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ESTIMATION OF THE SMOOTHING PARAMETERS IN THE HPMV FILTER

BY

A. DERMOUNE, B. DJEHICHE and N. RAHMANIA

Abstract. We suggest an optimality criterion, for choosing the best smoothing parameters for an extension of the so-called Hodrick-Prescott Multivariate (HPMV) filter. We show that this criterion admits a whole set of optimal smoothing parameters, to which belong the widely used noise-to-signal ratios. We also propose explicit consistent estimators of these noise-to-signal ratios, which in turn yield a new performant method to estimate the output gap.

Mathematics Subject Classification 2000: 62M10, 62G15.

Key words: adaptive estimation, Gaussian process, Hodrick-Prescott filter, noise-to-signal ratio, orthogonal parametrization, output gap, potential output, detrending, unobserved components models.

1. Introduction

To estimate unobserved economic variables such as the potential output, the non-accelerating inflation rate of unemployment (NAIRU), and the real interest rate, three approaches are usually advocated. These include structural methods which rely on economic modeling, filtering methods which apply statistical devices, and methods which combine both and to which belongs the so-called Hodrick-Prescott Multivariate (HPMV) filter, suggested in LAXTON and TETLOW [11]. The HPMV filter is the standard univariate Hodrick-Prescott (HP) filter (LESER [12]) augmented with relevant economic information, such that the Phillips curve, Okun's law, production function (see for example PROIETTI ET AL. [13]) and other indicators such as the output capacity utilization (see BELL [2], LAXTON and TETLOW [11], RICHARDSON ET AL. [14], COTIS ET AL. [6], BOONE [4], CHAGNY and

LEMOINE [5], BENES and N'DIAYE [3], TRIMBUR [16], and the references therein).

The univariate HP filter captures the output processes, without taking into account any structure or dynamics of other important macroeconomic variables such as inflation rate. Moreover, it suffers from a number of deficiencies such as end-points uncertainty in estimating the output gap. Indeed, since trends are estimated as two-sided moving averages of the data, with future outcomes used to condition the current value, at the end of the sample, where these future values are not available, the filter does not have the benefit of hindsight to infer the current trend value. This means that the precision of the trend estimates deteriorates markedly right when those estimates are needed most to prepare a forecast or make judgments as to appropriate settings of the policy instrument (see [4] and [13] for more details). The main idea behind the HPMV filter is thus to be able to correct this deficiency by including more information on the major macroeconomic variables than just the available data of the variable of interest.

More specifically, the HPMV filter seeks to estimate the unobserved variable $(y_t, t = 1, \dots, T)$ which minimizes

$$(1.1) \quad \sum_{t=1}^T (x_t - y_t)^2 + \alpha_1 \sum_{t=1}^{T-2} (y_{t+2} - 2y_{t+1} + y_t)^2 + \alpha_2 \xi_t^2$$

for appropriately chosen positive parameters α_1 and α_2 . This is a basic HP filter augmented with the residuals ξ from the following estimated economic relationship (the Phillips Curve): $x_t^* = \beta y_t + dX_t + \xi_t$, where, x_t^* is another explanatory variable that can be explained by the unobserved variable y , and X is an exogenous variable affected by the parameter d .

There are two different approaches to the question of choosing the smoothing parameters α_1 and α_2 . The first approach is to consider the free choice of the smoothing parameters as an advantage. The features of the data can be explored by varying them (see RICHARDSON ET AL. [14] and TRIMBUR [16] for instance). The other view often used by macroeconomists suggests that the choice of α_1 and α_2 should be on the basis of priors about the relative variance of each of the terms in (1.1); the higher the variance of the term the lower weight it receives in the minimization process (see LAXTON-TETLOW [11] and BOONE [4]).

The main result of this paper is an optimality criterion for selecting the *best* smoothing parameters and construct consistent estimators of them.

In turn, this will be the basis for improving estimation methods of the potential output gap which is a powerful concept in the models of many inflation targeting central banks, which rely on this "unobserved measure" in its policy making.

The optimal smoothing parameters for the HPMV filter are pairs of positive parameters (α_1, α_2) which minimize the difference (using the Euclidean norm) between $\hat{y}((\alpha_1, \alpha_2), (x, z))$, that solves (1.1), and $E[y|x]$,

$$\alpha^* = \arg \min_{(\alpha_1, \alpha_2)} \|E[y|(x, z)] - \hat{y}((\alpha_1, \alpha_2), (x, z))\|^2,$$

where, $z = x^* - dX$.

The organisation of this paper is as follows. Section 2 describes the HPMV filter, and suggests an optimality criterion for choosing the best smoothing parameters, where we show that there is a whole set of such optimal solutions. We also show that the noise-to-signal ratios used in the macroeconomic literature are optimal as well. In Section 3, we apply the recent results obtained in DERMOUNE ET AL. [7] and [8] to find consistent estimators of these noise-to-signal ratios. Finally, in Section 4 we present calibration and simulation results that highlight the performance of these estimators. We apply our estimators to analyse the performance of the potential output of the US GDP using US inflation as explanatory variable. The data sets correspond to the period 2002-2009. Compared with the values of the smoothing parameters suggested in the classical HP filter and the HPMV of LAXTON-TETLOW [11], our results show higher performance in estimating the potential output gap, notably at the end-points.

2. The Hodrick-Prescott Multivariate filter

As a general principle, a macroeconomic time series can be decomposed into its seasonal variation, a 'business-cycle' component, irregular short-term movements, and its long-term trend (or *potential output*) component. It is standard practice for economic series to be seasonally adjusted. The univariate HP filter is then applied to the seasonally adjusted series to obtain a cyclical residual, also called the *potential output gap*, which is an estimate of the combined cyclical and irregular component of the series.

Let $x = (x_1, \dots, x_T) \in \mathbb{R}^T$ be a time series of observables. The HP filter decomposes x into a nonstationary potential output $y \in \mathbb{R}^T$ and a cyclical component, the potential output gap (noise term) $u \in \mathbb{R}^T$:

$$(2.1) \quad x = y + u.$$

Given a smoothing parameter $\alpha > 0$, this decomposition of x is obtained by minimizing the weighted sum of squares

$$(2.2) \quad \|x - y\|^2 + \alpha \|D^2 y\|^2$$

with respect to y , where, for $a \in \mathbb{R}^T$, $\|a\|^2 = \sum_{i=1}^T a_i^2$. Here, $D^2 y$ is the potential output disturbance obtained by acting the second order forward shift operator D^2 on $y = (y_1, y_2, \dots, y_T)$: $D^2 y_t := (y_{t+2} - y_{t+1}) - (y_{t+1} - y_t)$, $t = 1, 2, \dots, T - 2$, or, equivalently, $D^2 y_t := 2(\frac{y_{t+2} + y_t}{2} - y_{t+1})$, $t = 1, 2, \dots, T - 2$, measuring the deviation between the value of the output at $t + 1$, y_{t+1} and the linear interpolation between y_t and y_{t+2} .

In vector form,

$$(2.3) \quad Py(t) = D^2 y_t, \quad t = 1, \dots, T - 2,$$

where, the shift operator P is the following $(T - 2) \times T$ -matrix

$$(2.4) \quad P := \begin{pmatrix} 1 & -2 & 1 & \dots & \dots & 0 \\ 0 & 1 & -2 & 1 & \dots & 0 \\ & \dots & \dots & \dots & \dots & \\ 0 & \dots & \dots & 1 & -2 & 1 \end{pmatrix}.$$

The first term in (2.2) measures a goodness-of-fit by minimizing the deviation between the potential output y_t and the observation x_t and the second term is a measure of the degree-of-smoothness which penalizes decelerations in growth rate of the trend component, by minimizing the deviation between the value y_{t+1} and the linear interpolation between y_t and y_{t+2} .

The Hodrick-Prescott Multivariate filter (HPMV in short) seeks to estimate the unobserved variable y as a solution to the following minimization problem

$$(2.5) \quad \arg \min_y (\|x - y\|^2 + \alpha_1 \|Py\|^2 + \alpha_2 \|\xi\|^2),$$

given the following dynamics:

$$(2.6) \quad \begin{cases} x = y + u, \\ z := x^* - dX = \beta y + \xi, \\ Py = v, \end{cases}$$

where, x^* is another explanatory variable and X is an exogenous variable affected by the parameter d . Moreover, the noise term u and the signal terms

v and ξ are independent Gaussian vectors with mean zero and respective covariance matrices $\sigma_u^2 I_T$, $\sigma_v^2 I_{T-2}$ and $\sigma_\xi^2 I_T$, where, I_T and I_{T-2} denote the $T \times T$ and $(T-2) \times (T-2)$ identity matrices, respectively:

$$(2.7) \quad \begin{pmatrix} u \\ \xi \\ v \end{pmatrix} \sim \mathcal{N}(0, \Sigma),$$

with covariance matrix

$$\Sigma := \begin{pmatrix} \sigma_u^2 I_T & 0 & 0 \\ 0 & \sigma_\xi^2 I_T & 0 \\ 0 & 0 & \sigma_v^2 I_{T-2} \end{pmatrix}.$$

Since P is of rank $T-2$, the signal $v = Py$ does not determine a unique y but rather yields an orthogonal parameterization given by the set of solutions $y := \{P'(PP')^{-1}v + Z\gamma; \gamma \in \mathbb{R}^2\}$, where, the $T \times 2$ -matrix Z satisfies

$$(2.8) \quad PZ = 0, \quad Z'Z = I_2.$$

In view of (2.6), the time series (x, z) can be represented in terms of (u, v, ξ) as

$$(2.9) \quad \begin{cases} x = u + P'(PP')^{-1}v + Z\gamma, \\ z = \xi + \beta P'(PP')^{-1}v + \beta Z\gamma, \end{cases}$$

for some $\gamma \in \mathbb{R}^2$.

Hence,

$$(2.10) \quad \begin{pmatrix} x \\ z \\ y \end{pmatrix} \sim \mathcal{N} \left(\begin{pmatrix} Z \\ \beta Z \\ Z \end{pmatrix} \gamma, \Sigma_{xzy} \right),$$

with covariance matrix

$$\Sigma_{xzy} = \begin{pmatrix} \sigma_u^2 I_T + \sigma_v^2 Q & \beta \sigma_v^2 Q & \sigma_v^2 Q \\ \beta \sigma_v^2 Q & \sigma_\xi^2 I_T + \beta^2 \sigma_v^2 Q & \beta \sigma_v^2 Q \\ \sigma_v^2 Q & \beta \sigma_v^2 Q & \sigma_v^2 Q \end{pmatrix},$$

where, $Q := P'(PP')^{-1}(PP')^{-1}P$ is a symmetric matrix that together with P and Z satisfies the following properties.

$$(2.11) \quad P'(PP')^{-1}P + ZZ' = I_T, \quad QZ = 0, \quad Z'Z = I_T,$$

and, for any $\lambda > 0$,

$$(2.12) \quad Q[\lambda I_T + Q]^{-1} = [\lambda I_T + Q]^{-1}Q.$$

This yields that

$$(2.13) \quad Q[\lambda I_T + Q]^{-1}Z = 0.$$

Moreover, the following identity holds.

$$(2.14) \quad (I_T + \lambda P'P)^{-1} = ZZ' + Q[\lambda I_T + Q]^{-1}.$$

Since the matrix $((1 + \alpha_2\beta^2)I_T + \alpha_1P'P)$ is positive definite, the unique solution $\hat{y} := y((\alpha_1, \alpha_2, \beta), (x, z))$ to Problem (2.5) is $\hat{y} = a(I_T + \lambda P'P)^{-1}w$, or, using (2.14),

$$(2.15) \quad \hat{y} = aZZ'w + aQ[bI_T + Q]^{-1}w,$$

where, we have set $a := 1/(1 + \alpha_2\beta^2)$, $b := \alpha_1/(1 + \alpha_2\beta^2)$ and $w := x + \alpha_2\beta z$.

Equation (2.15) defines the descriptive filter that associates a trend y to the time series (x, z) through w , depending on the smoothing parameters α_1 , α_2 , the disturbance operator P and the parameter β .

2.1. A criterion to find the best smoothing parameters

Following Schlicht (SCHLICHT [15], Theorem 1), a way to estimate the smoothing parameters α_1 and α_2 is to let the optimal solution \hat{y} in (2.15) be the best predictor of any trend y given the time series (x, z) , i.e.

$$(2.16) \quad \hat{y} \approx E[y | (x, z)].$$

In other words, the conditional expectation $E[y | (x, z)]$ which is usually not smooth, is 'equal' to the smooth trend \hat{y} .

In the macroeconomic literature dealing with the HPMV filter such as LAXTON and TETLOW [11], RICHARDSON ET AL. [14], BOONE [4] and CHAGNY and LEMOINE [5], it is suggested that the best smoothing parameters are the noise-to-signal ratios $\alpha_1^e = \sigma_u^2/\sigma_v^2$ and $\alpha_2^e = \sigma_u^2/\sigma_\xi^2$. Applying Schlicht's criterion to this filter we will show in Proposition 2.1 below, that there is a whole family of best smoothing parameters to which belong α_1^e and α_2^e .

Since \hat{y} is a (linear) function of w , adapting Schlicht's criterion, the best smoothing parameters of the HPMV filter, given a realization of (x, z) , are those positive parameters α_1^* and α_2^* for which

$$(2.17) \quad \hat{y} - Z\hat{\gamma} = E[y|w] - Z\gamma,$$

where, $\hat{\gamma}$ is the maximum likelihood estimator of γ which we determine using observations from $w = x + \alpha_2\beta z$:

$$\hat{\gamma} = \arg \min_{\gamma} (w - E(w))' \Sigma_w^{-1} (w - E(w))$$

Since, $E(w) = Z\gamma/a$ and $\Sigma_w = \frac{\sigma_v^2}{a^2} [cI_T + Q]$, we get,

$$(2.18) \quad \hat{\gamma} = aZ'w.$$

In view of (2.10), we have

$$(2.19) \quad E[y|w] = Z\gamma + aQ[cI_T + Q]^{-1}(w - Z\gamma),$$

where, $c := \frac{\sigma_u^2 + \alpha_2^* \beta^2 \sigma_\xi^2}{(1 + \alpha_2^* \beta^2) \sigma_v^2}$. From (2.15) and (2.19) we get

$$(2.20) \quad \begin{aligned} \hat{y} - E[y|w] &= Z(aZ'w - \gamma) + aQ\{[bI_T + Q]^{-1} \\ &\quad - [cI_T + Q]^{-1}(w - Z\gamma)\}. \end{aligned}$$

Hence, by Eq. (2.17), the best smoothing parameters are those positive α_1^* and α_2^* for which $b = c$, which yields $\alpha_1^* = \frac{\sigma_u^2 + \alpha_2^* \beta^2 \sigma_\xi^2}{(1 + \alpha_2^* \beta^2) \sigma_v^2}$, or, $\alpha_1^*/\alpha_1^e = 1 + \alpha_2^* \beta^2 (\alpha_2^*/\alpha_2^e - \alpha_1^*/\alpha_1^e)$, where, $\alpha_1^e = \sigma_u^2/\sigma_v^2$ and $\alpha_2^e = \sigma_u^2/\sigma_\xi^2$.

Furthermore,

$$(2.21) \quad \hat{y}^* - E[y|w] = Z(aZ'w - \gamma),$$

where, $\hat{y}^* := y((\alpha_1^*, \alpha_2^*), (x, z))$.

We have proved the following proposition

Proposition 2.1. *The best smoothing parameters of the HPMV filter according to the criterion (2.17) are those positive parameters α_1^* and α_2^* for which*

$$(2.22) \quad \alpha_1^*/\alpha_1^e = 1 + \alpha_2^* \beta^2 (\alpha_2^*/\alpha_2^e - \alpha_1^*/\alpha_1^e),$$

where, $\alpha_1^e = \sigma_u^2/\sigma_v^2$ and $\alpha_2^e = \sigma_u^2/\sigma_\xi^2$.

In particular,

- (1) $\alpha_1^* = \alpha_1^e$ if and only if $\alpha_2^* = \alpha_2^e$.
- (2) If $\alpha_1^*/\alpha_1^e < 1$, then necessarily $\alpha_2^*/\alpha_2^e < \alpha_1^*/\alpha_1^e$.
- (3) If $\alpha_1^*/\alpha_1^e > 1$, then necessarily $\alpha_2^*/\alpha_2^e > \alpha_1^*/\alpha_1^e$.

2.2. Optimality of the best smoothing parameters

In the main result of this section, Theorem 2.2, we will show that the best smoothing parameters α_1^* and α_2^* given in Proposition 2.1 minimize the gap between the descriptive HPMV filter $\hat{y} := y((\alpha_1, \alpha_2), (x, z))$ and the best predictor of the trend y given w , $E[y|w]$, i.e. $(\alpha_1^*, \alpha_2^*) = \arg \min_{\alpha_1 > 0, \alpha_2 > 0} \|\hat{y} - E[y|w]\|$, and give an explicit formula for the expected value of the optimal quadratic gap. In particular, it follows that the noise-to-signal ratios α_1^e and α_2^e are optimal.

Theorem 2.2. *We have*

$$(2.23) \quad (\alpha_1^*, \alpha_2^*) = \arg \min_{\alpha_1 > 0, \alpha_2 > 0} \|\hat{y} - E[y|w]\|,$$

Moreover, the optimal gap $\hat{y}^* - E[y|w] = Z(aZ'w - \gamma)$ is a centered Gaussian vector with covariance matrix $\text{cov}(Z(aZ'w - \gamma)) = \rho ZZ'$. where, $\hat{y}^* := y((\alpha_1^*, \alpha_2^*), (x, z))$ and $\rho := (\sigma_u^2 + \alpha_2^{*2} \beta^2 \sigma_\xi^2) / (1 + \alpha_2^{*2} \beta^2)$.

In particular, $E[\|\hat{y}^* - E[y|x]\|^2] = E[\|Z(\gamma - Z'x)\|^2] = \rho \text{trace}(ZZ')$.

Proof. Recall that, by (2.20), we have $\hat{y} - E[y|w] = Z(aZ'w - \gamma) + aQ\{[bI_T + Q]^{-1} - [cI_T + Q]^{-1}\}$. Now, since $Z'Q = 0$, we also have $(Z(aZ'w - \gamma))' aQ\{[bI_T + Q]^{-1} - [cI_T + Q]^{-1}\} = 0$. Hence, $\|\hat{y} - E[y|x]\|^2 = \|Z(aZ'w - \gamma)\|^2 + \|aQ\{[bI_T + Q]^{-1} - [cI_T + Q]^{-1}\}x\|^2$. This yields, using (2.21), $\|\hat{y} - E[y|x]\| \geq \|\hat{y}^* - E[y|x]\| = \|Z(\gamma - Z'x)\|$, for all positive α_1 and α_2 . Thus α_1^* and α_2^* are optimal.

The rest of the proofs is straightforward. \square

3. Estimation of the noise-to-signal ratios

In this section we propose consistent estimators of the noise-to-signal ratios $\alpha_1^e = \sigma_u^2 / \sigma_v^2$ and $\alpha_2^e = \sigma_u^2 / \sigma_\xi^2$, and the parameter β as ratios of explicit unbiased consistent estimators of the variances σ_u^2, σ_v^2 and σ_ξ^2 . To this end, we follow DERMOUNE ET AL. [7], Section 3.2, to estimate σ_u^2, σ_v^2 , using the fact that

$$(3.1) \quad Px = v + Pu \sim \mathcal{N}(0, \sigma_v^2 I_{T-2} + \sigma_u^2 PP')$$

We get the following unbiased consistent estimates of σ_u^2 and σ_v^2 .

$$(3.2) \quad \hat{\sigma}_u^2 = -\frac{1}{4(T-3)} \sum_{j=1}^{T-3} Px(j)Px(j+1).$$

and

$$(3.3) \quad \hat{\sigma}_v^2 = \frac{1}{T-2} \sum_{j=1}^{T-2} Px(j)^2 + \frac{3}{2(T-3)} \sum_{j=1}^{T-3} Px(j)Px(j+1).$$

Similarly, to estimate σ_ξ^2 , we use the fact that the time series

$$(3.4) \quad Pz \sim \mathcal{N}(0, \beta^2 \sigma_v^2 I_{T-2} + \sigma_\xi^2 PP').$$

is Gaussian and stationary, with the following covariance matrix $(V(i, j))_{ij}$: $V(i, j) = \beta^2 \sigma_v^2 \delta_i^j + \sigma_\xi^2 (PP')_{ij} = r_{|i-j|}$, where,

$$(3.5) \quad r_k = \begin{cases} \beta^2 \sigma_v^2 + 6\sigma_\xi^2, & \text{if } k = 0; \\ -4\sigma_\xi^2, & \text{if } k = 1; \\ \sigma_\xi^2, & \text{if } k = 2; \\ 0, & \text{otherwise,} \end{cases}$$

whose unbiased consistent estimator is explicitly given by

$$\hat{r}_k = \frac{1}{(T-2) - k} \sum_{j=1}^{T-2-k} Pz(j)Pz(j+k), \quad k = 0, 1, 2.$$

From that we derive the following consistent unbiased estimator of σ_ξ^2

$$(3.6) \quad \hat{\sigma}_\xi^2 = -\frac{1}{4} \hat{r}_1 = -\frac{1}{4(T-3)} \sum_{j=1}^{T-3} Pz(j)Pz(j+1).$$

Finally, in view of (3.5), combining (3.2), (3.3) and (3.6), we get the following consistent estimators of α_1^e , α_2^e and β .

Theorem 3.1. *The following statistics*

$$(3.7) \quad \hat{\alpha}_1^e = -\frac{1}{4} \left(\frac{3}{2} + \frac{(T-3) \sum_{j=1}^{T-2} Px(j)^2}{(T-2) \sum_{j=1}^{T-3} Px(j)Px(j+1)} \right)^{-1},$$

$$(3.8) \quad \hat{\alpha}_2^e = \frac{\sum_{j=1}^{T-3} Px(j)Px(j+1)}{\sum_{j=1}^{T-3} Pz(j)Pz(j+1)},$$

and

$$(3.9) \quad \hat{\beta} = \left(\frac{2(T-3) \sum_{j=1}^{T-2} Pz(j)^2 + 3(T-2) \sum_{j=1}^{T-3} Pz(j)Pz(j+1)}{2(T-3) \sum_{j=1}^{T-2} Px(j)^2 + 3(T-2) \sum_{j=1}^{T-3} Px(j)Px(j+1)} \right)^{1/2},$$

based on the time series of observations Px and Pz , are consistent estimators of the smoothing parameters α_1^e , α_2^e and the parameter β .

4. Simulation and calibration

4.1. Numerical examples

To illustrate the performance of the estimators $\hat{\alpha}_1^e$, $\hat{\alpha}_2^e$ and $\hat{\beta}$, we give a numerical example based on simulations drawn from the distributions of Px and Pz for different values of the time horizon T , and specified values of α_1^e , α_2^e and β .

The estimator $\hat{\alpha}_1^e$ of α_1^e is exactly the one obtained in DERMOUNE ET AL. [7] for the standard HP filter, thus suffering from the same deficiency in the sense that it may not be good enough to estimate values of α_1^e larger than 1. Therefore, we restrict our simulation exercise, reported in the numerical example below, to the case $\alpha_1^e \leq 1$.

In Tables 1, 2 and 3, we report on the performance of different estimates of the statistics $\hat{\alpha}_1^e$, $\hat{\alpha}_2^e$ and $\hat{\beta}$, based on 1000 simulations drawn from the distributions of Px and Pz for the values of $T - 2$ (the dimension of the matrix PP') ranging from 500 to 5000, and for the following sets of true values of the parameters; $(\alpha_1^e = 1, \alpha_2^e = 1, \beta = 0.5)$, $(\alpha_1^e = 1, \alpha_2^e = 0.5, \beta = 2)$ and $(\alpha_1^e = 1, \alpha_2^e = 16, \beta = 0.2)$, where, the last set of values is used in BOONE [4].

4.2. Calibration

The data used are from U.S. Department of Commerce, Bureau of Economic Analysis.

- The quarterly US real GDP (output) under the period January 1, 2002 – March 31, 2009 for the time series x .
- The quarterly US Inflation under the period January 1, 2002 – March 31, 2009 for the time series z .

T-2	500	1000	5000
(mean($\hat{\alpha}_1^e$), std($\hat{\alpha}_1^e$))	(1.13, 0.61)	(1.05, 0.33)	(1.00, 0.11)
(mean($\hat{\alpha}_2^e$), std($\hat{\alpha}_2^e$))	(1.01, 0.16)	(1.00, 0.11)	(1.00, 0.05)
(mean($\hat{\beta}$), std($\hat{\beta}$))	(0.47, 0.29)	(0.47, 0.22)	(0.49, 0.08)

Table 1: Performance of different estimates of the statistics on $\alpha_1^e = 1$, $\alpha_2^e = 1$ and $\beta = 0.5$ based on 1000 simulations.

T-2	500	1000	5000
(mean($\hat{\alpha}_1^e$), std($\hat{\alpha}_1^e$))	(1.13, 0.61)	(1.05, 0.33)	(1.00, 0.11)
(mean($\hat{\alpha}_2^e$), std($\hat{\alpha}_2^e$))	(0.50, 0.09)	(0.50, 0.06)	(0.50, 0.02)
(mean($\hat{\beta}$), std($\hat{\beta}$))	(2.06, 0.43)	(2.01, 0.23)	(2.00, 0.10)

Table 2: Performance of different estimates of the statistics on $\alpha_1^e = 1$, $\alpha_2^e = 0.5$ and $\beta = 2$ based on 1000 simulations.

T-2	500	1000	5000
(mean($\hat{\alpha}_1^e$), std($\hat{\alpha}_1^e$))	(1.13, 0.61)	(1.05, 0.33)	(1.00, 0.11)
(mean($\hat{\alpha}_2^e$), std($\hat{\alpha}_2^e$))	(16.30, 2.73)	(16.14, 1.84)	(15.96, 0.84)
(mean($\hat{\beta}$), std($\hat{\beta}$))	(0.19, 0.05)	(0.20, 0.03)	(0.19, 0.01)

Table 3: Performance of different estimates of the statistics on $\alpha_1^e = 1$, $\alpha_2^e = 16$ and $\beta = 0.2$ based on 1000 simulations.

In the sequel, we first estimate the potential output, y , with \hat{y} and then the output gap, u , which is the difference between the actual output, x , and potential output, \hat{y} .

4.2.1 Calibration of the potential output

In Table 4, we collect the estimated values of the statistics, α_1^e , α_2^e and β based on our estimators

$\hat{\alpha}_1^e$	$\hat{\alpha}_2^e$	$\hat{\beta}$
0.1708	53872.09	-0.231

Table 4: Estimated values of α_1^e , α_2^e and β .

As shown in Figure 1, we get a quite good fit of the trend by our method of the considered US real GNP time series. We note, in particular, that at the end of the sample our estimator is performant. The behavior of the HP(1600) filter under the period 2008Q4-2009Q1 is dramatic.

4.2.2 Calibration of the output gap

Figure 2 shows estimates of the output gap corresponding to the classical HP filter with $\alpha = 1600$, while Figure 3 displays estimates of the output gap obtained using our estimates of $(\alpha_1^e, \alpha_2^e, \beta)$. It appears clearly that our output gap estimator does significantly better in predicting important turning points of inflation and is more compatible and correlated with the inflation rate.

5. Conclusion

The HPMV filter is an alternative way of estimating unobserved variables. This method stems from the use of the standard HP filter augmented by relevant economic information. Intuitively, the estimation based on the HPMV filter is not only a simple average going through the observed series, but also allows for input from economic relationships. According to BASISTHA and NELSON [1] in the same spirit, we can use the Keynesian Phillips curve to estimate the smoothing parameters or another added economic condition. We can also use the state-space model as in HARVEY ET AL. [9] to estimate the parameters by maximum likelihood method. In this paper we suggest an optimality criterion for the smoothing parameters and explicit consistent estimators of the HPMV filter. We show that this criterion admits a whole set of optimal smoothing parameters to which belong the widely used noise-to-signal ratios and devices a rule to fix the choice of these parameters by estimating the noise-to-signal ratios which in turn yields a new performant method to estimate the potential output and the output gap.

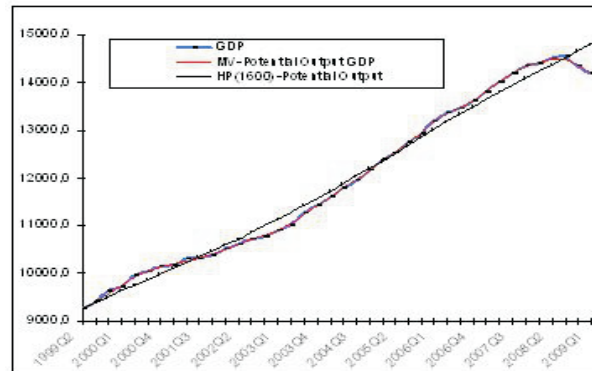


Figure 1: Potential Output Estimation.

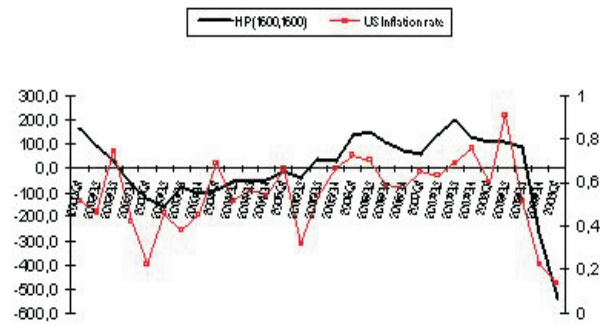


Figure 2: Potential Output Estimation.

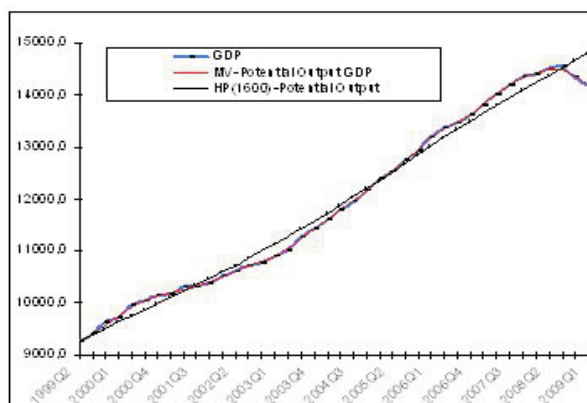


Figure 3: Multivariate Output Gap Estimation.

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