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Credit Rationing in Unsecured Debt Markets*

V. Filipe Martins-da-Rocha[†] Toan Phan[‡] Yiannis Vailakis[§]

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Abstract

In dynamic general equilibrium models featuring unsecured debt, the risk of default and the absence of collateral pose significant challenges to debt sustainability. Traditional models focus on the largest permissible debt limits consistent with repayment incentives and show that they inevitably form a bubble—current debt must be exactly rolled over indefinitely so that no effective repayment is ever made. This research challenges this paradigm by shifting the focus on equilibria with more restrictive debt limits. We establish that, under certain conditions, a credit rationing intervention can make everybody in the economy better off. By reassessing the choice and impact of debt ceilings in credit markets, we identify potential strategies to boost overall economic efficiency and welfare. Our findings shed light on the dynamics of unsecured debt markets and offer policymakers and economists a fresh perspective on managing debt without collateral.

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1 Introduction

Understanding whether competitive debt markets with financial frictions are vulnerable to potential inefficiencies or market failures is an important question in macroeconomics with many relevant implications, especially since the Global Financial Crisis (e.g., [Reinhart and Rogoff 2009](#), [Mian and Sufi 2009](#), [Rajan 2011](#)). It helps us understand if and when prudential policy interventions are warranted (e.g., [Holmstrom and Tirole 1997](#), [Dávila and Korinek 2018](#), [Jeanne and Korinek 2019](#)). However, asserting whether equilibria might be constrained inefficient turns out to be more nuanced than it may appear.

Two broad strands of the literature provide different answers and implications. On the one hand, a large and growing body of research analyses financial markets where debt is partly secured by collateral.¹ There, borrowing constraints depend on market prices of goods or assets. Private agents fail to internalize the general equilibrium effects of their individual decisions on market prices, and that failure could lead, for instance, to excessive borrowing in equilibrium.

On the other hand, standard general equilibrium models with unsecured debt have found it generally harder to show that competitive equilibria are constrained inefficient. In the well-known class of single-commodity models widely used in applications, debt constraints are endogenously determined by the threat of financial autarky, and competitive equilibria are indeed constrained efficient. This occurs even though the debt constraints depend on market prices. For instance, in the seminal work of [Alvarez and Jermann \(2000, 2001\)](#)²

¹A nonexhaustive list includes, among others, [Kiyotaki and Moore \(1997\)](#), [Lorenzoni \(2008\)](#), [Farhi et al. \(2009\)](#), [Bianchi \(2011\)](#), [Bianchi and Mendoza \(2011\)](#), [Aiyagari and Gertler \(1999\)](#), [Caballero and Krishnamurthy \(2001, 2003\)](#), [Jeanne and Korinek \(2010\)](#), and [Dávila and Korinek \(2018\)](#).

²See also [Kehoe and Levine \(1993, 2001\)](#), [Kocherlakota \(1996\)](#), [Bloise and Reichlin \(2011\)](#).

borrowing is subject to debt limits set at the largest possible levels such that the repayment value (depending on asset prices) equals the autarkic value.

The risk of default and the absence of collateral pose substantial challenges to debt sustainability when the default punishment is weaker than autarky. This is, in particular, the case in models with complete markets and lack of commitment where the default punishment induces a sort of restricted market participation. In a celebrated paper, [Bulow and Rogoff \(1989\)](#) showed that unless default involves direct sanctions (partial seizure of endowments) or leads to full exclusion from credit markets as in [Eaton and Gersovitz \(1981\)](#), it is impossible to sustain positive levels of debt. If agents are allowed to save upon default, so exclusion is only one-sided, borrowers with a large debt exposure would always prefer to declare bankruptcy and invest their saved repayments. Creditors anticipate debtors' incentives, and debt is therefore unsustainable without additional sanctions.³

[Bulow and Rogoff \(1989\)](#) focus primarily on the demand side of the credit market, assuming the interest rate as a given factor. In contrast, [Hellwig and Lorenzoni \(2009\)](#) adopt a broader perspective, examining debt sustainability within a general equilibrium framework. Their model incorporates borrowing constraints by means of “not-too-tight” debt limits, similar to those described in [Alvarez and Jermann \(2000\)](#), where interest rates and debt limits are determined endogenously. Remarkably, they challenge Bulow and Rogoff's findings by demonstrating that low-interest rates can, in equilibrium, support sustainable debt. Interest rates must be low because the not-too-tight debt limits inevitably form a bubble, enabling borrowers to perpetually roll over their debt limit each period without actual repayment. As not-too-tight debt limits are the maximum borrowing threshold that is still aligned with repayment incentives, [Hellwig and Lorenzoni \(2009\)](#)'s result suggests that these bubbly limits could be the most effective liquidity-provision mechanism to alleviate credit restrictions stemming from the lack of commitment.

The main contribution of this paper is to challenge this conventional wisdom by showing

³[Krueger and Uhlig 2006](#) provide a microfoundation of this default punishment in an optimal contracting model where defaulting agents can start a new credit relationship with a competing principal.

that restricting the amount of private credit more than what is traditionally deemed necessary might lead to Pareto improvement. More precisely, our focus is on policy interventions where regulators impose tighter, with respect to not-too-tight, debt constraints. We interpret such interventions as a parsimonious representation of regulatory or prudential policies constraining financial leverage. We illustrate that these policy interventions can increase the ex-ante welfare of all economic agents.

Intuitively, though all agents are fully rational and forward-looking, they fail to internalize how changes in the severity of credit restrictions affect equilibrium prices and, most crucially, feedback on the value of the default option. In particular, tightening the debt constraints at some period τ might increase bond prices or, equivalently, lower the implied interest rates at the same date. In our setting where defaulters are subject to exclusion from credit (à la [Bulow and Rogoff 1989](#), and [Hellwig and Lorenzoni 2009](#)), this intervention might reduce the value of the default in period τ since it is now more costly to smooth consumption over time by saving only. Consequently, the not-too-tight debt limit at period τ increases, relaxing the credit constraints at the previous period $\tau - 1$. This opens the possibility for Pareto improvement: the benefits from the relaxed debt constraint at period $\tau - 1$ may compensate for the costs of facing tighter constraints at period τ . This is the essence of the mechanism we explore in this paper.

To elucidate the underlying mechanism, we first compare the laissez-faire equilibrium with the allocations chosen by a constrained social planner who internalizes the broader market impact of tighter debt constraints. Specifically, we show that a planner with the flexibility to select among all possible equilibria with self-enforcing debt limits will not necessarily find it optimal to select an equilibrium with not-too-tight debt limits. The difference with laissez-faire is that the social planner internalizes the endogeneity of bond prices and the default option to the aggregate level of liquid wealth, which decentralized agents take as given.

We then show how Pareto improvement is obtained in a simplified example featuring an economy with two agents who face uncertainty only in the initial period. Once this uncertainty is resolved, the economy becomes deterministic, with endowments alternating between high and low values across periods. In this context, we first outline the symmetric

Markov laissez-faire equilibrium with zero interest rates and constant debt limits. In this equilibrium, the low-income agent borrows up to his debt limit while the high-income agent saves.

The policy intervention in our model tightens the debt limits by a fraction ε at a specific period τ while maintaining all other debt limits (before and after period τ , respectively) at their largest permissible values, consistent with repayment incentives. Our key contribution is examining the feedback effects of such a policy distortion on equilibrium prices and the default option. It is worth remarking on an essential feature of our policy experiment. The intervention in financial markets is not equivalent to modifying the default punishment, which remains fixed throughout the paper. The reallocation is induced by tightening the borrowing limits to lower levels than the largest self-enforcing ones.

In line with [Hellwig and Lorenzoni \(2009\)](#), our example with fully unsecured debt features a laissez-faire equilibrium where debt limits are maximal, and the interest rate is inevitably zero. One might assume that the inefficiencies we identify are inherently linked to these low interest rates. However, we demonstrate that Pareto improvement is robust to more stringent default punishments that give rise to a positive interest rate. Specifically, we replicate our policy experiment in a setting where default induces not only exclusion from credit but also deadweight endowment losses.⁴ To calculate the symmetric Markovian laissez-faire equilibrium in this modified setting poses additional challenges that we overcome by providing a novel and intuitive decomposition of not-too-tight debt limits into a fundamental and a bubble component.⁵ This characterization is particularly useful for our analysis, as it significantly streamlines the computation of laissez-faire equilibria, eliminating the

⁴In the context of sovereign default, these endowment losses correspond to a contraction in output, while in consumer and corporate defaults, they represent legal repercussions such as recourse and seized collateral.

⁵The fundamental component equates to the present value of endowment losses and the bubble component stands for the amount of credit agents can roll over indefinitely. Although this decomposition parallels the characterization result of [Hellwig and Lorenzoni \(2009\)](#) in our modified setting, our result is not a straightforward extension of their work. It relies on novel insights with no counterparts in models without output losses.

complexities of determining the fixed-point debt limits.

Regardless of how small the endowment loss might be, the laissez-faire equilibrium always exhibits positive interest rates. We demonstrate that subject to the same policy intervention—namely, tightening the debt limits at a specific period—the resulting new equilibrium still yields a consumption allocation that Pareto dominates the laissez-faire allocation. This confirms that a low-interest rate is not the reason for inefficiency in credit markets.

Related Literature. The idea that economies with limited commitment are prone to market failures dates back to [Kehoe and Levine \(1993\)](#). When there is more than one commodity and default cannot exclude agents from trading in spot markets, constrained efficiency might fail because private contracts cannot internalize their effect on relative prices and the default option. The logic there is conceptually the same as in incomplete markets economies where redistribution of asset holdings, through the induced price changes, affect the spanning properties of the limited assets ([Hart 1975](#), [Stiglitz 1982](#), [Geanakoplos and Polemarchakis 1986](#)). Similar results are also obtained in settings where contracting is subject to private information ([Greenwald and Stiglitz 1986](#)). In the single good model studied here, however, there are no spot markets or private information, and as a result, this mechanism is absent. Moreover, [Alvarez and Jermann \(2000, 2001\)](#) show that competitive equilibria are constrained efficient when the default option is autarky. We instead show that constrained inefficiency obtains in economies with a single commodity when debt enforcement relies on a weak form of exclusion (i.e., one-sided exclusion) from financial markets. Changes in the severity of credit restrictions induce price changes in bond markets. These price changes, in turn, affect the value of default and, therefore, the extent of risk sharing, potentially improving efficiency. This source of inefficiency is not present in Alvarez and Jermann’s framework since the value of default does not respond to changes in bond prices.

Our work is related to well-developed literature studying the emergence of pecuniary externalities in production economies with collateral constraints. [Gromb and Vayanos \(2002\)](#) show that both distributive and collateral externalities can emerge due to market segmentation. [Lorenzoni \(2008\)](#) shows that financial distress might lead to fire sales whose ef-

fects on asset prices are not internalized by highly leveraged investors. [Dávila and Korinek \(2018\)](#) characterize pecuniary externalities in dynamic settings subject to reduced-form, price-dependent collateral constraints. They distinguish between distributive and collateral externalities and show that these two types can be quantified as a function of sufficient intuitive statistics. In all of these works, because of capital accumulation, the reallocation of resources is induced by a change in the level of investment. A planner can overcome market failure by reducing aggregate investment ex-ante and, therefore, the size of the asset sales in bad states. In contrast, this channel is absent in our pure exchange setup as aggregate resources are fixed, and only their distribution can vary. The reallocation of resources is solely induced by the tightening of the endogenously determined debt constraints.

Our paper also relates to the work of [Gottardi and Kubler \(2015\)](#), which studies the effects of unexpected mandatory savings in an economy where agents can only take long positions on contingent trees, the work of [Guerrieri and Lorenzoni \(2017\)](#), which studies the effects of unexpected credit contractions in a heterogeneous-agent incomplete-market model with exogenous borrowing limits, and the work of [Aguiar et al. \(2022\)](#), which studies Pareto-improving fiscal policies in this kind of environment when the interest rate on the government bond is below the growth rate. Our work differs from these studies by placing limited commitment and the endogeneity of borrowing limits as a potential source of inefficiency in debt markets at the forefront and center.

Finally, our work is related to a complementary strand of literature that focuses on macroprudential controls that take the form of Pigouvian taxes or subsidies to reduce pecuniary externalities. [Park \(2014\)](#) studies optimal taxation in an Alvarez–Jermann production economy. There, a Ramsey planner internalizes how individual labor and saving decisions affect aggregate labor, capital supply, wages, and thus the value of autarky. In our setting, the planner internalizes how saving/borrowing choices (indirectly determined by consumption choices) affect asset prices, the default value, and credit constraints. [Jeanne and Korinek \(2010, 2019\)](#) and [Dávila and Korinek \(2018\)](#) provide a welfare rationale for the taxation of capital flows to mitigate the financial amplification effects of fire sales in economies with collateral constraints. In [Farhi and Werning \(2016\)](#), the focus is on demand externalities

associated with the presence of nominal price rigidities. Though such externalities are qualitatively different from the pecuniary externalities we study here, [Korinek and Simsek \(2016\)](#) argue that the two externalities interact and may mutually reinforce each other.

The plan of the paper is as follows. Section 2 introduces the baseline model environment and defines self-enforcing and not-too-tight debt limits. Section 3 compares the laissez-faire equilibrium with a social planner. Section 4 demonstrates that laissez-faire equilibria can be Pareto inferior to equilibria with tighter debt constraints. Section 5 incorporates endowment losses upon default and establishes that our findings on inefficiency are not contingent on interest rates being lower than growth rates. Section 6 concludes. Detailed proofs and technical arguments are available in the online appendix, where additional topics are also explored.

2 General Model

Consider an infinite-horizon endowment economy with a single nonstorable consumption good at each date. Time and uncertainty are both discrete. We use an event tree Σ to describe the revelation of information over an infinite horizon. There is a unique initial date-0 event $s^0 \in \Sigma$, and for each date $t \in \{0, 1, 2, \dots\}$, there is a finite set $S^t \subseteq \Sigma$ of date- t events s^t . Each s^t has a unique predecessor $\sigma(s^t)$ in S^{t-1} and a finite number of successors s^{t+1} in S^{t+1} for which $\sigma(s^{t+1}) = s^t$. The notation $s^{t+1} \succ s^t$ specifies that s^{t+1} is a successor of s^t . The event $s^{t+\tau}$ is said to follow event s^t , also denoted $s^{t+\tau} \succ s^t$, if $\sigma^{(\tau)}(s^{t+\tau}) = s^t$.⁶ The set $S^{t+\tau}(s^t) := \{s^{t+\tau} \in S^{t+\tau} : s^{t+\tau} \succ s^t\}$ denotes the collection of all date- $(t+\tau)$ events following s^t . Abusing notation, we let $S^t(s^t) := \{s^t\}$. The subtree starting at event s^t is then given by:

$$\Sigma(s^t) := \bigcup_{\tau \geq 0} S^{t+\tau}(s^t).$$

⁶Formally, σ is a mapping from $\Sigma \setminus \{s^0\}$ to Σ such that $\sigma(S^{t+1}) = S^t$ for every $t \geq 0$. We pose $\sigma^{(1)} := \sigma$ and $\sigma^{(\tau+1)} := \sigma \circ \sigma^{(\tau)}$ for every $\tau \geq 1$.

We use the notation $s^\tau \succeq s^t$ when $s^\tau \succ s^t$ or $s^\tau = s^t$. In particular, we have $\Sigma(s^t) = \{s^\tau \in \Sigma : s^\tau \succeq s^t\}$.

There is a finite set I of household types, each consisting of a unit measure of identical, infinitely lived agents who consume a single perishable good. Preferences over (nonnegative) consumption processes $c = (c(s^t))_{s^t \succeq s^0}$ are represented by the lifetime expected and discounted utility:

$$U(c) := \sum_{t \geq 0} \beta^t \sum_{s^t \in S^t} \pi(s^t) u(c(s^t)),$$

where $\beta \in (0, 1)$ is the discount factor, $\pi(s^t)$ is the agent's subjective belief of state s^t , and $u : [0, \infty) \rightarrow \mathbb{R}$ is a utility function that is strictly increasing, strictly concave, continuous on $[0, \infty)$, differentiable on $(0, \infty)$, and satisfies Inada's condition $\lim_{\varepsilon \rightarrow 0} [u(\varepsilon) - u(0)]/\varepsilon = \infty$. Given an event s^t , we denote by $U(c|s^t)$ the lifetime continuation utility conditional on s^t , as defined by:

$$U(c|s^t) := u(c(s^t)) + \sum_{\tau \geq 1} \beta^\tau \sum_{s^{t+\tau} \succ s^t} \pi(s^{t+\tau}|s^t) u(c(s^{t+\tau})),$$

where $\pi(s^{t+\tau}|s^t) := \pi(s^{t+\tau})/\pi(s^t)$ is the conditional probability of $s^{t+\tau}$ given s^t . Agents' endowments are subject to random shocks. We denote by $y^i = (y^i(s^t))_{s^t \succeq s^0}$ the process of positive endowments $y^i(s^t) > 0$ of a representative agent of type i . For notational convenience, we have written the primitives as if agents' preferences and beliefs are homogeneous. However, our arguments remain valid when agents have heterogeneous preferences and beliefs, and the only necessary change is to replace (u, β, π) with (u^i, β^i, π^i) .

2.1 Asset Markets with Self-Enforcing Debt Constraints

At any event s^t , agents can issue and trade state-contingent one-period bonds, each one promising to pay one unit of the consumption good contingent on the realization of a successor event $s^{t+1} \succ s^t$. Let $q(s^{t+1}) > 0$ denote the price, at event s^t , of the s^{t+1} -contingent bond (the inverse of $q(s^{t+1})$ is the interest rate between s^t and s^{t+1}). Agent i 's bond holdings are $a^i = (a^i(s^t))_{s^t \succeq s^0}$, where $a^i(s^t) \leq 0$ is a liability, and $a^i(s^t) \geq 0$ is a claim. Each agent's debt is observable and subject to certain (state-contingent and finite)

debt limits $D^i = (D^i(s^t))_{s^t \succeq s^0}$. Given an initial bond holding $a^i(s^0)$ and debt limits D^i , we denote by $B^i(D^i, a^i(s^0)|s^0)$ the budget set of an agent who never defaults. It consists of all pairs (c^i, a^i) of consumption and bond holdings satisfying the following budget flows and debt constraints: for all $s^t \succ s^0$,

$$c^i(s^t) + \sum_{s^{t+1} \succ s^t} q(s^{t+1})a^i(s^{t+1}) \leq y^i(s^t) + a^i(s^t), \quad (2.1)$$

and

$$a^i(s^{t+1}) \geq -D^i(s^{t+1}), \quad \text{for all } s^{t+1} \succ s^t. \quad (2.2)$$

We naturally restrict attention to allocations where the initial asset holdings clear the market, i.e., $\sum_{i \in I} a^i(s^0) = 0$, and satisfy the debt constraints, i.e., $a^i(s^0) \geq -D^i(s^0)$ for each i . Similarly, contingent on an event s^τ , we let $B^i(D^i, x|s^\tau)$ be the set of all plans (c^i, a^i) satisfying restrictions (2.1) and (2.2) at every successor node $s^t \succeq s^\tau$ with initial claim $a^i(s^\tau) = x$. Denote the contingent value function at event s^τ , when agent i starts with financial wealth x , by $V^i(D^i, x|s^\tau)$. It is defined as the largest continuation utility $U(c^i|s^\tau)$ among all budget feasible plans $(c^i, a^i) \in B^i(D^i, x|s^\tau)$. When $x = a^i(s^\tau)$, this will be the equilibrium value, i.e., the payoff to each agent i along the equilibrium path following any event s^τ .

So far, debt limits are arbitrary. We next move to the endogenous determination of the debt limits, which are a critical determinant of equilibrium allocations and payoffs. Debt limits represent the maximal amount of debt that borrowers can issue. In general equilibrium, they also represent the maximal amount of liquidity (or storage of value) that savers can access. We follow [Alvarez and Jermann \(2000\)](#) and provide a microfoundation for the level of debt limits by assuming that agents have limited commitment. We consider an environment where agents cannot commit to their financial contracts and may opt for default. We denote by $V_{\text{def}}^i(s^t)$ agent i 's value of the default option at event s^t , and we impose that the debt limits reflect the fact that repayment is always individually rational. Specifically, we say that debt limits D^i are *self-enforcing* if debtors prefer to repay even the maximum debt allowed, i.e.,

$$V^i(D^i, -D^i(s^t)|s^t) \geq V_{\text{def}}^i(s^t), \quad \text{for all } s^t \succeq s^0. \quad (2.3)$$

We say that D^i are *not too tight* if (2.3) always holds with equality, i.e., borrowers are indifferent between repaying and defaulting:

$$V^i(D^i, -D^i(s^t)|s^t) = V_{\text{def}}^i(s^t), \quad \text{for all } s^t \succeq s^0. \quad (2.4)$$

Given future debt limits $(D^i(s^\tau))_{s^\tau \succ s^t}$, the level $D^i(s^t)$ satisfying (2.4) is interpreted as the largest self-enforcing debt limit contingent on event s^t . We say that D^i are *too tight* if they are self-enforcing and (2.3) holds with strict inequality at some event $s^t \succ s^0$.

Definition 2.1. Given a family of default value functions $(V_{\text{def}}^i)_{i \in I}$, we call a *self-enforcing equilibrium* $(q, (c^i, a^i, D^i)_{i \in I})$ a collection of state-contingent bond prices q , a consumption allocation $(c^i)_{i \in I}$, a bond holdings allocation $(a^i)_{i \in I}$, and a family of debt limits $(D^i)_{i \in I}$ such that:

- (a) the plan (c^i, a^i) of agent i is optimal among budget feasible plans in $B^i(D^i, a^i(s^0)|s^0)$;
- (b) the debt limits D^i of agent i satisfy the self-enforcing condition (2.3);
- (c) markets clear: $\sum_{i \in I} c^i = \sum_{i \in I} y^i$ and $\sum_{i \in I} a^i = 0$.

When the debt limits of all agents satisfy condition (2.4), we use the term *not-too-tight equilibrium*. It is reasonable to expect that in a competitive market, competition among lenders should naturally lead them to offer as much credit as possible without violating borrowers' incentive to repay. Hence, we will also use the term *laissez-faire equilibrium* as a synonym for not-too-tight equilibrium. Similarly, when the debt limits are too tight, we use the term *too-tight equilibrium*.

The default value V_{def} is the key object determining the not-too-tight debt limits. We consider a framework where all assets are seized upon default, and debtors lose access to credit while retaining the ability to save (by purchasing other people's debt). As a consequence, the value of default for any agent i at any event s^t is given by:

$$V_{\text{def}}^i(s^t) = V^i(0, 0|s^t) := \sup\{U(c^i|s^t) : (c^i, a^i) \in B^i(0, 0|s^t)\}, \quad (2.5)$$

The condition (2.4) then reads as follows:

$$V^i(D^i, -D^i(s^t)|s^t) = V^i(0, 0|s^t), \quad \text{for all } s^t \succeq s^0. \quad (2.6)$$

The assumption that agents can save after default follows [Bulow and Rogoff \(1989\)](#) and [Hellwig and Lorenzoni \(2009\)](#) but contrasts with [Alvarez and Jermann \(2000\)](#) who assume that defaulting agents can neither borrow nor save. This weaker form of punishment captures the idea that it is much easier for market participants to coordinate on not purchasing the claims issued by a borrower with a “bad reputation” than to enforce an outright ban from financial markets. Most importantly, [Krueger and Uhlig \(2006\)](#) provided micro-foundations for this default punishment by analyzing dynamic equilibrium risk-sharing contracts between profit-maximizing intermediaries and agents facing income uncertainty. They showed that our default value coincides with the endogenously determined outside option in the model they study.

2.2 Weak and Exact Rollover

[Hellwig and Lorenzoni \(2009\)](#) proved that not-too-tight debt limits necessarily form a bubble that captures the possibility of exactly rolling over debt indefinitely. Formally, a process D^i of debt limits is not too tight (for the default punishment described by (2.5)) if, and only if,

$$D^i(s^t) = \sum_{s^{t+1} \succ s^t} q(s^{t+1})D^i(s^{t+1}), \quad \text{for all } s^t \succ s^0. \quad (2.7)$$

It is straightforward to verify that if there is a strict inequality in (2.7), then debt limits are self-enforcing and too tight.

Proposition 2.1. *Fix an event $s^\tau \in \Sigma$ and assume that*

$$D^i(s^t) \leq \sum_{s^{t+1} \succ s^t} q(s^{t+1})D^i(s^{t+1}), \quad \text{for all } s^t \succeq s^\tau \quad (2.8)$$

with a strict inequality in at least one successor event $s^t \succeq s^\tau$. Then $V^i(D^i, -D^i(s^\tau)|s^\tau) > V^i(0, 0|s^\tau)$.

The characterization of not-too-tight and too-tight debt limits, framed in terms of the exact or weak rollover, facilitates the computation of equilibria. It sidesteps the typical complexities associated with the fixed-point process of identifying self-enforcing debt limits: as the default values depend on prices (given that defaulting agents retain the ability to save), they are influenced by equilibrium allocations and, consequently, the debt limits themselves. The usefulness will become clear when we conduct our policy intervention experiments in Sections 4.

3 A Ramsey Program

In models where financial frictions stem from limited commitment, the prevailing view is that borrowing should adhere to the most lenient debt constraints that align with the financial friction. As articulated by [Krueger and Uhlig \(2006\)](#), not-too-tight debt limits should emerge organically in a competitive credit market, where competition among lenders would allow borrowers to take on the maximum debt that still aligns with repayment incentives. This perspective is bolstered by the notion that such debt limits facilitate optimal risk-sharing. While this may hold in a partial equilibrium context with static prices, its validity becomes dubious in general equilibrium scenarios where prices and debt limits are endogenously determined.

[Hellwig and Lorenzoni \(2009\)](#) demonstrated that debt can be sustained in a bubbly equilibrium where debt limits are not too tight. Our research investigates potential equilibrium alternatives to the conventional *laissez-faire* paradigm. We question whether it's possible to move beyond bubbly equilibria and identify an equilibrium characterized by *too-tight* debt limits that Pareto outperforms the *laissez-faire* equilibrium.

To motivate our investigation, we analyze a social planner problem that faces the same participation constraints as the markets. To formalize the planner's problem, we start by recalling standard properties satisfied by our market equilibrium concept.

Consider a self-enforcing equilibrium $(q, (c^i, a^i, D^i)_{i \in I})$. The value of default $V_{\text{def}}^i(s^t) = V^i(0, 0 | s^t)$ depends on market prices. To clarify this dependence, we make a slight abuse

of notation and write $V_{\text{def}}^i(q|s^t)$. The Principle of Optimality implies that $U^i(c^i|s^t) = V^i(D^i, a^i(s^t)|s^t)$. As debt limits are self-enforcing, we deduce that the consumption process c^i satisfies the participation constraints

$$U^i(c^i|s^t) \geq V_{\text{def}}^i(q|s^t), \quad \text{for every } s^t \succeq s^0.$$

Euler equations imply that

$$q(s^t) \geq q^i(s^t) \quad \text{where} \quad q^i(s^t) := \beta \pi(s^t|\sigma(s^t)) \frac{u'(c^i(s^t))}{u'(c^i(\sigma(s^t)))},$$

with an equality if the participation constraint is not binding: $U^i(c^i|s^t) > V^i(D, -D^i(s^t)|s^t)$.⁷

In particular, we can assume without any loss of generality that the equilibrium price must satisfy

$$q(s^t) = \max_{i \in I} q^i(s^t).$$

We use these equilibrium conditions to construct a constrained maximization problem for a social planner that faces the same participation constraints as the markets.

Formally, a social planner chooses a strictly positive consumption allocation $(c^i)_{i \in I}$ and a strictly positive price process $q = (q(s^t))_{s^t \succeq s^0}$ such that

(a) markets clear, that is, $\sum_{i \in I} c^i = \sum_{i \in I} y^i$;

(b) participation constraints are satisfied

$$U^i(c^i|s^t) \geq V^i(q|s^t), \quad \text{for all } s^t \succeq s^0;$$

(c) the price q is given by $q(s^t) = \max_{i \in I} q^i(s^t)$.

The pairs $(q, (c^i)_{i \in I})$ satisfying the above properties (a), (b), and (c) are called **socially feasible**. Given some strictly positive welfare weights $(\lambda^i)_{i \in I}$, the problem of the **constrained social planner** is to maximize

$$\sum_{i \in I} \lambda^i U^i(c^i|s^0)$$

⁷As debt limits are self-enforcing, we have $V^i(D, -D^i(s^t)|s^t) \geq V_{\text{def}}^i(q|s^t)$.

over all socially feasible pairs $(q, (c^i)_{i \in I})$.

When defining the default value, the social planner is constrained to take the asset prices as determined by the largest individual marginal rates of substitution. In contrast to individuals in a competitive equilibrium, the social planner internalizes that saving/borrowing choices (indirectly determined by consumption choices) affect asset prices, the default value, and credit constraints.

Our setup is equivalent to the problem of a constrained Ramsey planner who maximizes social welfare among all possible competitive equilibria with self-enforcing debt limits. This follows from the following implementation result.⁸

Proposition 3.1. *For every socially feasible pair $(q, (c^i)_{i \in I})$, we can find self-enforcing debt limits $(D^i)_{i \in I}$ and an allocation $(a^i)_{i \in I}$ of bond holdings such that $(q, (c^i, a^i, D^i)_{i \in I})$ is a self-enforcing competitive equilibrium.*

The striking observation is that the constrained social planner will not necessarily find optimal to select a competitive equilibrium with not-too-tight debt limits. Indeed, consider a social planner choice $(q, (c^i)_{i \in I})$. According to Proposition 3.1, this choice can be implemented as a self-enforcing competitive equilibrium $(q, (c^i, a^i, D^i)_{i \in I})$. Consider an agent i and an event s^t such that $U^i(c^i|s^t) > V_{\text{def}}^i(q|s^t)$. If debt limits were not-too-tight, we would necessarily have $a^i(s^t) > -D^i(s^t)$, and therefore $q(s^t) = q^i(s^t)$. This means that when we have simultaneously

$$U^i(c^i|s^t) > V_{\text{def}}^i(q|s^t) \quad \text{together with} \quad q(s^t) > q^i(s^t),$$

then the debt limits are too tight as we must have $a^i(s^t) = -D^i(s^t)$ and therefore

$$V^i(D^i, -D^i(s^t)|s^t) = U^i(c^i|s^t) > V_{\text{def}}^i(q|s^t).$$

To clarify this point, we analyze the first-order conditions of the constrained social planner problem. Consider a solution $(q, (c^i)_{i \in I})$. Given an arbitrary event s^t , we let $I^{\text{NB}}(s^t)$ the set

⁸The proof is constructive and postponed to Appendix ??.

of agents whose participation constraint does not bind, i.e.,

$$I^{\text{NB}}(s^t) = \{i \in I : U^i(c^i|s^t) > V^i(q|s^t)\},$$

and we let $I^*(s^t)$ the set of agents whose individual valuation $q^i(s^t)$ coincides with the price $q(s^t)$, i.e.,

$$I^*(s^t) = \{i \in I : q(s^t) = q^i(s^t)\}.$$

Let $\mu(s^t)$ be the Lagrange multiplier for the market clearing condition, $\xi^i(s^t) \geq 0$ the Lagrange multiplier for the participation constraint, and $\chi(s^t)$ the Lagrange multiplier for the definition of the implied price. The equality constraint $q(s^t) = \max_{i \in I} q^i(s^t)$ can be written as

$$0 = q(s^t) - \beta\pi(s^t|\sigma(s^t)) \max_{i \in I} f(c^i(s^t), c^i(\sigma(s^t)))$$

where the function $(x, y) \mapsto f(x, y)$ is defined by

$$f(x, y) := \frac{u'(x)}{u'(y)}.$$

The mapping $H(\cdot|s^t) : (0, \infty)^I \rightarrow \mathbb{R}$ defined by, for any $x = (x^i)_{i \in I}$,

$$H(x|s^t) := \max_{i \in I} h^i(x|s^t) \quad \text{where} \quad h^i(x|s^t) := f(x^i, c^i(\sigma(s^t)))$$

is not necessarily differentiable. However, it follows from Danskin's Theorem (see Theorem 10.22 in [Clarke \(2013\)](#)) that $H(\cdot|s^t)$ is a regular locally Lipschitz function whose generalized gradient is given by

$$\partial H(x|s^t) = \text{co}\{\nabla h^i(x|s^t) : i \in I^*(s^t)\}$$

Similarly, for every $s^{t+1} \succ s^t$, the mapping $G(\cdot|s^{t+1}) : (0, \infty)^I \rightarrow \mathbb{R}$ defined by, for any $x = (x^i)_{i \in I}$,

$$G(x|s^{t+1}) := \max_{i \in I} g^i(x|s^{t+1}) \quad \text{where} \quad g^i(x|s^{t+1}) := f(c^i(s^{t+1}), x^i)$$

is not necessarily differentiable. Nonetheless, the function $G(\cdot|s^{t+1})$ is a regular locally Lipschitz function whose generalized gradient is given by

$$\partial G(x|s^{t+1}) = \text{co}\{\nabla g^i(x|s^{t+1}) : i \in I^*(s^{t+1})\}.$$

This implies that, for every s^t , there exists $\alpha(s^t) \in \Delta(I)$ with support in $I^*(s^t)$, and for every $s^{t+1} \succ s^t$, there exists $\gamma(s^{t+1}) \in \Delta(I)$ with support in $I^*(s^{t+1})$ such that the first-order condition associated with the choice $c^i(s^t)$ is

$$\begin{aligned} & \beta^t \pi(s^t) u'(c^i(s^t)) \left[\lambda^i + \xi^i(s^0) + \frac{\xi^i(s^1)}{\beta \pi(s^1)} + \dots + \frac{\xi^i(s^t)}{\beta^t \pi(s^t)} \right] \\ & + \frac{u''(c^i(s^t))}{u'(c^i(s^t))} \left[\sum_{s^{t+1} \succ s^t} \chi(s^{t+1}) q(s^{t+1}) \gamma^i(s^{t+1}) - \chi(s^t) q(s^t) \alpha^i(s^t) \right] \\ & = \mu(s^t). \end{aligned} \quad (3.1)$$

The first-order condition associated with the choice $q(s^t)$ is

$$\chi(s^t) = \sum_{i \in I} \xi^i(s^0) \frac{\partial V^i(\cdot | s^0)}{\partial q(s^t)}(q) + \dots + \xi^i(s^t) \frac{\partial V^i(\cdot | s^t)}{\partial q(s^t)}(q). \quad (3.2)$$

As the RHS of (3.2) is nonpositive, we must have $\chi(s^t) \leq 0$.

In a laissez-faire equilibrium, if the participation does not bind, $U^i(c^i | s^t) > V^i(q | s^t)$, then $q(s^t) = q^i(s^t)$. Equivalently, we have

$$I^{\text{NB}}(s^t) \subseteq I^*(s^t). \quad (3.3)$$

If $\chi = 0$,⁹ then the solution of the social planner problem satisfies $I^{\text{NB}}(s^t) \subseteq I^*(s^t)$ (and implements a competitive equilibrium with not-too-tight debt limits). Indeed, assume $\chi = 0$ and assume $i \in I^{\text{NB}}(s^t)$, i.e., agent i is unconstrained at event s^t . This implies that $\xi^i(s^t) = 0$ and

$$\begin{aligned} \mu(s^t) &= \beta^t \pi(s^t) u'(c^i(s^t)) \left[\lambda^i + \xi^i(s^0) + \frac{\xi^i(s^1)}{\beta \pi(s^1)} + \dots + \frac{\xi^i(\sigma(s^t))}{\beta^t \pi(\sigma(s^t))} \right] \\ &= \beta \pi(s^t) u'(c^i(s^t)) \left[\frac{\mu(\sigma(s^t))}{\pi(\sigma(s^t)) u'(c^i(\sigma(s^t)))} \right] \\ &= q^i(s^t) \mu(\sigma(s^t)). \end{aligned}$$

For any other agent $j \neq i$, as $\xi^j(s^t) \geq 0$, we have $\mu(s^t) \geq q^j(s^t) \mu(\sigma(s^t))$. We have thus proved that

$$q^i(s^t) = \frac{\mu(s^t)}{\mu(\sigma(s^t))} \geq \max_{j \in J} q^j(s^t) = q(s^t)$$

⁹This occurs if the value of the default option does not depend on q , as in the case where autarky is the default punishment .

and we get the desired result: $i \in I^*(s^t)$.

When $\chi \neq 0$, it is conceivable that the social planner chooses a solution $(q, (c^i)_{i \in I})$ such that $q(s^t) > q^i(s^t)$ even if $U^i(c^i|s^t) > V^i(q|s^t)$. Indeed, although $\xi^i(s^t) = 0$ and $\alpha^i(s^t) = 0$, there is still the term

$$\frac{u''(c^i(s^t))}{u'(c^i(s^t))} \left[\sum_{s^{t+1} \succ s^t} \chi(s^{t+1}) q(s^{t+1}) \gamma^i(s^{t+1}) \right]$$

that can be strictly positive.¹⁰ The objective of the next section is to illustrate, using a simple example, that the constrained social planner can implement an allocation Pareto superior to a laissez-faire allocation by choosing too-tight debt limits.

4 Tightening Debt Constraints

We propose to show that a policy intervention to tighten debt constraints can lead to Pareto improvements over the laissez-faire allocation. Our model envisions a scenario where a credit agency or the government enforces these too-tight debt limits. We interpret these interventions as a streamlined representation of regulatory or prudential policies crafted to curtail financial leverage.

What is the intuition for our (constrained) inefficiency results? As the value of default depends on market prices, there is a pecuniary externality that agents do not internalize in a competitive environment. In particular, we will show that reducing the borrowing capacity at a period τ reduces the credit volume, increases bond prices, or, equivalently, lowers the implied interest rates. This impact on prices has a negative feedback effect on the value of the default option at periods $t < \tau$ since it is now more costly to smooth consumption over time by saving only. This implies that the not-too-tight debt limits at periods $t < \tau$ can be looser than their level before the intervention. Pareto improvement can be obtained when the benefits from the relaxed credit conditions at periods $t < \tau$ compensate the costs of the tighter credit conditions in period τ .

¹⁰Recall that $\chi(s^{t+1}) \leq 0$, $\gamma^i(s^{t+1}) \geq 0$, $u'(\cdot) > 0$ and $u''(\cdot) < 0$.

To illustrate this intuition in the most straightforward possible manner, we consider a simpler version of the standard example presented in [Hellwig and Lorenzoni \(2009\)](#). There are two agents facing uncertainty only during the initial period. The economy is thereafter a deterministic one, in which, every other period, agents' endowments switch from a high value to a low value. Within this setting, we perform the following exercise. We first construct a Markov laissez-faire equilibrium $(q, (c^i, a^i, D^i)_{i \in I})$, in which after the realization of uncertainty, the economy settles in a cyclical and symmetric steady-state equilibrium where debt limits are not too tight. We then construct another equilibrium $(\hat{q}, (\hat{c}^i, \hat{a}^i, \hat{D}^i)_{i \in I})$, supported by the same allocation of initial financial claims, but with debt constraints being too tight at a single period τ . We then show that the consumption allocation $(\hat{c}^i)_{i \in I}$ Pareto dominates the consumption allocation $(c^i)_{i \in I}$ of the laissez-faire equilibrium.

4.1 Primitives of the Example

There are two agents $I = \{a, b\}$ who enter the market with an identical endowment $y_0 > 0$ and no financial claims (i.e., $a^a(s^0) = a^b(s^0) = 0$). There is uncertainty only at the initial period $t = 0$, described by two possible states $z^a \neq z^b$. After realizing the state z^i , the economy becomes deterministic where agents' endowments switch between a high value y_H and a low value y_L with $y_H > y_L$. Realizing state z^i means that the agent i starts with the high endowment at $t = 1$. The beliefs are homogeneous, with each agent assigning the probability $\pi_H = 1/2$ ($\pi_L := 1 - \pi_H$) of getting the high (low, respectively) endowment at $t = 1$. Since there is uncertainty only at the initial period, we simplify notation by writing a generic process $(x(s^t))_{s^t \succeq s^0}$ as follows: $x(s^0) = x_0$ and $x(s^t) = x_t(z)$ if $s^t \succeq (s^0, z)$ with $z \in \{z^a, z^b\}$.¹¹ The representation of the event tree is as in [Figure 4.1](#).

For future reference, we point out that the *symmetric first-best allocation* of this economy obtains when both agents consume their endowment at $t = 0$ and the perfect risk-sharing consumption level $c^{\text{fb}} := (y_H + y_L)/2$ at every period $t \geq 1$.

¹¹The event tree Σ can be formally defined as follows: $S^0 := \{s^0\}$ and for every $t \geq 1$, $S^t = \{(z^a, t), (z^b, t)\}$. The binary relation \succ is defined as follows: $(z, 1) \succ s^0$ and $(z, \tau) \succ (\zeta, t)$ when $z = \zeta$ and $\tau > t$.

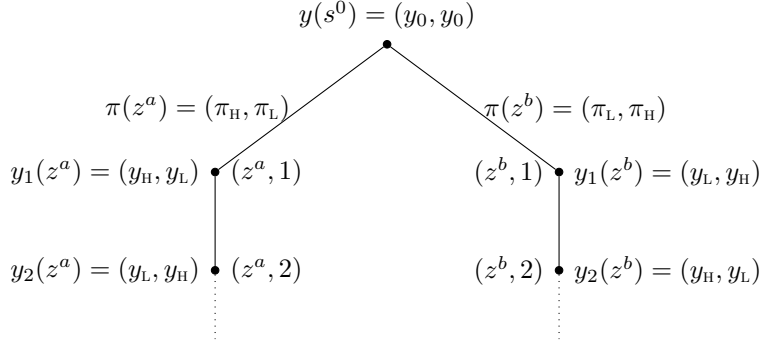


Figure 4.1: Event tree and endowments.

4.2 Laissez-Faire Equilibrium

We recall the characteristics of the laissez-faire equilibrium presented in [Hellwig and Lorenzoni \(2009\)](#). It is a symmetric Markov equilibrium: at period $t = 0$, both agents borrow up to the debt limit d^{lf} against their high-income state and save contingent on their low-income state. After resolving the uncertainty at period $t = 1$, the economy settles in a cyclical steady-state where the low-income agent borrows up to the not-too-tight debt limit d^{lf} while the high-income agent saves.

Proposition 4.1. *Assume interest rates at autarky are negative, i.e., $\beta u'(y_L) > u'(y_H)$. Then, there exists an equilibrium $(q, (c^i, a^i, D^i)_{i \in I})$ with not-too-tight debt limits where for each $z \in \{z^a, z^b\}$ and every $i \in I$:*

(i) *debt limits equal $D_t^i(z) = d^{\text{lf}}$, for $t \geq 1$;*

(ii) *consumption is: $c_0^i = y_0$ at $t = 0$, $c_t^i(z^i) = c_H^{\text{lf}}$ and $c_t^i(z^j) = c_L^{\text{lf}}$, for $t \geq 1$;*

(iii) *net asset positions are $a_t^i(z) = -d^{\text{lf}}$ (i.e., the debt limit binds) if $y_t^i(z) = y_H$, and $a_t^i(z) = d^{\text{lf}}$ if $y_t^i(z) = y_L$, for $t \geq 1$;*

(iv) *prices are given by $q_1(z) = \beta \pi_L u'(c_L^{\text{lf}})/u'(y_0)$ and $q_{t+1}(z) = q^{\text{lf}} = 1$, for $t \geq 1$.*

The proof of the proposition is standard (and, therefore, omitted). Debt limits are trivially not-too-tight as $d^{\text{lf}} = q^{\text{lf}} d^{\text{lf}}$. The equilibrium net trade $x^{\text{lf}} := (1 + q^{\text{lf}})d^{\text{lf}} = 2d^{\text{lf}}$ is

determined by the first order condition

$$1 = q^{\text{lf}} = \beta \frac{u'(c_{\text{L}}^{\text{lf}})}{u'(c_{\text{H}}^{\text{lf}})} = \beta \frac{u'(y_{\text{L}} + x^{\text{lf}})}{u'(y_{\text{H}} - x^{\text{lf}})}.$$

At $t = 1$, the continuation utilities are $U_{\text{H}}^{\text{lf}} = [u(y_{\text{H}} - x^{\text{lf}}) + \beta u(y_{\text{L}} + x^{\text{lf}})]/(1 - \beta^2)$ for the high-income agent and $U_{\text{L}}^{\text{lf}} = [u(y_{\text{L}} + x^{\text{lf}}) + \beta u(y_{\text{H}} - x^{\text{lf}})]/(1 - \beta^2)$. As the equilibrium is symmetric, both agents share the same time 0 expected utility $U_0^{\text{lf}} := u(y_0) + (\beta/2)(U_{\text{H}}^{\text{lf}} + U_{\text{L}}^{\text{lf}})$.

We let $A_{\text{H}} := -u''(c_{\text{H}}^{\text{lf}})/u'(c_{\text{H}}^{\text{lf}})$ and $A_{\text{L}} := -u''(c_{\text{L}}^{\text{lf}})/u'(c_{\text{L}}^{\text{lf}})$ denote the absolute risk-aversion at the laissez-faire consumption levels c_{H}^{lf} and c_{L}^{lf} respectively. When u is the CRRA utility function with risk-aversion coefficient $\gamma > 0$, the contingent consumption and the net trade values satisfy

$$c_{\text{H}}^{\text{lf}} = \alpha_{\text{H}}(y_{\text{H}} + y_{\text{L}}), \quad c_{\text{L}}^{\text{lf}} = \alpha_{\text{L}}(y_{\text{H}} + y_{\text{L}}) \quad \text{and} \quad x^{\text{lf}} = \alpha_{\text{L}}y_{\text{H}} - \alpha_{\text{H}}y_{\text{L}}$$

where the coefficients α_{H} and α_{L} are given by

$$\alpha_{\text{H}} = \frac{\beta^{-1/\gamma}}{1 + \beta^{-1/\gamma}} \quad \text{and} \quad \alpha_{\text{L}} = \frac{1}{1 + \beta^{-1/\gamma}}.$$

We let $\zeta := y_{\text{H}}/y_{\text{L}}$ denote income dispersion. Observe that interest rates at autarky are negative if, and only if, $\beta^{1/\gamma}\zeta > 1$.

4.3 Pareto-Improving Policy Intervention

We are interested in equilibria where debt limits are self-enforcing but not necessarily not-too-tight at every period. For clarity in our presentation, we center our discussion on equilibria where agents borrow up to their debt limits contingent on high income, denoted by d_t , and save contingent on low income. Such equilibria are uniquely determined by the sequence $(q_{t+1}, d_t)_{t \geq 1}$ where q_{t+1} is the asset price at date t .¹² Here, net trade is given by $x_t := d_t + q_{t+1}d_{t+1}$. Consumption for the high-income agent is given by $c_{\text{H},t} = y_{\text{H}} - x_t$, while for the low-income agent, it is $c_{\text{L},t} = y_{\text{L}} + x_t$. At $t = 0$, both agents consume their endowment y_0 , and the two contingent bonds share the same price q_1 .

¹²The debt limit contingent on low income need not be specified since our focus is on equilibria where agents save to hedge against the low-income shock.

The conditions ensuring the optimality of individual choices are, at $t = 0$,

$$q_1 = \beta\pi_L \frac{u'(y_L + x_1)}{u'(y_0)} \geq \beta\pi_H \frac{u'(y_H - x_1)}{u'(y_0)}; \quad (4.1)$$

and, for any $t > 1$,¹³

$$q_{t+1} = \beta \frac{u'(y_L + x_{t+1})}{u'(y_H - x_t)} \geq \beta \frac{u'(y_H - x_{t+1})}{u'(y_L + x_t)}. \quad (4.2)$$

We propose a targeted policy intervention that tightens debt constraints for a single period, thereby achieving an equilibrium allocation that is Pareto superior to the laissez-faire allocation. Specifically, we set $\tau = 2$ and construct an equilibrium where the low-income agent faces a too-tight debt limit d_3 at date $t = \tau$. Formally, the equilibrium has the following features:

- (a) For periods $t \in \{0, 1\}$, there is no intervention, and the debt limit d_{t+1} is not too tight.
- (b) At period $t = 2$, a policy intervention imposes a debt limit $d_3 := (1 - \varepsilon)q_4 d^{\text{lf}}$.
- (c) At period $t = 3$, the high-income agent starts with the too-tight debt level d_3 . Since the intervention is over, the debt limit reverts to its laissez-faire level d^{lf} , which is not too tight given future debt limits.¹⁴
- (d) For $t \geq 4$, the laissez-faire equilibrium prevails: $d_t = d^{\text{lf}}$ and $q_{t+1} = q^{\text{lf}}$.

By construction all debt limits are self-enforcing. In particular, Proposition 2.1 assures that this is the case for d_3 . It follows that d_3 is too tight when $\varepsilon > 0$.

We refer to the above as an *equilibrium with ε -tight debt constraints*, and we use the notation $(q_{t+1}(\varepsilon), d_t(\varepsilon))$ to emphasize the dependence on the tightening coefficient ε . Our contribution amounts to identifying a set of parameter values for which such equilibria exist and feature a consumption allocation that is Pareto superior to the laissez-faire allocation. We summarize our main result below.

¹³To ensure that consumption is positive, we also need that $-y_L < x_t < y_H$ for every $t \geq 1$.

¹⁴The not-too-tight debt limits d_1 and d_2 though may differ from d^{lf} as d_3 is too-tight.

Theorem 4.1. *If the interest rate at autarky is negative (i.e., $\beta u'(y_L) > u'(y_H)$), then, for sufficiently small $\varepsilon > 0$, there exists an equilibrium with ε -tight debt constraints that Pareto dominates the laissez-faire equilibrium provided that the following condition holds:*

$$0 < A_L x^{\text{lf}} - 1 < \beta(A_H x^{\text{lf}} + 1). \quad (4.3)$$

For the CRRA utility function characterized by a risk-aversion coefficient $\gamma > 0$, negative interest rates at autarky occur if, and only if, $\gamma > f(\beta|\zeta)$, where $f(\beta|\zeta)$ is the value of γ that satisfies $\beta^{1/\gamma}\zeta = 1$. The sufficient condition (4.3) can be rewritten as:

$$\zeta + 1 < \gamma(\zeta - \beta^{-1/\gamma}) < \frac{1 + \beta}{1 - \beta^{1+1/\gamma}}(\zeta + 1). \quad (4.4)$$

Both functions $\gamma \mapsto \gamma(\zeta - \beta^{-1/\gamma})$ and $\gamma \mapsto \gamma(\zeta - \beta^{-1/\gamma})(1 - \beta^{1+1/\gamma})$ are strictly increasing. Therefore, condition (4.4) is equivalent to:

$$g(\beta|\zeta) < \gamma < h(\beta|\zeta),$$

where $g(\beta|\zeta)$ is the value of γ that solves $\zeta + 1 = \gamma(\zeta - \beta^{-1/\gamma})$ and $h(\beta|\zeta)$ the value of γ that solves $\gamma(\zeta - \beta^{-1/\gamma})(1 - \beta^{1+1/\gamma}) = (\zeta + 1)(1 + \beta)$.

In Figure 4.2(a), we plot the functions $f(\cdot|\underline{\zeta})$, $g(\cdot|\underline{\zeta})$, and $h(\cdot|\underline{\zeta})$ when income dispersion is $\underline{\zeta} = 2$. The green area represents the set of primitive values (β, γ) where the intervention is Pareto improving.¹⁵

Denote by $U_0(\varepsilon)$ the common expected and discounted utility at $t = 0$.¹⁶ We let $ce(\varepsilon)$ be the certainty equivalent of the expected continuation utility at $t = 1$. This consumption level is determined by the equation $U_0(\varepsilon) = u(y_0) + \beta u(ce(\varepsilon))$. Figure 4.2(b) offers an alternative illustration when $\bar{\beta} = 0.9$ by plotting the difference of the certainty equivalent consumption, $\varepsilon \mapsto ce(\varepsilon) - ce(0)$. Pareto improvement obtains if, and only if, $ce(\varepsilon) - ce(0) > 0$.

¹⁵The red area represents the set of primitive values (β, γ) where a laissez-faire equilibrium with trade does not exist.

¹⁶It is formally defined as $U_0(\varepsilon) = u(y_0) + \frac{1}{2} \sum_{t \geq 1} \beta^t [u(y_L + x_t) + u(y_H - x_t)]$.

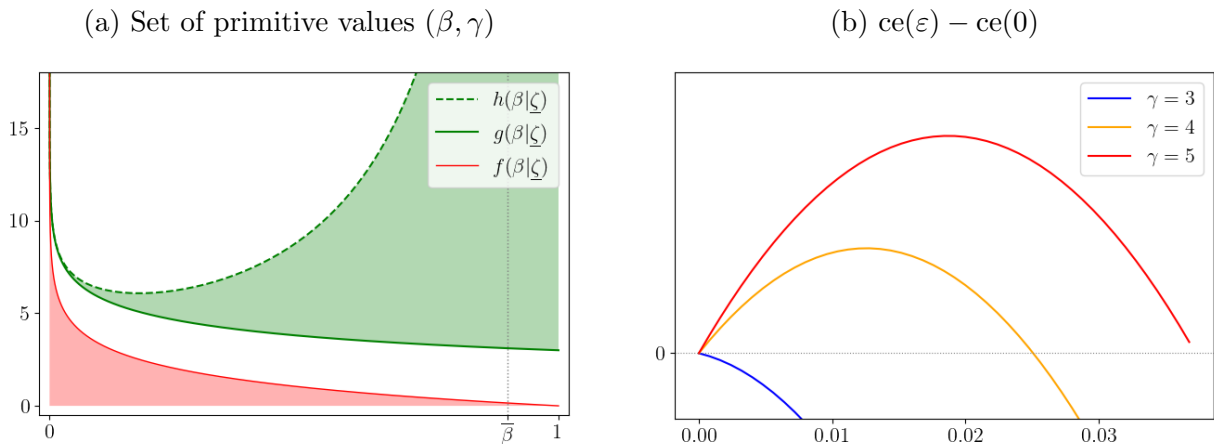


Figure 4.2: Tightening Debt Constraints at $\tau = 2$ is Pareto improving.

In the following sections, we elucidate the intuition and furnish the analytical underpinnings for Theorem 4.1. In the equilibrium scenario with ε -tight debt constraints, the economy undergoes a transitional phase before reverting to the laissez-faire regime at period $t = 4$. A crucial step in this analysis is determining the not-too-tight debt limits d_1 and d_2 that prevail during this intervention period. This task is particularly challenging since the powerful characterization result of Hellwig and Lorenzoni (2009) cannot be directly applied. Specifically, the condition that d_1 and d_2 should not be too tight does not automatically imply that they permit the exact rollover of debt. Therefore, their determination cannot be solely based on the equilibrium price specifications. This deviation from the standard approach is a direct consequence of our policy experiment, which prescribes a specific debt limit d_3 that disrupts the usual not-too-tight conditions for future debt limits.

To overcome this issue, it becomes necessary to compute the value functions for periods $t \in \{1, 2\}$ that correspond to both equilibrium and out-of-equilibrium paths. The debt limits d_1 and d_2 then emerge as non-trivial solutions to the not-too-tight condition, as expressed in Equation 2.6.

4.4 Equilibrium Construction

To streamline the exposition, we break down the construction of the equilibrium into several steps. We begin by identifying the equilibrium variables for period $t = 3$ and then proceed backward to ascertain the corresponding variables for periods $t \in \{2, 1\}$. At $t = 4$ the economy settles down at the laissez-faire equilibrium described in Proposition 4.2.

The intervention forces the high-income agent to start at period $t = 3$ with a level of debt

$$d_3(\varepsilon) = (1 - \varepsilon)q_4(\varepsilon)d^{\text{lf}}. \quad (4.5)$$

To support the laissez-faire steady state next period, this agent must save d^{lf} while the low-income agent must find it optimal to borrow the same amount. In this case, the net trade and equilibrium price at period = 3 are jointly determined by the following two equations

$$q_4(\varepsilon) = \beta \frac{u'(y_L + x^{\text{lf}})}{u'(y_H - x_3(\varepsilon))} \quad \text{and} \quad x_3(\varepsilon) = (2 - \varepsilon)q_4(\varepsilon)d^{\text{lf}}. \quad (4.6)$$

The first equation is the FOC of the high-income agent, while the second equation is obtained by substituting $d_3(\varepsilon)$ in $x_3(\varepsilon) = d_3(\varepsilon) + q_4(\varepsilon)d^{\text{lf}}$ that stands for net trade.

It follows from a straightforward application of the Implicit Function Theorem that there exists $\bar{\varepsilon} > 0$ and continuously differentiable functions $q_4 : [0, \bar{\varepsilon}] \rightarrow (0, \infty)$ and $x_3 : [0, \bar{\varepsilon}] \rightarrow (0, \infty)$ satisfying (4.6) and such that $q_4(0) = q^{\text{lf}} = 1$ and $x_3(0) = x^{\text{lf}}$. To be optimal for the low-income agent to issue the maximum debt level d^{lf} , we must have that

$$q_4(\varepsilon) \geq \beta \frac{u'(c_H^{\text{lf}})}{u'(y_L + x_3(\varepsilon))}. \quad (4.7)$$

As $x_3(0) = x^{\text{lf}}$ and $q_4(0) = 1 > \beta u'(c_H^{\text{lf}})/u'(c_L^{\text{lf}})$, we can reduce $\bar{\varepsilon} > 0$ so that (4.7) is also satisfied for every $\varepsilon \in [0, \bar{\varepsilon}]$. Differentiating the FOC in (4.6) at $\varepsilon = 0$, we get that

$$q_4'(0) = \frac{A_H x^{\text{lf}}}{2(A_H x^{\text{lf}} + 1)} > 0 \quad \text{and} \quad x_3'(0) = -\frac{1}{A_H} q_4'(0) < 0.$$

Figure 4.3 plots the functions $\varepsilon \mapsto q_4(\varepsilon)$ and $\varepsilon \mapsto x_3(\varepsilon)$ for CRRA utility with different values of γ . The income dispersion coefficient and discount factor values are set to $\zeta = 2$ and $\bar{\beta} = 0.9$. Observe that the higher the tightening coefficient ε , the lower the consumption

smoothing (i.e., the function $\varepsilon \mapsto x_3(\varepsilon)$ is decreasing), and the higher is the asset price $q_4(\varepsilon)$ (or, equivalently, the lower is the implied interest rate). The contraction of net trade reflects the lower debt ceiling as ε increases. Indeed, $d_3(0) = d^{\text{lf}}$ and $d'_3(0) = (q'_4(0) - 1)d^{\text{lf}} < 0$.

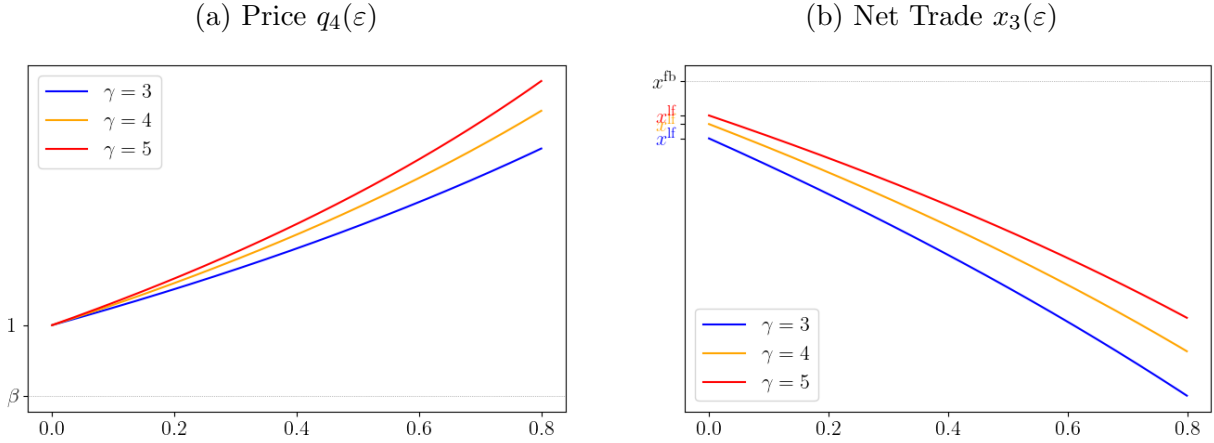


Figure 4.3: Equilibrium variables at $t = 3$ as functions of the tightening coefficient ε

Tightening debt constraints at period $t = 2$ reduces net trade at $t = 3$. This reduction proves, at first instance, to be disadvantageous when viewed from an ex-ante standpoint. Specifically, the time 0 expected utility is given by:

$$U_0(\varepsilon) := u(y_0) + \frac{1}{2} \sum_{t \geq 1} \beta^t \varphi(x_t(\varepsilon)) \quad \text{where} \quad \varphi(x) := u(y_H - x) + u(y_L + x).$$

When $x_t < x^{\text{fb}}$, it holds that $\varphi'(x_t) > 0$. However, these changes produce secondary effects through their feedback on the equilibrium variables of previous periods. In particular, as we illustrate below, these changes alter the default value in periods $t \in \{1, 2\}$, affecting the not-too-tight debt limits d_1 and d_2 . If default values reduce, d_1 and d_2 might exceed the laissez-faire limit d^{lf} . This might lift net trade over the laissez-faire value x^{lf} , increasing ex-ante utility. There is scope for Pareto improvement when the reduction in utility due to the contraction of net trade at period $t = 3$ is offset by the increase in utility due to the expansion of net trade in periods $t \in \{1, 2\}$.

4.4.1 Computing the Not-Too-Tight Debt Limit at $t = 2$

We now turn our attention to the equilibrium variables for period $t = 2$, specifically focusing on establishing the not-too-tight debt limit d_2 . This is equivalent to determining the net trade x_2 .

The high-income agent starts with debt $d_2(\varepsilon)$ and saves the amount $d_3(\varepsilon)$. This position is accommodated by the low-income agent who borrows the same amount. Net trade at this period therefore equals $x_2(\varepsilon) = d_2(\varepsilon) + q_3(\varepsilon)d_3(\varepsilon)$. The FOC associated with these saving and borrowing decisions are, respectively,

$$q_3(\varepsilon) = \beta \frac{u'(y_L + x_3(\varepsilon))}{u'(y_H - x_2(\varepsilon))} \quad \text{and} \quad q_3(\varepsilon) \geq \beta \frac{u'(y_H - x_3(\varepsilon))}{u'(y_L + x_2(\varepsilon))}. \quad (4.8)$$

Since we require the debt limit $d_2(\varepsilon)$ to be not too tight, condition (2.4) reads as follows

$$u(y_H - x_2(\varepsilon)) + \beta u(y_L + x_3(\varepsilon)) + \beta^2 U_H^{\text{lf}} = V_{\text{H},2}^{\text{def}}(\varepsilon), \quad (4.9)$$

where U_H^{lf} is the continuation utility of the high-income agent in the laissez-faire equilibrium and $V_{\text{H},2}^{\text{def}}(\varepsilon)$ is the out-of-equilibrium value function of the high-income agent at $t = 2$.

The existence and behavior of functions $\varepsilon \mapsto x_2(\varepsilon)$ and $\varepsilon \mapsto q_3(\varepsilon)$ compatible with (4.8) and (4.9) depend on the behavior of the default value function $\varepsilon \mapsto V_{\text{H},2}^{\text{def}}(\varepsilon)$. The latter is affected by the equilibrium prices. Therefore, its determination relies on an educated guess about the out-of-equilibrium path. We postulate the following default value function:

$$V_{\text{H},2}^{\text{def}}(\varepsilon) = u(y_H - q_3(\varepsilon)\theta_3(\varepsilon)) + \beta u(y_L + \theta_3(\varepsilon)) + \beta^2 U_H^{\text{lf}}, \quad (4.10)$$

that is derived by assuming that the defaulting agent only saves when income is high. In particular, the agent with high income at period $t = 2$ will not save at period $t = 3$ when income becomes low, and the out-of-equilibrium continuation value at period $t = 4$ will then coincide with the laissez-faire continuation utility U_H^{lf} . This is because d^{lf} is not-too-tight so that $V_{\text{H},4}^{\text{def}}(\varepsilon) = U_H^{\text{lf}}$. The necessary and sufficient FOCs to support the out-of-equilibrium guess are

$$q_3(\varepsilon) = \beta \frac{u'(y_L + \theta_3(\varepsilon))}{u'(y_H - q_3(\varepsilon)\theta_3(\varepsilon))} \quad \text{and} \quad q_4(\varepsilon) \geq \beta \frac{u'(c_H^{\text{lf}})}{u'(y_L + \theta_3(\varepsilon))}. \quad (4.11)$$

Replacing the expression of the default option from (4.10) in the not-too-tight condition (4.9) gives

$$u(y_H - x_2(\varepsilon)) + \beta u(y_L + x_3(\varepsilon)) = \underbrace{u(y_H - q_3(\varepsilon)\theta_3(\varepsilon)) + \beta u(y_L + \theta_3(\varepsilon))}_{=: \psi_2(\varepsilon)}. \quad (4.12)$$

Reducing $\bar{\varepsilon} > 0$ if necessary, it follows from the Implicit Function Theorem that there exist continuously differentiable functions $q_3, x_2, \theta_3 : [0, \bar{\varepsilon}] \rightarrow (0, \infty)$ satisfying the equations in (4.8) and (4.11) as well as equation (4.12). Moreover, we have $q_3(0) = q_4(0) = 1$, $x_2(0) = x_3(0) = x^{\text{lf}}$ and $\theta_3(0) = x^{\text{lf}}$. Therefore, for $\varepsilon = 0$, the weak inequalities in (4.8) and (4.11) read as $q_3(0) = q_4(0) = 1 > \beta u'(c_H^{\text{lf}})/u'(c_L^{\text{lf}})$, so they are both satisfied for ε small enough. The not-too-tight debt limit $d_2(\varepsilon)$ is then obtained as the difference $x_2(\varepsilon) - q_3(\varepsilon)d_3(\varepsilon)$.

A closer inspection of (4.12) reveals that a necessary condition for net trade $x_2(\varepsilon)$ to be increasing in ε is that the value of the default option $\psi_2(\varepsilon)$ is itself decreasing in ε . Nevertheless, a decreasing default option is still compatible with $x_2(\varepsilon)$ decreasing since net trade $x_3(\varepsilon)$ decreases unambiguously with ε . Differentiating the equation in (4.8) and the not-too-tight condition (4.12) at $\varepsilon = 0$ produces

$$q'_3(0) = -(x'_2(0)A_H + x'_3(0)A_L) \quad \text{and} \quad x'_2(0) = x'_3(0) + q'_3(0)x^{\text{lf}}. \quad (4.13)$$

Solving the above system, we deduce that

$$q'_3(0) = -x'_3(0) \frac{A_L + A_H}{A_H x^{\text{lf}} + 1} > 0 \quad \text{and} \quad x'_2(0) = -x'_3(0) \frac{A_L x^{\text{lf}} - 1}{A_H x^{\text{lf}} + 1}. \quad (4.14)$$

Given that $u'(c_H^{\text{lf}}) = \beta u'(c_L^{\text{lf}})$, a similar exercise yields $\psi'_2(0) = -q'_3(0)u'(c_H^{\text{lf}})x^{\text{lf}} < 0$.

Our first-order analysis reveals that, for ε small enough, the asset price $q_3(\varepsilon)$ increases. The higher price at $t = 2$ reduces the value of the default option $\psi_2(\varepsilon)$. As $d_2(\varepsilon)$ is required to be not-too-tight, the left-hand side of the not-too-tight condition (4.12) has to adjust accordingly. The adjustment occurs by the unambiguous fall of net trade $x_3(\varepsilon)$ at period $t = 3$. However, when parameters are such that $A_L x^{\text{lf}} > 1$, the decrease of $x_3(\varepsilon)$ is insufficient to compensate for the decrease of the default option.¹⁷ This, in turn, forces the net trade

¹⁷The restriction corresponds to the first inequality of condition (4.3) in Theorem 4.1.

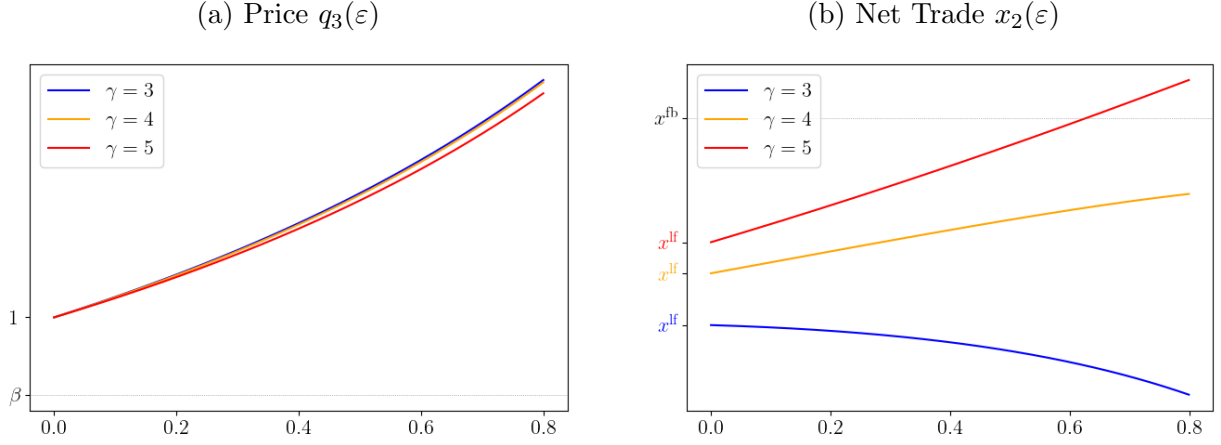


Figure 4.4: Equilibrium variables at $t = 2$ as functions of the tightening coefficient ε

$x_2(\varepsilon)$ that prevails at period $t = 2$ to increase. The expansion of net trade reflects that $d_2(\varepsilon)$ exceeds the laissez-faire level d^{lf} . Indeed, we have $d_2(0) = d^{\text{lf}}$ and $d'_2(0) = (q'_4(0) + q'_3(0))d^{\text{lf}} > 0$.

Figure 4.4 plots the functions $\varepsilon \mapsto q_3(\varepsilon)$ and $\varepsilon \mapsto x_3(\varepsilon)$ for CRRA utility with different values of γ . The income dispersion and the discount factor equal $\zeta = 2$ and $\bar{\beta} = 0.9$. For $\gamma = 3$, we have $A_L x^{\text{lf}} < 1$ and $\varepsilon \rightarrow x_2(\varepsilon)$ is decreasing. For $\gamma \in \{4, 5\}$, we have $A_L x^{\text{lf}} > 1$ and $\varepsilon \rightarrow x_2(\varepsilon)$ is increasing.

4.4.2 Computing the Not-Too-Tight Debt Limit at $t = 1$

Determining the equilibrium variables at period $t = 1$ follows similar reasoning, so we only discuss the key differences with respect to period $t = 2$.

The high-income agent starts the period with liabilities $d_1(\varepsilon)$ and saves the amount $d_2(\varepsilon)$. The FOC determines the equilibrium price

$$q_2(\varepsilon) = \beta \frac{u'(y_L + x_2(\varepsilon))}{u'(y_H - x_1(\varepsilon))}. \quad (4.15)$$

As the debt level $d_1(\varepsilon)$ is required to be not-too-tight, the participation constraint reads as

follows

$$\begin{aligned}
& u(y_H - x_1(\varepsilon)) + \beta u(y_L + x_2(\varepsilon)) + \beta^2 u(y_H - x_3(\varepsilon)) + \beta^3 U_L^{\text{lf}} = \\
& u(y_H - q_2(\varepsilon)\theta_2(\varepsilon)) + \beta u(y_L + \theta_2(\varepsilon)) + \beta^2 u(y_H - q_4(\varepsilon)\theta_4(\varepsilon)) + \beta^3 u(y_L + \theta_4(\varepsilon)) + \beta^4 U_H^{\text{lf}}.
\end{aligned} \tag{4.16}$$

The right-hand side stands for the value of the default option $V_{H,1}^{\text{def}}(\varepsilon)$ obtained by guessing that out-of-equilibrium, there is saving only when income is high. The variables $\theta_2(\varepsilon)$ and $\theta_4(\varepsilon)$ correspond to the optimal saving decisions at $t = 1$ and $t = 3$ and are determined by the following FOCs

$$q_2(\varepsilon) = \beta \frac{u'(y_L + \theta_2(\varepsilon))}{u'(y_H - q_2(\varepsilon)\theta_2(\varepsilon))} \quad \text{and} \quad q_4(\varepsilon) = \beta \frac{u'(y_L + \theta_4(\varepsilon))}{u'(y_H - q_4(\varepsilon)\theta_4(\varepsilon))}. \tag{4.17}$$

The corresponding marginal price at $\varepsilon = 0$ is $q'_2(0) = -(x'_1(0)A_H + x'_2(0)A_L)$. Differentiating the not-too-tight condition (4.16) at $\varepsilon = 0$, we obtain

$$-x'_1(0) + x'_2(0) - \beta^2 x'_3(0) = -q'_2(0)x^{\text{lf}} - \beta^2 q'_4(0)x^{\text{lf}}.$$

Using the expressions for $q'_4(0)$ and $q'_2(0)$, we deduce that

$$x'_1(0)(A_H x^{\text{lf}} + 1) = -x'_2(0)(A_L x^{\text{lf}} - 1) - \beta^2 x'_3(0)(A_H + 1).$$

As $x'_2(0) = -x'_3(0)(A_L x^{\text{lf}} - 1)/(A_H x^{\text{lf}} - 1)$, we get that $x'_1(0) > 0$ if, and only if, $\beta(A_H x^{\text{lf}} + 1) > (A_L x^{\text{lf}} - 1)$.¹⁸ To illustrate this possibility, we plot in Figure 4.5 the functions $\varepsilon \mapsto q_2(\varepsilon)$ and $\varepsilon \mapsto x_1(\varepsilon)$ for CRRA utility with different values of γ , setting the values of the income dispersion and discount factor equal to $\underline{\zeta} = 2$ and $\bar{\beta} = 0.9$.

4.5 Pareto Improvement

Since the equilibrium is symmetric, for each agent $i \in I$, the ex-ante (expected and discounted) utility satisfies $U^i(c^{i,\varepsilon} | s^0) = U_0(\varepsilon)$ where¹⁹

$$U_0(\varepsilon) = u(y_0) + \frac{1}{2} \sum_{t \geq 1} \beta^t \varphi(x_t(\varepsilon)), \quad \text{with} \quad \varphi(x) = u(y_H - x) + u(y_L + x).$$

¹⁸The restriction corresponds to the second inequality of condition (4.3) in Theorem 4.1.

¹⁹We recall that $\pi_L = \pi_H = 1/2$.

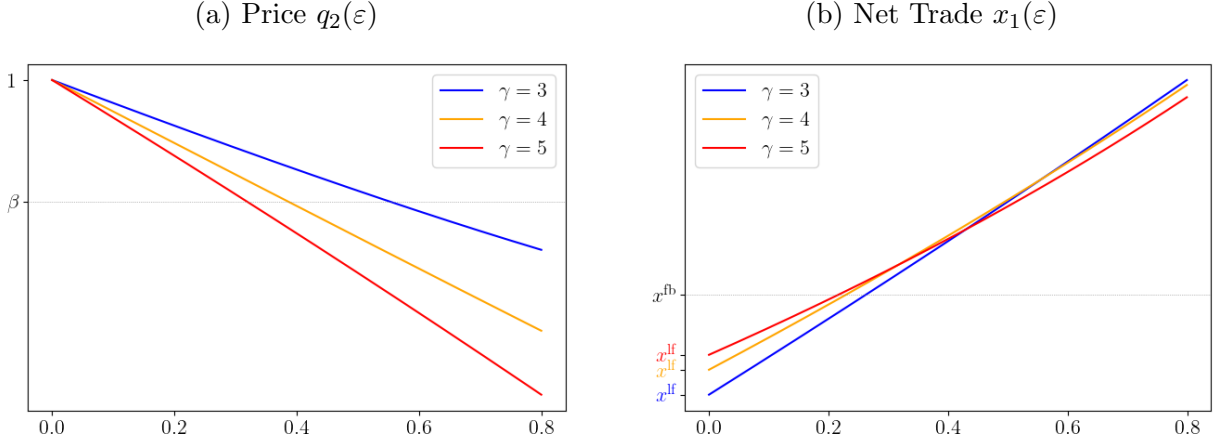


Figure 4.5: Equilibrium variables at $t = 1$ as functions of the tightening coefficient ε .

It is straightforward to verify that for $\varepsilon = 0$, then we recover the laissez-faire equilibrium with not-too-tight debt limits, that is $(q^0, (c^{i,0}, a^{i,0}, D^{i,0})_{i \in I}) = (q, (c^i, a^i, D^i)_{i \in I})$ and we deduce that

$$U^i(c^i | s^0) = U_0(0) = u(y_0) + \frac{1}{2} \sum_{t \geq 1} \beta^t \varphi(x^{\text{lf}}).$$

Therefore, to show that the consumption allocation $(c^{i,\varepsilon})_{i \in I}$ Pareto dominates the laissez-faire consumption allocation $(c^i)_{i \in I}$, it is sufficient to show that $U'_0(0) > 0$. The variation of the ex-ante utility is given by

$$U_0(\varepsilon) - U_0(0) = \frac{\beta}{2} [\varphi(x_1) + \beta \varphi(x_2) + \beta^2 \varphi(x_3)] - \frac{\beta}{2} [\varphi(x^{\text{lf}}) + \beta \varphi(x^{\text{lf}}) + \beta^2 \varphi(x^{\text{lf}})]$$

As $\varphi'(x^{\text{lf}}) = (1 - \beta)u'(c^{\text{lf}})$, we deduce that

$$\begin{aligned} \frac{2U'_0(0)}{\beta(1 - \beta)u'(c^{\text{lf}})} &= x'_1(0) + \beta x'_2(0) + \beta^2 x'_3(0) \\ &= -x'_2(0) \frac{A_L x^{\text{lf}} - 1}{A_H x^{\text{lf}} + 1} - \beta^2 x'_3(0) + \beta x'_2(0) + \beta^2 x'_3(0) \\ &= \frac{x'_2(0)}{A_H x^{\text{lf}} + 1} [\beta(A_H x^{\text{lf}} + 1) - (A_L x^{\text{lf}} - 1)] \\ &= \frac{-x'_3(0)}{A_H x^{\text{lf}} + 1} (A_L x^{\text{lf}} - 1) [\beta(A_H x^{\text{lf}} + 1) - (A_L x^{\text{lf}} - 1)]. \end{aligned}$$

As a direct consequence of the above arguments, we deduce that there is Pareto improvement for ε sufficiently small if the two inequalities in (4.3) of Theorem 4.1 are satisfied.

5 High Interest Rates

In the policy experiment discussed in Section 4, the laissez-faire equilibrium and the equilibrium with ε -tight debt constraints feature low interest rates—specifically, rates that are either zero or negative.²⁰ This comes at no surprise since we know from [Bulow and Rogoff \(1989\)](#) and [Hellwig and Lorenzoni \(2009\)](#) that, in complete markets, unsecured debt is not sustainable at high interest rates.

However, one might wonder whether there is room for Pareto improvement in scenarios where debt remains sustainable despite high interest rates. To address this question, the logical first step is to conduct our policy experiment in a relevant environment that closely resembles the one discussed in previous sections. To this end, we align with the extensive quantitative literature on default and assume that default results in deadweight endowment losses in addition to credit exclusion.²¹ These losses could manifest as output contraction in the case of sovereign default or as seized collateral and legal recourse in the context of consumer and corporate default.

The key modification from the model outlined in Section 2 is the treatment of default. Specifically, if an agent i defaults at s^τ , her endowments for all future events $s^t \succeq s^\tau$ will be reduced to $y^i(s^t) - \ell^i(s^t)$, where $\ell^i(s^t)$ is an exogenously given value within the range $[0, y^i(s^t)]$. Given that this agent is also barred from the credit markets, her outside option is now defined as:

$$V_{\text{def}}^i(s^t) = V_{\ell^i}^i(0, 0|s^t) := \sup\{U(c^i|s^t) : (c^i, a^i) \in B_{\ell^i}^i(0, 0|s^t)\}, \quad (5.1)$$

where $B_{\ell^i}^i(0, 0|s^t)$ denotes the budget set for any agent i who has zero liabilities, is unable to borrow, and has resources equal to $y^i - \ell^i$. The condition analogous to the not-too-tight

²⁰Formally, $q^{\text{lf}} = 1$ and $q_t(\varepsilon) > 1$ for $t \in \{3, 4, \dots\}$.

²¹In consumer credit contexts, endowment loss serves as a simplified proxy for the legal consequences of default, as elaborated in works such as [Chatterjee et al. 2007](#), [Livshits et al. 2007](#), [Livshits 2015](#). In the realm of sovereign debt, it encapsulates the negative impact of default on domestic production (e.g., [Eaton and Gersovitz 1981](#), [Bulow and Rogoff 1989](#), [Cole and Kehoe 2000](#), [Aguiar and Gopinath 2006](#), [Arellano 2008](#)).

condition (2.4) is then articulated as:

$$V^i(D^i, -D^i(s^t)|s^t) = V_{\ell^i}^i(0, 0|s^t), \quad \text{for all } s^t \succeq s^0.$$

To conduct a policy experiment akin to the one in Section 4, the first step involves extending the characterization of not-too-tight debt limits as presented by [Hellwig and Lorenzoni \(2009\)](#). This extension is not merely a procedural necessity; it significantly simplifies the computation of the laissez-faire equilibrium and the equilibrium with ε -tight debt constraints.²²

Moreover, this extended characterization has implications for the restrictions on the model’s primitives—in our case, the properties of the endowment loss—that can sustain equilibria with high interest rates, pre- and post-intervention.

5.1 Characterizing Not-Too-Tight Debt Limits

The following result serves as the counterpart to the characterization provided by [Hellwig and Lorenzoni \(2009\)](#), adapted for our modified context. It posits that not-too-tight debt limits can be broken down into two components: a fundamental component and a credit bubble component. The latter encapsulates the potential for indefinitely rolling over a portion of the debt. The proof of this result is relegated to the online Appendix.

Theorem 5.1. *A process D^i of debt limits is not too tight (for the default punishment described by (5.1)) if, and only if,*

$$D^i(s^t) = \ell^i(s^t) + \sum_{s^{t+1} \succ s^t} q(s^{t+1})D^i(s^{t+1}), \quad \text{for all } s^t \succ s^0. \quad (5.2)$$

²²The extension eliminates the typical complexities associated with the fixed-point process of determining not-too-tight debt limits. In this process, the default value is contingent on prices (since defaulting agents can still save), which depend on equilibrium allocations and, consequently, on the debt limits. It’s worth noting that this extension is of independent interest. Although it serves as the direct analog of [Hellwig and Lorenzoni \(2009\)](#)’s result in an augmented setting, the proof cannot be derived through a simple adaptation of their arguments. Instead, it relies on novel insights with no counterparts in scenarios without output losses.

It is straightforward that a process of debt limits D^i satisfies property (5.2) if, and only if, it can be decomposed into a *fundamental* and a *bubble* component:

$$D^i(s^t) = \underbrace{\text{PV}(\ell^i|s^t)}_{\text{fundamental}} + \underbrace{M^i(s^t)}_{\text{bubble}}, \quad \text{for all } s^t \succeq s^0. \quad (5.3)$$

Here, the fundamental component is simply the present value of endowment losses:

$$\text{PV}(\ell^i|s^t) := \frac{1}{p(s^t)} \sum_{s^\tau \succeq s^t} p(s^\tau) \ell^i(s^\tau),$$

where $p(s^t)$ is the date-0 price of consumption at event s^t .²³ The bubble component of D^i is a nonnegative process satisfying the following *exact rollover* property:

$$M^i(s^t) = \sum_{s^{t+1} \succ s^t} q(s^{t+1}) M^i(s^{t+1}), \quad \text{for all } s^t \succ s^0.$$

Intuitively, the bubble component reflects that credit beyond the fundamental component is sustainable only if agents can roll over their debt.

How can we support equilibria with high interest rates? The following result shows that this is the case when the aggregate losses amount to a *nonnegligible* fraction of aggregate resources. Indeed, since the present value of endowment losses is always finite at equilibrium, under this restriction, we necessarily have that interest rates are high.

Proposition 5.1. *If endowment losses are a nonnegligible fraction of aggregate resources, in the sense that there exists $\varepsilon > 0$ such that:*

$$\sum_{i \in I} \ell^i(s^t) \geq \varepsilon \sum_{i \in I} y^i(s^t), \quad \text{for all } s^t \succ s^0, \quad (5.4)$$

then in any equilibrium with not-too-tight debt limits, the bubble component is necessarily zero. As a consequence, $D^i = \text{PV}(\ell^i)$ for every agent i .

5.2 Laissez-Faire Equilibrium

Consider the same simplified economy as in Section 4 and assume that the endowment loss upon default is time-invariant and identical for both agents, i.e., $\ell^i(s^t) = \ell$ for all agent i and event s^t .

²³Formally, $p(s^t)$ is defined recursively by $p(s^0) = 1$ and $p(s^{t+1}) = q(s^{t+1})p(s^t)$ for all $s^{t+1} \succ s^t$.

The (symmetric Markov) laissez-faire equilibrium is characterized by the asset price $q^{\text{lf}}(\ell)$ and the not-too-tight debt level $d^{\text{lf}}(\ell)$. Our characterization result (Theorem 5.1) implies that $d^{\text{lf}}(\ell) = \ell + q^{\text{lf}}(\ell)d^{\text{lf}}(\ell)$, or, equivalently,

$$d^{\text{lf}}(\ell)(1 - q^{\text{lf}}(\ell)) = \ell.$$

When $\ell > 0$, we must have $q^{\text{lf}}(\ell) < 1$, which is consistent with Proposition 5.1. The net trade defined by $x^{\text{lf}}(\ell) = d^{\text{lf}}(\ell) + q^{\text{lf}}(\ell)d^{\text{lf}}(\ell)$ and the asset price $q^{\text{lf}}(\ell)$ are determined by the following equations:²⁴

$$x^{\text{lf}}(\ell) = \ell \times \frac{1 + q^{\text{lf}}(\ell)}{1 - q^{\text{lf}}(\ell)} \quad \text{and} \quad q^{\text{lf}}(\ell) = \beta \frac{u'(y_L + x^{\text{lf}}(\ell))}{u'(y_H - x^{\text{lf}}(\ell))}. \quad (5.5)$$

Risk aversion implies that the asset price must decrease with ℓ , and the net trade must increase. Indeed, as the mapping $q \mapsto (1 + q)/(1 - q)$ is increasing, if $\ell \mapsto q^{\text{lf}}(\ell)$ were increasing, then $\ell \mapsto x^{\text{lf}}(\ell)$ would also be increasing by the first equation in (5.5). However this would contradict the second equation in (5.5) as the mapping $x \mapsto u'(y_L + x)/u'(y_H - x)$ is decreasing by concavity of u . Therefore, the mapping $\ell \mapsto q^{\text{lf}}(\ell)$ must decrease and the mapping $\ell \mapsto x^{\text{lf}}(\ell)$ must increase. This is confirmed in Figure 5.1 that plots the equilibrium variables as functions of the endowment loss ℓ for CRRA utility for different values of γ when the income dispersion and the discount factor equal to $\zeta = 2$ and $\bar{\beta} = 0.9$. We denote by ℓ^{fb} the level of endowment loss that implements perfect risk-sharing, i.e., $x^{\text{lf}}(\ell^{\text{fb}}) = x^{\text{fb}}$. This level is given by $\ell^{\text{fb}} = x^{\text{fb}}(1 - \beta)/(1 + \beta)$.

²⁴The first equation corresponds to our characterization of not-too-tight debt limits. The second equation is the FOC of the high-income agent's saving decision.

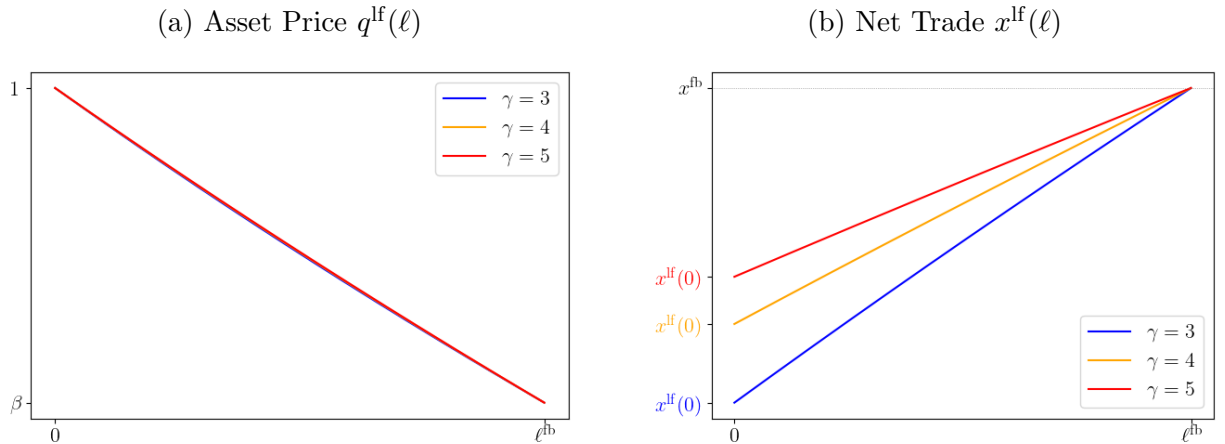


Figure 5.1: Laissez-faire equilibrium variables as functions of the endowment loss ℓ .

5.3 Tightening Debt Constraints

For any arbitrary $\ell \in (0, \ell^{\text{fb}})$, the laissez-faire interest rate is positive. We perform the same policy experiment: we tighten the high-income borrowing constraint at $t = 2$ by imposing the too-tight debt limit

$$d_3(\varepsilon) := (1 - \varepsilon)[\ell + q_4(\varepsilon)d^{\text{lf}}(\ell)].$$

All the arguments of Section 4 remain qualitatively valid. To illustrate our policy intervention, we set the following arbitrary value $\ell^* = \ell^{\text{fb}}/2$ for the endowment loss and plot the equilibrium prices for each period.

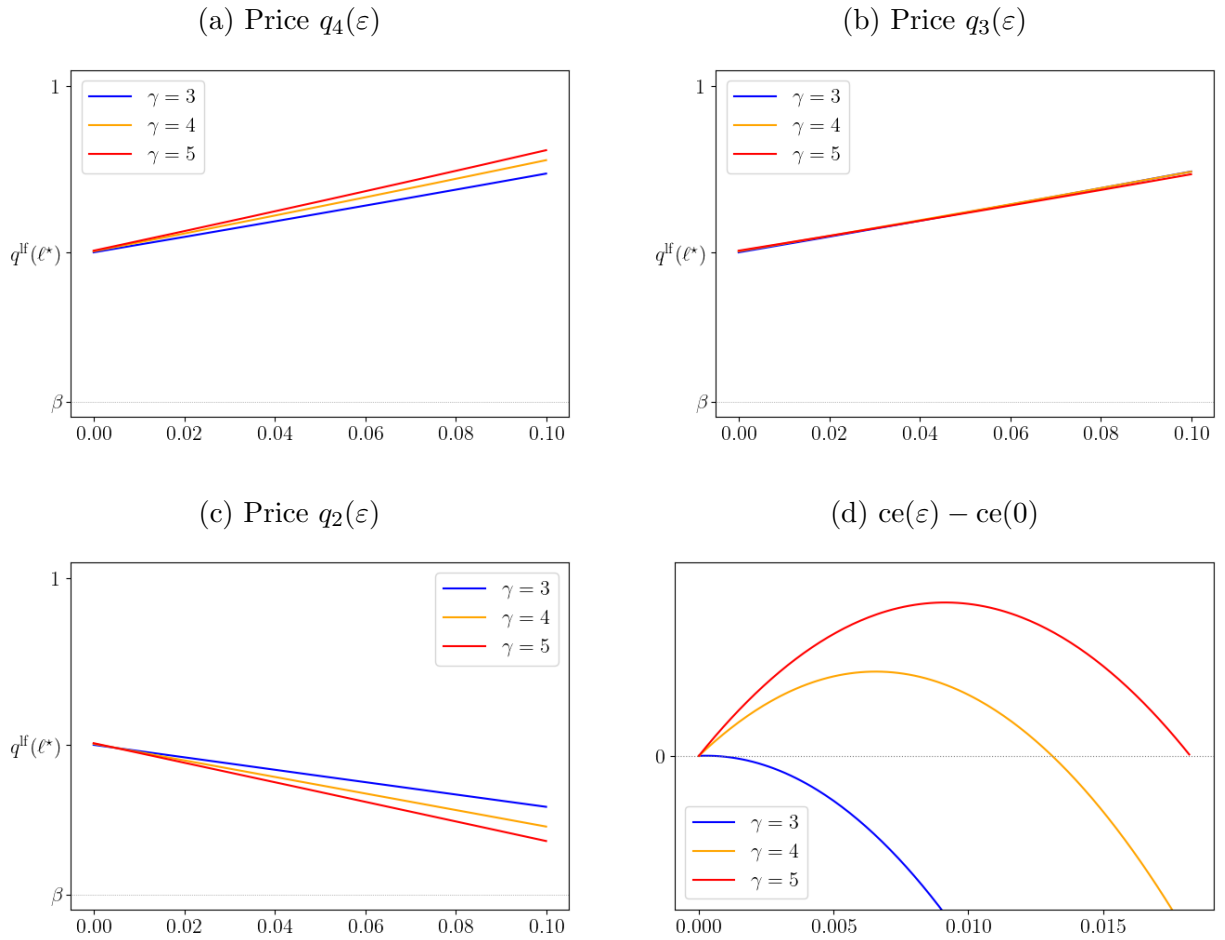


Figure 5.2: Equilibrium variables as functions of the tightening coefficient ε when the endowment loss is $\ell^* = \ell^{\text{fb}}/2$.

6 Conclusion

This study presents a groundbreaking approach to the complex dynamics of unsecured debt markets, challenging traditional models often focused on maximizing permissible debt limits. Utilizing a dynamic general equilibrium framework, we demonstrate that imposing tighter debt constraints can paradoxically lead to Pareto improvements, thereby enhancing economic efficiency and societal welfare.

One of the most salient findings of this research is that debt bubbles, commonly con-

sidered mechanisms to ensure efficient liquidity provision, are not necessarily the optimal solution. Our results indicate that a more nuanced approach to debt management could yield superior outcomes, particularly in markets lacking collateral.

Our research has far-reaching implications, especially for policymakers and financial practitioners grappling with debt management challenges in environments devoid of collateral. It suggests that regulatory or prudential policies aimed at reducing financial leverage could be beneficial—contrary to the prevailing notion that such measures are overly restrictive and stifle economic activity.

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