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Explore a Way to Extend the Weak Equivalence Principle to Massless Bodies

Tong Wang

Abstract

The validity of the Weak Equivalence Principle (WEP) for massive bodies has been empirically confirmed to extremely high precision. The WEP states that every massive body falls at the same rate in a gravitational field. However, can this principle be applied beyond massive bodies? Theoretically, a body can be massive, massless, or negative massive. Does a massless body fall at the same rate as a massive body within the same field? We aim to extend the WEP to massless bodies. For massless bodies, we argue that only a type of massless body—with zero net mass—has a role in the WEP. Furthermore, we investigate the possibility that such massless bodies can be constructed in theory. To accomplish this goal, we adopt the concept of negative mass. Hence, a massless body can consist of mass and negative mass. We derive that the WEP applies to this type of massless body based on its configuration. Therefore, we extend the WEP beyond massive bodies satisfactorily.

Keywords: Equivalence Principle; Weak Equivalence Principle; Extended Weak Equivalence Principle; Massless Body; Massless Configuration; Negative Mass

1. Introduction

The Weak Equivalence Principle (WEP) emerged in the 17th century when Galileo [1] experimentally derived that the acceleration of a massive body due to gravitation is independent of the amount of mass being accelerated.

In the 20th century, Einstein [2] used the following mathematical formulation to describe the WEP:

$$\text{acceleration} = \frac{\text{gravitational mass } (m_g)}{\text{inertial mass } (m_i)} (\text{intensity of the gravitational field}). \quad (1)$$

If the acceleration is always the same for a given gravitational field regardless of the nature and condition of a massive body, then the ratio of the gravitational mass (m_g) to the inertial mass (m_i) must likewise be the same for all massive bodies, usually one by a suitable choice of units.

In Newtonian mechanics, this principle is commonly described as the gravitational mass of a massive body being equal to its inertial mass or the ratio of the gravitational mass of a body to its inertial mass being one.

$$\frac{m_g}{m_i} = 1. \quad (2)$$

There are variations in the interpretation of the WEP [3-5]. The followings all describe the same principle:

Every object falls at the same rate—undergoes the same acceleration—in a gravitational field, independent of the composition of the object.

The inertial mass and gravitational mass of any object are equal.

In small enough regions of spacetime, the motion of freely falling particles is the same in a gravitational field and in a uniformly accelerated frame.

It would not be able to distinguish—by performing mechanical experiments—a frame that is accelerating in free fall in a uniform gravitational field from one that is inertial and without a gravitational field.

Experimentally this principle has been verified with extremely high accuracy by Eötvös [6] for the first time. Later experiments have reconfirmed it or improved the precision further [7-14]. However, the principle states that it is only applicable to massive bodies.

Few studies have investigated the possibility of extending the WEP to massless bodies. In theory, can the WEP be applied to a massless body? This principle indicates that every massive body falls at the same rate in a gravitational field regardless of the amount of mass being accelerated. Intuitively, if the amount approaches zero, the massive body becomes massless. Hence, it is possible that the principle also applies to a massless body.

We aim to extend the WEP to massless bodies. Nevertheless, what is the meaning of a massless body in the context of the WEP? In classical physics if a body is truly empty and has nothing, then there is no meaning in its role in the WEP. In this study, we consider a massless body as a body with zero net mass. In other words, massless bodies can have zero net mass, which makes it possible for them to consist of parts—not empty. However, if the net mass of all parts is zero, then the mass of some parts must be negative.

The concept of negative mass is relatively new. It is abstract and has a sign opposite to that of a normal mass. In his 1928 hypothesis of electrons and positrons, Dirac [15] speculated on the existence of negative energy in a vacuum. Later, modern investigations of negative mass began in the 1950s. Ferrell [16], within the framework of Newtonian mechanics, discussed a possible way to shield the gravitational effect by applying a negative mass. The idea behind it assumed that negative mass could substitute for mass in Newton’s gravitational law and second law, but with a negative sign. He studied the interaction between mass and negative mass. One result was that mass could attract negative mass—negative mass moves toward mass—in the same way as mass attracts mass. Bondi [17], within the framework of General Relativity (GR), studied the uniform acceleration in a two-body system consisting of a body of normal mass and a body of negative mass separated by empty space. He concluded that the uniform acceleration did not violate GR. To date, there is no conclusive physical evidence that a negative mass has ever existed in a vacuum. However, this has not hindered people’s interest in studying it [18-25]. Recently, other possible applications of negative mass have been proposed in areas such as cosmology [26-27]. A conclusion from this research is that the existence of a negative mass in a vacuum does not violate the laws of Newtonian mechanics or GR. In our study, we adopt the concept of negative mass to construct massless bodies.

In the following sections, we first derive that the WEP applies to negative massive bodies in the same manner as massive bodies. Upon it, we further derive that the WEP applies to massless bodies which consist of masses and negative masses. Finally, we revisit Einstein’s thought experiment regarding the indistinguishable behaviors of massive bodies within an accelerating frame and in a gravitational field. Instead of studying the motion of massive bodies, we elaborate on the motion of massless bodies in the thought experiment.

2. Methodology

Based on previous studies, we can make three assumptions.

Postulate 1: A negative massive object can exist in a vacuum.

Postulate 2: Physical meanings are the same for a negative massive object as for a massive object — negative mass can substitute for mass in physical meanings (with the same magnitude but opposite sign).

Postulate 3: Physics equations are the same for negative mass as for mass—negative mass can substitute for mass in physics equations (with the same magnitude but opposite sign).

First, from postulate 2, we can say that a negative massive body has a negative gravitational mass ($-m_g$) and negative inertial mass ($-m_i$).

From Eq. (2) it follows

$$\frac{-m_g}{-m_i} = \frac{m_g}{m_i} = 1. \quad (3)$$

Next, from postulate 3, we can substitute $-m_g$ and $-m_i$, in Eq. (1), for m_g and m_i . We obtain

$$acceleration = \frac{gravitational\ mass\ (-m_g)}{inertial\ mass\ (-m_i)} (intensity\ of\ the\ gravitational\ field), \quad (4)$$

where $-m_g$ can be interpreted as the gravitational mass of a negative massive body, and $-m_i$ as the inertial mass of the negative massive body. Therefore, Eq. (4) describes the acceleration of a negative massive body falling in a gravitational field.

Plug Eq. (3) into Eq. (4), we can see that the acceleration does not change. That is, the acceleration is always the same for a given gravitational field regardless of the nature and condition of a negative massive body.

Hence, we conclude that the accelerations of all the negative massive bodies are the same in a gravitational field. That is, the WEP is applicable to negative massive bodies. Moreover, according to Eq. (1) and Eq. (4), the acceleration of falling negative massive bodies is the same as that of massive bodies. We summarize the first conclusion of the extended WEP as follows:

Conclusion 1: Negative massive bodies fall at the same rate as massive bodies in a given gravitational field.

2.1. Composing Massless Body

Next, let us explore the possibility of applying the WEP to massless bodies—with zero net mass. Theoretically, can such a massless body exist? We will start with a simple case. In the following thought experiment, we construct a massless body S_0 , which consists of two negative masses and a mass, as depicted in Fig. 1. We consider those masses as point particles in this case. Hence, we only consider themselves and the interactions between them.

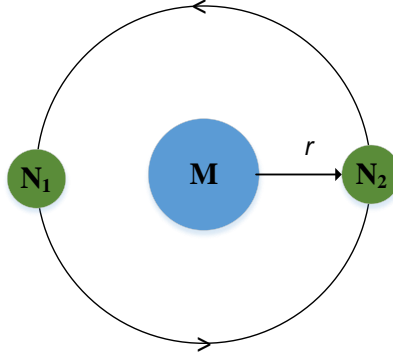


Fig. 1 A body consists of two negative masses (N_1, N_2) and a mass (M). The two negative masses have equal quantity ($-N$), and rotate around the barycenter of the body, which is located at the position of the mass. During their revolution, they are always on opposite sides of the orbit. r is the distance between a negative mass and the mass. With a certain relationship between the magnitude of M and N , the body is massless in its entirety

Fig. 1 shows a configuration in which a mass (M) is at the center, and two negative masses (N_1, N_2) move around it in a circular orbit. r is the distance between a negative mass and the mass, and both N_1 and N_2 have the same quantity $-N$. The two negative masses are always on opposite sides of the orbit. The barycenter of the entire body is located at the position of the mass.

Let us examine whether such a configuration is stable. We draw an axis from N_1 to N_2 , connecting M in the middle. We denote the vector pointing to N_2 from M by \hat{r} . Based on symmetry, the mass at the center receives the exact same magnitude of force from both negative masses, but with opposite directions. Therefore, it will be stationary in this configuration.

According to postulate 3, the force exerted on N_2 by N_1 is

$$F_{N_2 N_1} = -G \frac{-N(-N)}{4r^2} \hat{r}, \quad (5)$$

and the force exerted on N_2 by M is

$$F_{N_2 M} = -G \frac{-NM}{r^2} \hat{r}. \quad (6)$$

By adding Eq. (5) and Eq. (6) we have the net force on N_2

$$F_{N_2} = -G \frac{-N(-N)}{4r^2} \hat{r} - G \frac{-NM}{r^2} \hat{r} = -G \frac{N}{r^2} \left(\frac{N}{4} - M \right) \hat{r}, \quad (7)$$

by Newton's second law

$$F_{N_2} = -G \frac{N}{r^2} \left(\frac{N}{4} - M \right) \hat{r} = -N a_{N_2}, \quad (8)$$

where a_{N_2} is the acceleration of N_2 . We have

$$a_{N_2} = -\frac{G}{r^2} \left(M - \frac{N}{4} \right) \hat{r}. \quad (9)$$

If $M > \frac{N}{4}$, then $a_{N_2} < 0$. It is the centripetal acceleration of N_2 toward the barycenter of the body. That is, it keeps the negative mass (N_2) in circular motion around mass (M).

Similarly, the acceleration of N_1 is

$$a_{N_1} = \frac{G}{r^2} \left(M - \frac{N}{4} \right) \hat{r}. \quad (10)$$

If $M > \frac{N}{4}$, then $a_{N_1} > 0$. It is the centripetal acceleration of N_1 toward the barycenter of the body. That is, it keeps the negative mass (N_1) in circular motion around mass (M).

From Eq. (9) and Eq. (10) we notice that the magnitude of a_{N_1} equals that of a_{N_2} . That is, both N_1 and N_2 rotate (accelerate) around M at the same rate, which we call a_N

$$|a_N| = |a_{N_1}| = |a_{N_2}| = \frac{G}{r^2} \left(M - \frac{N}{4} \right). \quad (11)$$

Therefore, when $M > \frac{N}{4}$, the body S_0 is stable—all its constituents are bound together.

Now, let us determine if it is possible for S_0 to become massless in its entirety. In addition to the mass (M) and negative masses (N_1, N_2) considered individually, there is binding energy E_b between them inside S_0 . From the mass-energy equivalence principle, E_b can be converted into mass $\frac{E_b}{c^2}$. Hence, we have

$$M - 2N = M_s + \frac{E_b}{c^2}, \quad (12)$$

where M_s is the mass of S_0 (from the vantage point externally). The massless condition of S_0 requires its mass to be zero— M_s equals 0. By rearranging Eq. (12) we have

$$M - 2N - \frac{E_b}{c^2} = 0. \quad (13)$$

Furthermore, the binding energy E_b equals the negative sum of the total gravitational potential energy among the mass and negative masses, and the total kinetic energy of negative masses.

The total potential energy includes the potential energies between mass and two negative masses, and that between the two negative masses:

$$U = -G \frac{-NM}{r} - G \frac{-NM}{r} - G \frac{-N(-N)}{2r} = G \frac{4NM-NN}{2r}, \quad (14)$$

the total kinetic energy includes the kinetic energies for the two negative masses:

$$KE = \frac{1}{2}(-N)v^2 + \frac{1}{2}(-N)v^2 = -N|a_N|r = -G \frac{4NM-NN}{4r}, \quad (15)$$

therefore, the binding energy is:

$$E_b = -(U + KE) = -G \frac{4NM-NN}{2r} + G \frac{4NM-NN}{4r} = -G \frac{4NM-NN}{4r}, \quad (16)$$

plugging Eq. (16) into Eq. (13) we have

$$M - 2N + G \frac{4NM-NN}{4rc^2} = 0, \quad (17)$$

solving Eq. (17) we have

$$r = \frac{G}{4c^2} \cdot \frac{4NM-NN}{2N-M}. \quad (18)$$

From Eq. (9) and Eq. (10), we conclude that if S_0 is stable, it requires $M > \frac{N}{4}$. Hence, $4NM > NN$. From Eq. (18), if $2N > M$, then $r > 0$. Therefore, in order for S_0 to be stable and massless, the conditions $2N > M > \frac{N}{4}$ should be met. We can conclude that with some relationship of M and N , there exists a distance r between each negative mass and the mass, to make S_0 stable and massless in its entirety.

2.2. Motion of Massless Body

Thus far, we have constructed a massless object S_0 . Let us study its free fall in a gravitational field. From Eq. (13), we can interpret that S_0 consists of a mass (M), two negative masses (N_1, N_2), and an equivalent mass of $\frac{E_b}{c^2}$ corresponding to the binding energy inside S_0 . The WEP indicates that all massive objects fall at the same rate in a given gravitational field, and *Conclusion 1* says that all negative massive objects fall at the same rate as that of massive objects in the same gravitational field. Therefore, the mass (M) and the two negative masses (N_1, N_2) fall at the same rate within the same field. Since all the constituents of S_0 fall at a uniform rate, during the fall their relative positions remain the same in the same field.

From Eq. (16), the binding energy (E_b) depends on the relative positions of the constituents inside S_0 . In this case it is the r which is the distance between a negative mass and the mass. Since all its constituents (M, N_1, N_2) keep the same relative positions during the fall in a given gravitational field, the binding energy (E_b) remains the same. Hence, during the fall, Eq. (13) is still true. That is, S_0 remains massless. In fact, S_0 falls at the same rate as any of its constituents.

S_0 is not unique in the configuration of a massless object. As long as an object consists of masses and negative masses, with a zero net mass as a whole, it will qualify as a massless object. We can generalize the aforementioned reasoning for S_0 for any massless object.

Suppose a massless object C_0 consists of m masses: $a_1, a_2, a_3, \dots, a_m$; and n negative masses: $b_1, b_2, b_3, \dots, b_n$. Like Eq. (13) the massless condition of C_0 requires its net mass to be zero:

$$\sum_{j=1}^m a_j - \sum_{j=1}^n b_j - \frac{E_b}{c^2} = 0. \quad (19)$$

The first term in Eq. (19) is the total mass inside C_0 ; the second term is the total negative mass inside C_0 —the sign is negative since the sum of negative masses is negative; and the third term is the equivalent mass caused by the binding energy (E_b) inside C_0 . Therefore, we say that C_0 is composed of m masses and n negative masses as well as an equivalent mass caused by binding energy.

The WEP indicates that all massive objects fall at the same rate in a given gravitational field, and *Conclusion 1* says that all negative massive objects fall at the same rate as that of massive objects in the same gravitational field. Therefore, all the masses ($a_1, a_2, a_3, \dots, a_m$) and all the negative masses ($b_1, b_2, b_3, \dots, b_n$) fall at the same rate within the same field. Since all the constituents of C_0 fall at a uniform rate, during the fall, their relative positions remain the same in the same field.

The binding energy (E_b) depends on the relative positions of the constituents of C_0 . If the relative positions do not change, then the binding energy does not change either. Hence, during the fall, Eq. (19) is still true. That is, C_0 remains massless. In fact, C_0 falls at the same rate as any of its constituents.

Therefore, we conclude that if a massless body consists of masses and negative masses, then the massless body falls at the same rate as a massive body in a gravitational field. We summarize the second conclusion of the extended WEP as follows:

Conclusion 2: Massless bodies fall at the same rate as massive bodies in a given gravitational field.

3. Results

We have studied all possible types of bodies freely falling in a gravitational field. The WEP indicates that all massive bodies fall at the same rate in a gravitational field. Based on three postulates, we have derived that negative massive bodies fall at the same rate as massive bodies within the same field. Considering that massless bodies consist of masses and negative masses, we have derived that massless bodies fall at the same rate as massive bodies within the same field. Table 1 summarizes the motion of a body in a gravitational field with different amounts of mass.

Table 1. Motion of a body in a gravitational field

Amount of mass	Massive	Massless	Negative massive
Fall at the same rate as a massive body in a gravitational field	Yes	Yes	Yes

4. Discussion

Following Einstein's original elevator thought experiment, let us explore the motion of massless objects in thought experiments conducted in a uniform gravitational field and in a uniformly accelerating frame.

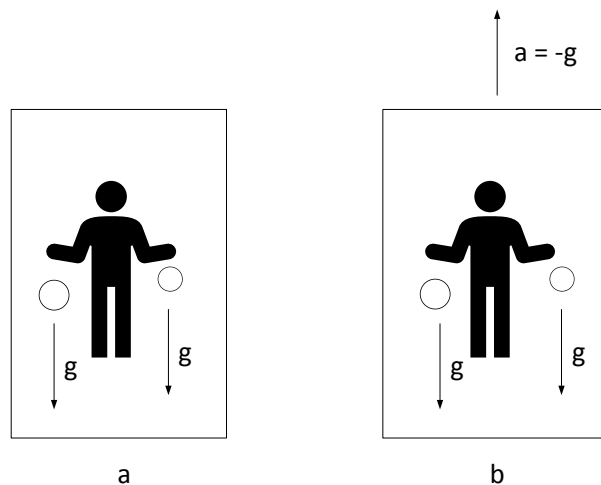


Fig. 2 An observer performs experiments involving the motion of test particles by dropping massless objects within two frames. (a) An observer stands in a closed case on Earth, which generates a downward acceleration of g . (b) The same observer is in an identical closed case, far from any gravitational fields, accelerating upward in a straight line at a rate of $a = -g$. For the same observer, the trajectories of the massless objects are measured the same in both cases

Suppose that an observer stands in a closed case on Earth, as depicted in frame (a) of Fig. 2. The case is in a downward gravitational field of g generated by the earth. If the observer drops massless objects, according to Table 1, the objects fall toward the floor of the case at an acceleration of g , similar to any massive object. Next, the same observer moves into an identical closed case, frame (b) of Fig. 2, far from any gravitational fields. By means of a rocket engine, the case is accelerating upward in a straight line at a rate of $a = -g$. If the observer drops the same massless objects, then due to the upward acceleration of g for the case, the objects—have no attachment to the case—appear to fall toward the floor of the case at an acceleration of g . Hence, the trajectories of the massless objects in both experiments appear to be the same to the observer. As a result, the observer cannot distinguish a uniform gravitational field from a uniformly accelerating frame by performing mechanical experiments involving massless objects.

This thought experiment, alongside Einstein’s original experiment, implies that in small enough regions of spacetime, the motion of freely falling objects is the same in a gravitational field as in a uniformly accelerated frame regardless of whether they are massive or massless.

Furthermore, we change the situation of frame (a) in Fig. 2 to the situation in which the frame falls freely in the gravitational field of the earth, as depicted in Fig. 3.

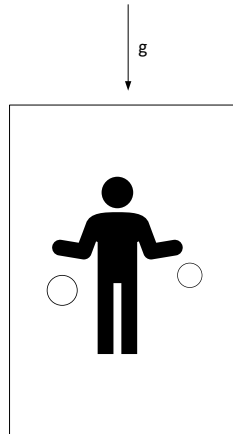


Fig. 3 An observer is in a free-falling frame in the gravitational field of the earth generating a downward acceleration of g . The observer performs experiments involving the motion of test particles by dropping massless objects. The observer would see those objects seemingly floating or moving at constant velocity, just like in an inertial frame

Fig. 3 shows there exists a frame in which massless objects maintain a uniform velocity. This is a frame which falls (accelerates) freely in the gravitational field of the earth. If an observer is inside the frame but not attached to it, the observer accelerates at the same rate (g) as the frame (g). According to Table 1, the observer falls at the same rate as the free massless objects inside

the frame. To the observer, those massless objects seem to be at rest or move at constant velocity—the same phenomenon the observer would expect in an inertial frame. Hence, the freely falling frames are equivalent to the inertial frames for performing mechanical experiments involving massless objects.

Finally, we can describe the extended WEP in a variety of formulations:

Every object falls at the same rate—undergoes the same acceleration—in a gravitational field, independent of the composition of the object, regardless of whether it is massive, massless, or negative massive.

In small enough regions of spacetime, the motion of freely falling bodies—regardless of whether they are massive, massless, or negative massive—is the same in a gravitational field and in a uniformly accelerated frame.

It would not be able to distinguish—by performing mechanical experiments, regardless of whether test objects are massive, massless, or negative massive—a frame that is accelerating in free fall in a uniform gravitational field from one that is inertial and without a gravitational field.

5. Conclusion

The WEP has played a key role in the formation of GR. However, many studies have only focused on how massive bodies obey the WEP. In this article, we have satisfactorily extended the WEP to massless bodies. Furthermore, we have elaborated that it is impossible to distinguish a uniformly accelerated frame from a uniform gravitational field by performing mechanical experiments involving massless bodies.

The WEP implies that once gravity is included, every observer of any accelerated reference frame is on equal footing to study the motion of massive bodies. In addition, the extended WEP implies that for any types of bodies involved, regardless of whether they are massive or not, any observer can be on equal footing to study their motion.

We feel satisfied that the extended WEP applies to all types of bodies. Naturally, we ask the following question: can the Einstein Equivalence Principle also be extended to all types of bodies? We would like to further investigate how massless bodies fit into the Einstein Equivalence Principle.

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Data availability statement

The datasets used and/or analysed during the current study available from the corresponding author on reasonable request.

Author's contribution statement

All authors contributed to the study conception and design.

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