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A fast Neuber-type finite element simulator to enable deep-learning-based fatigue life predictions from thermographic data

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Résumé :

Cet article aborde les aspects numériques d'une nouvelle méthode développée pour accélérer les calculs sur des structures poreuses à comportement non linéaire. L'hypothèse de proportionnalité est utilisée en conjonction avec une méthode de Neuber modifiée afin de réduire la complexité dimensionnelle typique des méthodes de Neuber multiaxiales. Cette méthode est utilisée afin d'intégrer numériquement les lois matérielles non linéaires en fonction de la réponse élastique EF de la structure, pour obtenir la dissipation intrinsèque en champ complet via les composantes déviatoriques élasto-plastiques approchées de contrainte et de déformation. Les erreurs generés en utilisant de cette méthode sont illustrées. Cette méthode permet la génération rapide de grands ensembles de données de la réponse thermique des structures avec des populations de pores variables ayant un comportement non linéaire. Des approches d'apprentissage automatique basées sur ces données pour lier la réponse thermique des structures à leur durée de vie en fatigue sont donc rendues possibles.

Abstract :

This paper addresses the numerical aspects of a new method developed to accelerate calculations on porous structures with non-linear material behavior. The hypothesis of proportionality is used in conjuction with a modified Neuber method in order to reduce the dimensional complexity that the multi-axial Neuber methods intrinsically have. This method is used in order to numerically integrate the non-linear material laws as a function of the FE elastic response of the structure, to obtain the full-field intrinsic dissipation via the approximated elasto-plastic deviatoric components of stress and strain. The errors generated by using this method are illustrated. This method enables rapid generation of large data-sets of the thermal response of structures with varying pore populations having a non-linear behavior. Data-driven deep-learning approaches for linking the thermal response of structures to their fatigue life are therefore made possible.

Keywords : Model Reduction, Neuber's rule, Finite Element Method, Accelerated Calculations

Nomenclature

$\hat{\sigma}_d, \hat{\epsilon}_d$	Approximated total deviatoric stress and strain tensors
$\hat{\sigma}_{d,o}, \hat{\epsilon}_{d,o}$	Approximated total deviatoric stress and strain tensors at the last peak of cyclic loading
$\bar{\overline{\mathcal{Q}}}_d, \bar{\overline{\mathcal{E}}}_d$	Finite-Element elastic stress and strain deviatoric tensors for a point at a fixed global loading
$ar{arphi}^e_{d},ar{arepsilon}^e_{d}$	Elastic deviatoric stress and strain tensors
$ar{\sigma}^e_{d,o},ar{\epsilon}^e_{d,o}$	Elastic deviatoric stress and strain tensors at the last peak of cyclic loading
$\widetilde{\mathcal{Q}}_d, \widetilde{\mathcal{L}}_d$	Reference total deviatoric stress and strain tensors
e	Proportionality ratio of the approximated total deviatoric strain tensor to the elastic deviatoric strain tensor
e _o	Proportionality ratio of the approximated total deviatoric strain tensor to the elastic deviatoric strain tensor at the last peak of cyclic loading
f	Loading function
f_o	Loading function at the last peak of cyclic loading
8	Proportionality ratio of the approximated total deviatoric stress tensor to the elastic deviatoric stress tensor
S ₀	Proportionality ratio of the approximated total deviatoric stress tensor to the elastic deviatoric stress tensor at the last peak of cyclic loading

1 Introduction

Fatigue life characterization of industrial components has traditionally required a large number of experiments, in depth investigation by SEM and more recently tomographic analysis to understand the mechanisms of damage especially in presence of defects, to identify a fatigue lifetime law based on a criterion and to describe the scatter in lifetimes [1], [2]. This approach is both costly and time consuming, therefore, a new method of fatigue lifetime characterization is to be developed, consisting of a threefold approach, including experiments, simulations and machine learning. The main objective is to reach a real-time criterion from measurements to assess residual lifetime of a material subjected to fatigue loading in presence of defects.

A deep-learning based tool will be developed that can be used in real-time to aid in the decision making regarding the service life of structures bearing fatigue loading in presence of a population of defects. The fatigue lifetime will be predicted based on image data that can be easily and inexpensively acquired via non contact measurement techniques. The study is based around laboratory specimens made from cast aluminium alloys, manufactured by the lost foam method, that is commonly used in the automotive industry. The manufacturing method causes formation of pores and defects in the material, which adds to the uncertainty in the fatigue lifetime characterization [1], [4]. The proposed approach aims to exploit the thermal response of these porous specimens under cyclic mechanical solicitation in order to predict their fatigue lifetime. The thermal response includes thermoelastic coupling as well as intrinsic dissipation

in the structures, which causes a temperature change in the structures identifiable via IR thermography that reduces the ill-posedness of the inverse problem to be solved.

Deep-learning approaches require a large and representative data-set for learning the parameters responsible for failure. As experiments remain scarce, simulations will form a large part of the data-set. A Monte-Carlo approach is adopted to simulate varying pore populations. Nevertheless, nonlinear FE calculations on structures with pores remain computationally expensive and highly time consuming and are not feasible in large quantities. Thus, a method of acceleration is needed.

In this paper, a modified Neuber type method is developed and solved in conjunction with an elastoplastic model [8] to accelerate the calculation by using inexpensive elastic finite element calculations. This permits fast calculation of internal plastic variables like the intrinsic dissipation for a range of structures with varying pore populations.

2 Literature

2.1 Neuber-Type methods

Neuber [9] originally derived a way to approximate the unidimensional plastic stress and strain at a notch root under monotonic loading from an elastic calculation. The Neuber rule states that the Hookian or elastic stress concentration factor k_t is the geometric mean of the plastic stress and strain concentration factors k_{σ} and k_{ϵ} , i.e.

$$k_t = (k_\sigma k_\epsilon)^{\frac{1}{2}} \tag{1}$$

which can also be written as

$$\sigma\epsilon = \sigma^e \epsilon^e \tag{2}$$

where σ^e and ϵ^e are the elastic stress and strain at the notch and σ and ϵ are the approximated real stress and strain at the notch. This equation represents the equality of the total work at the notch tip.

This concept was extended for cyclic uni-axial loading by assuming a zero mean stress at the notch root and relating the elastic local stress and strain ranges $\Delta \sigma^e$ and $\Delta \epsilon^e$ to the real local stress and strain ranges $\Delta \sigma$ and $\Delta \epsilon$ [10]:

$$\Delta \sigma \Delta \epsilon = \Delta \sigma^e \Delta \epsilon^e \tag{3}$$

The authors in [11] suggest taking into account the mean stress by including a change of origin every time a peak in the uni-axial cycling loading is reached :

$$(\sigma - \sigma_o) : (\epsilon - \epsilon_o) = (\bar{\sigma}^e - \bar{\sigma}_o^e) : (\bar{\epsilon}^e - \bar{\epsilon}_o^e)$$
(4)

The Neuber method has been generalized for proportional multi-axial loadings, with the extension being [13] :

$$\sigma_{ij}\epsilon_{ij} = \sigma^e_{ij}\epsilon^e_{ij} \tag{5}$$

Where i,j = 1,2,3 and summation of the components is applied. The authors note that this formulation applies when the deviatoric stresses of the notch tip are proportional during loading. The authors also assume that the fractional contribution of the largest principal notch tip stress and strain to the total notch tip strain energy density is virtually equal to the corresponding fraction calculated elastically. Together with the material constitutive equations, the stresses and strains at the notch tip are thereby calculated.

The investigations in [14] report that the estimated elastic-plastic solution seems insensitive to the assumed path for proportional nominal loadings, but becomes very sensitive to the assumed path for nonproportional nominal loadings. Thus, an incremental version of the Neuber method has been developed to account for non-proportional cyclic loading via the application of incremental plasticity. The authors in [15] suggest that in order to formulate the set of necessary equations for a multiaxial non-proportional analysis of elastic-plastic stresses and strains at the notch tip, increments of the total distortional strain energy density contributed by each pair of stress and strain components are equal :

$$\sigma_{d_{ij}}\Delta\epsilon_{d_{ij}} + \Delta\sigma_{d_{ij}}\epsilon_{d_{ij}} = \sigma^e_{d_{ij}}\Delta\epsilon^e_{d_{ij}} + \Delta\sigma^e_{d_{ij}}\epsilon^e_{d_{ij}}$$
(6)

Therefore, the incremental deviatoric elastoplastic stress and strain components at the notch tip are therefore calculated via increments of the hypothetical linear elastic stresses and strains due to nonproportional loading and the material constitutive equations (the authors use the Prandtl-Reuss flow rule).

3 Methodology

3.1 Modified Neuber hypothesis

The Neuber-type hypothesis used for our case considers the deviatoric parts of the stress and strain tensors :

$$(\hat{\underline{\sigma}}_d - \hat{\underline{\sigma}}_{d,o}) : (\hat{\underline{\epsilon}}_d - \hat{\underline{\epsilon}}_{d,o}) = (\bar{\underline{\sigma}}_d^e - \bar{\underline{\sigma}}_{d,o}^e) : (\bar{\underline{\epsilon}}_d^e - \bar{\underline{\epsilon}}_{d,o}^e)$$
(7)

Where $\hat{\sigma}_d$, $\hat{\varepsilon}_d$ stand for the approximated total deviatoric stress and strain tensors and $\hat{\sigma}_{d,o}$, $\hat{\varepsilon}_{d,o}$ stand for the approximated total deviatoric stress and strain tensors at the last peak. Similarly, $\bar{\sigma}_d^e$, $\bar{\varepsilon}_d^e$ stand for the hypothetical elastic deviatoric stress and strain tensors and $\bar{\sigma}_{d,o}^e$, $\bar{\sigma}_{d,o}^e$ stand for the hypothetical elastic deviatoric stress and strain tensors and $\bar{\sigma}_{d,o}^e$, $\bar{\sigma}_{d,o}^e$ stand for the hypothetical elastic deviatoric stress and strain tensors and $\bar{\sigma}_{d,o}^e$, $\bar{\sigma}_{d,o}^e$ stand for the hypothetical elastic deviatoric stress and strain tensors at the last peak. The hydrostatic part of the total stress and strain tensors is assumed to be directly equal to the elastic hydrostatic stress and strain.

It is important to note that stresses and strains in a structure depend on its geometry, the material behavior and the external loading applied. When there is a change in the direction or ratio of the stresses developed in the material, these local stresses are said to be non-proportional. If there is plastic flow anywhere in the structure, local non-proportionality can appear even if the external loading is proportional.

We assume a hypothesis of proportionality, that is, while the local evolution of the actual deviatoric stress and strain tensors at a point can be arbitrarily complex, it stays in the direction of the elastic deviatoric stress and strain, i.e. there is no shift in the direction in which plasticity develops. All the components of

Parameter	E MPa	σ_y MPa	b	Q MPa	C MPa	D MPa
Value	200000	100	10	100	40000	400

TABLE 1 - Parameters of the elasto-plastic model

the total deviatoric stress have the same ratio s with the respective components of the elastic deviatoric stress that does not change in direction or magnitude over the loading history. Similarly, the ratio between the total and elastic components of the deviatoric strain is defined as e. According to this hypothesis :

$$\hat{\sigma}_d = s\bar{\sigma}_d \tag{8}$$

$$\hat{\epsilon}_d = e\bar{\epsilon}_d \tag{9}$$

where s and e are scalar functions of time.

The elastic tensors vary linearly with respect to the loading, and therefore, if an elastic calculation via the finite-element method for a point at a fixed global loading is denoted by $\overline{\sigma}$ and $\overline{\epsilon}$, then the following holds true :

$$\bar{\varphi}_d^e = f\bar{\varphi}_d \tag{10}$$

$$\bar{\epsilon}^e_d = f\bar{\epsilon}_d \tag{11}$$

where the loading function f is chosen to be a sawtooth function to represent cyclic loading.

The main advantage of this hypothesis allows for a reduction in the number of dimensions that the Neuber equation needs to be solved in, i.e. the number of equations is reduced from six to one :

$$(s\bar{\underline{\sigma}}_d - s_o\bar{\underline{\sigma}}_d) : (e\bar{\underline{\epsilon}}_d - e_o\bar{\underline{\epsilon}}_d) = (f\bar{\underline{\sigma}}_d - f_o\bar{\underline{\sigma}}_d) : (f\bar{\underline{\epsilon}}_d - f_o\bar{\underline{\epsilon}}_d)$$
(12)

$$\bar{\mathcal{Q}}_d: \bar{\mathcal{E}}_d(s-s_o)(e-e_o) = \bar{\mathcal{Q}}_d: \bar{\mathcal{E}}_d(f-f_o)(f-f_o)$$
(13)

$$(s - s_o)(e - e_o) = (f - f_o)^2$$
(14)

This equation is now scalar as there is only one component to solve for. The evolution of this component is shown in Figure 1.

3.2 Material law

While any model of plasticity can be used to solve the modified Neuber equations, the following elastoplastic Chaboche model with kinematic and isotropic hardening [8] is used in our case :



FIGURE 1 – An illustration of the modified Neuber method during the (a) first branch of loading (b) second branch of loading, with s_o and e_o updated to their respective values at the last peak

The total strain tensor is the sum of the elastic and plastic strain tensors :

$$\underline{\epsilon} = \underline{\epsilon}^e + \underline{\epsilon}^p \tag{15}$$

The stress tensor is given by :

$$\underline{\sigma} = 2\mu(\underline{\epsilon} - \underline{\epsilon}^p) + \lambda tr(\underline{\epsilon} - \underline{\epsilon}^p) \tag{16}$$

The total stress tensor can be split into its deviatoric and hydrostatic parts :

$$\underline{\sigma} = \underline{\sigma}_d + \underline{\sigma}_h \tag{17}$$

The deviatoric stress tensor is given by :

$$\underline{\sigma}_d = 2\mu(\underline{\epsilon}_d - \underline{\epsilon}_d^p) \tag{18}$$

The total hydrostatic stress tensor is assumed to be equal to the elastic hydrostatic stress, as they are close [12] :

$$\underline{\sigma}_h = \frac{1}{3} tr(\underline{\sigma}^e) \tag{19}$$

The Von-Mises stress is :

$$\sigma_{\rm VM} = \sqrt{\frac{3}{2}} \underline{\sigma}_d : \underline{\sigma}_d \tag{20}$$

Kinematic hardening :

$$\dot{X} = \frac{2}{3}C\dot{\epsilon}^{p} - DX\dot{p}$$
⁽²¹⁾

Isotropic hardening :

$$R = Q(1 - e^{-bp}) \tag{22}$$

The accumulated plastic strain depends only on the deviator of the plastic strain :

$$\dot{p} = \sqrt{\frac{2}{3}} \dot{\underline{\xi}}_d^p : \dot{\underline{\xi}}_d^p \tag{23}$$

where $\dot{\epsilon}_{d}^{p}$ is the time derivative of the deviatoric plastic strain tensor. The yield surface is defined by :

$$f(\sigma; X, p) = \sigma_{eq}(\sigma; X) - \sigma_y - R$$
where $f(\sigma; X, p) <= 0$
(24)

With the hypothesis of proportionality, these equations change accordingly from six components to one component. The system of equations can thus be solved for the scalar functions s and e. Together with the modified equations describing the material behavior and the modified Neuber equation, s and e are calculated, which result in a full approximate mechanical solution of the non-linear structural problem. This can be used to calculate the intrinsic dissipation over a cycle [16] :

$$\phi_1 = (f(\sigma; X, p) + \sigma_y + \frac{R^2}{2Q} + \frac{D}{2C}J^2(\tilde{X}))\dot{p}$$
(25)

This equation is suitably simplified to a variant containing scalar functions that arise from the hypothesis of proportionality. The parameters chosen for this study are summarized in table 1.

3.3 Finite Element simulator and interpretability

The previous section focused on obtaining the intrinsic dissipation, a value that depends heavily on the form of the non-linear stress-strain curves, from elastic calculations at a cyclically loaded point undergoing multi-axial stress. It is desired to apply this approach to not just a point, but all points within a larger domain of interest, for example, all points within a geometry with an explicitly modelled distribution of defects, while also keeping the calculation time short.

To make interpretation of developing plasticity easier in the reduced dimensions, some steps are needed : A structure without pores with elastic behavior is modelled, and the global loading is chosen such that the Von-Mises stress at all the points in the region of interest is equal to the yield stress of the material.

Next, this global loading is applied to the same geometry, with the same elastic behavior, but this time with explicitly modelled spherical pores. Due to the elastic behavior, the points near the pores develop a Von-Mises stress higher than the yield stress, and are therefore hypothetically elastic. The plasticity at these points will be approximated via the modified Neuber method. The quantities \overline{a} and \overline{e} are therefore obtained for all the points in the material, and the quantity $\overline{a} : \overline{e}$ captures the elastic information for all the points in the structure, including hypothetically elastic points near the pores.

The loading function f defined earlier captures the extent of loading of the structure. As $\overline{\sigma}$ and $\overline{\epsilon}$ represent a calculation at the yield limit, the function f will vary between 0 and 1 for a load that is below the yield limit of the material, and be exactly 1 when the yield limit of the material is reached. Therefore, choosing the amplitude of f to be 0.8 would be equivalent to loading the material to 80% of its yield limit. Locally, due to explicitly modelled pores, some points would cross the yield limit due to localized stresses, i.e. plasticity occurs when $\sigma_{VM} > \sigma_y$.

Information about these variably stressed local points is extracted in the form of $\bar{\sigma} : \bar{\epsilon}$ from the elastic FE calculation. This quantity, which captures the elastic stress and strain values at these points, are used



FIGURE 2 – (a) Elastic FE calculation at the yield limit without pores, with $\sigma_{VM} = \sigma_y$ (b) Loading function as a proportion of the yield limit (c) The Von-Mises stress obtained by applying the loading function to the elastic FE calculation, shown for a peak (d) The Gaussian process to predict the dissipation given by the modified Neuber method for the chosen load cycle (e) Full-field dissipation calculated via the modified Neuber method, sampled from the Gaussian process for all the points in the structure

by the modified Neuber equations and material laws to solve for the scalar functions representing the actual stress and strain evolution, and thereby the intrinsic dissipation.

Next, a 1-D Gaussian process is fitted on this range of variably stressed points and their corresponding intrinsic dissipation. This allows for rapid sampling - the entirety of the elastic field of the porous structure is given as an input, the full-field intrinsic dissipation is directly sampled as the output. Instead of doing a lengthy cyclic non-linear FE calculation on a structure to obtain the dissipation, the FE simulator thus created needs only a call to an elastic FE calculation of the structure to predict the full-field intrinsic dissipation in a cycle for a given loading sequence (illustrated in Figure 2).

4 Results

4.1 Full-field intrinsic dissipation

Part of the full-field dissipation obtained by the modified Neuber approach and the reference dissipation obtained by a full elasto-plastic FE calculation is shown in Figure 3(a). A comparison of all the points is shown in Figure 4(c). A good overall match is found in the values of the dissipation obtained by the modified Neuber method and the reference dissipation. However, there exists some scatter, which is expected due to the cumulative errors of the assumptions of the modified Neuber method. The next section aims to go in depth of the nature of these errors.



FIGURE 3 - (a) Full-field dissipation using the modified Neuber approach, with the highlighted purple cell showing the point of comparison of the curves (b) Full-field reference dissipation as a result of a complete nonlinear FE calculation in the Z-Set [17] software (c) The evolution of the proportionality ratios evaluated by the modified Neuber method at the highlighted cell (d) Comparison between the cumulative dissipation obtained by the two methods at the same point, showing an under-estimation of the dissipation at this cell via the modified Neuber method





FIGURE 4 - (a) Dissipation using the modified Neuber approach, with purple cells showing examples of some outlier points where the method over-estimates (b) Reference dissipation as a result of a complete nonlinear FE calculation in the Z-Set [17] software (c) Comparison between the modified Neuber and reference values; the outlier points corresponding to the cells are circled in purple.



FIGURE 5 – (a) Relative error and cumulated plastic strain of an element near a pore, over time (b) Percentage of elements in the FE calculation below 15% relative error, over time, alongside the loading history

4.2 Errors due to the hypothesis of proportionality

The relative error ξ_{rel} between the approximated deviatoric stress tensor using the hypothesis of proportionality and the reference deviatoric stress tensor, calculated via a full non-linear FE calculation can be calculated :

Error,
$$\xi = \sigma_d - \hat{\sigma}_d$$
 (26)

$$\hat{\underline{\sigma}}_d = s\bar{\underline{\sigma}}_d \tag{27}$$

$$(\bar{\mathfrak{g}}_d:\hat{\mathfrak{g}}_d) - s(\bar{\mathfrak{g}}_d:\bar{\mathfrak{g}}_d) = 0 \tag{28}$$

Combining equations 26 and 28 :

$$\xi = \underline{\sigma}_d - [(\bar{\sigma}_d : \hat{\sigma}_d) / (\bar{\sigma}_d : \bar{\sigma}_d)] \bar{\sigma}_d \tag{29}$$

$$\xi_{rel} = \frac{\|\xi\|_F}{\|g_d\|_F}$$
(30)

The relative error ξ_{rel} is calculated for all the loading history. Figure 5(a) shows the evolution of the relative error of an element near a pore (shown in Figure 3), alongside the accumulated plastic strain. The load reversal naturally causes the error due to the hypothesis of proportionality to skyrocket, but due to elasticity during reversal, the accumulated plastic strain is zero and thereby there is no accumulated dissipation during load reversal. In other words, the error during load reversal does not contribute to the error in plasticity or dissipation calculations.

5 Conclusion

Neuber's rule was modified to obtain the full-field intrinsic dissipation of structures with non-linear behavior subjected to cyclic loading. The modifications to the Neuber method consist of :

- ♦ Equivalency between the deviatoric stress-strain tensors
- ◊ Incorporation of full load reversal, and
- ◊ A hypothesis of proportionality

These assumptions lead to certain errors, which are non-negligible during load reversal due to the hypothesis of proportionality. However, as demonstrated, the bulk of the errors occur when there is no accumulating plasticity, therefore the dissipation is reasonably well estimated. While the overall match between the dissipation calculated via this method and the reference dissipation from a complete non-linear FE calculation is rather good, certain points remain over and under-estimated. The main advantage in using this method is in its speed : the calculation time is accelerated manifold by using this method, as the modified Neuber method relies on cheap elastic calculations at a fixed global loading. Thus, the developed finite element simulator allows generation of big data-sets within a reasonable time.

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