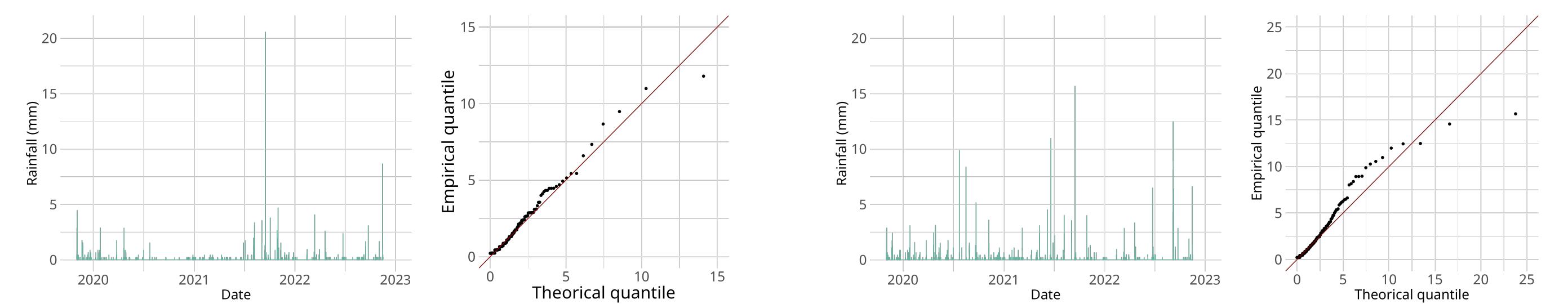
where  $a_+ = \max(a, 0)$ ,  $\sigma > 0$ ,  $\xi \in \mathbb{R}$  and  $y > 0$ .

**Extended Generalized Pareto Distribution (EGPD) [3]** for the entire distribution

The cumulative distribution function (cdf) of the EGPD is given by

$$F_Y(y) = G \left( H_\xi \left( \frac{y}{\sigma} \right) \right),$$

with  $G(x) = x^\kappa$ ,  $\kappa > 0$ .



EGPD fit with  $\hat{\kappa} = 0.56$ ,  $\hat{\sigma} = 0.26$  and  $\hat{\xi} = 0.51$

**Spatio-temporal**

$$\chi(v, h) = 2 \left( 1 - \phi \left( \sqrt{\frac{1}{2} \delta(v, h)} \right) \right)$$

**Transformation:**

$$\eta(x) = 2 \log \left( \phi^{-1} \left( 1 - \frac{1}{2} x \right) \right)$$

**Spatial**

$$\eta(\chi(v, 0)) = \log \theta_1 + \alpha_1 \log v, \quad v > 0$$

$$:= c_1 + \alpha_1 x_v$$

**Temporal**

$$\eta(\chi(0, h)) = \log \theta_2 + \alpha_2 \log h, \quad h > 0$$

$$:= c_2 + \alpha_2 x_h$$

**Weighted Least Squares Estimation (WLSE)**

$$\begin{pmatrix} \hat{c}_i \\ \hat{\alpha}_i \end{pmatrix} = \operatorname{argmin}_{c_i, \alpha_i} \sum_x w_x (\eta(\hat{\chi}) - (c_i + \alpha_i x))^2$$

## Temporal extremogram

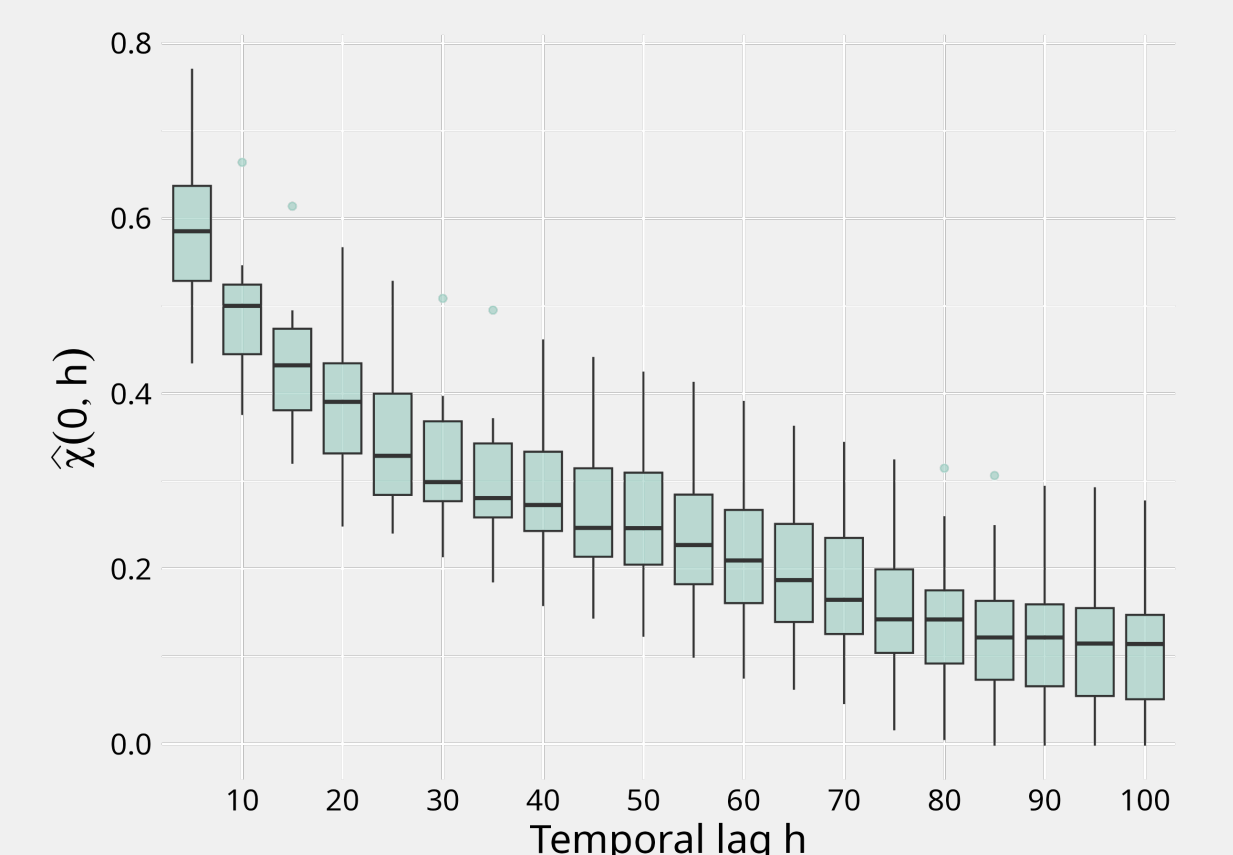
For a site  $s$  and for any  $t$

$$\chi_{t,q}^{(s)}(0, h) = \frac{\mathbb{P}(X(s, t) > q, X(s, t+h) > q)}{\mathbb{P}(X(s, t) > q)}$$

**Estimator:**

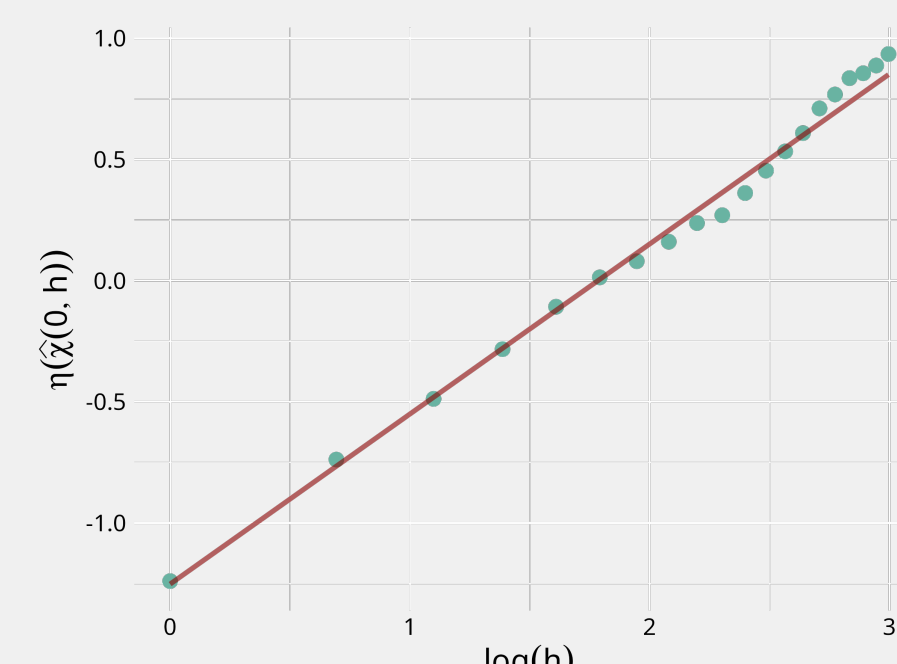
$$\hat{\chi}_q^{(s)}(0, h) = \frac{\frac{1}{T-h} \sum_{k=1}^{T-h} \mathbb{1}\{X(s, t_k) > q, X(s, t_k+h) > q\}}{\frac{1}{T} \sum_{k=1}^T \mathbb{1}\{X(s, t_k) > q\}}$$

with  $q$  a high quantile (99.8%) and  $t_k \in \{t_1, \dots, t_T\}$ .



With  $\Delta h = 5$  minutes

WLSE



	Estimate	Std. Error
$\hat{c}_2$	-1.252***	0.023
$\hat{\alpha}_2$	0.702***	0.012

\*\*\*p-value < 0.001

## Future works

- ▶ More complex variograms, including space-time non-separability
- ▶ Anisotropic structure and advection
- ▶ Combination of small-scale and larger-scale spatio-temporal modeling using different data sources

## References

- [1] Pascal Finaud-Guyot et al. *Rainfall data collected by the HSM urban observatory (OMSEV)*. 2023.
- [2] Stuart Coles et al. *An introduction to statistical modeling of extreme values*. Vol. 208. Springer, 2001.
- [3] Philippe Naveau et al. "Modeling jointly low, moderate, and heavy rainfall intensities without a threshold selection". In: *Water Resources Research* (2016).
- [4] Sven Buhl et al. "Semiparametric estimation for isotropic max-stable space-time processes". In: *Bernoulli* 25.4A (2019), pp. 2508-2537.



Author portfolio website