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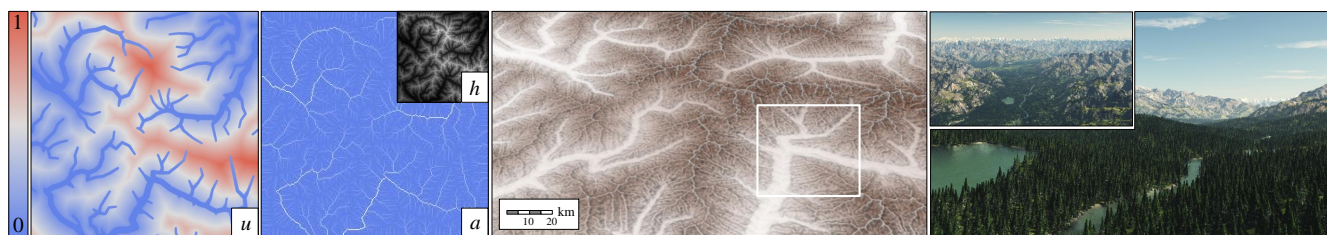
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# Large-scale terrain authoring through interactive erosion simulation

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**Figure 1:** Given an input uplift field defined as a hierarchical construction tree, we automatically generate the large-scale terrain elevation by simulating stream power erosion using a parallel and iterative drainage area algorithm inlined in the simulation.

## Abstract

Large-scale terrains are essential in the definition of virtual worlds. Given the diversity of landforms and the geomorphological complexity, there is a need for authoring techniques offering hydrological consistency without sacrificing user control. In this paper, we bridge the gap between large-scale erosion simulation and authoring into an efficient framework. We set aside modelling in the elevation domain in favour of the uplift domain, and compute emerging reliefs by simulating the stream power erosion. Our simulation relies on a fast yet accurate approximation of drainage area and flow routing to compute the erosion interactively. Our model provides landscape artists with tools for shaping mountain ranges and valleys, such as copy-and-paste operations; warping for imitating folds and faults; point and curve elevation constraints to precisely sculpt ridges or carve river networks. It also lends itself to inverse procedural modelling by reconstructing the uplift from an input digital elevation model and allows hydrologically consistent blending between terrain patches.

**Keywords:** Erosion simulation, interactive authoring

## 1. Introduction

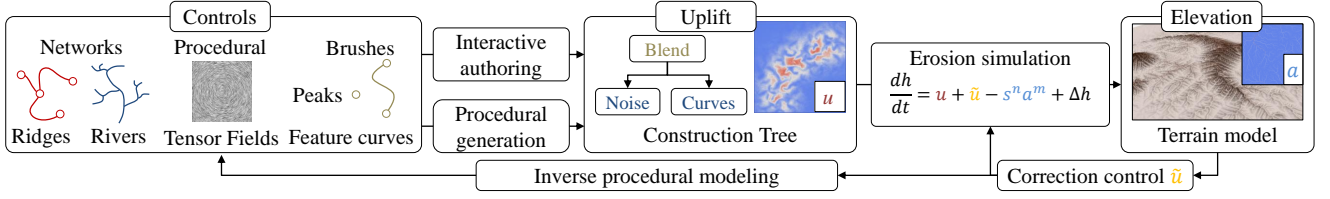
Modelling realistic and controllable large-scale terrain models is essential for creating virtual worlds. The challenge stems not only from the complexity of landforms, the variety of details forming patterns at different scales, the need for geomorphological and hydrological realism, but also from the need to control the shape and location of landforms to follow the designer's intent.

An overview of existing generation methods can be found in [GGP\*19]. We improve the otherwise computationally intensive stream power simulation developed in [CCB\*18] by accelerating the drainage area computation with a parallel algorithm suitable for graphics hardware. Moreover, we provide ways for the designer to define or impose the location, the orientation, and the distribution of specific mountain ranges while preserving the global hydrological consistency of the final terrain (Figure 1). More precisely, our contributions are as follows: 1) We propose a model allowing to author

the terrain in the elevation domain or in the uplift domain. 2) We propose an iterative approximation of the drainage area, allowing for a parallel implementation and embedding in the stream power erosion simulation, which produces a coherent drainage computation. 3) We introduce the Uplift Tree, a hierarchical vector-based construction tree taking its inspiration from the implicit models construction trees, but with the particularity that it can represent complex uplift scalar fields.

## 2. Overview

The elevation of the terrain results from an equilibrium between uplift and stream power erosion (also called fluvial erosion). We depart from tectonic-based computation and propose to rely on a construction tree to control the uplift that indirectly defines the shape and elevation of the mountain ranges. Our authoring framework rests on the foundations of a fast erosion simulation based on



**Figure 2:** Our method computes the elevation  $h$  from the uplift  $u$  using a modified Stream Power law. Control is achieved by prescribing  $u$  from a user-defined or procedurally-generated Uplift Tree.

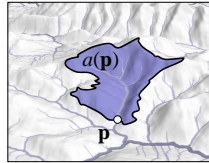
an incremental and parallel drainage area approximation. Figure 2 presents a synthetic overview of our method. The user constructs the uplift  $u$  using brushes and the erosion simulation carves the mountain ranges. At any time, the user can modify  $u$ , while the stream power erosion simulation ensures that the resulting elevation is coherent and flows towards the borders of the map.

The shaping of mountainous landscapes by the Stream Power law has been extensively studied in geomorphology [WT99]. The simplified governing equation can be written as:

$$\frac{\partial h}{\partial t} = u - s^n a^m + \Delta h \quad (1)$$

The term  $u - s^n a^m$  states that the rate of change of surface topography  $h$  is controlled by the balance between the uplift  $u$  and the fluvial erosion, which is a function of the local slope  $s$  and the drainage area  $a$  that represents the amount of water flowing through a point. Geomorphology studies [TK20] agree that only the ratio  $m/n$  is relevant and should be set to  $\approx 1/2$ . The hillslope term  $\Delta h$  smoothes the terrain by elevating concavities and lowering convexities by using the Laplacian.

The drainage area  $a$  of a point  $\mathbf{p}$  represents the amount of water that flows through  $\mathbf{p}$ . It is the most computationally intensive part of the stream power erosion, as it requires processing cells in descending elevation order. Our work relies on an efficient and accurate approximation of  $a$  (Section 3).



**Figure 3:** Drainage area of a point and its watershed.

The uplift defines the speed at which the ground is elevating due to tectonic processes and plays a crucial part in the shape of large-scale mountain ranges. It is defined using a hierarchical construction tree, the *Uplift Tree*, an efficient vector-based model for constructing and interactively authoring uplift maps. Contrary to elevation models, it does not require continuity.

User-control is central to our framework and involves a variety of tools for authoring complex large-scale terrains. In the uplift domain, it is achieved by directly modifying  $u$  using brushes that operate on the underlying Uplift Tree model (Section 4).

### 3. Interactive erosion simulation

We address the incremental and interactive resolution of the stream power equation. Let  $h_i, a_i, u_i, s_i$  denote the elevation, drainage area, uplift and steepest slope of a cell  $c_i$  respectively. Let  $\Delta t$  denote the time step, at iteration  $k + 1$ , we have:

$$h_i(k+1) = \Delta t (u_i(k) - s_i^n(k) a_i^m(k) + \Delta h_i(k)) \quad (2)$$

We propose an approximation to the computation of the drainage area which can be directly integrated in the explicit integration scheme of equation 2.

**Flow routing.** The efficient computation of the drainage area, based on flow routing between cells, is crucial in the stream power erosion simulation. We choose to always propagate the flow in the steepest slope direction. Let  $\mathcal{V}_i$  the set of 1-neighbouring cells of  $c_i$ . Let  $\mathcal{A}_i = \{j \in \mathcal{V}_i \mid i = \arg \max_{k \in \mathcal{V}_j} s_{kj}\}$  denote the neighbouring cells with a steepest downward slope leading to  $c_i$ . Overall, in the discrete case, the drainage area  $a_i$  of a cell  $c_i$  can be recursively defined as the weighted sum of the drainage area  $a_j$  of the cells  $\mathcal{A}_i$ , summed with the local precipitation (taken as 1 for simplification).

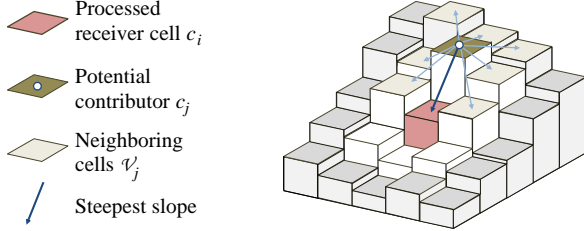
$$a_i = 1 + \sum_{j \in \mathcal{A}_i} a_j \quad (3)$$

**Parallel approximation of the drainage area.** In the general case, the computation of the drainage area proceeds as follows. Cells are first sorted according to their elevation and assigned with their local precipitation. Cells are then processed in descending order, incrementally computing the drainage area by propagating the values to lower cells according to the flow routing equation. This algorithm has  $O(n^2 \ln n)$  complexity (with  $n$  the width of the terrain in cells) and cannot be easily parallelized.

One particular challenge in accelerating the erosion simulation is to compute the drainage area in parallel. Our solution consists of an iterative approximation of the drainage area that progressively converges to the exact value and can be used with the erosion algorithm. At every step, we test which of the 8 neighbouring cells  $\mathcal{V}_i$  are flowing on  $c_i$  (see Figure 4), and update the approximated drainage value  $\tilde{a}_i$ . The values  $\tilde{a}_i(k)$  obtained at iteration  $k$  are used to compute the flow contributions at the next iteration  $k + 1$ :

$$\tilde{a}_i(k+1) = 1 + \sum_{j \in \mathcal{A}_i} \tilde{a}_j(k) \quad (4)$$

The convergence rate depends on the length of the longest flow path in the terrain. A terrain with a pyramidal shape with its peak at the centre is optimal, with a maximum flow path length of  $n/2$ . In the worst case, a flow path is meandering through the terrain [HAP20], producing flow paths with  $O(n^2)$  length. The drainage area network progressively improves through the first iterations and becomes rapidly accurate over the entire domain.



**Figure 4:** A neighbouring cell contributing to the processed cell  $c_i$ .

Using an iterative solver rather than computing the exact solution is largely compensated by the parallel execution. The approximation proves to be sufficiently fast and accurate to be inlined with the erosion process. The stream power erosion algorithm converges to the steady-state elevation in a few seconds for  $n \leq 1024$ .

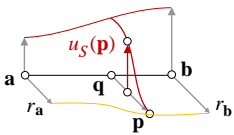
#### 4. Uplift construction tree

To build uplift maps efficiently, we introduce a hierarchical construction tree, the *Uplift Tree*, that is synthesized and updated interactively. The leaves are primitives defined over a compact region in  $\mathbb{R}^2$ . The nodes are either unary operators (deformations or affine transformations) or binary operators (minimum, maximum, blending) that combine values calculated by their sub-trees. The value at a given point is evaluated by recursively traversing the tree.

**Primitives.** The base function of a primitive, denoted as  $u_{\mathcal{P}}$ , combines the distance  $d$  to a skeleton  $\mathcal{S}$  with a falloff function  $g$  parameterized by a radius  $r$  and a maximum value  $u_{\mathcal{S}}$  on the skeleton:

$$u_{\mathcal{P}}(\mathbf{p}) = g \circ d(\mathbf{p}) \quad g(d) = \begin{cases} u_{\mathcal{S}} (1 - d^2/r^2)^3 & \text{if } d \leq r \\ 0 & \text{otherwise} \end{cases} \quad (5)$$

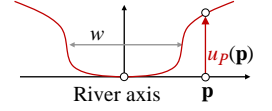
We describe here two typical primitives useful for creating large-scale landscapes: the first used for mountain range modelling, the second designed to carve valleys.



**Figure 5:** Mountain range uplift profile.

The segment mountain range primitive is based on segment-skeleton  $\mathcal{S} = [\mathbf{a}, \mathbf{b}]$  associated with a varying radius  $r : [0, 1] \rightarrow \mathbb{R}_+$  and profile  $u_{\mathcal{S}} : [0, 1] \rightarrow \mathbb{R}_+$  along the skeleton. For any point  $\mathbf{p}$  in the plane, we compute its projection  $\mathbf{q}$  on the segment. Let  $x = \|\mathbf{a} - \mathbf{q}\| / \|\mathbf{a} - \mathbf{b}\|$  clamped between 0 and 1, we compute the corresponding radius  $r(x)$  and value on the skeleton  $u_{\mathcal{S}}(x)$ . We finally compute  $u_{\mathcal{P}}(\mathbf{p})$  by evaluating the distance  $\|\mathbf{p} - \mathbf{q}\|$  and applying the falloff function of equation (5) with radius  $r(x)$  and skeleton  $u_{\mathcal{S}}(x)$ .

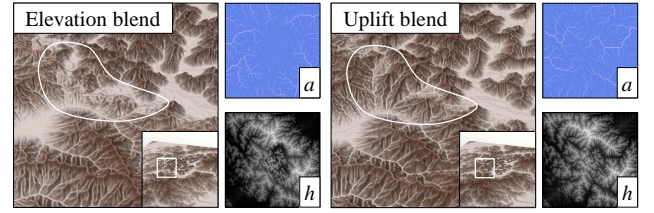
The valley primitive is also based on a segment-skeleton  $\mathcal{S} = [\mathbf{a}, \mathbf{b}]$ , whereas the classic falloff is replaced by a *valley profile* curve, with low values in the valley zone, and high values outside (see Figure 6). The main parameter of the profile is the varying width of the valley  $w : [0, 1] \rightarrow \mathbb{R}_+$  along the segment. The user may edit specific profile curves, keeping in mind that high slopes are required to produce valleys during the simulation.



**Figure 6:** Valley uplift profile curve.

**Operators** offer a flexible way to transform a subtree or combine several ones. Blending or warping in the elevation often yields unrealistic artificial-looking terrains with inconsistent drainage network featuring numerous pits, and a loss of the characteristic geometric proportions of landforms.

Working in the uplift domain alleviates this limitation. Extreme deformations or even discontinuities are correctly handled by the erosion process and do not violate the geomorphological and hydrological coherence of the resultant terrain. Figure 7 illustrates the advantages of uplift operators, through an example of terrain blending.



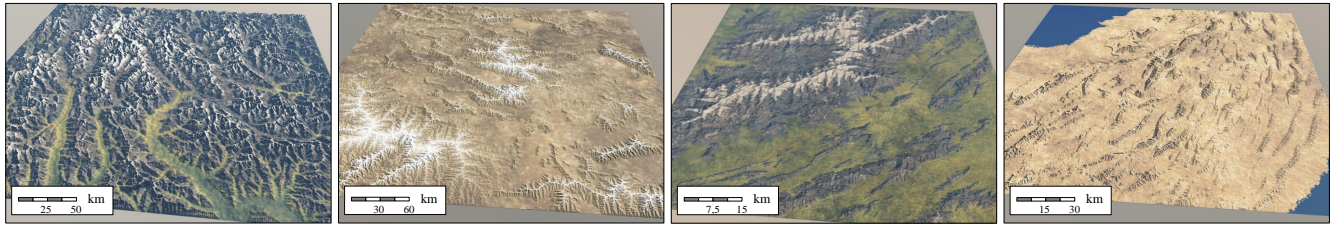
**Figure 7:** Two terrains obtained from the erosion simulation are blended together using their elevation or their uplift. The loss of landmarks such as dendrites is emphasised in white. The drainage areas show that the second method preserves a consistent flow.

#### 5. Results

We implemented our method in C++, the drainage area and stream power erosion algorithms were coded as GLSL compute shaders. Experiments were performed on a desktop computer equipped with Intel® Core i7, clocked at 4GHz with 16GB of RAM, and an NVidia GTX 1080ti graphics card.

**Control.** Although indirect, the Uplift Tree provides control over the location and the average elevation of mountains and valleys; and over the trajectories of ridges and rivers. Users can model large-scale terrains featuring different types of valleys and landforms in a few minutes, as demonstrated in the accompanying video.

**Performance.** Erosion simulations are a key modelling tool for virtual terrains and achieving interactive visualisation helps artists and geologists understand the processes and tune parameters accordingly. While the exact graph-based drainage area computations of [CCB\*18] allows for fewer steps, our method still converges

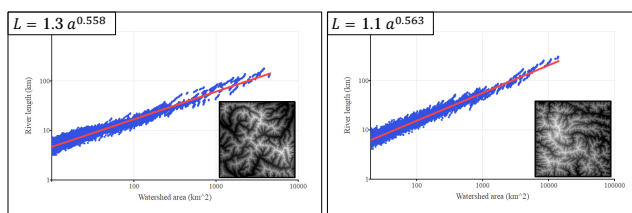


**Figure 8:** Various large scale landscapes obtained through the erosion simulation, inspired from Himalayas, Nevada, Appalachians and Iran (from left to right)

faster, thanks to the parallel implementation on graphics hardware. This more efficient stream power erosion achieves interactive frame rates on large grids up to  $2048 \times 2048$ .

**Validation.** The consistency of the surface flow of water is a crucial aspect of digital landscape realism. We qualify a river network as consistent when the flow is directed to the boundary of the domain, without pit-cells retaining water. Most terrain synthesis techniques do not guarantee the consistency of the water flow, and require a pit-removal process: most often depression breaching or filling. In contrast, our erosion simulation based on the stream power equation converges to a consistent water flow and a more natural river network.

Evaluating the plausibility of a generated terrain is not a simple assignment. One solution consists in assessing the geomorphological coherency by verifying if observations made by geologists remain valid for synthesized models. We rely on Hack's river scaling power law observed in geomorphology [Hac57]. For a given point on the terrain, Hack's law correlates the area of its watershed (fundamentally its drainage area)  $a$  to the length of its longest upstream river  $l$  as  $l = ca^n$ . The terms  $c$  and  $n$  are constants referred to as Hack's coefficient and Hack's exponent, studies [SSCF18] report a range of  $[1, 6]$  for  $c$  and  $[0.45, 0.7]$  for  $n$ .



**Figure 9:** Hack's Law validation for two terrains generated with the erosion framework.

Figure 9 reports statistics for two different terrains generated using the stream power erosion simulation. The obtained Hack's coefficients  $c$  and  $n$  lie in a valid range which demonstrates that our model conforms to this power law. The red line was fitted on the log-scaled terrain data using a least square method. We tested all our generated models, and the results indicate that they validate Hack's law.

In addition to realism, an authoring framework is also characterized by the variety of the terrains it can produce. Figure 8 shows

four different outputs of the erosion simulation and demonstrates that our model is capable of producing various types of large scale landscapes.

## 6. Conclusion

Combining the stream power erosion simulation with the uplift construction tree provides foundations for interactive large-scale terrains authoring. Our method authorises painting, sketching, and copy-and-paste of terrain fragments with automatic generation of hydrologically consistent reliefs. The controlled erosion formalism bridges the gap between simulation and authoring. Our fast parallel drainage area computation algorithm allows designers to interactively compose the large-scale shape of complex landforms with a broad spectrum of procedural and modelling tools, including landform feature primitives and ridge or river networks.

This work opens avenues for future research. In particular, improving the stream power erosion model to take into account sediments or even glacial erosion would be a direction worth investigation for modelling a broader range of landforms.

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