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Antoine Benady, Emmanuel Baranger, Ludovic Chamoin

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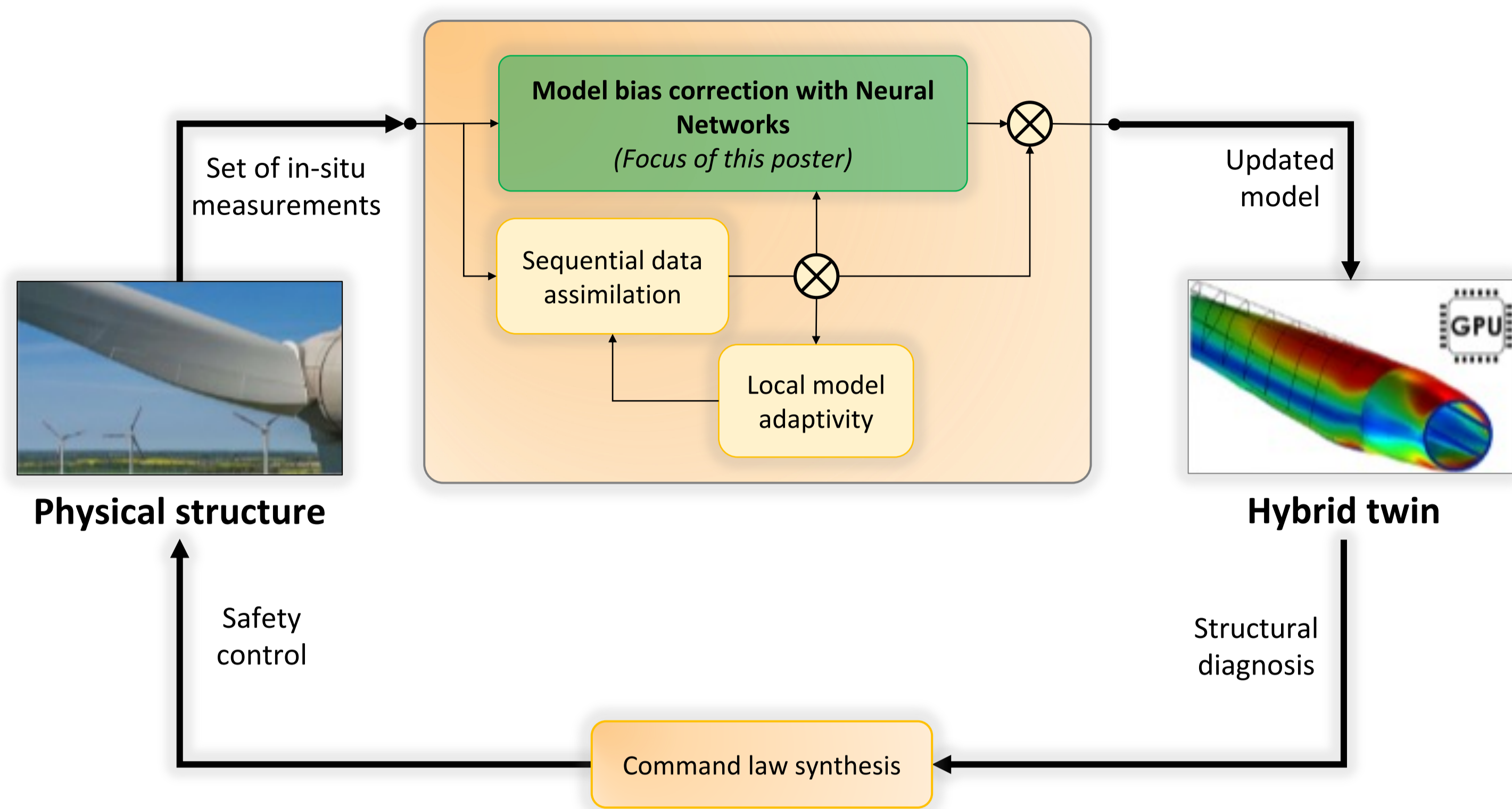
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# PHYSICS-AUGMENTED NEURAL NETWORKS FOR CONSTITUTIVE MODELING: TRAINING WITH THE MODIFIED CONSTITUTIVE RELATION ERROR (MCRE)

Antoine Benady, Emmanuel Baranger, Ludovic Chamoin  
antoine.benady@ens-paris-saclay.fr

**Keywords:** Constitutive modeling, Physics-augmented Neural Network, Unsupervised learning, Constitutive relation error, Plasticity

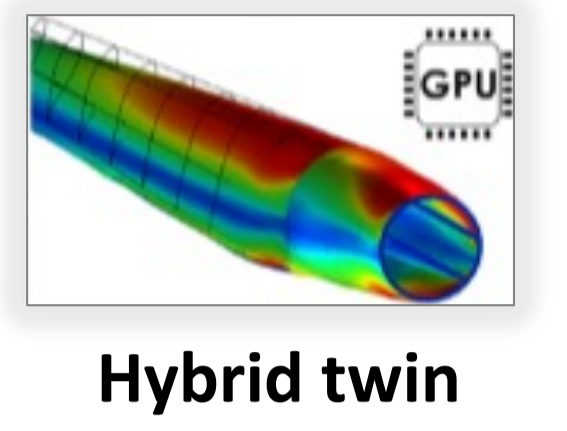
Context: hybrid-twin for real-time control



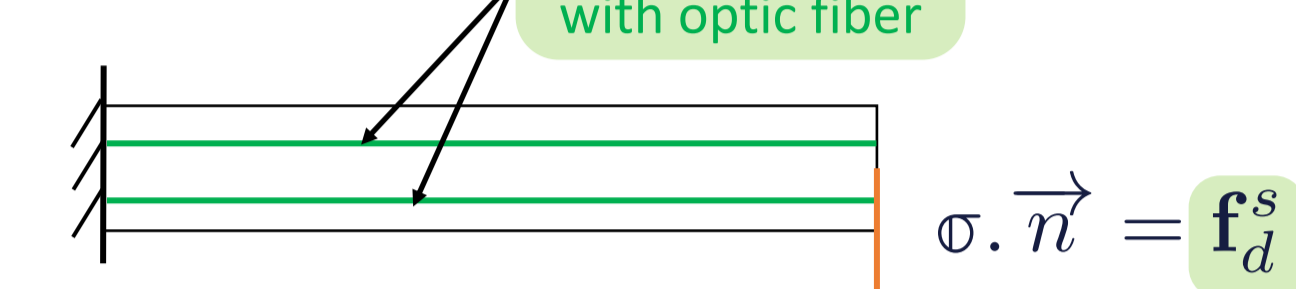
Problem definition



- Information available to the hybrid twin
- To learn (information not available)



$$\mathbf{u}|_{\partial\Omega_1} = \mathbf{u}_d$$



Unknown constitutive relation

Reliable information:

- Admissibility
- Thermodynamics consistency

Noisy information:

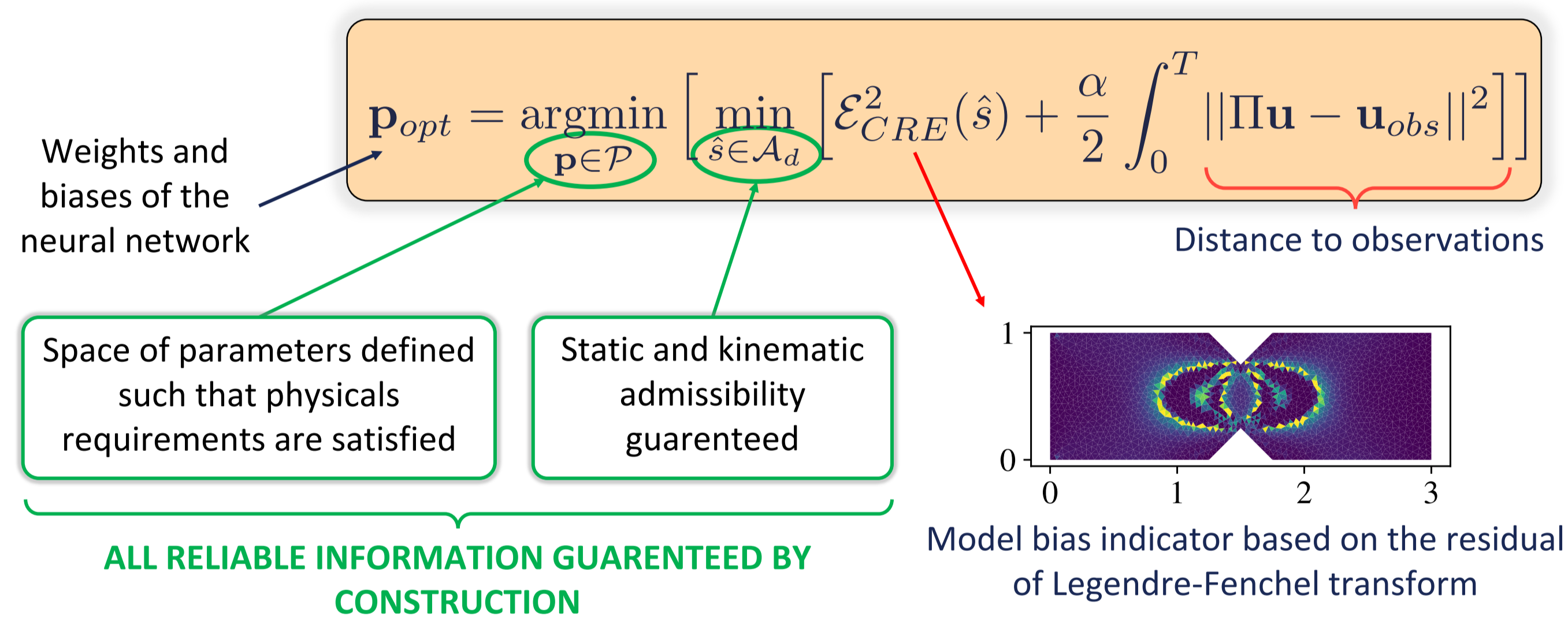
Measurements noise level known a priori

Unreliable information: initial idea on the constitutive relation. To relax the model form, the constitutive relation is represented by a thermodynamics consistent neural network.

A way to automatically satisfy thermodynamics is to established the constitutive model with:

- A convex Helmholtz free energy  $\psi$  involved in set the state laws,
- A convex, non-negative, zero at origin pseudo-potential  $\varphi$  involved in the set of evolution laws.

The modified Constitutive Relation Error (mCRE)



Learning of nonlinear state laws

While mCRE has not converged:

1. We look for admissible fields that minimize mCRE:

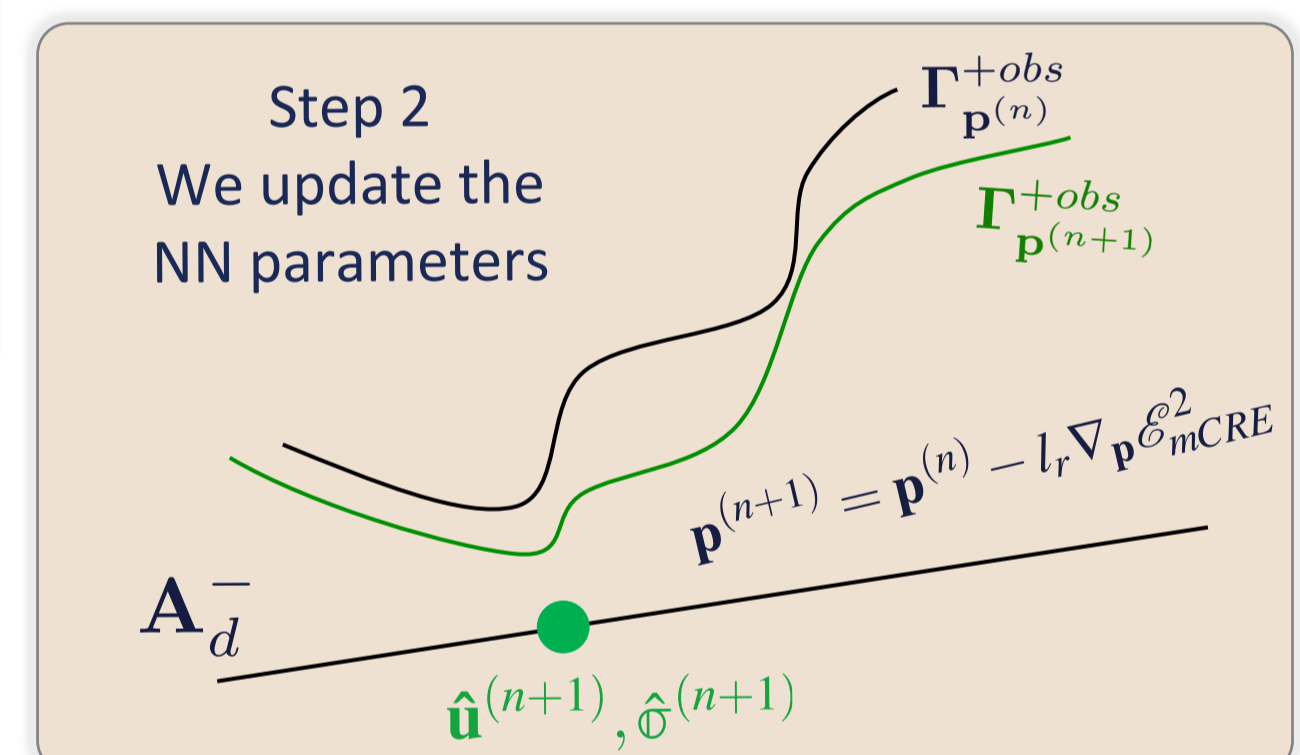
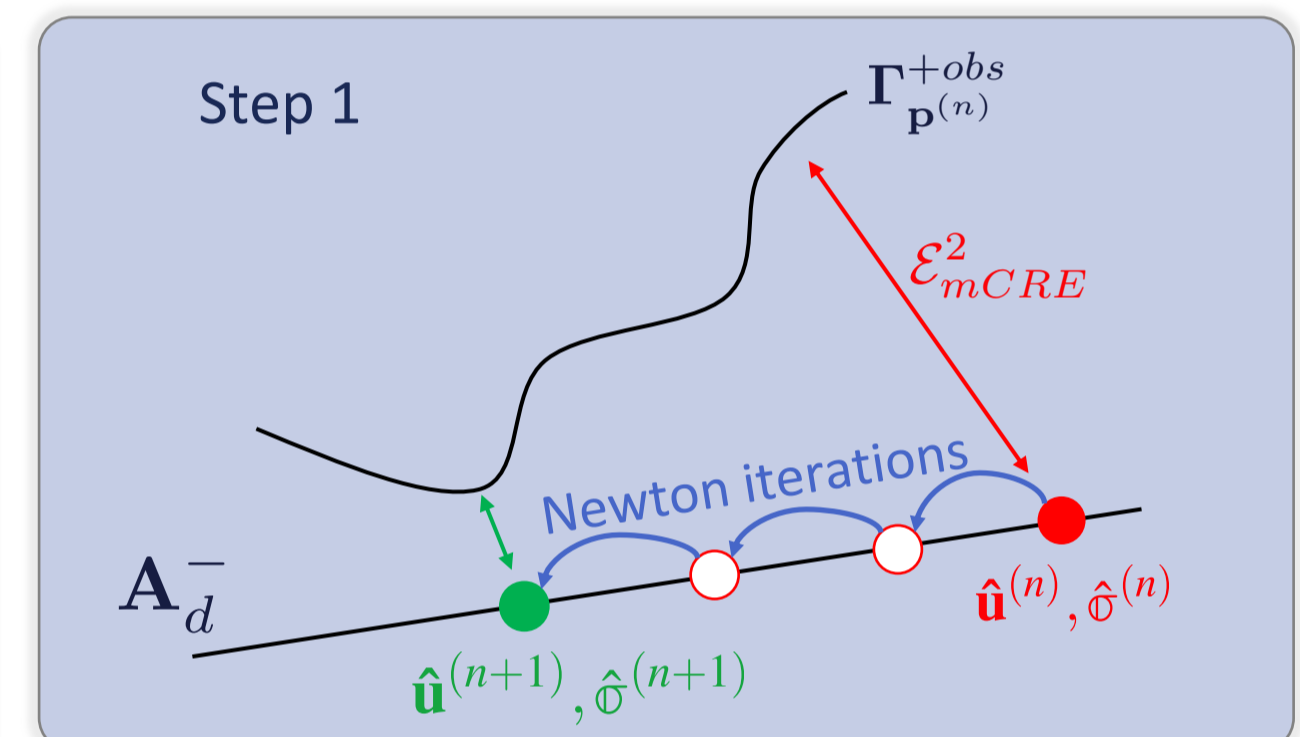
$$(\hat{\mathbf{u}}^{(n+1)}, \hat{\sigma}^{(n+1)}) = \operatorname{argmin}_{(\hat{\mathbf{u}}, \hat{\sigma}) \in (\mathbf{A}_d^-)} \mathcal{E}_{mCRE}^2(\hat{\mathbf{u}}, \hat{\sigma}; \mathbf{p}^{(n)})$$

- Kinematic admissibility enforced in the search space
- Equilibrium imposed by a Lagrangian
- Stationarity of Lagrangian through a Newton method

2. We update parameters with gradient descent step:

$$\mathbf{p}^{(n+1)} = \mathbf{p}^{(n)} - l_r \nabla_{\mathbf{p}} \mathcal{E}_{mCRE}^2(\hat{\mathbf{u}}^{(n+1)}, \hat{\sigma}^{(n+1)}; \mathbf{p}^{(n)})$$

→ Gradient computed with adjoint state method



High robustness to noise level is achieved: up to 40% noise

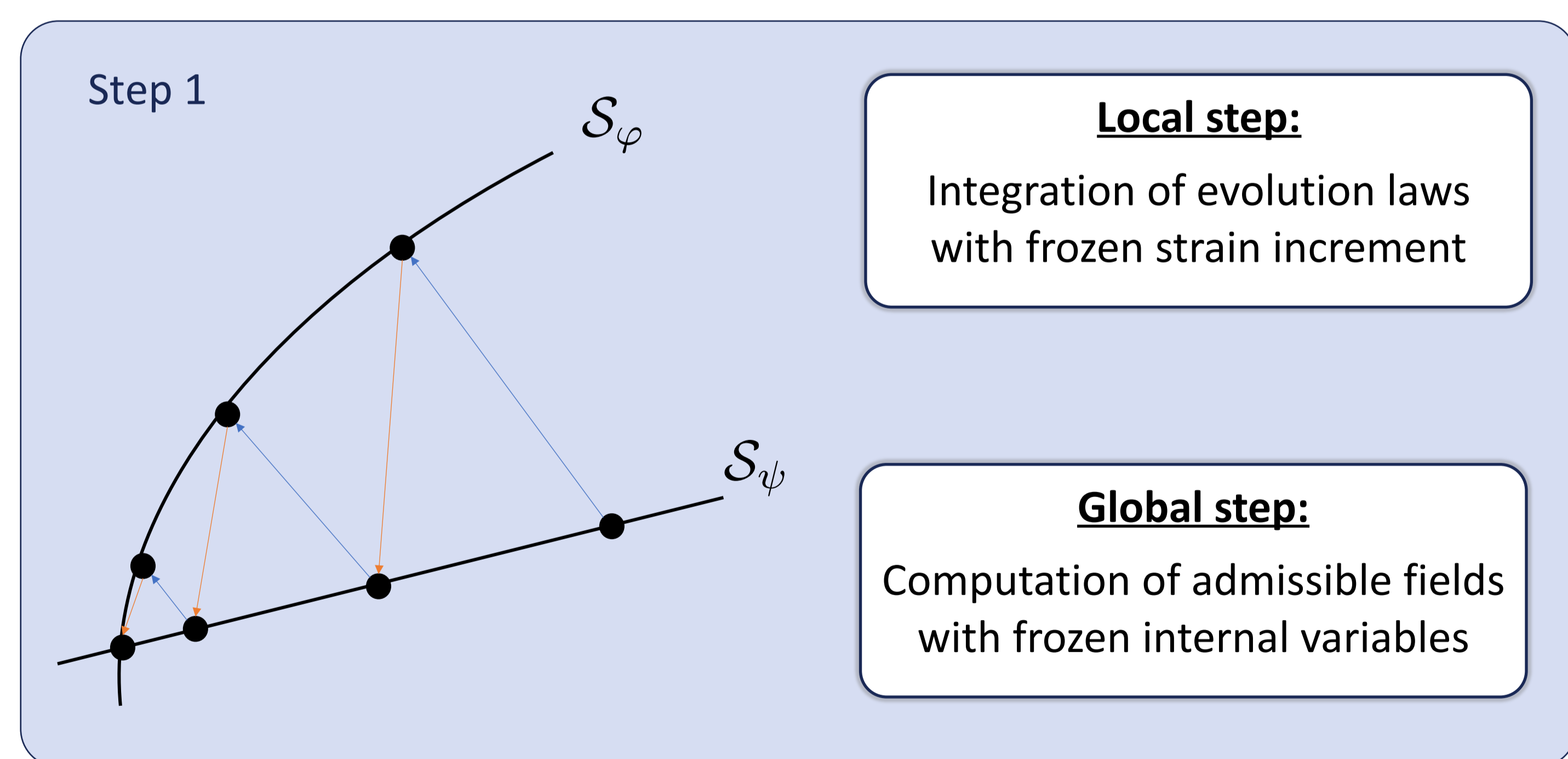
Good generalization when database is rich enough

Method also tested with Mooney-Rivlin hyperelastic model:

$$\psi(\mathbf{E}) = A(\bar{\mathbf{I}}_1 - 3) + B(\bar{\mathbf{I}}_2 - 3) + C(\bar{\mathbf{I}}_1 - 3)(\bar{\mathbf{I}}_2 - 3)$$

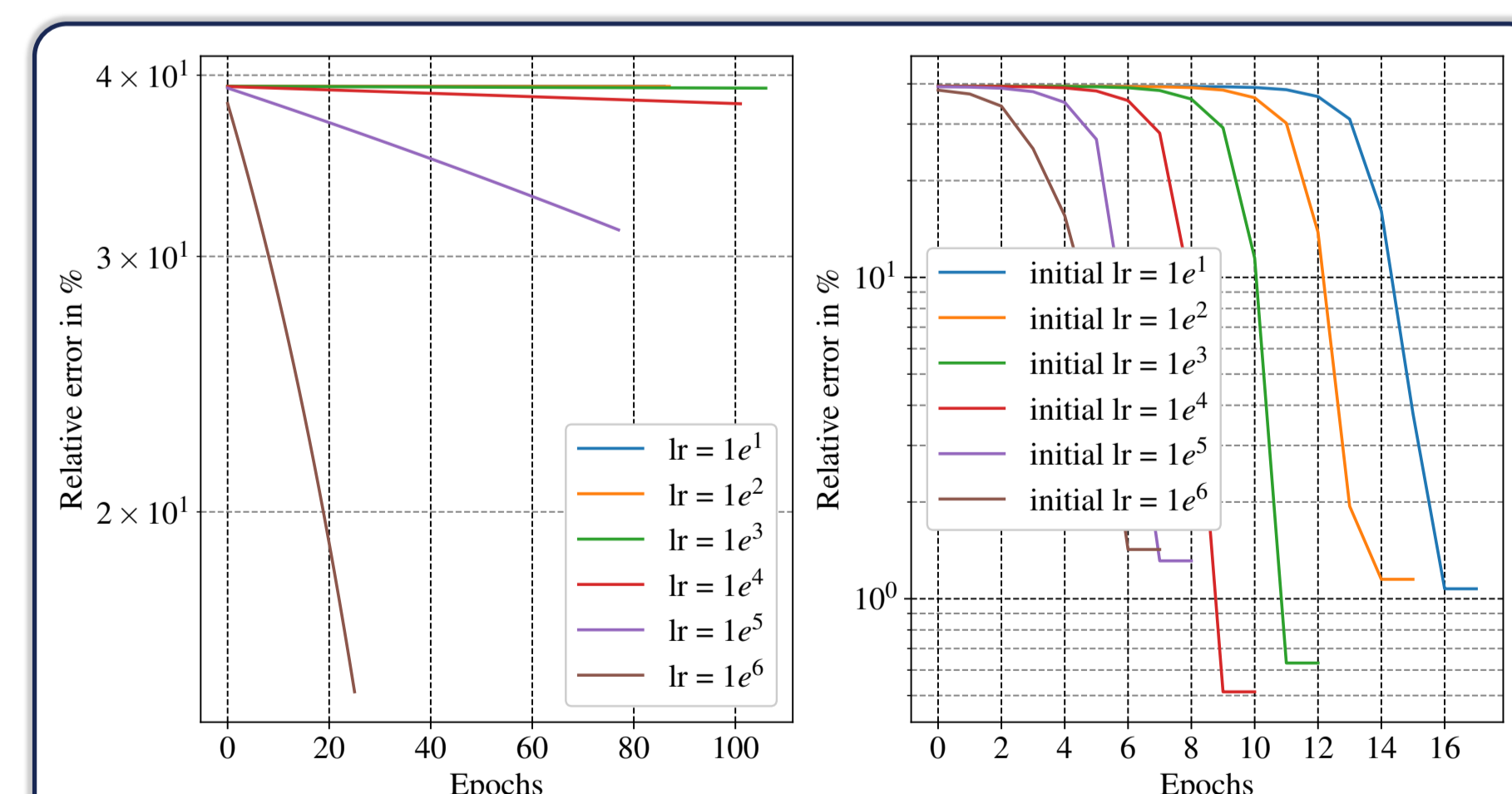
Learning of history-dependent behavior

For history-dependent behavior, the minimization of the mCRE is slightly different. The Step 1 is performed with a LATIN-inspired method composed of the alternance of a local step (integration of evolution law) and a global step (computation of admissible fields).



What about hyperparameters tuning ?

Parameters	Tuning
Learning rate	Adaptive learning rate during training
Weighing between losses	Morozov-based automatic tuning
Number of epochs	mCRE-based stop criteria
Network initialization	Initialization with simple model
Batch size	Fixed by mesh



Convergence without (left) and with (right) adaptive learning rate  
The training is no longer sensitive to user-defined hyperparameters.

Conclusion

Neural network trained to learn constitutive model with observable data

All reliable information is guaranteed by construction

No need for user to tune hyperparameters

Model used to generate data	Model to train
Elastic part of the free energy: linear elasticity	Form and parameters known
Plastic part of the free energy: $\psi_p(\epsilon_p) = A p + \frac{A}{B}(e^{-Bp} - 1)$	Neural network initialized with linear hardening law
Von Mises criterion for the limit of elasticity domain	Form and parameters known

