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# Parameters Extraction of Unknown Radar Signals Using Change Point Detection

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**Abstract**—Many applications like seismic data processing or navigation system monitoring require detection of abrupt changes. This paper considers the problem of signal detection and recognition in the context of radar application applied to a segmentation of an unknown received signal. Therefore a well known algorithm in change point detection is being implemented, the Cumulative Sum algorithm (CuSum), in order to detect and extract signals of interest in radar recording. It enables the localization of the beginning and ending of unknown waveforms in a pulse train making its extraction from a noisy background and its characterisation possible. Different models of the problems are considered and compared. The capacity of the algorithm to correctly retrieve the temporal parameters of the signal is tested on both simulations and acquisitions.

**Index Terms**—Cumulative sum, change point detection, waveform, parameters extraction, recognition, signal processing.

## I. INTRODUCTION

Detection and recognition of several signals with unknown parameters in the presence of noise is a challenging task. Being capable to detect and extract fundamental parameters from an unknown emission is becoming ever more difficult. Indeed, the increasing number of users and applications combined with the implementation of new types of waveforms induces an increase in signal emissions that complexifies the electromagnetic environment. The need to quickly localize and identify with precision a received signal is then becoming of the utmost importance. Concerning radar applications, the detection of unknown signals can help providing countermeasures, avoiding interference and localizing potential threats and hostile emissions. For known signals, a matched filter is typically used. However, it requires knowing the transmitted signal. For unknown ones, other methods must hence be considered and they are numerous. The most classic one is the energy detection [1]. Commonly, a network of antennas is also a good way to detect and localize signals of interest [2]. More recently, the combination of time-frequency methods (Short-Time Fourier Transform, Wigner-Ville Distribution or Continuous Wavelet Transform) with machine learning has been expanding. The main idea proposed in [3] - [4] is to construct a mask around the detected component using time-frequency methods and the recognition is processed by a convolutional neural network (CNN).

This article deals with detection using a single antenna, and mainly focuses on time of arrival (TOA) and duration estimation, and treats this objective as a change detection problem. In radar applications, change detection is mostly encountered when trying to detect modifications between a series of Synthetic Aperture Radar (SAR) images [5]. Among the proposed methods, we found the Cumulative Sum (CuSum) algorithm attractive for its simplicity, efficiency and its processing speed. Usually, one of the difficulties in change detection arises when several change points are to be identified [6]. In this case, the computational cost of the proposed method usually explodes. In the considered problem, we want to detect several signals and to estimate for each one their TOA and duration. We will propose a rather simple framework, based on the search for local minima and maxima on the CuSum output, which allows a correct segmentation of a pulse train. We evaluate the potential of this method for unknown signals detection and parameters extraction, using different models of the received data. In particular, we show that the proposed method is suitable to detect several change points (the beginning and the end of several pulses). A proposition on how to set the thresholds to detect those change points is also made.

This paper is organized as follows. Section II provides an introduction to the CuSum algorithm and the signal distribution being considered for change point detection. Section III presents results obtained via the CuSum algorithm on simulated data. In Section IV, tests on real signals are being discussed. Lastly, section V draws the conclusion.

## II. CHANGE POINT DETECTION THEORY

### A. CUSUM basis

The Cumulative Sum (CuSum) algorithm was firstly introduced by Page [7]. CuSum is among the most commonly used algorithm in change point detection theory where the objective is to identify moments when the probability distribution of a time-series changes. The algorithm searches for changes in mean or variance. In this paper, it is applied to detect the beginning and ending of unknown waveforms. This can be considered as an initial step for further processing such as signal classification or parameter estimation.

The following model describes the elementary problem in change point detection: the detection of a single change point

corresponding in our case to the search for a beginning of a signal. It is later easily generalized to the detection of several change points. Statistically, let the received data be modeled by a discrete random variable  $x[n]$  ( $n \in [0, N]$  where  $N$  is the duration of the data) with a given probability density function  $p(x[n], \theta)$ ,  $\theta$  being a deterministic parameter that has two possible values,  $\theta_0$  and  $\theta_1$ , depending whether the signal of interest is absent or present. Two hypotheses are possible:

$$H_1 : \theta = \theta_1, \text{ The signal is present.}$$

$$H_0 : \theta = \theta_0, \text{ Only observation noise is present.}$$

Under each hypothesis, the probability density functions are given by

$$p_{x|H_0} = \prod_{n=0}^N p(x[n], \theta_0), \quad (1)$$

$$p_{x|H_1} = \prod_{n=0}^{\tau-1} p(x[n], \theta_0) \prod_{n=\tau}^N p(x[n], \theta_1), \quad (2)$$

where  $\tau \in [0 : N - 1]$  is the time when the signal starts. This configuration means that for  $n < \tau$ , samples are only noise components. Following the detection theory, the log-likelihood ratio (LLR)  $\Delta_\chi$  is the best test (in the statistical sense) to decide between the two hypotheses. Hypothesis  $H_1$  is validated if  $\Delta_\chi > \eta$ , where  $\eta$  is a predefined threshold. Otherwise we are still under the hypothesis  $H_0$

$$\Delta_\chi = \ln \left( \frac{p_{x|H_1}}{p_{x|H_0}} \right) \underset{H_0}{\overset{H_1}{\gtrless}} \eta. \quad (3)$$

Due to the impossibility to compute this LLR because  $\theta_0$ ,  $\theta_1$  and  $\tau$  are unknown, we define a generalised log-likelihood ratio (GLLR) by taking the maximum-likelihood estimate of each unknown (ML estimates). Letting  $\hat{\theta}_0(\tau)$  and  $\hat{\theta}_1(\tau)$  the ML estimates of  $\theta_0$  and  $\theta_1$  for a fixed  $\tau$ , the ML estimate  $\hat{\tau}$  of  $\tau$  is the value maximizing the likelihood  $p(x_{0:N-1} | \tau, \hat{\theta}_0(\tau), \hat{\theta}_1(\tau))$ , where  $p(x_{0:N-1})$  denotes  $p(x_0, x_1, \dots, x_{N-1})$ . It can be checked that

$$\hat{\tau} = \arg \max_{0 \leq \tau \leq N-1} \sum_{n=\tau}^N \ln \left( \frac{p(x[n], \hat{\theta}_1)}{p(x[n], \hat{\theta}_0)} \right). \quad (4)$$

Now let us introduce a few more notations. According to [7], letting the instantaneous LLR (ILLR) at time  $n$  be defined by

$$s[n] = \ln \left( \frac{p(x[n], \hat{\theta}_1)}{p(x[n], \hat{\theta}_0)} \right), \quad (5)$$

the cumulative sum of  $s$  from 0 to  $N$  is then

$$S[N] = \sum_{n=0}^N s[n] = S[N-1] + s[N] \quad (6)$$

and the change point can be defined as

$$\hat{\tau} = \arg \max_{0 \leq \tau \leq N-1} (S[N] - S[\tau - 1]), \quad (7)$$

$$= \arg \min_{0 \leq \tau \leq N-1} S[\tau - 1]. \quad (8)$$

The previous equation means that the algorithm tries to find a minimum in the sum from which the next samples will have a sufficient change of slope. The figure 1 illustrates this result with a signal present after sample  $n = 10000$ . The change of variation in the sum indicates the change point where the signal starts.

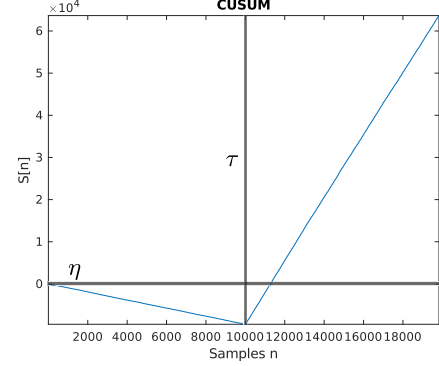


Fig. 1: Results of the CuSum algorithm for the detection of a single change point.

The elementary problem behind the CuSum algorithm thus described, it is clear that the search for the beginning of one signal leads to  $N$  tests. Yet in our case, we will focus in the detection of  $NI$  pulses, which means detecting  $NI$  beginning but also  $NI$  end of pulses. To obtain such results, we first need to tackle the distribution assumed for each probability density function under hypotheses  $H_0$  and  $H_1$  for the elementary change detection. We will then propose a procedure to detect the beginning and end of several pulses.

### B. Signal distribution

In this section, we consider different models and calculate the ILLR accordingly. For the data being analyzed, hypothesis  $H_0$  describes portions with noise only and hypothesis  $H_1$ , the portions containing signal of interest. Usually, noise is described as a random variable following a complex normal distribution with zero mean  $\mu_0 = 0$  and variance  $E[|x[n]|^2] = \sigma_0^2$ :

$$p(x[n], \theta_0) = \frac{1}{\pi \sigma_0^2} e^{-\frac{|x[n]|^2}{\sigma_0^2}}. \quad (9)$$

Concerning the signal, the familiar distribution in a radar framework would be to consider the signal as an unknown but deterministic variable. In that case, we assume that the signal follows a Gaussian probability density function with a certain unknown mean  $\mu_1 = u[n]$  ( $u$  the noise-free signal) and a variance  $\sigma_1^2 = \sigma_0^2$ :

$$p(x[n], \theta_1) = \frac{1}{\pi \sigma_1^2} e^{-\frac{|x[n] - \mu_1|^2}{\sigma_0^2}}. \quad (10)$$

After estimation of  $\hat{\mu}_1 = x[n]$ , it leads to a classic energy detection scheme where (5) becomes

$$s[n] = \frac{|x[n]|^2}{\sigma_0^2}. \quad (11)$$

Obviously, the Cumulative Sum using this ILLR will be constantly increasing, leading to difficulties finding the change points with the ML estimates (8). A solution could be to use a slope detection, another detection algorithm with the disadvantage of being time-consuming when used to find several change points because of the number of hypotheses to test [6]. A second strategy could be to use the Akaike Information Criterion (AIC) [8], which adds a penalty to the log-likelihood. The classic energy detection scheme (11) becomes

$$s[n] = 2 \frac{|x[n]|^2}{\sigma_0^2} - 2C. \quad (12)$$

here,  $C = 2$  is the number of parameters of our model (real and imaginary part of the signal). The figure 2 shows the CuSum results for the deterministic case with or without the AIC penalty term compared to the random variable one with Rayleigh-Rice (RR) distribution (introduced later in this paper). As expected, the non-penalized deterministic case gives an ever increasing CuSum value not allowing the detection of a local minimum. For high enough SNR, the AIC enables the CuSum to exhibit a strong change of slope along with a change of sign of slope. This change of sign of slope is not longer present at low SNR (bottom left graph in figure 2).

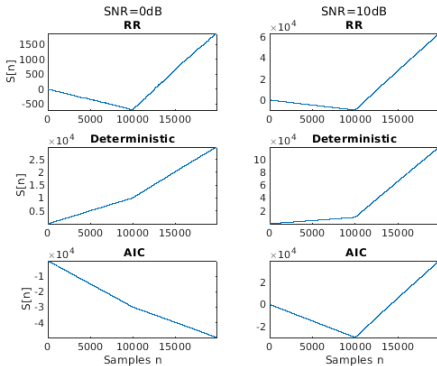


Fig. 2: Illustration of CuSum results for the RR distribution, the deterministic case with and without AIC for a signal at  $n = 10000$  samples.

Since even with the AIC penalty, the deterministic model leads to an unsatisfying test statistic, we consider another approach. This time, the signal is modeled as a random variable with unknown distribution. In fact it is difficult to assume a particular distribution for every possible signal because we are looking for unknown signals and the task becomes even more difficult when looking for several different signals. In order to be more general, this paper considers that the event we are looking for has an unknown mean  $\mu_1$ . This choice comes from the fact that the CuSum algorithm is designed to detect changes in mean and/or variance [9] and the signal of interest can in fact have an impact on both parameters of the recorded signal. Moreover it could be of interest for signals such as the ones used in noise radar or non-coded pulses. Working with complex time series, we can

adopt the Rayleigh-Rice distribution. The noise and the signal can be interpreted in terms of a real and imaginary part, each following a normal distribution  $\mathcal{N}(0, \frac{\sigma_0^2}{2})$  for the noise,  $\mathcal{N}(\nu \cos \phi, \frac{\sigma_1^2}{2})$  and  $\mathcal{N}(\nu \sin \phi, \frac{\sigma_1^2}{2})$  for the signal. It follows that the variable  $r = |x| = \sqrt{\text{Re}(x)^2 + \text{Im}(x)^2}$  is Rayleigh distributed for the noise and Rice distributed for the signal of interest.

$$p(r[n], \theta_0) = \frac{2r[n]}{\sigma_0^2} e^{-\frac{r^2[n]}{\sigma_0^2}}, \quad (13)$$

$$p(r[n], \theta_1) = \frac{2r[n]}{\sigma_1^2} e^{-\frac{(r^2[n] + \nu^2)}{\sigma_1^2}} I_0\left(\frac{2r[n]\nu}{\sigma_1^2}\right), \quad (14)$$

where  $I_0(z)$  is the Bessel function. It gives the following ILLR (5) :

$$s[n] = \frac{r^2[n]}{\sigma_0^2} - \frac{r^2[n] + \nu^2}{\sigma_1^2} + \ln\left(\frac{\sigma_0^2}{\sigma_1^2} I_0\left(\frac{2r[n]\nu}{\sigma_1^2}\right)\right). \quad (15)$$

The CuSum results using this model is illustrated in figure 2, along with the previous deterministic model. We can see that even at low SNR, this model could allow to detect a change point with procedure (8).

Finally, in the case where we do not not assume that the samples follow a RR distribution, we can find in [10] that Tartakovsky proposed a more general ILLR of linear-quadratic form with  $y[n] = \frac{|x[n]| - \mu_0}{\sigma_0}$  the centered scaled observation at time  $n$ :

$$s[n] = C_1 y^2[n] + C_2 y[n] - C_3, \quad (16)$$

where

$$C_1 = \frac{1 - q^2}{2}, \quad C_2 = \delta q^2, \quad C_3 = \frac{\delta^2 q^2}{2} - \ln q. \quad (17)$$

with  $q = \frac{\sigma_0}{\sigma_1}$ ,  $\delta = \frac{\mu_1 - \mu_0}{\sigma_0}$ .

These two models performance will be tested in section III. In the next subsection, we will propose a procedure to detect several change points.

### C. Generalization of the algorithm to the detection of multiple change points

To compute the CuSum algorithm, we estimate the noise in the record to get the variance of the noise  $\sigma_0^2$ , then the mean and variance of the entire record  $(\mu_1, \sigma_1^2)$ . It gives coarse estimates of the parameters, but we will see in section III that they are sufficient. The ILLR is calculated for each time  $n$  according to the chosen distribution and added to the cumulative sum  $S$  in equation (6). This sequence enables a layout like the one visible in figure 3 where a zoom of  $S(t)$  obtained for a chirp pulse train with parameters  $NI = 20$  pulses,  $SNR = 0$  dB,  $B_1 = 26.73$  MHz,  $T_1 = 19.66$   $\mu$ s is displayed. The first two pulses are visible, and unlike figure 1, each pulse is indicated by two change points. The beginning and the end of a pulse correspond to local minima and maxima. As mentioned earlier, the elementary problem of detecting a single change point seems to become more complex because we usually have to test all the possible combinations of change points. However, in our case, we do not need to change the

algorithm and complexify it that much but just need to adapt the thresholds. Intuitively, and as it can be observed in figure 3, the portions of the data containing noise only will have similar behaviors related to the noise variance. They result in having similar decreasing slopes on the CuSum output. On the contrary, distinct signals can display variable evolutions of the CuSum. They will alternate with increasing slopes when signal of interest is present in data. With coarse estimates of  $\mu_1$  and  $\sigma_1^2$ , the procedure "simply" consists in finding alternating minimum and maximum. This procedure could be refined in a second step, but this is not the scope of the paper.

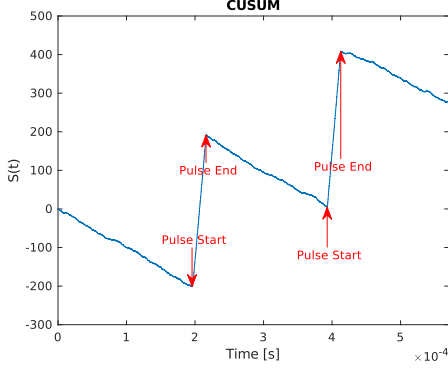


Fig. 3: Results of the CuSum algorithm for a pulse train. Zoom on the first two pulses.

The remaining difficulty is setting the thresholds. Two thresholds  $\eta_{min}$  and  $\eta_{max}$  are defined to detect local minimum and maximum. Time  $\tau$  is evaluated to be a change point if:

$$\Gamma_{min,\chi}[N] \triangleq |S[N] - \min_{1 \leq \tau \leq N-1} S[\tau - 1]| > \eta_{min}. \quad (18)$$

$$\Gamma_{max,\chi}[N] \triangleq |S[N] - \max_{1 \leq \tau \leq N-1} S[\tau - 1]| > \eta_{max}. \quad (19)$$

We implemented two distinct thresholds based on the estimation of noise in the signal and adapted to the search of a minimum or a maximum.

To detect a minimum, we want to avoid false alarms. The threshold is designed to be an empirical constant-threshold inspired from the one given in [11]. Before applying the CuSum on the studied signal, the CuSum algorithm is computed with the estimated  $\sigma_0^2$ ,  $\mu_1$  and  $\sigma_1^2$  for  $L$  noise series of  $M$  samples to estimate a threshold to guaranty a certain false alarm. For each noise serie, we analyze a local score

$$S_i^j = \max(0, S_{i-1}^j + s_i^j), \text{ where } j \in [1, L] \text{ and } i \in [1, M]. \quad (20)$$

And the maximum of this local score is saved  $m^j = \max S^j$ . Finally, the threshold is set as :

$$\eta_{min} = q_{1-M\alpha}[m_{1 \leq j \leq L}^j]. \quad (21)$$

where  $\alpha$  is the false alarm rate and then  $q_{1-M\alpha}$  represents the quantile of order  $(1 - M\alpha)$ . Figure 4 gives an example of the setting of threshold  $\eta_{min}$ . We generated  $L = 100$  noise series of  $M = 10000$  samples and chose to take the quantile with  $\alpha = 1 \times 10^{-5}$ .

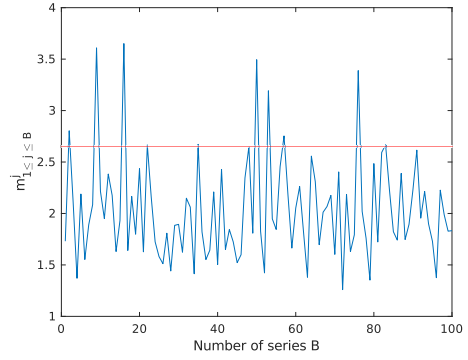


Fig. 4: Setting of the minimum threshold. The blue curve represents the maximum of each noise serie. The red line is the obtained empirical threshold.

To detect a maximum, we need to set a threshold high enough to avoid subdividing a single pulse into multiple pieces. As the maximum detects the end of a pulse, what follows is only noise and the CuSum shows a notable behavior in the noise hypothesis. The sum is decreasing following a linear behavior (visible on figure 3 before each pulse start) and therefore can be described by an average slope factor  $a$ . Again with  $L$  noise series of  $M$  samples, the CuSum algorithm is computed and a mean slope factor  $a_{mean}$  is calculated. To validate a maximum, we need to have this linear decreasing slope for at least  $K$  points which means that the threshold can be defined by:

$$\eta_{max} = K a_{mean} \quad (22)$$

Considering a slope on  $K$  points should guaranty that the CuSum is indeed decreasing because the signal is no longer present and not just because of some unlucky noise trials.

### III. SIMULATIONS

#### A. Simulated signals

Along this paper, we consider pulse trains of  $NI = 20$  pulses with linear (Chirp) or hyperbolic frequency modulations (HFM) and the following sets of parameters:

- Bandwidth  $B_1 = 26.73MHz$  or  $B_2 = 68.96MHz$ ,
- Pulse duration  $T_1 = 19.66\mu s$  or  $T_2 = 2.56\mu s$ ,
- Pulse Repetition Interval (PRI)  $T_{pi} = 10T_i$ .

Figure 5 shows the kind of pulses used in this section which the CuSum algorithm has to find in a noisy recording

#### B. Performance indicators

To evaluate our algorithm, several performance indexes are examined :

- The false alarm rate  $FAR$  being the ratio between the number of false alarms and the number of samples.
- The mean decision delay  $D_d$  which evaluates the delay needed for the algorithm to evaluate a change point.
- The mean time between false alarms  $MTBFA$  which according to [11] is the inverse of the false alarm rate  $\alpha$ .

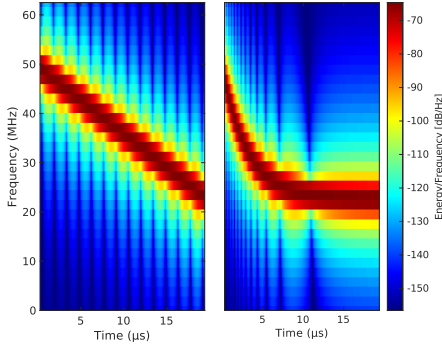


Fig. 5: Spectrograms of a Chirp and a Hyperbolic pulse with set 1 of parameters  $B_1 = 26,73\text{MHz}$ ,  $T_1 = 19,66\mu\text{s}$ .

- The root mean squared error (RMSE) on the estimation of  $T$  the pulse duration.

The algorithm has been tested for varying SNR (0 to 30dB with 100 simulations at each SNR), for the different configurations described earlier.

### C. Results

Simulations for each distribution (RR, Tartakovsky) and sets of parameters have been conducted and the figures 6 to 9 show the results for a false alarm rate  $\alpha = 5 \times 10^{-5}$  and thresholds set from  $L = 100$  noise series of  $M = 10000$  samples (sampling frequency is  $F_s = 0.5\text{GHz}$ ).

It is visible on figure 6 that the FAR is close to  $\alpha$  for the Chirp and HFM pulse train with set 1 of parameters. Slightly higher deviations can be observed with the pulse train of HFM with the second set of parameters. In this case the FAR is lower which is not wrong but it is under evaluated because the signal is in this case too short compared to the other two.

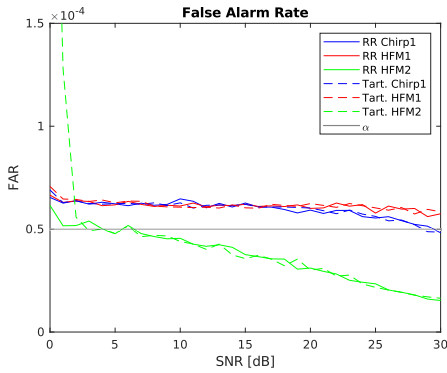


Fig. 6: False Alarm Rate against SNR.

The next performance index being evaluated is the mean decision delay  $D_d$  for the detection of the start and end points of a pulse respectively shown in figures 7 and 8. For a better visualization, the y-axis is in logarithmic scale. It can be seen that considering either a RR distribution or an unknown one with the Tartakovsky model does not play an important role

according to the graphs. By looking at the evolution of those mean decision times, we see that the higher the SNR, the shorter the mean decision time to detect the beginning of a signal. As the noise is less and less impacting the signal, the CuSum value increases faster after the beginning of a pulse, exceeding the threshold rapidly. On the contrary, the mean time decision for end detection is increasing and it is again linked to the threshold. Defined by (22), at high SNR the algorithm reaches the  $K$  samples defined for the maximum threshold before taking the decision hence in our case the mean time decision push towards  $\frac{K}{F_s} = 4 \times 10^{-7}\text{s}$ . The choice of  $K$  is thus of importance to correctly estimate the end (and thus the duration) of a signal. Moreover it is better to have a  $K$  large enough which involves longer mean time decision for the end detection but ensures to catch the end of the signal with a few additional samples. To conclude on the first three performance indexes, the correct detection and estimation of the start and the end of a pulse are strongly dependent on the false alarm allowed  $\alpha$  and the number  $K$  of samples before making a decision about the end of the signal.

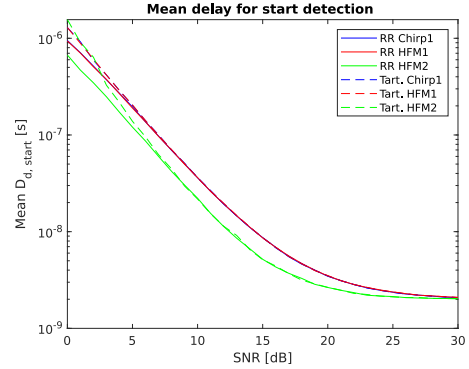


Fig. 7: Mean decision delay for start detection against SNR.

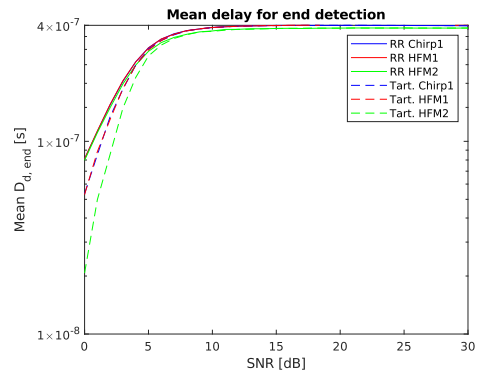


Fig. 8: Mean decision delay for end detection against SNR.

The last index to check is the RMSE on the pulse duration which requires a correct estimation of both the beginning and the end of a pulse. Here again, it is visible in figure 9 that the distribution does not play a role. Moreover the algorithm gives good general accuracy for each type of pulse train. The

matched filter will indeed have better results on TOA even at low SNR (note that for the CuSum algorithm it is quite promising) but we can only estimate this TOA and the PRI because the searched signal is known. The CuSum algorithm offers the possibility to detect the TOA but also the end of a pulse without having preliminary information.

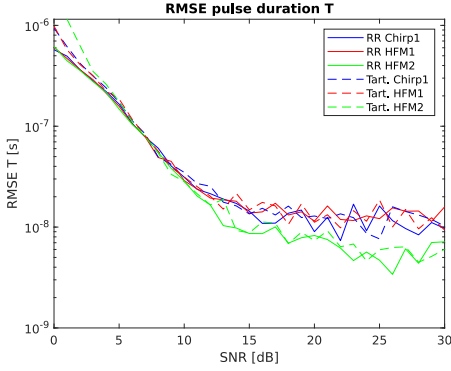


Fig. 9: RMSE for pulse duration against SNR.

#### IV. TESTS ON REAL DATA

The CuSum algorithm has been tested on a real radar signal to check its capacities in a real situation. The tested signal was composed of 13 pulse trains of 10 up- or down-chirp with different durations and PRI. The figure 10 shows a portion of this signal where the end and beginning of 2 distinct pulse trains are visible with the black lines indicating the time indices detected with the CuSum algorithm (RR distribution,  $SNR = 20dB$ ). The preliminary analyses show very good results in estimating pulse duration and PRI for real signal. It has also been tested on simulated OFDM signals with realistic parameters which present amplitude modulation and gave promising results as visible in figure 11.

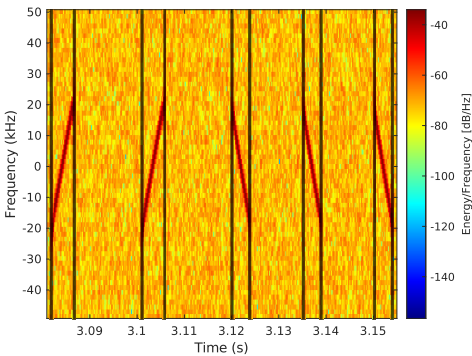


Fig. 10: Real radar signal and CuSum splitting.

#### V. CONCLUSION

In the context of unknown radar signal detection and parameters extraction, this article has proposed a change point detection algorithm based on the CuSum along with the

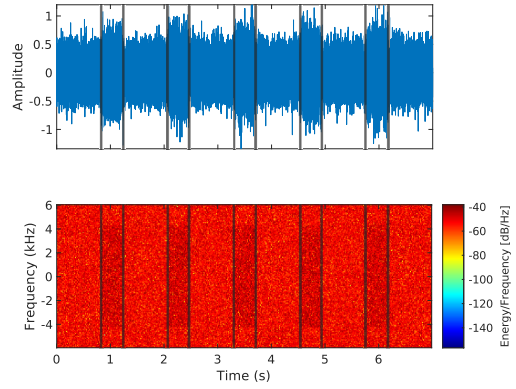


Fig. 11: OFDM and CuSum splitting. Top figure corresponds to the temporal representation of the signal, where the vertical lines show the segmentation resulting from our algorithm. Bottom figure represents the spectrogram of the signal.

definition of thresholds. Mainly it is capable to detect signal without preliminary knowledge, contrary to classical cross-correlation radar receivers, and to achieve estimate of both its time of arrival and duration. Moreover even at low SNR the results are quite encouraging.

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