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| Pierre Gaillard. Another rational solutions to the Gardner equation. 2024. hal-04369040

**HAL Id: hal-04369040**

**<https://hal.science/hal-04369040v1>**

Preprint submitted on 2 Jan 2024

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# Another rational solutions to the Gardner equation

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## Abstract

Rational solutions to the Gardner (G) equation are constructed in terms of a quotient of determinants involving certain particular polynomials. This gives a very efficient method to construct solutions to this equation. We construct very easily explicit expressions of these rational solutions for the first orders for  $n = 1$  until 8.

**Key Words :** Gardner equation, rational solutions.

**PACS numbers :**

33Q55, 37K10, 47.10A-, 47.35.Fg, 47.54.Bd

## 1 Introduction

We consider the Gardner equation in the following normalization

$$u_t + 6u(u - 2)u_x + u_{xxx} + 6u_x = 0, \quad (1)$$

where the subscripts  $x$  and  $t$  stand for partial derivatives.

This equation was introduced by Gardner [1] in 1968. He considered this equation as an auxiliary mathematical tool in the derivation of the infinite set of local conservation laws of the Korteweg de Vries equation. This equation is used to describe nonlinear wave effects in several physical contexts: for example, in plasma physics [2, 3], fluid flows [4], quantum fluid dynamics [5], in ocean and atmosphere [6].

A lot of research has been done to solve this equation. We can quote many methods as the Hirota method [8], the series expansion method [9], the mapping method [10] or the method of leading-order analysis [11].

Here, we used particular polynomials to construct rational solutions to the Gardner equation. We obtain rational solutions as a quotient of polynomials in  $x$  and  $t$ . We give explicit solutions for the first orders.

## 2 Rational solutions to the Gardner equation

We consider the following polynomials  $p_n(x, t)$  defined by

$$\begin{cases} p_n(x, t) = \sum_{l=0}^n \frac{x^k t^{\binom{n-k}{3}}}{k! \binom{n-k}{3}!} \left( 1 - \left[ \frac{1}{2}(n-k+1) - 3 \left\lfloor \frac{n-k}{3} \right\rfloor \right] \right), & k \geq 0, \\ p_n(x, t) = 0, & n < 0, \end{cases} \quad (2)$$

with  $[x]$  denoting the largest integer less or equal to  $x$ .

We denote  $A_n(x, t)$  the determinant defined by

$$A_n(x, t) = \det(p_{n+1-2i+j}(x, t))_{\{1 \leq i \leq n, 1 \leq j \leq n\}} \quad (3)$$

With these notations we have the following result

**Theorem 2.1** *The function  $v_n(x, t)$  defined by*

$$v_n(x, t) = 1 - \partial_X \left( \ln \frac{A_{n+1}(X, T)}{A_n(X, T)} \right)_{|X=ix, T=4it} \quad (4)$$

is a rational to the Gardner equation (1)

$$u_t + 6u(u-2)u_x + u_{xxx} + 6u_x = 0,$$

## 3 Explicit construction of rational solutions to the Gardner equation for the first orders

The solutions we present in the following are singular. There is therefore no interest in constructing figures in the plan  $(x, t)$  of the coordinates.

### 3.1 First order rational solution

The function  $v_1(x, t)$  defined by

$$v_1(x, t) = -\frac{-x^4 - 12xt - 2ix^3 + 12it}{x(x^3 + 12t)} \quad (5)$$

is a rational to the Gardner equation (1).

### 3.2 Second order rational solution

The function  $v_2(x, t)$  defined by

$$v_2(x, t) = \frac{-i(-72ix^6t + 8640it^3 - ix^9 + 72x^5t + 4320x^2t^2 + 3x^8)}{(-60tx^3 + 720t^2 - x^6)(x^3 + 12t)} \quad (6)$$

is a rational to the Gardner equation (1).

### 3.3 Third order rational solution

The function  $v_3(x, t)$  defined by

$$v_3(x, t) = \frac{n(x, t)}{d(x, t)} \quad (7)$$

with

$$n(x, t) = -x^{16} - 4ix^{15} - 240tx^{13} - 600ix^{12}t - 10080t^2x^{10} - 36000ix^9t^2 - 172800t^3x^7 + 2419200it^3x^6 - 18144000t^4x^4 + 36288000it^4x^3 + 217728000t^5x + 217728000it^5$$

and

$$d(x, t) = (x^9 + 180tx^6 + 302400t^3)(-x^6 - 60tx^3 + 720t^2)x$$

is a rational to the Gardner equation (1).

### 3.4 Rational solution of order four

The function  $v_4(x, t)$  defined by

$$v_4(x, t) = \frac{n(x, t)}{d(x, t)} \quad (8)$$

with

$$n(x, t) = -i(-ix^{25} + 5x^{24} - 600ix^{22}t + 2280tx^{21} - 100800ix^{19}t^2 + 352800t^2x^{18} - 6955200ix^{16}t^3 + 4838400t^3x^{15} - 254016000ix^{13}t^4 + 2794176000t^4x^{12} + 39626496000it^5x^{10} + 259096320000t^5x^9 - 365783040000ix^7t^6 + 5120962560000t^6x^6 + 76814438400000it^7x^4 - 153628876800000t^7x^3 + 460886630400000it^8x + 460886630400000t^8)$$

and

$$d(x, t) = (-x^{15} - 420tx^{12} - 25200t^2x^9 - 2116800t^3x^6 + 254016000t^4x^3 + 1524096000t^5)(x^9 + 180tx^6 + 302400t^3)x$$

is a rational to the Gardner equation (1).

### 3.5 Rational solution of order five

The function  $v_5(x, t)$  defined by

$$v_5(x, t) = \frac{n(x, t)}{d(x, t)} \quad (9)$$

with

$$\begin{aligned} n(x, t) = & -i(ix^{36} - 6x^{35} + 1260itx^{33} - 6300tx^{32} + 544320it^2x^{30} - 2419200t^2x^{29} + \\ & 110073600it^3x^{27} - 381024000t^3x^{26} + 11430720000it^4x^{24} - 50295168000t^4x^{23} + \\ & 329204736000it^5x^{21} + 192036096000t^5x^{20} + 119062379520000it^6x^{18} + 1371137725440000t^6x^{17} + \\ & 7189831434240000it^7x^{15} + 129970029772800000t^7x^{14} + 1393721170329600000it^8x^{12} - \\ & 116143430860800000t^8x^{11} - 59620294508544000000it^9x^9 + 319394434867200000000t^9x^8 - \\ & 4292661204615168000000it^{10}x^6 + 2146330602307584000000t^{10}x^5 \\ & + 12877983613845504000000t^{11}x^2 + 12877983613845504000000it^{12}) \end{aligned}$$

and

$$\begin{aligned} d(x, t) = & (-x^{21} - 840tx^{18} - 166320t^2x^{15} - 16934400t^3x^{12} + 1397088000t^4x^9 - \\ & 352066176000t^5x^6 - 14082647040000t^6x^3 + 84495882240000t^7)(-x^{15} - 420tx^{12} - \\ & 25200t^2x^9 - 2116800t^3x^6 + 254016000t^4x^3 + 1524096000t^5) \end{aligned}$$

is a rational to the Gardner equation (1).

### 3.6 Rational solution of order six

The function  $v_6(x, t)$  defined by

$$v_6(x, t) = \frac{n(x, t)}{d(x, t)} \quad (10)$$

with

$$\begin{aligned} n(x, t) = & -i(ix^{49} - 7x^{48} + 2352itx^{46} - 14448tx^{45} + 2116800it^2x^{43} - 11733120t^2x^{42} + \\ & 979655040it^3x^{40} - 4792435200t^3x^{39} + 259756761600it^4x^{37} - 1190674598400t^4x^{36} + \\ & 40391592192000it^5x^{34} - 148315878144000t^5x^{33} + 4855696699392000it^6x^{31} - \\ & 8843902341120000t^6x^{30} + 61513002270720000it^7x^{28} - 7215948343296000000t^7x^{27} - \\ & 86555891849011200000ix^{25}t^8 - 2329982402356224000000t^8x^{24} \\ & - 29124513867423744000000ix^{22}t^9 - 1965877005200080896000000t^9x^{21} \\ & + 125918062002044928000000it^{10}x^{19} - 19426796003252994048000000t^{10}x^{18} - \\ & 26702499023308652544000000ix^{16}t^{11} + 573430854357312602112000000t^{11}x^{15} - \\ & 4809282980590603468800000000ix^{13}t^{12} + 89978213950266259537920000000t^{12}x^{12} - \\ & 13953068593089999903129600000000ix^{10}t^{13} + 6295896804199146297753600000000t^{13}x^9 - \\ & 12251474862225365768601600000000ix^7t^{14} - 171520648071155120760422400000000t^{14}x^6 - \\ & 8576032403557756038021120000000000ix^4t^{15} - 1715206480711551207604224000000000t^{15}x^3 + \\ & 51456194421346536228126720000000000it^{16}x - 51456194421346536228126720000000000t^{16}) \end{aligned}$$

and

$$\begin{aligned} d(x, t) = & (-x^{27} - 1512tx^{24} - 680400t^2x^{21} - 139708800t^3x^{18} - 5029516800t^4x^{15} - \\ & 3168595584000t^5x^{12} + 604145558016000t^6x^9 + 108746200442880000t^7x^6 \\ & + 60897872248012800000t^9)(-x^{21} - 840tx^{18} - 166320t^2x^{15} - 16934400t^3x^{12} + \\ & 1397088000t^4x^9 - 352066176000t^5x^6 - 14082647040000t^6x^3 + 84495882240000t^7)x \end{aligned}$$

is a rational to the Gardner equation (1).

### 3.7 Rational solution of order seven

The function  $v_7(x, t)$  defined by

$$v_7(x, t) = \frac{n(x, t)}{d(x, t)} \quad (11)$$

with

$$\begin{aligned} n(x, t) = & x^{64} + 8ix^{63} + 4032tx^{61} + 29232itx^{60} + 6652800t^2x^{58} + 44331840it^2x^{57} + \\ & 6008325120t^3x^{55} + 36696844800it^3x^{54} + 3329540121600t^4x^{52} + 18820451865600it^4x^{51} + \\ & 1197306651340800t^5x^{49} + 6162707171174400it^5x^{48} + 293930755795353600t^6x^{46} + \\ & 1373247357176217600it^6x^{45} + 46643754897653760000t^7x^{43} + 260442131005292544000it^7x^{42} + \\ & 5156738132927496192000t^8x^{40} + 7389254129539645440000it^8x^{39} \\ & + 1123297792337944903680000t^9x^{37} - 28613498613600193413120000it^9x^{36} \\ & + 621308836469927647641600000t^{10}x^{34} - 9639939765218166551347200000it^{10}x^{33} + \\ & 96034709039084017916313600000t^{11}x^{31} - 1768604112551541393063936000000it^{11}x^{30} + \\ & 12747529948908855776772096000000t^{12}x^{28} - 103264051547071763510722560000000it^{12}x^{27} + \\ & 300223914988325881595822080000000t^{13}x^{25} - 3265018050783059977332326400000000it^{13}x^{24} - \\ & 498353242970741203369407283200000000t^{14}x^{22} \\ & - 2066613083889788928871869972480000000it^{14}x^{21} \\ & - 104928442540121429745671444889600000000t^{15}x^{19} \\ & - 464599008554226302534480717414400000000it^{15}x^{18} \\ & - 5587991432946635075558051099443200000000t^{16}x^{16} \\ & + 1181438340409670179520687741337600000000it^{16}x^{15} \\ & - 211387398127111390864912290938880000000000t^{17}x^{13} \\ & - 853337549281549667333619616579584000000000it^{17}x^{12} \\ & + 12515617389462728454226421043167232000000000t^{18}x^{10} \\ & - 77427124528032133657502435267051520000000000it^{18}x^9 \\ & - 76366478986552241415618840263393280000000000t^{19}x^7 \\ & - 1069130705811731379818663763687505920000000000it^{19}x^6 \\ & + 8018480293587985348639978227656294400000000000t^{20}x^4 \\ & + 16036960587175970697279956455312588800000000000it^{20}x^3 \\ & + 32073921174351941394559912910625177600000000000t^{21}x \\ & - 32073921174351941394559912910625177600000000000it^{21} \end{aligned}$$

and

$$\begin{aligned} d(x, t) = & (-x^{36} - 2520tx^{33} - 2162160t^2x^{30} - 884822400t^3x^{27} - 163459296000t^4x^{24} - \\ & 30207277900800t^5x^{21} + 4833164464128000t^6x^{18} - 891718843631616000t^7x^{15} - \\ & 831255956185374720000t^8x^{12} - 34833582925863321600000t^9x^9 \\ & - 2194515724329389260800000t^{10}x^6 + 131670943459763355648000000t^{11}x^3 + \\ & 526683773839053422592000000t^{12})(-x^{27} - 1512tx^{24} - 680400t^2x^{21} - 139708800t^3x^{18} - \\ & 5029516800t^4x^{15} - 3168595584000t^5x^{12} + 604145558016000t^6x^9 + 108746200442880000t^7x^6 + \\ & 60897872248012800000t^9)x \end{aligned}$$

is a rational to the Gardner equation (1).

### 3.8 Rational solution of order eight

The function  $v_8(x, t)$  defined by

$$v_8(x, t) = \frac{n(x, t)}{d(x, t)} \quad (12)$$

with

$$\begin{aligned} n(x, t) = & -i(-ix^{81} + 9x^{80} - 6480itx^{78} + 54000tx^{77} - 17962560it^2x^{75} + 139708800t^2x^{74} - \\ & 28440720000it^3x^{72} + 206689190400t^3x^{71} - 28894574016000it^4x^{69} + 196729549632000t^4x^{68} - \\ & 20027877904742400it^5x^{66} + 127548405221529600t^5x^{65} - 9845156013428736000it^6x^{63} + \\ & 58650450772193280000t^6x^{62} - 3496233842718769152000it^7x^{60} + 19742220213202206720000t^7x^{59} - \\ & 917522577057504337920000it^8x^{57} + 4584204344380840058880000t^8x^{56} \\ & - 186047506160225456947200000it^9x^{54} + 705970885405282482585600000t^9x^{53} - \\ & 25933156618443791880683520000it^{10}x^{51} + 379336840256361840468295680000t^{10}x^{50} + \\ & 6395652736671085473890304000000it^{11}x^{48} + 297650067947402651243642880000000t^{11}x^{47} + \\ & 5207428862069036677958467584000000it^{12}x^{45} + 129905635334268499051254644736000000t^{12}x^{44} + \\ & 1702586892784983334684867952640000000it^{13}x^{42} \\ & + 29620639540599344304800019775488000000t^{13}x^{41} \\ & + 396994691532759366938073508085760000000it^{14}x^{39} \\ & + 4232360525213696990146612942602240000000t^{14}x^{38} \\ & - 43491550690438395253575322089553920000000it^{15}x^{36} \\ & + 583120649740812817456115098224427008000000t^{15}x^{35} \\ & - 202466214174573975549688586510008320000000it^{16}x^{33} \\ & + 28159381522332939098979530493552230400000000t^{16}x^{32} \\ & + 5518612694466199647008064688942649180160000000it^{17}x^{30} \\ & - 31710925844235863073697671356573181542400000000t^{17}x^{29} \\ & + 1253795797863202772049566358144828624076800000000it^{18}x^{27} \\ & - 3779312133537331858253773128621109542912000000000t^{18}x^{26} \\ & + 127205858675790678540997355173101824901120000000000it^{19}x^{24} \\ & - 454698326162344677609910772685294048116736000000000t^{19}x^{23} \\ & + 1234128242496442531104836094950737862197248000000000it^{20}x^{21} \\ & - 28547550061967276275702055938476943173746688000000000t^{20}x^{20} \\ & + 446177931156674484955283193242186295080386560000000000it^{21}x^{18} \\ & + 3908572835159623913132810908575137703584071680000000000t^{21}x^{17} \\ & + 13662139312759241693421665342506948154118635520000000000it^{22}x^{15} \\ & + 370790999888692634519688589966263523867203993600000000000t^{22}x^{14} \\ & + 2648353159088714543647892051009039180644535500800000000000it^{23}x^{12} \\ & - 1471307310604841413137717806116132878135853056000000000000t^{23}x^{11} \\ & - 42483998593714795804351601651603336856172756992000000000000it^{24}x^9 \\ & + 333802846093473395605619727262597646727071662080000000000000t^{24}x^8 \\ & - 3058847898747465297913315318915440253644438503424000000000000it^{25}x^6 \\ & + 1019615966249155099304438439638480084548146167808000000000000t^{25}x^5 \\ & + 6117695797494930595826630637830880507288877006848000000000000t^{26}x^2 \\ & + 4078463864996620397217753758553920338192584671232000000000000it^{27}) \end{aligned}$$

and

$$d(x, t) = (x^{45} + 3960tx^{42} + 5821200t^2x^{39} + 4324320000t^3x^{36} + 1743565824000t^4x^{33} +$$

$$\begin{aligned}
& 455986052121600 t^5 x^{30} + 33630769396224000 t^6 x^{27} + 18378107874846720000 t^7 x^{24} + \\
& 3172344159319695360000 t^8 x^{21} - 6261336530923932057600000 t^9 x^{18} \\
& - 1088291697919577411420160000 t^{10} x^{15} - 98937546915666185433907200000 t^{11} x^{12} + \\
& 5121473016810955481284608000000 t^{12} x^9 - 645305600118180390641860608000000 t^{13} x^6 - \\
& 19359168003545411719255818240000000 t^{14} x^3 + 77436672014181646877023272960000000 t^{15} \\
& (-x^{36} - 2520 t x^{33} - 2162160 t^2 x^{30} - 884822400 t^3 x^{27} - 163459296000 t^4 x^{24} - \\
& 30207277900800 t^5 x^{21} + 4833164464128000 t^6 x^{18} - 891718843631616000 t^7 x^{15} - \\
& 831255956185374720000 t^8 x^{12} - 34833582925863321600000 t^9 x^9 \\
& - 2194515724329389260800000 t^{10} x^6 + 131670943459763355648000000 t^{11} x^3 + \\
& 526683773839053422592000000 t^{12})
\end{aligned}$$

is a rational to the Gardner equation (1).

### 3.9 Rational solution of order nine

The function  $v_9(x, t)$  defined by

$$v_9(x, t) = \frac{n(x, t)}{d(x, t)} \quad (13)$$

with

$$\begin{aligned}
n(x, t) = & -i(-ix^{100} + 10x^{99} - 9900ix^{97}t + 93060tx^{96} - 43243200ix^{94}t^2 + 383961600t^2x^{93} - \\
& 111198700800ix^{91}t^3 + 934226092800t^3x^{90} - 189254729664000ix^{88}t^4 + 1506596547456000t^4x^{87} - \\
& 227051874137088000ix^{85}t^5 + 1713219123103488000t^5x^{84} - 199652557934269440000ix^{82}t^6 + \\
& 1428145956200291328000t^6x^{81} - 131959599374218936320000ix^{79}t^7 \\
& + 895581632952719032320000t^7x^{78} - 66760303356563349012480000ix^{76}t^8 \\
& + 428022377616371217039360000t^8x^{75} - 26201771314573067845632000000ix^{73}t^9 + \\
& 158789776023912561028300800000t^9x^{72} - 7982051135095428620156928000000ix^{70}t^{10} + \\
& 48867778365532304849672601600000t^{10}x^{69} - 1845794510242697463355146240000000ix^{67}t^{11} + \\
& 10500563590228982713863477657600000t^{11}x^{66} - 4369386748577714555583890718720000000ix^{64}t^{12} - \\
& 3509125889298278795399059734528000000t^{12}x^{63} \\
& - 229767750578469269555278604402688000000ix^{61}t^{13} \\
& - 6153349357243245450710701786005504000000t^{13}x^{60} \\
& - 144981592655468437732757578564239360000000ix^{58}t^{14} \\
& - 3665293540829092709903367569229742080000000t^{14}x^{57} \\
& - 65471826215870206689389630766270382080000000ix^{55}t^{15} \\
& - 1304243412541510083295718221838737735680000000t^{15}x^{54} \\
& - 15948204848317269786001172326369984512000000000ix^{52}t^{16} \\
& - 321470231876680131350631334233741176340480000000t^{16}x^{51} \\
& - 3263409627924212769117367114275659710464000000000ix^{49}t^{17} \\
& - 50226666131933164197071036030534503366656000000000t^{17}x^{48} \\
& - 1109034109153602138195454830018333524361216000000000ix^{46}t^{18} \\
& - 1648734229738630070957192326433181511188480000000000t^{18}x^{45} \\
& + 27768030541570534401038610181385410880471040000000000it^{19}x^{43} \\
& - 1744391947969980802122124052001220299533058048000000000t^{19}x^{42} \\
& + 1763536190793961642216834855538817392014818017280000000000it^{20}x^{40} \\
& - 984738559457980653135258698718009747572885815296000000000t^{20}x^{39}
\end{aligned}$$



$$\begin{aligned}
&+ 60347123414229766954512942201852809682107215380480000000000 it^{21}x^{37} \\
&- 2763086941422691151555415080335470118499484287631360000000000 t^{21}x^{36} \\
&+ 7917139362968738240500030294776989889075583679201280000000000 it^{22}x^{34} \\
&- 39520943879481443272459668542895966001157408009748480000000000 t^{22}x^{33} \\
&+ 861918461622108393759032387159223091252121997064273920000000000 it^{23}x^{31} \\
&- 201812703802412530614295675464043415116483567382691840000000000 t^{23}x^{30} \\
&+ 37114684499860893874540947963358435349516821785162547200000000000 it^{24}x^{28} \\
&- 111009714025675678910786955318908309101987512458202316800000000000 t^{24}x^{27} \\
&- 7902927875606052270399741122952866937188248122353831116800000000000 it^{25}x^{25} \\
&- 128446740342524481121949490167970826891662327175606330982400000000000 t^{25}x^{24} \\
&- 140297230639136534260525403388901069495050322358460796108800000000000 ix^{22}t^{26} \\
&- 845178584186676456943949690755485022602870712303714985574400000000000 t^{26}x^{21} \\
&- 1814073251355131043450164895241263140377527325959580444262400000000000 ix^{19}t^{27} \\
&- 6967217887900169109541511035767629813488068102366890809082400000000000 t^{27}x^{18} \\
&+ 203415316089806637919266623219086948457478531340576986916454400000000000 it^{28}x^{16} \\
&+ 491221933119161257564277504087443703234881277678491513179340800000000000 t^{28}x^{15} \\
&- 60739601253277288355525834226779370109343487463001759466651648000000000000 ix^{13}t^{29} \\
&+ 90140977115035873726599882809767598370180035284697126453379072000000000000 t^{29}x^{12} \\
&- 13516110758478004753195200305107052180199062273973580301724549120000000000000 ix^{10}t^{30} \\
&+ 6536479793034444921627187032797672775670038312823288834440560640000000000000 t^{30}x^9 \\
&- 79767211033640683789348723112107193194617416698860473911817011200000000000000 ix^7t^{31} \\
&- 1116740954470969573050882123569500704724643833784046634765438156800000000000000 t^{31}x^6 \\
&- 4187778579266135898940807963385627642717414376690174880370393088000000000000000 ix^4t^{32} \\
&- 8375557158532271797881615926771255285434828753380349760740786176000000000000000 t^{32}x^3 \\
&+ 16751114317064543595763231853542510570869657506760699521481572352000000000000000 it^{33}x \\
&- 16751114317064543595763231853542510570869657506760699521481572352000000000000000 t^{33})
\end{aligned}$$

and

$$\begin{aligned}
d(x, t) = &(-x^{54} - 5940 tx^{51} - 13899600 t^2x^{48} - 17254036800 t^3x^{45} - 12586365792000 t^4x^{42} - \\
&5851581261926400 t^5x^{39} - 1623511727405568000 t^6x^{36} - 400077271429355520000 t^7x^{33} - \\
&28551097433877258240000 t^8x^{30} + 50926698237612176179200000 t^9x^{27} \\
&- 42318665025272343181393920000 t^{10}x^{24} - 24094202601815176923316224000000 t^{11}x^{21} - \\
&4866679734224610446090698752000000 t^{12}x^{18} - 89328732359216685504566132736000000 t^{13}x^{15} - \\
&56277101386306511867876663623680000000 t^{14}x^{12} \\
&+ 4292066932395643305123393545699328000000 t^{15}x^9 \\
&+ 772572047831215794922210838225879040000000 t^{16}x^6 \\
&+ 216320173392740422578219034703246131200000000 t^{18})(x^{45} + 3960 tx^{42} + 5821200 t^2x^{39} + \\
&4324320000 t^3x^{36} + 1743565824000 t^4x^{33} + 455986052121600 t^5x^{30} + 33630769396224000 t^6x^{27} + \\
&18378107874846720000 t^7x^{24} + 3172344159319695360000 t^8x^{21} - 6261336530923932057600000 t^9x^{18} - \\
&1088291697919577411420160000 t^{10}x^{15} - 98937546915666185433907200000 t^{11}x^{12} + \\
&5121473016810955481284608000000 t^{12}x^9 - 645305600118180390641860608000000 t^{13}x^6 - \\
&19359168003545411719255818240000000 t^{14}x^3 + 77436672014181646877023272960000000 t^{15})x
\end{aligned}$$

is a rational to the Gardner equation (1).

### 3.10 Rational solution of order ten

The function  $v_{10}(x, t)$  defined by

$$v_{10}(x, t) = \frac{n(x, t)}{d(x, t)} \quad (14)$$

with

$$\begin{aligned} n(x, t) = & -i(-ix^{121} + 11x^{120} - 14520ix^{118}t + 151800tx^{117} - 95135040ix^{115}t^2 + 948261600t^2x^{114} - \\ & 375171825600ix^{112}t^3 + 3570850483200t^3x^{111} - 1001486566080000ix^{109}t^4 + 9111786780096000t^4x^{108} - \\ & 1928677978260633600ix^{106}t^5 + 16783754184791193600t^5x^{105} - 2788145214059132928000ix^{103}t^6 + \\ & 23214265448682147840000t^6x^{102} - 3107387740090288951296000ix^{100}t^7 + 24758895105773203415040000t^7x^{99} - \\ & 2720957040436266106675200000ix^{97}t^8 + 20739568173400178020638720000t^8x^{96} \\ & - 1897998004600345592443699200000ix^{94}t^9 + 13838512826996381618660966400000t^9x^{93} \\ & - 1065031427670194137372455075840000ix^{91}t^{10} + 7441832235420948554463447613440000t^{10}x^{90} - \\ & 484359826252919742513781250457600000ix^{88}t^{11} \\ & + 3216285223633520093028577443840000000t^{11}x^{87} \\ & - 180973477671370782253997564834611200000ix^{85}t^{12} \\ & + 1094007551086362182820948800962560000000t^{12}x^{84} \\ & - 55662624659826717571311231389663232000000ix^{82}t^{13} \\ & + 366409302919024594307060726369353728000000t^{13}x^{81} \\ & - 10629210508837371167598914804499087360000000ix^{79}t^{14} \\ & + 287719612082605280136521050805250293760000000t^{14}x^{78} \\ & + 4280030807361509169541614389644146769920000000it^{15}x^{76} \\ & + 30453897698296387702699431050813489807360000000t^{15}x^{75} \\ & + 6885670109355945389014278608796839313408000000000it^{16}x^{73} \\ & + 228041011956866466611552032321683792592896000000000t^{16}x^{72} \\ & + 4453721520274708640987473009279000243077120000000000it^{17}x^{70} \\ & + 117762805264914466257694778298876385933918208000000000t^{17}x^{69} \\ & + 1928767694961190051291693412300125238030499840000000000it^{18}x^{67} \\ & + 42939910195222368442373794245841509282389950464000000000t^{18}x^{66} \\ & + 654735071078952292481291250564426986112984023040000000000it^{19}x^{64} \\ & + 11100640725265289005976333178685141575591650131968000000000t^{19}x^{63} \\ & + 1189838851002926830068328643726287392805959373946880000000000it^{20}x^{61} \\ & + 2548058689427082387186572761563839262072167014072320000000000t^{20}x^{60} \\ & - 7153367620935353165234889584513031133610669475102720000000000ix^{58}t^{21} \\ & + 501342812165200991524948146064415704363708955523809280000000000t^{21}x^{57} \\ & + 112209811111614321838747913725512417362386128599619993600000000000it^{22}x^{55} \\ & - 53280631666435839568505888704723585563633395222917939200000000000t^{22}x^{54} \\ & + 18666731499151103582697435263135280572769467169281723596800000000000it^{23}x^{52} \\ & - 111900720524349972607748089158539708346975177833478881280000000000000t^{23}x^{51} \\ & + 8211604920987036524942436185081467249913411096884077920256000000000000it^{24}x^{49} \\ & - 47235626798516616642910711000941244534600220241264883689062400000000000t^{24}x^{48} \\ & + 2074576890338308833679718042344039076057947005326331519455723520000000000it^{25}x^{46} \\ & - 8871281053769860654870984551097381234761018804136244822535045120000000000t^{25}x^{45} \\ & + 310982628871002671709704390044044172790555050597378687653799526400000000000it^{26}x^{43} \\ & - 131022646464880460717563747042605482741701249383376515624232550400000000000t^{26}x^{42} \end{aligned}$$

$$\begin{aligned}
& + 275930186232651073615404657875346905303154392267848934894157692928000000000000 \, it^{27}x^{40} \\
& - 27039621086618360278262036106275044892571367413617428368798724915200000000000000 \, t^{27}x^{39} \\
& + 34612681307949155341638944237023314224694049592402208339361215283200000000000000 \, it^{28}x^{37} \\
& + 1694434980827419820710309790828491267205454289096273137837241112985600000000000000 \, t^{28}x^{36} \\
& + 8678751568702817160964557066872837900711573699983466823073470087168000000000000000 \, it^{29}x^{34} \\
& + 121522198287550125954273147621213608370048486986445078771138702606336000000000000000 \, t^{29}x^{33} \\
& + 14271641456187722670288912029985750088687753762567256258118473598841651200000000000000 \, it^{30}x^{31} \\
& + 246871757359120966531869566383627805947941153995961672091633926202510540800000000000000 \, t^{30}x^{30} \\
& + 720658962466513411447262837580091939472208367744484447636696763765620736000000000000000 \, it^{31}x^{28} \\
& + 1413883752438650284231654702297033818274673941794291466132393298271141888000000000000000 \, t^{31}x^{27} \\
& + 27799878737936339943148580809710650916551459703971532219498795609657389875200000000000000 \, it^{32}x^{25} \\
& + 2186347006955808854323160657473222559728478424077556347724728600882262835200000000000000 \, t^{32}x^{24} \\
& - 90840229532843846042412137018193738079990797936534294316472658867592888320000000000000000 \, ix^{22}t^{33} \\
& + 401214645216852730269554616818765276029818689646851534038793223244097362329600000000000000 \, t^{33}x^{21} \\
& - 306852163904191313442393275460829953540472987251272901063075672251181469532160000000000000000 \, ix^{19}t^{34} \\
& + 157627886390273508530607586958466768093765832388950910966517947023627068112896000000000000000 \, t^{34}x^{18} \\
& - 1319763410247538789741875435615067271281418483444308067430020702793061740262195200000000000000 \, ix^{16}t^{35} \\
& - 5918012425651081958909283390487510842459526487065474871915514450141068056854528000000000000000 \, t^{35}x^{15} \\
& - 4477548163610674123084866747367973396012649914513256688701752774725820931165388800000000000000 \, ix^{13}t^{36} \\
& + 10924830585531309938447683246683672598696811228092701642565578266175477211201536000000000000000 \, t^{36}x^{12} \\
& + 1639196163250654676807661020092335796535400683259722197534875541865868724581433344000000000000000 \, it^{37}x^{10} \\
& + 98559262980260882466283415765045506753710800575742790358109605365352866351415296000000000000000 \, t^{37}x^9 \\
& - 746975466797766688165516414219292261712334488574050621661462272242674355505463296000000000000000 \, ix^7t^{38} \\
& + 10457656535168733634317229799070091663972682840036708703260471811397440977076486144000000000000000 \, t^{38}x^6 \\
& + 5228828267584366817158614899535045831986341420018354351630235905698720488538243072000000000000000 \, it^{39}x^4 \\
& - 10457656535168733634317229799070091663972682840036708703260471811397440977076486144000000000000000 \, t^{39}x^3 \\
& + 15686484802753100451475844698605137495959024260055063054890707717096161465614729216000000000000000 \, it^{40}x \\
& + 15686484802753100451475844698605137495959024260055063054890707717096161465614729216000000000000000 \, t^{40}) \\
& \text{and} \\
& d(x, t) = (x^{66} + 8580tx^{63} + 30270240t^2x^{60} + 58853995200t^3x^{57} + 70523605152000t^4x^{54} + 55594337313715200t^5x^{51} + \\
& 29373557030737920000t^6x^{48} + 11136392862316001280000t^7x^{45} + 2669301014056143667200000t^8x^{42} \\
& + 278616413032517777817600000t^9x^{39} + 593719639698671917622231040000t^{10}x^{36} \\
& - 143134362516421637127025459200000t^{11}x^{33} - 437557180951607550386188949913600000t^{12}x^{30} \\
& - 208542888399944642126271001657344000000t^{13}x^{27} - 30762135562543449987207439130296320000000t^{14}x^{24} \\
& - 5486487844439459470606303635273940992000000t^{15}x^{21} \\
& + 384097069780086119375492488404632862720000000t^{16}x^{18} \\
& + 11977957029575169684473956835854028636160000000t^{17}x^{15} \\
& - 37241464731120798410643610795476150701260800000000t^{18}x^{12} \\
& - 1198993498660474485415843079268988266479616000000000t^{19}x^9 \\
& - 60429272332487914064958491195157008630572646400000000t^{20}x^6 \\
& + 2417170893299516562598339647806280345222905856000000000t^{21}x^3 \\
& + 72515126798985496877950189434188410356687175680000000000t^{22}) \\
& (-x^{54} - 5940tx^{51} - 13899600t^2x^{48} - 17254036800t^3x^{45} - 12586365792000t^4x^{42} - 5851581261926400t^5x^{39} - \\
& 1623511727405568000t^6x^{36} - 400077271429355520000t^7x^{33} - 28551097433877252840000t^8x^{30} + \\
& 50926698237612176179200000t^9x^{27} - 42318665025272343181393920000t^{10}x^{24} - 24094202601815176923316224000000t^{11}x^{21} \\
& - 4866679734224610446090698752000000t^{12}x^{18} - 89328732359216685504566132736000000t^{13}x^{15} -
\end{aligned}$$

5627710138630651186787666362368000000  $t^{14}x^{12} + 4292066932395643305123393545699328000000 t^{15}x^9$   
 $+ 77257204783121579492221083822587904000000 t^{16}x^6 + 216320173392740422578219034703246131200000000 t^{18}x$

is a rational to the Gardner equation (1).

We could go on for greater orders but the expressions of the solutions become too long to be given in this text.

## 4 Conclusion

We have constructed rational solutions to the Gardner equation and we obtain an infinite hierarchy of families of rational solutions of this equation as a quotient of a polynomials in  $x$  and  $t$ .

At order  $k$  the numerator is a polynomial of degree  $(k+1)^2$  en  $x$  and of degree  $\left\lceil \frac{(k+1)^2}{3} \right\rceil$  in  $t$ , where as usual,  $[x]$  denotes the bigger integer less or equal to  $x$ .

The denominator is also a polynomial of degree  $(k+1)^2$  en  $x$  and of degree  $\left\lceil \frac{(k+1)^2}{3} \right\rceil$  in  $t$  at order  $k$ , with the same notation as previously.

It will relevant to study in details the structure of these polynomials.

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