A micro foundational episode of the early history of macroeconomics: a 1932 debate on Walrasian economics and multiple equilibria

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Abstract:

This paper documents an early fork in the development of macroeconomics, by examining a debate between the Dutch economists Jan Tinbergen and Johan Koopmans. In a 1932 paper, Tinbergen argued that two firms could be stuck in a "bad" equilibrium in the absence of a coordinated action to increase employment. Koopmans replied with a paper demonstrating that multiple equilibria in an exchange economy could not be ranked on the basis of their productive efficiency. This debate contributed to a larger turn away from dynamizing the general equilibrium model, towards the new field of macrodynamics, with long-ranging consequences for the field.

Keywords: coordination, Jan Tinbergen, multiple equilibria, macroeconomics, general equilibrium
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In 1932, at the request of the Dutch Vereeniging voor de Staathuishoudkunde en de Statistiek (Association for Political Economy and Statistics), Tinbergen was invited to give a talk "to explain the position of socialism with respect to the question of the regulation of production by private individuals."[2] Tinbergen, a member of the board of the Dutch magazine *The Socialist Guide* (*De Socialistische Gids*), had already had several occasions to express his view, which he saw as one possible approach rather than "the" view of socialists. He took up the socialist argument that "free competition" of individuals leads to a waste of resources, because in the absence of a planning device, economic activity was prone to recurrent crises and loss of "productive forces and thus to a lower level of prosperity than would be possible" (Tinbergen, 1932: 50). One way of representing this was to view crises as the manifestation of "unstable economic processes," making the return to equilibrium difficult or impossible. This raised for him the question of how to represent such a situation, and Tinbergen presented two possible avenues of research in this pre-advice

The first one was to extend various existing models such as the cobweb model, to identify "theoretical (as well as experimental) cases of economic processes that do not automatically return to the equilibrium position" (Tinbergen, 1932: 58). The second one was to consider that crises would leave the economy stuck in a position of "bad" equilibrium. Tinbergen suggested that such a situation could be represented in the framework of a Walrasian model of the economy, where multiple equilibria could appear under certain general conditions. Along that line, he explored the possibility that the

[2] See Knoester and Wellink (1993: 19-20) on this address and the rest of the paper as well as their book on the wider context of the talk, one of several given by Tinbergen in front of the Association, which became the Netherland Economic Association in 1950 and received the "Royal" title in 1987. This talk was published along with the one given by R. van Genechten on the same subject. See also Dekker (2023) on other preadvices written by Tinbergen during this period.

severity of the depression of the 1930s may be the result of a coordination failure which would be responsible for the fact that the economy may reach a stable, but "bad" equilibrium (with mass unemployment) from which no individual agent could unilaterally escape. This argument was sketched in broad lines with reference to Walras, Cassel, and the unpublished works of Johan G. Koopmans, and supported by a diagram without an explicit system of equations.[3]

Spurred by Tinbergen's reference to his work, in a three-part paper published in *De Economist* in 1932, Koopmans recognized that Tinbergen's system was determinate and could lead to non-trivial equilibria, but argued that Tinbergen misrepresented the meaning of the multiple equilibria arising from a Walrasian system. He emphasized that Tinbergen's approach was different from his own. Koopmans' point was that a Walrasian system was neither able to define a state of unemployment (in the absence of price and wage rigidities), nor to show that an equilibrium may be better than another.

This debate has only been mentioned in passing by previous research on the two Dutch economists (Fase, 1998: 128; Dekker, 2023: 10, Rodenburg, 2010: 10). It is not referred to by other economists of the time, and might appear at first sight purely academic, a "micro" event in the larger history of macroeconomics. This present paper, by reconstructing the debate, will show that it takes a foundational importance when it is reinserted into a larger shift of the study of economic crises and instability, towards the study of macrodynamic systems.

[3] Johan Gerbrand Koopmans (1900-1958), not to be confused with the Nobel laureate Tjalling Koopmans, was a Dutch economist, professor of economics in Rotterdam, and in Amsterdam towards the end of his career; his life and publications, especially with respect to monetary theory, have been retraced in Fase (1998).

Historians of econometrics (Morgan, 1990; Epstein, 1987), of macroeconomics (De Vroey, 2016) and of mathematical economics (Boumans, 2005), along with economic methodologists (Maas, 2014), have produced long-range histories which showed how the field of macroeconomics developed from the first works of Tinbergen and Frisch (or from Keynes in the case of De Vroey), towards the Cowles Commission approach of simultaneous equations modeling. They have shown how economic practices evolved from inductive statistics to building and testing models to inform decision-making, and how economists struggled to bridge gaps between economic theorizing and model-building, leading to conflicting approaches of macroeconomics at the end of the 20th century. These histories are important, because they trace the genealogy of tools and ideas, and sometimes of the context in which they have evolved (Dekker, 2021). But they also tend to erase the forks in the roads, and to leave unexplained the choice to go down one road rather than another.

For instance, Morgan (1990: 67) talks of a “lull” between Moore’s work of the 1920s and the “development of small macrodynamic models of the business cycle” on the basis of which Frisch and Tinbergen (and many others) developed process analysis in the 1930s. Rather than a “lull,” we see in the turn of the 1920s-1930s a pivotal moment when two alternative approaches of economic modeling were tested and debated, and the discussion between Koopmans and Tinbergen were a part of this debate. The importance of Koopmans’ reply to Tinbergen in this respect is that it illuminates the difficulties of what became called the “microdynamic” approach by Frisch, who put a name on two existing tracks, one of which was abandoned by the leading econometricians of the day (Carret, 2022). The seemingly obscure debate examined

here illuminates those two possible tracks, and the reasons for which Tinbergen and other econometricians made an informed choice to abandon Walrasian general equilibrium theory in favor of small macrodynamic models, setting the stage for a decades-long struggle of macroeconomics, peaking in the 1970s with the problem of microfoundations.

After this debate, Tinbergen focused on the development of macrodynamic models and moved away from Walrasian systems. It took him several more years to develop a model that he used between 1934 and 1937 to illustrate unstable macroeconomic processes and express the view that a disturbance such as the 1929 stock crash could push the economy over the hill, into an abrupt and bottomless precipice (Assous and Carret 2022b; 2023). On that basis, he continued to emphasize the importance of multiple equilibria, using it to justify public interventions to reach a better equilibrium, but he was now doing it in the framework of a macrodynamic model. Thus, it was after this debate that Tinbergen began to emphasize the possibility of an economic collapse, turning away from an analysis of multiple and suboptimal equilibria which became so central in Keynes' *General Theory*. The relationship between these different views of multiple equilibria will however not be addressed in this article, as it had no bearing on this debate, which took place four years before the publication of Keynes' book.

I. Two approaches of economic crises

Tinbergen's 1932 pre-advice was an answer to the question "is national wealth maximized in a system of free competition?" The main interest of this policy paper lies for us in the fact that he clearly presented two ways to approach this question: one

based on the study of dynamic processes, and the other based on the interaction between two profit-maximizing (representative) firms. In 1932 there was not yet an agreement as to which track should be pursued to talk about crises and unemployment. For Tinbergen, these two tracks might even have been complementary, as he argued that limiting the uncertainty brought on by the cyclical character of economic life would have the added benefit of eliminating strategic calculations between firms.

With respect to the first track, Tinbergen noted that in June 1932, the end of the disruption that had begun in 1929 was nowhere in sight.[4] This led him to discuss the possibility that there existed an unstable economic process, and to reflect on the economic policy needed in a section on “stable and unstable economic processes”. Tinbergen argued that although previous crises were usually followed by a return to equilibrium, there were examples of unstable processes that did not lead to an automatic recovery, such as the inflation that plagued European countries in the early 1920s. This begged the question: “are there, among dynamic economic processes taught by theory, some that lead to an ever-increasing distance from the equilibrium state?” (Tinbergen, 1932: 58). Although such mechanisms were not discussed in the theoretical literature, Tinbergen was confident that he could find dynamic models accounting, for instance, for exponential movements away from the equilibrium.

It was with this goal in mind that he reviewed the existing work on market-level dynamic processes, starting with the hog cycle. In this case, he noted that fluctuations usually led back to equilibrium, but that if the elasticity of production became too great it

[4] “The crisis which broke out in 1929 led to a disruption of production which did not occur in any previous crisis … And still nothing can be noticed of an end of this disruptive process … The question has been posed whether we are not dealing here with a movement that no longer returns automatically to an equilibrium state, or whether we are not dealing here with unstable states.” (Tinbergen, 1932: 56).

could lead to fluctuations of increasing amplitude.[5] He also referred to his own model of the "shipbuilding" cycle, published in 1931, where he had observed that its solutions could yield fluctuations of increasing amplitude, depending on the values of certain parameters like the sensitivity of the supply of ships to the price of freight, and that "increasingly violent fluctuations" could arise. Although he was concerned with the shipbuilding cycle in this article, Tinbergen already viewed in 1931 the potential of such mechanisms to describe the whole economy as their mathematical formulation "gives us a clue to a method of judging the stability of an *economic system in general*" (Tinbergen 1931: 162, our emphasis). In this context, he underlined the importance of building "a theory on endogenous trade cycles" whose "basic problem" boils down to answer to the following question: "how can an economic system show fluctuations which are not the effect of exogenous, oscillating forces, that is to say fluctuations due to some 'inner' cause?" (Tinbergen, 1931: 152).

The exploration of small dynamic models at the beginning of the 1930s was a decisive step from the study of market dynamics to the study of what Frisch came to call "macrodynamic" models studying economic aggregates instead of markets and individual choices (Boumans, 2005: Chapter 2, esp. 33ff; Assous and Carret, 2022a: Chapter 2; Carret, 2022). In 1932, Tinbergen had already started to build up toward a more intricate model of the economy and its interdependencies. In all those cases, he was chiefly interested in the periodicity of the oscillations and always used hypotheses

[5] The hog cycle was first studied by Arthur Hanau of the Berlin Business Cycle Institute, and Tinbergen continued to emphasize the importance of his work almost sixty years later in his interview with Morgan and Magnus (Tinbergen et al., 1987).

on the parameters that would make those oscillations self-sustained.[6] However, the shipbuilding cycle as well as the cobweb mechanism were still one step removed from macrodynamics; they remained focused only on one market and not the whole economy. Tinbergen pointed out in 1932 that if those models helped to identify new economic processes, they remained of little help to provide an appropriate framework to make sense of the working of the whole economy and the factors responsible for the ongoing crisis, in part because they did not explain satisfactorily a mechanism of "removal from the equilibrium state" (Tinbergen, 1932: 58).

In 1932, he claimed that the solution to account for aggregate phenomena may also come from another track, the "Walras-Cassel" model of the economy.[7] According to him, this approach left ample room for multiple equilibria and instability to appear, and he described a possible mechanism illustrated with a diagrammatic representation of the interactions between two representative firms. In this model with two sectors and two firms, Tinbergen claimed that the economy could get stuck in a "bad" equilibrium with high unemployment. Plotting the "profit lines" of each firm as a function of their employment levels a_1 and a_2, that is, all the combinations of employment for which the

[6] This was precisely the approach adopted by Kalecki (1935) that was criticized by Frisch and Holme (1935). Tinbergen had prepared the ground for Kalecki's model, both through this approach of endogenous fluctuations, and his 1931 shipbuilding model, which was used by Kalecki to solve his own model. Following Boumans' (2005) idea that Kalecki's model was like a recipe, this shows that the ingredient coming from Tinbergen was more than a mathematical mold. The type of solutions favored by Tinbergen had a direct impact on Kalecki.

[7] "Further investigation of the equilibrium equations (of, for example, Cassel or Walras) which determine the size of production, the level of prices, etc. in a stationary society, shows that there is not one equilibrium (in the sense of economic science), but that in general different equilibria are possible with the same technical and psychological data. It is quite possible, and even most likely, that with some form of utility functions and technical coefficients, two equilibria at least exist, both stable, one for instance with significant unemployment and the other without" (Tinbergen, 1932: 60). Rodenburg (2010: 10) mentions Tinbergen's argument in the context of his debate with the Dutch economist Goudriaan on dynamics and equilibrium but without contrasting it with the arguments raised by Koopmans.

profits of firms 1 and 2 are constant, Tinbergen provided the following diagram (Tinbergen, 1932: 62).

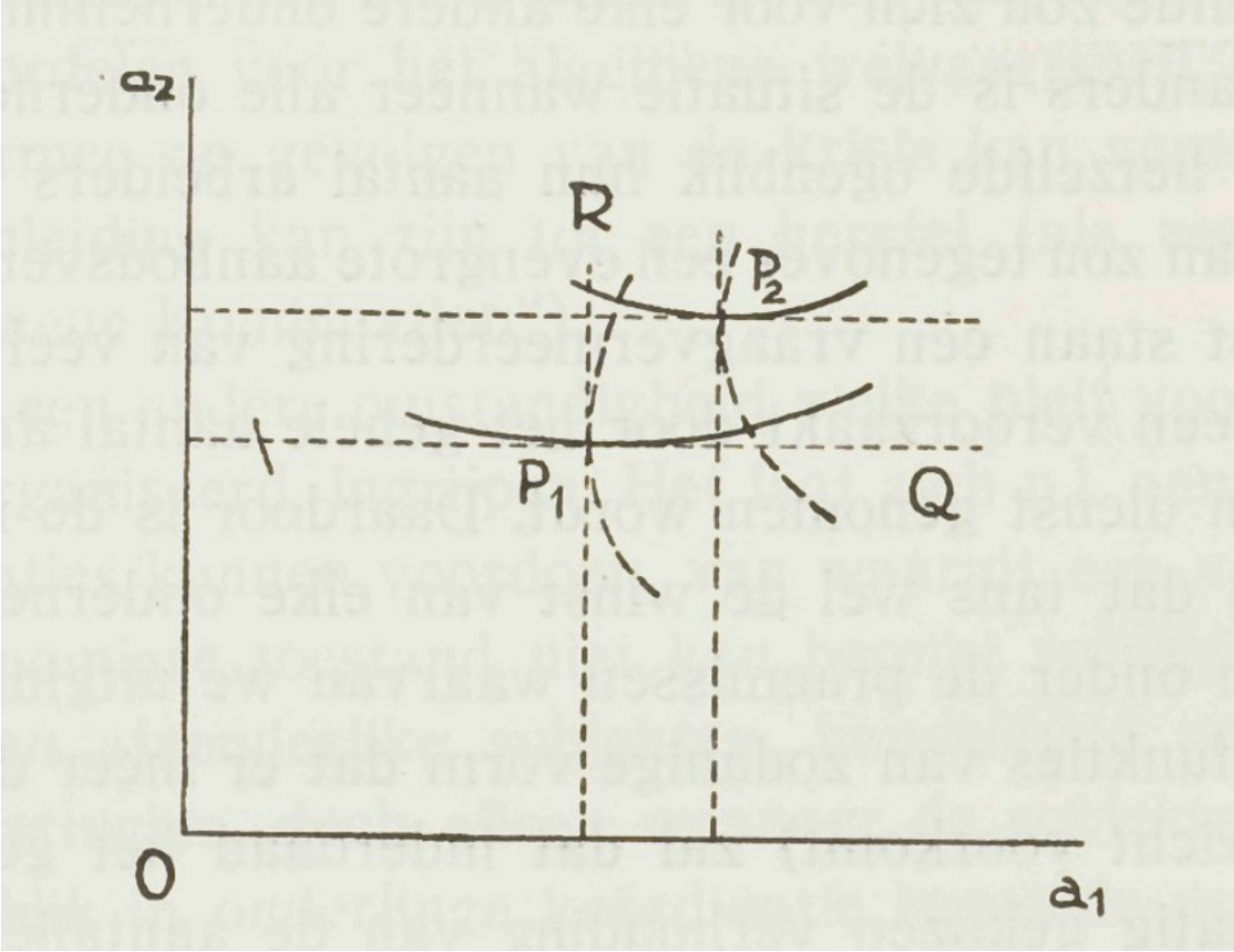

Figure 1: Tinbergen's illustration of a coordination problem (1932: 62)

Tinbergen underlined that an increase of employment from only one firm will correspond to a displacement along a vertical or horizontal line. Because the curves meet at their minimum point, this means that a firm increasing its employment will always see its profits go down if the other firm does not increase employment as well. Tinbergen argued that the configuration where both firms have their profit curves crossing at their minimum points will thus correspond to stable equilibria, because there

will be no incentive to deviate alone from such a situation (Tinbergen, 1932: 62). In Figure 1, there are two such equilibria: P_1 and P_2. We can readily see that in the second of these equilibria, there is more aggregate employment than in the first; this begs the question of how to move the economy from the "bad," low equilibrium, to the "better," high equilibrium (Tinbergen, 1932: 61).

If we denote by R and Q the two other points where the profit lines intersect, we can see that the profits of both firms will only increase together in the $P_1\ Q\ P_2\ R$ quadrilateral. But in order for the economy to move in this direction, it is necessary that both firms increase employment at the same time, so that neither has the incentive to reduce employment to go back to a higher level of profits. The bottom line is that a new stable equilibrium can be obtained for a specific ratio between a_1 and a_2, and that only an organized action may help to reach it: "In other words, it has been shown that there are cases in which competition cannot get out of a certain (low) situation, while organized action will make it possible" (Tinbergen, 1932: 61).

Unfortunately, Tinbergen did not explicitly state the set of equations that he might have used to derive the two profit lines, making it difficult to identify the exact mechanisms at work in the move of the economy from one equilibrium to the other. He described some potential mechanisms explaining how such a situation could arise, mainly from the mismatch in the increase of demand and supply when firms act alone, while when they act together this mismatch is construed to have a positive effect on profits because of a potential reduction in costs.

As he pointed out, in the case of a firm acting alone from equilibrium P_1, “the increase in demand which would arise as a result would by no means outweigh the increase in the supply of the commodity produced by that company” (Tinbergen, 1932: 61). But with no other information on the consumption of the workers of the two firms and on the way both firms decide to invest, the argument is a bit enigmatic. The possibility that demand may eventually adjust to supply and reach the “good” equilibrium would in fact be due to the evolution of cost: “Expansion of the number of workers [...] also lowers the unit cost of production” (Tinbergen, 1932: 62), which, for reasons also left unexplained, is assumed to have a positive effect on the demand for both goods.

Whatever the limits of Tinbergen’s argument, it is clear that he was thinking not only about stability and dynamic issues, but about how they could fit in the existing models of the economy, by examining two different possible approaches to the same problem: one focused on dynamic processes, the other on coordination issues and multiple equilibria. Koopmans’ response, published a few months later, was focused on the second track, and provided some decisive arguments against the possibility of representing unemployment crises through a Walrasian model.

II. Koopmans’ criticism of a Walrasian theory of crises and unemployment

As we have seen, Tinbergen associated the idea of multiple equilibria with a Walrasian system of equations. In a footnote, he admitted that he was puzzled to see that while the idea was mentioned in passing in the literature, it had “not been elaborated anywhere” except in “some unpublished studies of Mr. J. G. Koopmans,” suggesting

that his analysis derived from it (Tinbergen, 1932: 60). A couple of months later, Koopmans published a series of three papers with the aim to clarify his position with respect to Tinbergen's analysis. While he acknowledged in the first paragraph of his paper "the much appreciated help" which "as a non-mathematician" he received from Tinbergen (Koopmans, 1932: 679), he felt the need to underline the differences between his and Tinbergen's analysis. In fact, Koopmans made clear how far Tinbergen's analysis departed from his own approach and this for two main reasons.

First and foremost, Koopmans emphasized that neither in Walras's nor in Cassel's works, one can show that there might be an equilibrium let alone a "stable equilibrium" as far as a "significant degree of unemployment is assumed" (Koopmans, 1932: 682).[8] Accounting for unemployment means that one market equation expressing the equality between demand and supply will have to disappear, leaving the system undetermined with the result that many potential solutions become possible: "this means that the whole complex of economic phenomena, as expressed by the equilibrium equations, becomes undetermined, that is to say, there are not only two or a few, but even, at least in principle, infinitely many alternative solutions of the remaining system of equations" (Koopmans, 1932: 683). In particular, as soon as one assumes that wages are fixed and known, it becomes possible to define a set of equilibria, each parametrized by a specific level of money wage. It is therefore no surprise that with the same technical and

[8] "After all, one of the essential conditions for the existence of a state of equilibrium in the sense of these two authors [Walras and Cassel] is precisely that, at the prices applicable to that situation, the supply of all end products and production factors, including human labor, is fully absorbed by demand. The equations in which this condition is expressed are therefore an indispensable constituent of the whole system of "equilibrium equations" through which, in the thinking of the aforementioned authors, the economic phenomena are examined" (Koopmans, 1932: 682).

psychological data, it is possible to define a single “equilibrium state” for any given level of money wage.

Koopmans pointed out that it was unlikely that Tinbergen had understood the issue in this way: “it should be appreciated that, according to the continuation of his argument, the author certainly could not have meant the above statement in such a simple and almost trivial sense. For in reality he also speaks of a system in which the number of equations indeed corresponds to that of unknown quantities” (Koopmans, 1932: 683). The unknowns chosen by Tinbergen are simply not the same as those chosen by Cassel and Walras. The problem however remains similar as long as one can show when a system comprising as many unknowns as equations has multiple solutions. In that case, “the problem can indeed be regarded as “determined” in a similar sense as in the thinking of Walras and Cassel [...] from the formal-theoretical point of view, [the possibility of multiple equilibria] is indeed just as remarkable and of the same fundamental significance and scope as the corresponding possibility in a system of equations built up entirely in accordance with the method of Walras and Cassel, and in which all prices, including wages, figure as unknowns” (Koopmans, 1932: 684).

In addition, Koopmans doubted that on the basis of his graphical illustration, Tinbergen had managed to prove the existence of multiple stable equilibria. If it was clear to him that the two “lines of constant profit” were derived under the assumption that wages are rigid, it remained unclear why both lines could “run according to the starting point adopted by the author in the manner indicated by him in his drawing: a possibility which I do not want to dispute, but of which the proof has not been fully provided here on the spot” (Koopmans, 1932: 684). Koopmans argued that the rigidity

of wages acted on the stability of equilibria but in an obscure manner. While he agreed with Tinbergen's concern with the possibility of defining a stable equilibrium, he regretted that he did not clarify the mechanisms allowing to identify which equilibrium among all the equilibria defined by a given level of money wage was likely to be reached.

The second issue raised by Koopmans concerned the possibility of ranking equilibria. His point was that the equilibria displayed by Walrasian systems could not be ordered. From the policy point of view, this was an important difference between Tinbergen's and his approach of multiple equilibria, because he claimed that in the absence of nominal rigidities and unemployment there was no way to choose among the different equilibria:

> such an appraisal judgment can no longer be justified with the same certainty when it comes to the comparison between two (or more) alternative equilibrium states … which, after all, is characterized precisely by the fact that in neither of them there is a surplus of unused production factors … The remarkable thing about a double or multiple economic equilibrium in this last sense … consists precisely in the fact *that the various alternative equilibrium states are in every respect, thus also viewed from a normative point of view, completely "equivalent"*, or in others words, that none of these equilibria can be given a "preference" over one or more of the others on the basis of any objective criterion. (Koopmans, 1932: 685, original emphasis)

Thus Koopmans underlined that the different equilibria did not imply that some factors of production were underemployed, and they did not justify a socialist planning policy in

order to obtain a higher employment equilibrium. It was necessary to introduce another criterion from which to establish that an equilibrium was better or worse than the other. Koopmans sought to illustrate this equivalence of different equilibria in a Walrasian system with a numerical example exhibiting three equilibria.

III. Koopmans' multiple equilibria in a Walrasian system

While Koopmans started with a criticism of Tinbergen, he also made it clear that he had a limited knowledge of mathematics and that he relied mainly on others for his mathematical exposition, in particular on Tinbergen himself. But he still felt that he had something important to say about the importance of multiple equilibria in even the simplest exchange economy, and he did not think that Tinbergen had captured his ideas on the subject in his model.

The end of the first part of his paper was dedicated to setting up his model. First, Koopmans pointed out that the issue at hand had nothing to do with "the existence of so-called "indifference zones" generated by the discontinuity of supply and demand curves. It also had nothing to do with firms operating, as a result of a relatively high percentage of "fixed" cost in the production process (Koopmans, 1932: 690), in the decreasing part of the U-shaped supply curve. He acknowledged that both these cases allowed for an exploration of the possibility to "represent a position of 'stable' and one of 'unstable' equilibrium" (Koopmans, 1932: 690).

Koopmans emphasized that his own approach was based on the idea that supply curves could have inverse U-shapes: they started as normal, upward sloping schedules, before bending towards the bottom right in a price-quantity diagram: "which means that, even apart from decreasing costs, etc., as the price per unit increases, the volume of

the supply no longer continues to increase beyond a certain point, but on the contrary decreases further and further" (Koopmans, 1932: 690). These supply (and demand) functions arose from the utility functions used by Koopmans, and the problem he sets up is that of a simple exchange economy with two people and two goods.

The framework was similar to the one used by Tinbergen in his article "Mathematical Psychology'" which appeared originally in the Dutch journal *Mens en Maatschappij* in 1930 and was recently translated in 2021 in the *Erasmus Journal for Philosophy and Economics* (Tinbergen, 2021). The article starts by presenting the essential ingredients of early welfare economics as developed by "the very fine expositions in Bowley's *Mathematical Groundwork of Economics*, Pareto's *Cours d'Economie Politique* (1896-1897) and Lenoirs' *Etudes sur la formation et le mouvement des prix* (1913)" (Tinbergen, 2021: 211). Following these three authors, Tinbergen introduces the methodology of analyzing two situations: exchange between many individuals, each considering the price to be fixed and "isolated exchange," when "there is one individual on each side." When agents have ordinal preferences over bundles of goods, Tinbergen reaches the conclusion (illustrated with the help of indifference curves and what is today known as Edgeworth's box) that "without a further organizing principle this exchange has no fixed outcome; through economic forces alone no equilibrium is established" (Tinbergen, 2021: 214). In his view, it was precisely that point that Schumpeter had put forward in his 1928 study "the Instability of Capitalism" (Tinbergen, 2021: 215). This shows that Tinbergen had worked along this line of thought before, and that he was

well-placed to counsel Koopmans on his exploration of an exchange economy with multiple equilibria in an Edgeworth box.[9]

There are three types of equations in Koopmans' system: equilibrium equations relating the ratio of marginal utility of the two goods and their (relative) price, "budget" equations relating the aggregate value of goods given up in the exchange with the aggregate value of goods received, and balancing equations for the total stock of physical units of each goods. The known quantities are endowments (for good I and good II respectively, agent alpha has k and l and agent beta has m and n) and preferences (in the form of linear marginal utility functions with parameters A, a, B, b for alpha and C, c, D, d for beta, respectively for good I and good II). The unknowns of the problem are the final quantity held by participants after the exchange (x and y for alpha, and z and u for beta), and the rate of exchange (price p) between the goods. The equations determining the three equilibria are presented in Table 1.

Agent	Alpha	Beta
Marginal utility	$\frac{A-ax}{B-by} = p$	$\frac{C-cz}{D-du} = p$
Budget	$p(k - x) = y - l$	(determined by the others)
Quantity	$x + z = k + m$	$y + u = l + n$

Table 1: Equilibrium equations of Koopmans' system

In the case with two agents and two goods, at the equilibrium points the relative utility of each good is equal to the same (relative) price, one budget constraint will be given from the other budget constraint and the physical constraints, so that there are five

[9] Tinbergen, on a suggestion by Ehrenfest, contributed in his 1930 paper the idea of "no-envy" as a fairness criterion, which has been compared by Heilman and Wintein to other formulations of the same idea (2021).

equations and five unknowns, determining an optimal economic equilibrium. Koopmans argued that a solution existed because there were as many unknowns as equations in his system. But he also added that this did not mean that there was necessarily a unique solution, and in fact “even with the minimum number of two persons exchanging only two goods, … as a rule not one single, but three different solutions of the equations concerned are possible” (Koopmans, 1932: 694). To prove this, he gave a numerical example and obtained the following equations (Table 2), whose solution exhibits three equilibria, with each equilibrium being established at a different price (Table 3 gives the equilibria solving this system).

Agent	Alpha	Beta
Marginal utility	$\frac{7-x}{10-y} = p$	$\frac{66-12z}{11-u} = p$
Budget	$p(7 - x) = y$	(determined by the others)
Quantity	$x + z = 7$	$y + u = 10$

Table 2: Equilibrium equations of Koopmans’ numerical example

	Relative Price I / II	Alpha		Beta	
		Good I	Good II	Good I	Good II
First Equilibrium	1	2	5	5	5
Second Equilibrium	2	3	8	4	2
Third Equilibrium	3	4	9	3	1

Table 3: Equilibrium solutions of Koopmans’ numerical example

These three equilibria can be represented in an Edgeworth box diagram, which seems to be what Koopmans had in mind when he described in a footnote the application of Bowley's and Pareto's "graphical method," which "had to be scrapped due to lack of space" (Koopmans, 1932: 772-773). The following figure produces such a diagram, based on Koopmans' numerical example.

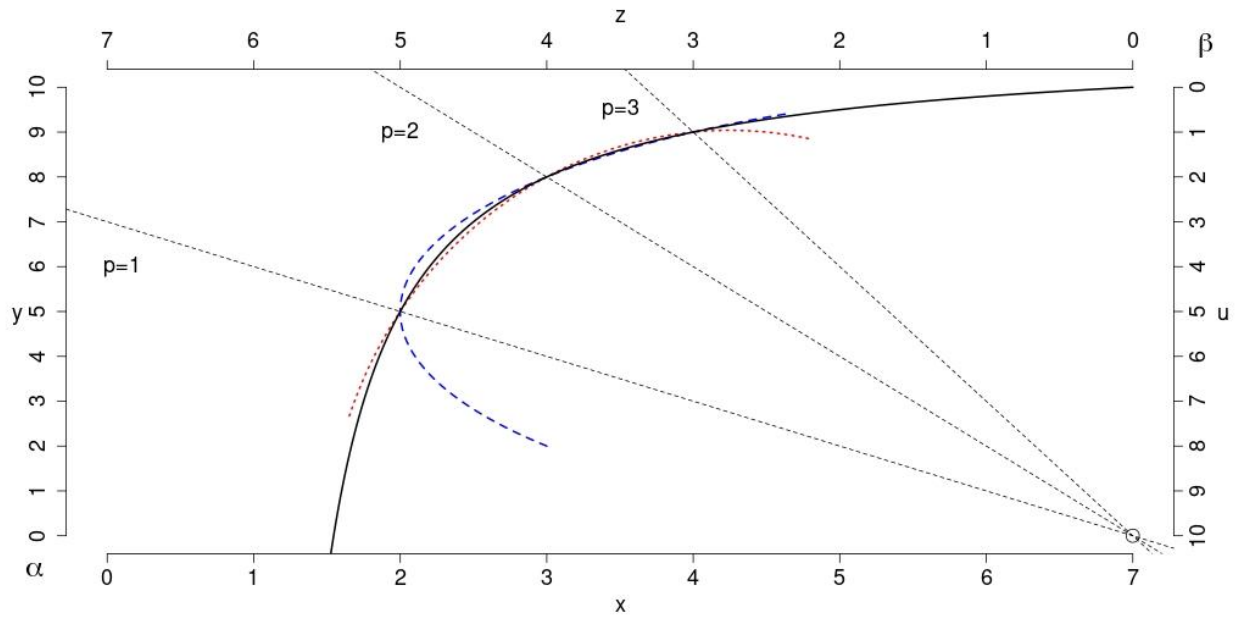

Figure 2: Edgeworth Box derived from Koopmans' numerical example

The thick black line is the Pareto "contract curve," all the points where the indifference curves of both agents are tangent to each other, mathematically expressed as the equality between the ratio of marginal utilities of alpha and beta, $\frac{7-x}{10-y} = \frac{66-12z}{11-u}$. While there are four unknowns in this equation, we also know from the start the physical limitations of the economy: $x + z = 7$ and $y + u = 10$. This subsystem of four unknowns and three equations defines the Pareto curve traced out in the diagram above. The blue (dashed) and red (dotted) lines interlacing around the contract curve

are the offer curves of alpha and beta respectively; they are determined by the point of tangency between each agents' indifference curves, and their budget line for different prices. This budget line is determined once endowments are chosen and a new variable is introduced, the relative price p between the two goods. The adjustment of this price rotates the budget line around the point of initial endowments, which Koopmans called the "zero point of the exchange" (Koopmans, 1932: 773). Economic equilibrium happens when both offer curves are compatible, that is, when both participants are ready to exchange compatible amounts of goods I and II, or in other terms, they have the same relative price for the two goods: the two indifference curves and the budget line are tangent at the same point, which lies on the contract curve.

Three economic equilibria are possible in this system, and they all lie on the contract curve, as is made formal by the first fundamental theorem of welfare economics. This theorem appears trivial in this case, as the economic equilibrium is defined by taking the four unknowns and three equations defining the Pareto curve, and adding one unknown (the price p) and two equations (the equality of price and relative marginal utility $p = \frac{7-x}{10-y} = \frac{66-12z}{11-u}$, and the budget equation $p(7 - x) = y$), so that the system becomes determinate with five unknowns and five equations.

But the fact that the system is determinate does not mean that there is only one equilibrium, as was the point of Koopmans' numerical example; a nonlinear system whose reduced form is a polynomial of degree more than one will in general have more than one solution (Koopmans discussed the problem introduced by complex solutions in the third part of his paper). Koopmans himself gave a form of his system reduced to one

equation with the price as an unknown, showing that it was a polynomial of degree three, with three equilibrium solutions. This was underlined by Koopmans to show that this example had nothing exceptional, or special, but could appear systematically; this led him to wonder why previous writers had not underlined this possibility. It is likely that Tinbergen influenced him in this direction, as Koopmans had underlined his debt towards him for the solution of his model; Tinbergen himself had adopted this approach of reducing his systems to one equation already in Tinbergen (1931), and continued this approach in his later models, such as the 1939 League of Nations model.

Because the marginal utility conditions are fulfilled in each of the three equilibria, Koopmans argued that “there is indeed no reason here to consider one of the three possible equilibrium states with respect to the two others as the ‘best’ or the ‘worst’” (Koopmans, 1932: 702). This was the brunt of his critique of Tinbergen, and he argued in a footnote that the utility of each person in each equilibrium was irrelevant because incomparable.

While Koopmans is convincing in showing the possibility of multiple equilibria for simple, nonpathological preferences, he stops short of analyzing the coordination problem underlying his setup, as a result of the methodology he adopted. The problem of his analysis is that it is fundamentally an equilibrium analysis, and Koopmans says nothing of the way in which the two agents got to one equilibrium in particular, focusing instead on other numerical examples with more goods or more agents. But while interpersonal comparisons are not possible, it is apparent, without even looking at their utility function, that agent alpha prefers the third equilibrium while agent beta will prefer the first equilibrium. This result is a simple consequence of the starting endowments, as

alpha owns the whole stock of good I while beta owns the whole stock of good II: in an equilibrium with a lower price for good I, beta will be able to buy more of it while alpha will not be able to obtain as much good II as in an equilibrium with a higher price for good I. But Koopmans did not say how one equilibrium was chosen instead of the other, even though this seems like a fundamental problem: whoever can influence prices can obtain a favorable equilibrium.

In his original example, Koopmans examined the three situations arising from three different price systems, taking the price as given each time. The only hint toward the dynamic problem underlying the approach of equilibrium is found in the second part of the paper, where Koopmans described a change in price from the starting point of the second, unstable equilibrium: “a slight rise in price beyond this point has the effect of decreasing supply more than demand, so that a shortage arises in the market, the consequence of which is that the price will rise even further” (Koopmans, 1932: 771). Koopmans noted that Walras himself had underlined this possibility of having three equilibria, with the middle one unstable (Walras, 1954: 112).

To prove the instability of the middle equilibrium, Koopmans relied on Walras’s argument which was based on tracing out offer curves in a price quantity diagram, and looking at their intersections with demand curves. It is interesting to note that the implied dynamic analysis is not spelled out in terms of differential or difference equations, something that became one of Tinbergen’s main contributions to the macrodynamic program whose goal, as for for Frisch (Dupont-Kieffer, 2003), was precisely to specify these dynamic adjustment processes (Assous and Carret, 2022, in particular chapter 8).

The model that Koopmans ended up building thus exhibited multiple equilibria that were very different in character from those of Tinbergen: while Tinbergen imagined a situation where maximality conditions could be satisfied *at different levels of production*, Koopmans built numerical examples showing that *for a given amount of goods* in the economy, different equilibria could be possible. They all satisfied marginal utility conditions, but they were impossible to rank without a normative criterion to choose an equilibrium. This later problem was raised briefly, but left unexplored by Koopmans, who kept his attention on marginal utility functions.

IV. Tinbergen's turn away from Walrasian economics

In the 1930s, Tinbergen argued that the main deficiency of the Walrasian system was its static features (Tinbergen, 1935: 241; 1936: 199). This move away from Walras also followed his debate with Koopmans, which made clear that Walrasian systems were difficult to reconcile with macroeconomic policy issues, and most importantly with an analysis of unemployment, even with the device of multiple equilibria. Nevertheless, he did not abandon the idea that there could exist coordination issues, which he now modeled in a macrodynamic framework.

This is clear in Tinbergen's 1934 macrodynamic model, which he built with the aim to account for various economic trajectories, some stable, others unstable and to show how policy could be used to change them. On the basis of an analysis of the determinants of aggregate purchasing power, Tinbergen was led to a nonlinear equation describing the relation between prices, production and the value of consumer goods sold, as well as the movement of employment. Using that model in a series of articles

published in 1935 and 1936, Tinbergen wondered how a change in initial conditions resulting from temporary shocks may change the trajectory of the economy and showed a concern for policy intervention likely to throw the economy over its stability zone, into a region of instability that could potentially trigger a complete collapse of the economy. At the same time, he drew attention to the importance of designing an economic policy likely to prevent such dangerous paths.

It is however only in a 1937 article that he really returned to the idea of coordination failures in the framework of a reflection on Kahn's multiplier and the effect of a rise in public expenditures. His basis was a simplified version of a Dutch model he had presented at the Namur meeting of the Econometric society in 1935, and which he went on to expand while working for the League of Nations in 1936-38. From his 1934 paper, Tinbergen kept the non-linear relationship between prices, production, the value of consumer goods sold and employment, but also introduced a speculative dimension through the effects of the evolution of current profits on expected profits. This model resulted in different stable equilibria, one with high employment and high output and one with low output and low employment. This was precisely the idea exposed in his 1932 pre-advice, but instead of referring to the Walras-Cassel general equilibrium model, the whole analysis was macrodynamic.

The role of economic policy in this respect was to give the initial impulse, which had to be large enough to send the economy from the bad to the good equilibria; in this respect, obtaining a higher level of employment and profits did not necessarily imply maintaining the initial expenditure at the source of the shock. Tinbergen continued to discuss the idea of multiple equilibria, and how macroeconomic policy may improve the

level of production and employment, in different papers published between 1938 and 1943. In his 1938 *Econometrica* article “On the Theory of Business Cycle Control”, he built a small model in which economies could move from one equilibrium to another: “In the case of a nonlinear final equation there exist other equilibrium positions or developments than the one chosen as reference developments and much depends on the situation of these other equilibria, which may be stable or unstable” (Tinbergen 1938, 33). The main message was that the final position reached by the economy was as important as the trajectory leading to it and that economic policies could play a pivotal role in leading the economy to various stationary states.

One of his last publications in this line was a paper in Dutch published in 1943 and subsequently translated in English (Tinbergen, 1943; 1959). Although Tinbergen examined for the first time the implications of multiple equilibria in a Keynesian cross diagram, his approach consisted in outlining the conditions for which multiple equilibria could occur and remained conceptually the same. Again, the issue was to make clear, depending this time on the slope of the expenditure demand curve relative to the 45° line, when multiple equilibria may appear. In view of the possibility that the expenditure may be non linear and S-shaped, Tinbergen considered the case in which three equilibria may be defined: two stable for extreme values of activity for which the slope of the expenditure curve is lower than 1 and one unstable for which the expenditure line is higher than one. With reference to Goudriaan with whom Tinbergen had debated several times in the 1930s (Rodenburg, 2010), Tinbergen could argue that the importance of the multiplier effect would dramatically change depending on whether public expenditures are increased by the appropriate amount. In particular, Tinbergen

could argue that, from a state of low equilibrium, it is only when changes in expenditures shift the expenditure line so that it becomes tangential with the 45° line that the economy will eventually stabilize in a state of high equilibrium and public expenditures have a full impact on employment.

In 1932, Tinbergen tried to represent a “bad” equilibrium, that is, one with low employment, in the context of the strategic interactions between profit maximizing firms. This was his last attempt to use microeconomic theory to demonstrate the varying efficiency of multiple equilibria. Subsequently, he developed a series of nonlinear models that successfully showed how the economy could be stuck in a low employment equilibrium; but all references to individual choices had now disappeared. The whole analysis was one of dynamic relationships between aggregate variables, in accordance with the macrodynamic program set out by Frisch. After Koopmans’ criticism, Tinbergen thus did not abandon the idea of multiple equilibria, but he abandoned the profit-maximizing framework of a general equilibrium model. He saw in the development of macrodynamics and process analysis a more fruitful avenue of research to talk about multiple equilibria, and the economy in general (Morgan, 1990: Chap. 4; 1991).

V. On the long-term effects of normal science: Challenges and adjustments of economic models

Other scholars of Tinbergen have argued that his “work and the way he developed his mathematic[s] owed significantly to the Germanic Staatswissenschaft tradition” (Dekker, 2023: 4). This present article showed that his approach to modeling crises and

unemployment owed at least as much to his debate with other economists, in the context of the development of two possible ways to tackle the problem of modeling economic crises: one based on microeconomics and multiple equilibria, the other on dynamic processes. There is no argument here that his writings “retained a political dimension which means that we should not reduce it to the mindset of an engineer” (Dekker, 2023: 20), as his beliefs along with his involvement in various policy circles clearly guided the questions he was tackling. And yet, there is something strange in thinking that “studying the forging of the tools will make us easily lose sight of the political-economic dimensions of this project” (Dekker, 2023: 22). Rather than make us lose sight of his larger project, we see in the specific way Tinbergen used his tools a possibility to highlight how far he went to integrate his political and scientific views and to add support to the fact that policy was primary for Tinbergen (a point put forward by Boogaard [1999] and further developed in Dekker [2021]).

As we hope this paper has shown, ignoring the way in which economic debates advance the econometricians’ understanding of their models deprives one of a central piece of information on scientific development. It is much like focusing only on the external impulse, the clubbing of the horse, and claiming that the endogenous propagation mechanism is irrelevant to understanding business cycles or the rocking of the wooden toy, to take up Frisch’s well-known metaphor (Frisch, 1933). The debate between Koopmans and Tinbergen is part of the process of “proofs and refutations” described by Lakatos (1976), which forms the core of normal scientific research; ignoring this slow process of challenges and adjustments is done at the risk of a “naïve falsificationism” (Kuhn, 1970: 14). Replaced by a sole focus on extra-scientific drivers, it

can lead to teleological or functionalist theories of how science develops and is able to inform policies. In the extreme, it leads to producing histories of economics without economics.

When Tinbergen, Frisch and others were writing at the beginning of the 1930s, the future was not written yet, and the distinction between microdynamics and macrodynamics was still in flux before being conceptualized by Frisch. There was a fork in the road between two approaches, one that was explicitly built on dynamics and transported at the aggregate level, the other which struggled to dynamise a general equilibrium model based on microeconomic principles. The second approach, which Frisch called microdynamics, all but disappeared from the work of the leading econometricians in the ensuing years, and the reasons for this have been largely overlooked, in spite of their long-term consequences. This present paper showed how the debate between Tinbergen and Koopmans fits into this larger transformation, and how it contributed to redirect the early econometricians towards macrodynamics, and away from microdynamics.

Koopmans' series of papers sought to show that multiple equilibria in a Walrasian model of the type envisioned by Tinbergen were a dead-end to say anything useful about crises and unemployment, because the different equilibria could not be ordered without a normative criterion. By embracing the development of macrodynamics, Tinbergen managed to provide an answer to the objections raised by Koopmans. Alongside the argument advanced in the early 1930s by Tinbergen and Frisch that if Walrasian systems were determinate, they suffered from the fact they were static (Frisch, 1933; Tinbergen, 1935), a second reason, underlined here, is that Walrasian

theory was an inappropriate framework to address issues related to unemployment and instability, which ultimately led Tinbergen to rethink the role of state interventions through his macrodynamic models and other “process analysis” (Morgan, 1991).

We have argued elsewhere that mathematical models of the economy embody the visions of the economists building them (Assous and Carret, 2022a). The debate presented here is a case in point: faced with different modeling strategies, Tinbergen chose one that was able to represent the economic crisis of underemployment he was living through. While Tinbergen has been depicted as the pioneer of independent economic expertise, finding the “means that would best serve to achieve a stated goal” (Maas, 2014: 59), it is not so easy to separate the instrument-model and the goal. Entering into the black box of the instrument allows us to question this position. It shows how much the instrument, developed to assess policy, is itself the result of a subjective position on the state of the world, a position that is never value-free, especially in the case of “the principled, socialist and pacifist Tinbergen” (Maas, 2014: 59).

These links between the tool and the engineer’s view of the world may explain why it took five more decades for macroeconomists to return to the connection between multiple equilibria, normative statements and optimization (Silvestre, 1999). In a new context, this connection was made possible by the implications of imperfect competition, where the first fundamental theorem of welfare economics no longer applied. The market power wielded by some agents in these models questioned the idea that the public interest could be served without coordination or cooperation. A major research effort in the early 1980s resulted in the development of a new class of models based on noncooperative game theory, in which a universally better outcome could, in some

cases, be achieved only by coordinating the activities of otherwise non-cooperative agents. In the end, in Tinbergen's case, as in the 1980s, it was by moving away from pure Walrasian economics that different research programs were opened.

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