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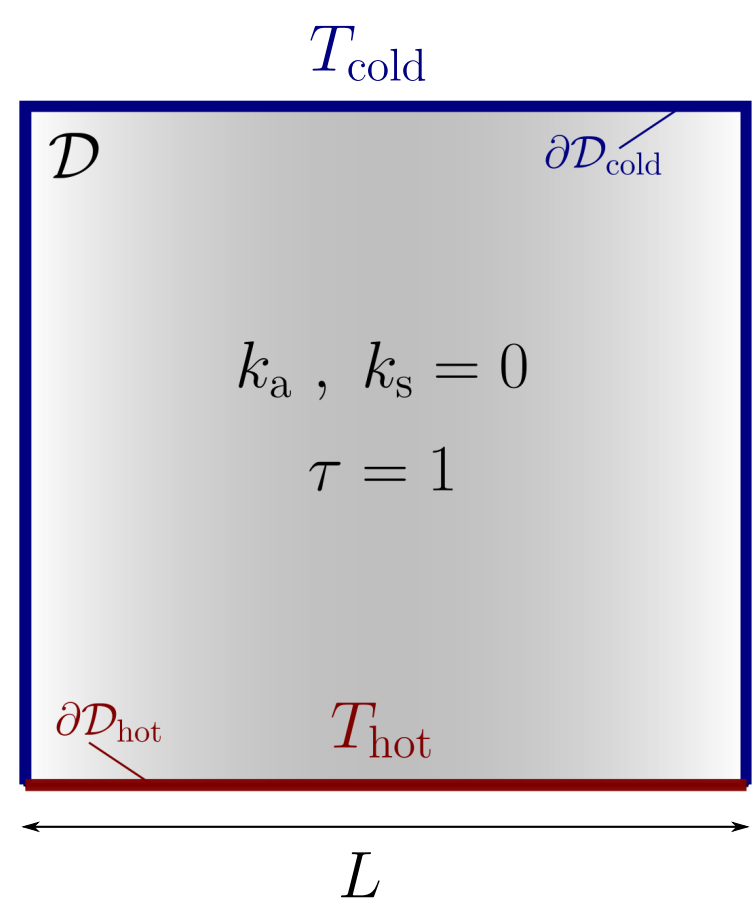
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# RESOLUTION OF CONDUCTIVE-RADIATIVE HEAT TRANSFER IN A SEMI-TRANSPARENT MEDIUM: FUNCTIONAL ESTIMATION OF THE RADIATIVE SOURCE TERM WITH MONTE CARLO METHOD

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**Abstract** - This study investigates using the Monte Carlo method to solve a conduction-radiation model in a semi-transparent material in a reduced computational time. Transient conduction in an absorbing/emitting and non-scattering grey medium in an academic configuration [1] is investigated where the radiative source term of the heat equation is estimated as a function of the temperature field to the power 4.

## PROBLEM DEFINITION



Heat transfer equation, boundary and initial conditions:

$$\begin{cases} \rho C_p \frac{\partial T}{\partial t} - \lambda \Delta T = -\vec{\nabla} \cdot \vec{q}_R & \forall \vec{x} \in \mathcal{D}, \forall t > 0 & (1) \\ T(\vec{x}, 0) = T_{\text{cold}} & \forall \vec{x} \in \mathcal{D}, t = 0 & (2) \\ T(\vec{x}, t) = T_{\text{hot}} & \forall \vec{x} \equiv \vec{y}_{\text{hot}} \in \partial \mathcal{D}_{\text{hot}}, \forall t > 0 & (3) \\ T(\vec{x}, t) = T_{\text{cold}} & \forall \vec{x} \equiv \vec{y}_{\text{cold}} \in \partial \mathcal{D}_{\text{cold}}, \forall t > 0 & (4) \end{cases}$$

Volumetric radiative source term:

$$\vec{\nabla} \cdot \vec{q}_R = \int_{\mathcal{D}} d\vec{x} \int_{4\pi} d\vec{\omega} k_a (\mathcal{I}^{\text{eq}}(\vec{x}) - \mathcal{I}(\vec{x}, \vec{\omega})) \quad \text{with} \quad \mathcal{I}^{\text{eq}}(\vec{x}) = \frac{\sigma_{\text{SB}}}{\pi} T^4(\vec{x}) \quad (5)$$

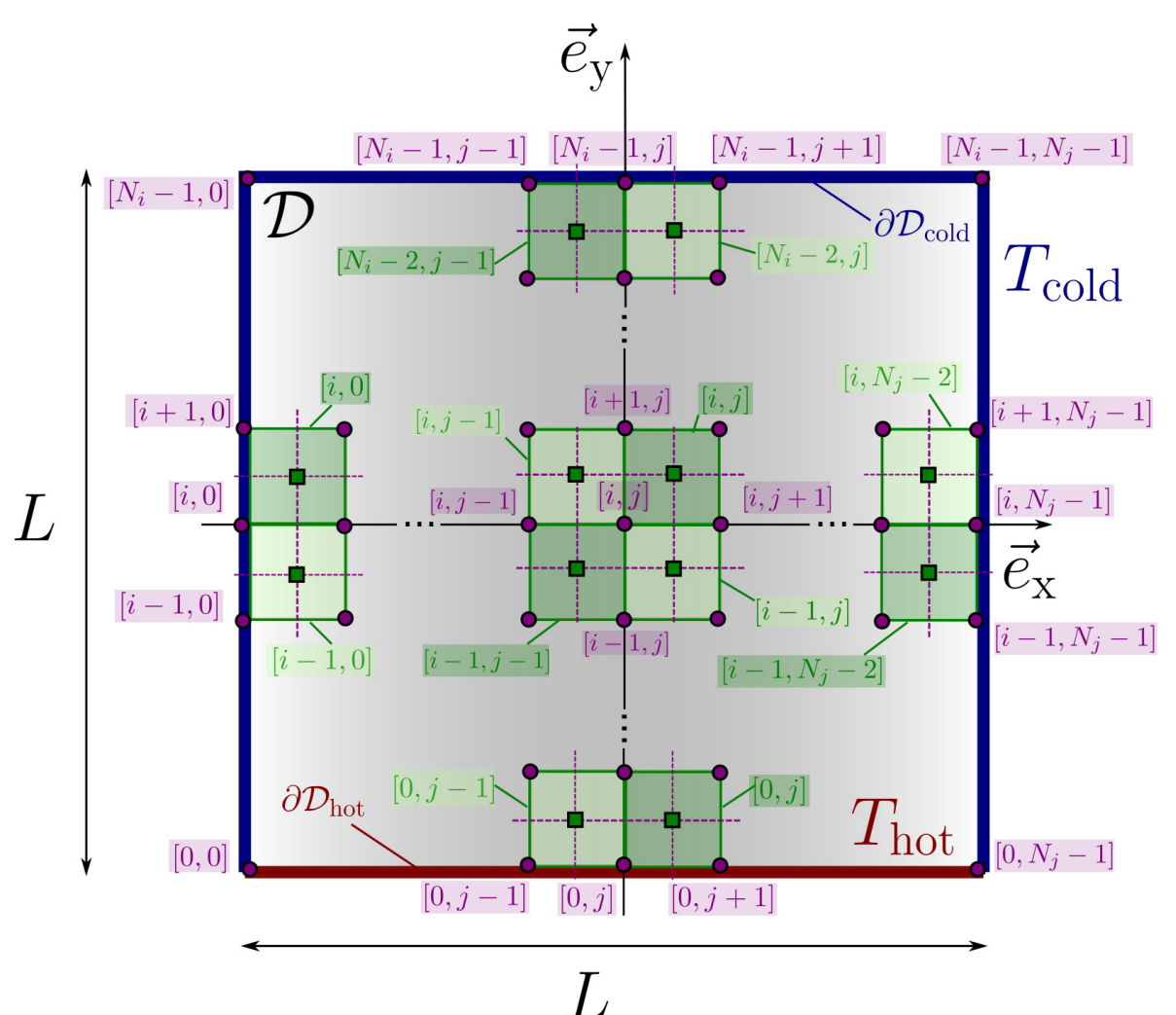
Radiative Transfer Equation:

$$\vec{\omega} \cdot \vec{\nabla} \mathcal{I}(\vec{x}, \vec{\omega}) = -(k_a + k_s) \mathcal{I}(\vec{x}, \vec{\omega}) + k_a \mathcal{I}^{\text{eq}} + k_s \int_{4\pi} \phi(\vec{x}, \vec{\omega} | \vec{\omega}') \mathcal{I}(\vec{x}, \vec{\omega}') d\vec{\omega}' \quad (6)$$

## HEAT CONDUCTION RESOLUTION

The domain  $\mathcal{D}$  is discretized in :

- $N_i \times N_j$  conductive nodes (represented in purple)
- $(N_i - 1) \times (N_j - 1)$  conductive nodes (represented in green)



Discretization of Eq. (1) with **Finite Differences** method:

$$T_{i,j}^n = \left(1 - 4 \frac{\alpha \delta_t}{\delta_x^2}\right) T_{i,j}^{n-1} + \frac{\alpha \delta_t}{\delta_x^2} (T_{i-1,j}^{n-1} + T_{i+1,j}^{n-1} + T_{i,j-1}^{n-1} + T_{i,j+1}^{n-1}) - \frac{\delta_t}{\rho C_p} \mathcal{I}_{\text{ray},i,j}^{n-1} \quad (7)$$

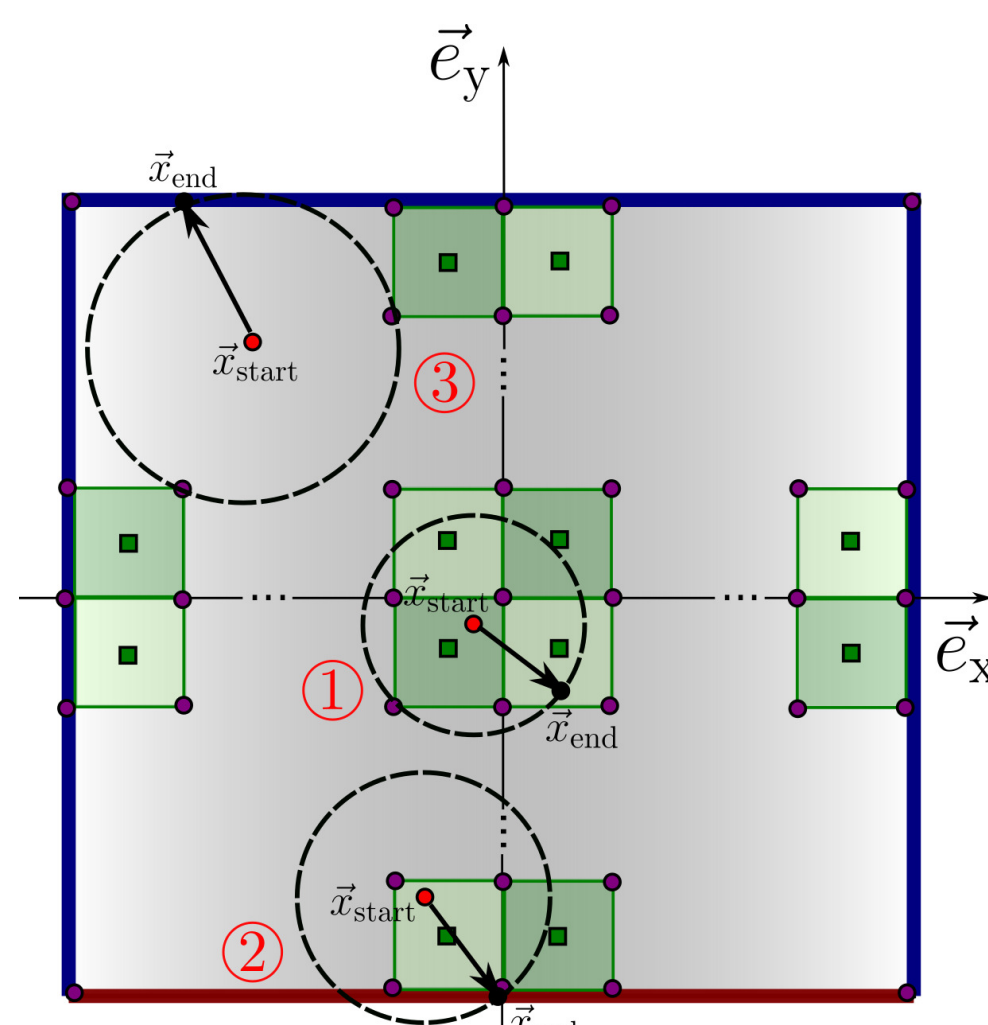
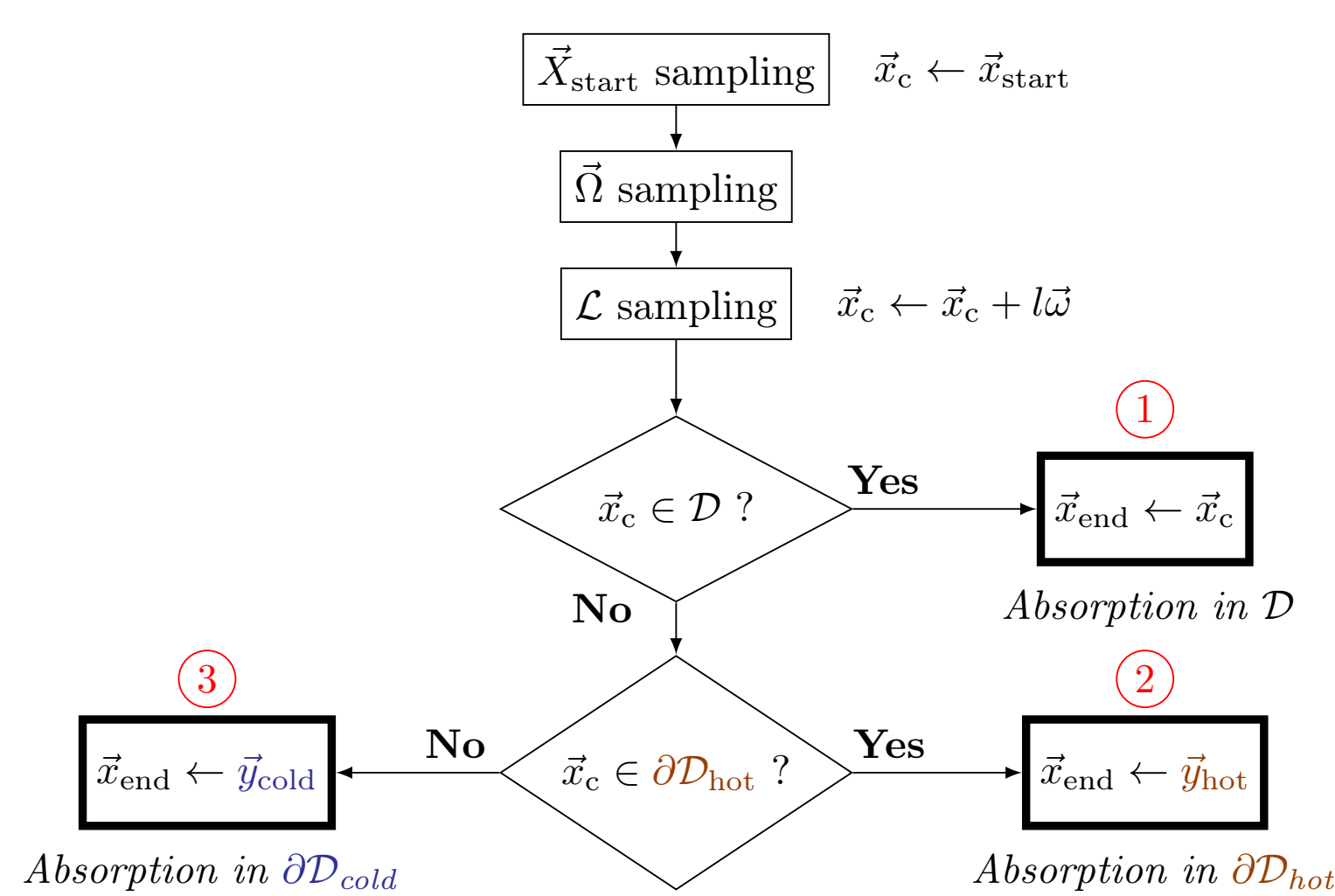
with:

$$\mathcal{I}_{\text{ray},i,j}^{n-1} = \frac{\vec{\nabla} \cdot \vec{q}_{R,i-1,j-1}^{n-1} + \vec{\nabla} \cdot \vec{q}_{R,i-1,j}^{n-1} + \vec{\nabla} \cdot \vec{q}_{R,i,j}^{n-1} + \vec{\nabla} \cdot \vec{q}_{R,i,j-1}^{n-1}}{4\delta_x^2} \quad (8)$$

## THE RADIATIVE SOURCE TERM

Estimation of the radiative source term with the **Monte Carlo** method:

$$\vec{\nabla} \cdot \vec{q}_R = \int_{\mathcal{D}} p_X(\vec{x}) d\vec{x} \int_{2\pi} p_\Phi(\varphi) d\varphi \int_0^\pi p_\Theta(\theta) d\theta \underbrace{4\pi \delta_x^2 k_a (\mathcal{I}^{\text{eq}} - \mathcal{I}(\vec{x}, \vec{\omega}))}_{\text{Monte Carlo weight } w_k} \quad (9)$$



(left) - Path sampling algorithm corresponding to a  $k^{\text{th}}$  realization of the Monte Carlo algorithm to estimation  $\vec{\nabla} \cdot \vec{q}_R$ ; (right) - Examples of random paths inside the square enclosure.

Functional estimation of the radiative source term:

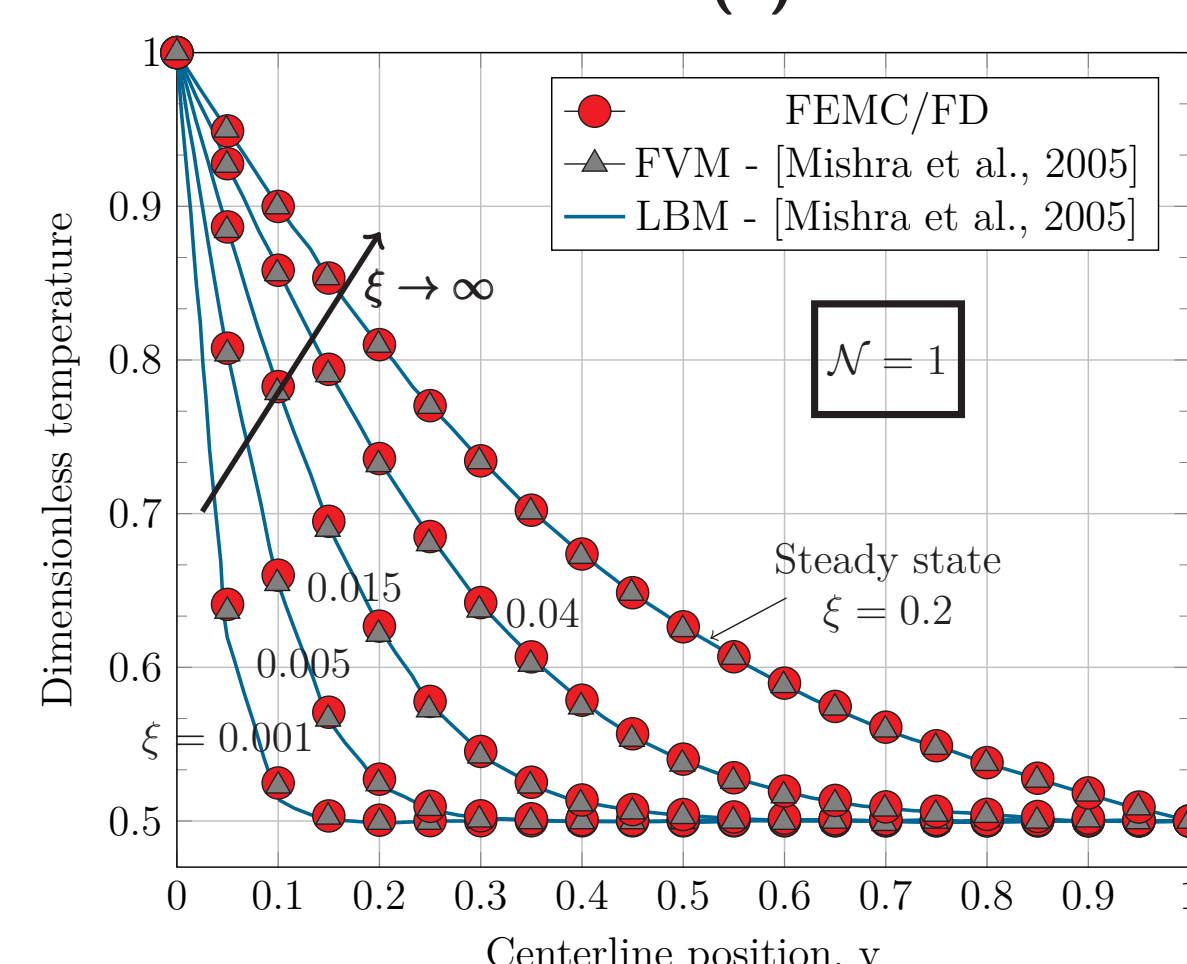
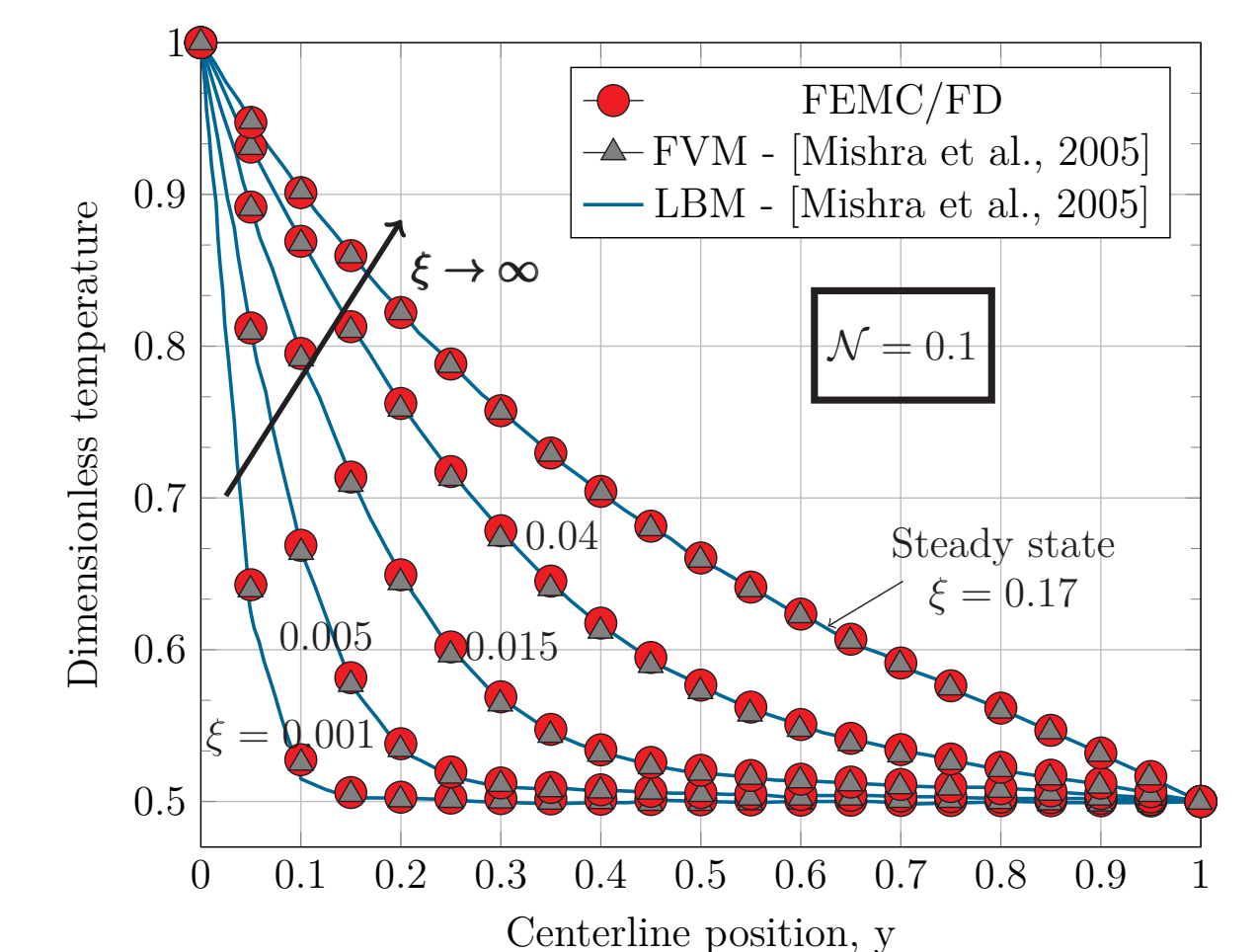
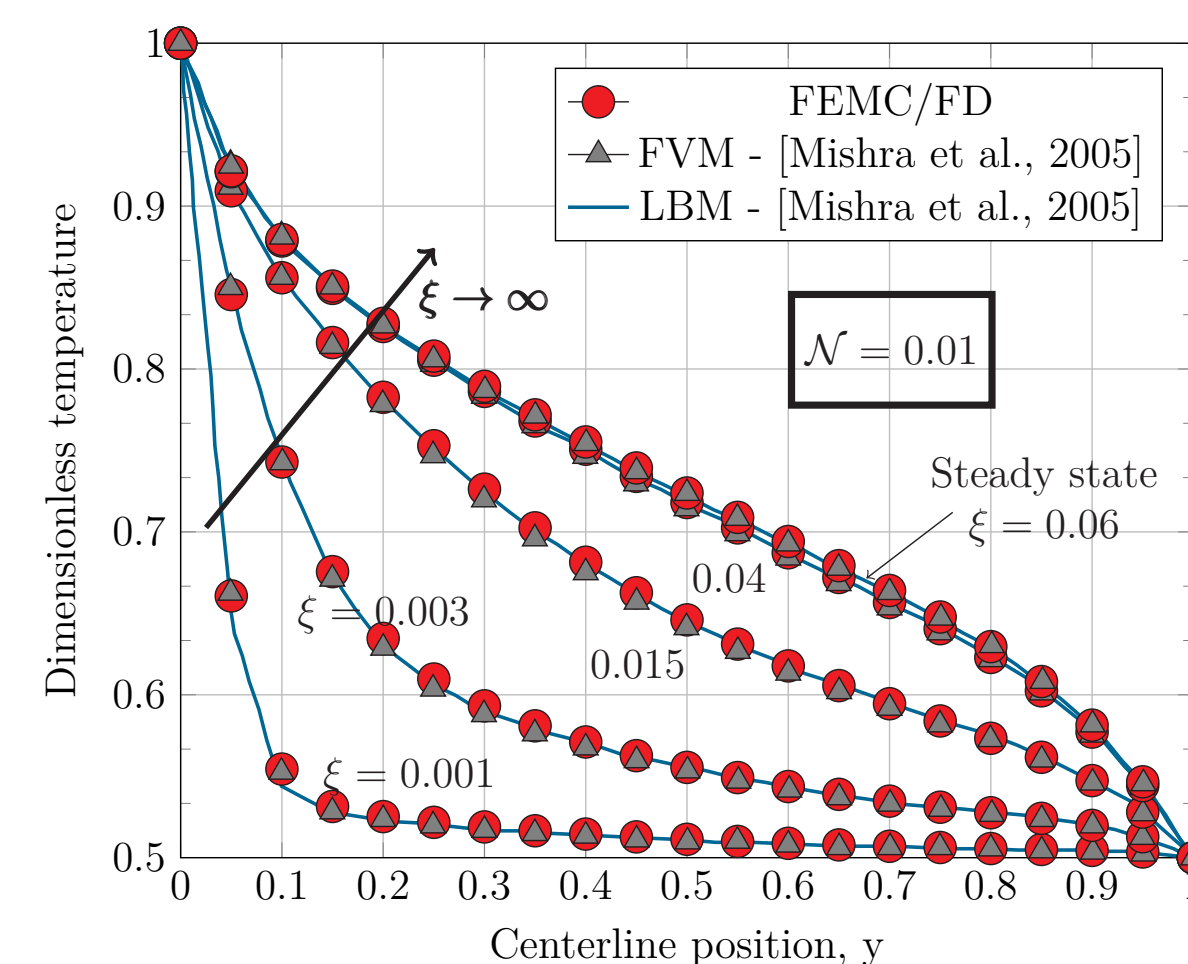
$$\vec{\nabla} \cdot \vec{q}_R[i_{\text{start}}, j_{\text{start}}] \approx 4\sigma_{\text{SB}} \delta_x^2 k_a \left( T[i_{\text{start}}, j_{\text{start}}]^4 - \sum_{i=0}^{N_i-1} \sum_{j=0}^{N_j-1} \frac{N_{i,j}}{N_{\text{MC}}} T[i, j]^4 \right) \quad (10)$$

## CONCLUSIONS

- **Proof of concept** in an academic case proved the **reduction of the computation time** required for the resolution of the coupled problem.
- Function obtained **without any assumption** on the radiative behaviour or the physics (no linearization) and not limited to grey media.
- Development of the method may be **extended to complex geometries** (2D/3D).

→ Considering the development and implementation of the method in **inversion processes** becomes practicable (sensitivity study to influencing parameters before inverting parallel hot wire measurements [2]).

## RESULTS



Dimensionless time $\xi$ (-)	Stark number $\mathcal{N}$ (-)		
	0.01	0.1	1
0.001		1.7	
0.005		8.2	
0.015		24.4	
0.04		65.9	
0.06	99.7		
0.17		284	
0.2			340

(a)-(c) - Dimensionless temperature distribution along the centerline position at different dimensionless times and for three different values of the Stark number  $\mathcal{N}$ ,  $N_{\text{MC}} = 10^7$  with  $21 \times 21$  conductive nodes; (d) - <sup>1</sup> The CPU time to obtain the functional Monte Carlo coefficients is 4h on an Intel®Xeon™E5 2.10 GHz; <sup>2</sup> Computing times are given for an Intel®Core™i5-10210U CPU 1.60GHz

[1] Mishra, S. C., Lankadasu, A. and Beronov, K. N., *Application of the lattice Boltzmann method for solving the energy equation of a 2-D transient conduction-radiation problem*, International Journal of Heat and Mass Transfer (2005).

[2] Penazzi, L., Jannot, Y., Meulemans, J., Farges, O. and Schick, V., *Influence of radiation heat transfer on parallel hot-wire thermal conductivity measurements of semi-transparent materials at high temperature*, International Journal of Thermal Sciences (2022).

