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# Design and Dimensioning of Natural Gas Pipelines with Hydrogen Injection

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## ABSTRACT

The global focus on reducing air pollution and dependence on fossil fuels has led to efforts to shift to renewable energy sources. Hydrogen is a promising alternative due to its high energy capacity and ability to regulate electricity production through electrolysis. In this context, the problem of designing and sizing natural gas pipelines with hydrogen injection is presented. The objective is to establish the network topology and diameter dimensions of each pipeline section for hydrogen distribution, in order to cover the demand at a minimum cost.

To address the proposed problem, we consider the dimensioning as the selection of a diameter from a set of available measures, i.e., a discrete diameter approach, and we compare it with a continuous diameter approach from the literature, including a mixed integer nonlinear programming (MINLP) formulation of degree six. In our discrete diameter approach, we propose a non-convex quadratic (MIQLP) model, and we derive a mixed-integer quadratic convex relaxation (MIQCP). Finally, we adapt a Delta Change heuristic to this context.

We implement several solution methods for a real case study in France. These include solving the dimensioning problem on a fixed Minimum Spanning Tree topology, considering both continuous and discrete diameters, employing the Delta Change heuristic for both cases, continuous and discrete, and solving the MIQCP relaxation problem. The strengths and weaknesses of each of these proposals are demonstrated through the study.

## KEYWORDS

MINLP, MIQP, hydrogen, Pipeline design

## 1 INTRODUCTION

One of the biggest problems facing humanity in the last century is air pollution, which is responsible for numerous respiratory diseases, as well as for global warming and the climate crisis we are experiencing. Humanity's reliance on fossil fuels is considered a major source of carbon emissions, which is widely blamed for air pollution [23]. In an attempt to curb the damage caused to the environment and to improve the quality of life of those living in industrialized and developing countries, numerous public policies

have been put forward all over the planet to favor other forms of energy, therefore, the power system of all countries in the world will inevitably develop towards a high proportion of renewable energy power systems [21], and hydrogen has emerged as a sustainable alternative.

As an energy source, hydrogen stands out for having the highest specific energy content, producing 2.75 times more energy than hydrocarbon fuels, and also, it's the most abundant element in the universe [8]. One of its strengths is its production, since this gas can be produced by electrolysis of water using renewable energies. The hydrogen produced in this way is called Green Hydrogen, and it has recently captured the attention of many researchers [10].

As an energy carrier, hydrogen stands out for its storage capacity and its relationship with electricity, being able to store excess renewable energy production, thus helping to mitigate the difference between renewable energy production and demand [14].

Thus, in 2019 the international energy agency presented in its report "The future of hydrogen" [1] the opportunities that hydrogen provides, both environmental and monetary, as well as the challenges faced by those who wish to exploit it, highlighting uncertainty in government policies and regulations, production standards, acceptance of the product in the markets and, the lack of technology and infrastructure.

We address the problem of the design and dimensioning of natural gas pipelines with hydrogen injection. This article is organized as follows, first, in Section 1, we present a literature review, showing how this problem has been addressed over the years. Then, the statement of the problem is presented. Finally, we analyze a mathematical formulation proposed in the literature. In Section 2 we make the realistic assumption that diameters of the pipelines must be chosen within a set of possible values, and we present mathematical formulations to address this problem. In Section 3, we present numerical experiments over a test example randomly generated instance, and over a real case study in France.

### 1.1 State of the Art

In recent years, interest in hydrogen has grown considerably, mainly due to its potential as a non-CO<sub>2</sub> emitting energy source and its great versatility in terms of production, storage and transportation [3], however, its development has been held back by the lack of infrastructure for its production and distribution. Thus, many authors have proposed to address the optimal design of these facilities [4].

On one hand, some researchers have opted for the design of a complete supply chain, considering different parts of this system. Wickham, Hawkes and Jalil-Vega [20] consider the production, transmission, distribution, reconversion and refueling in a mathematical model for the design of a complete hydrogen supply chain, as well as Tlili et al. [18], they present a study of the supply chain in the north of France, considering the production, the storage, and delivery methods for refueling hydrogen stations for fuel cell electric vehicles. Johnson and Ogden [13] propose a model for hydrogen production and transmission, where they seek to identify the optimal structure design to produce hydrogen and connect these production facilities to distribution centers, optimizing interconnected regional pipeline networks to link multiple production facilities and demand points. On the other hand, some authors have focused on specific components of the hydrogen supply chain, such as transmission and distribution, centering their studies on the optimal design of pipeline networks. Thus, one of the challenges that arises in this framework, is the fluid mechanics, being equation (1) one of the most important for piping design, which describes the flow within a pipe as a function of the pressure drop from one end to the other [12].

$$Q = CD^{2.5} \sqrt{\frac{p_1^2 - p_2^2}{\lambda Z_m T_m L \delta}} \quad (1)$$

Where  $Q$  is the flow in a pipe [ $m^3/h$ ],  $C$  is the proportionality constant [ $m^{10^9}/(barK^{1/2})$ ],  $D$  is the inner diameter [ $mm$ ],  $p_1$  is the inlet pressure [ $bar$ ], and  $p_2$  is the outlet pressure,  $\delta$  is the relative density compared to air,  $T_m$  is the gas temperature [ $K$ ],  $Z_m$  is the dimensionless compressibility factor,  $\lambda$  the dimensionless coefficient of friction, and  $L$  is the pipe length [ $km$ ], and the subindex  $m$  indicates that the average value of that parameter is considered.

The works that consider fluid mechanics in pipeline design can be classified into three main groups according to their approach to pipe diameter, (i) continuous diameter, where the diameter is considered as a continuous variable, (ii) split-pipe diameter, where a pipe is made up of several segments of different diameters joined together, and (iii) discrete diameter, where the diameter is selected from a set of commercially available sizes. Among the authors who use the continuous diameter approach are André et al. [2] who present a MINLP to design and dimension the pipelines, and also, they propose a Delta change heuristic, based on the reformulation of the model on a fixed topology of the network in the form of a tree. While Da Silva et al. [9] approach the sizing problem over a fixed topology from a multi-objective and multi-period perspective, including the location of compressor stations. In the group of those who consider the split-pipe diameter, Wu et al. [22] develop a global optimization algorithm for a MINLP problem based on the relaxed primal-dual decomposition method. Within the last group, where discrete diameters are considered, Wang et al. [19] propose an MILP to reformulate existing gas pipelines, considering the mixture of gas with hydrogen. In this work, the main decision is whether or not to keep a pipeline, and the dimension needed in each reformed pipeline, which is selected from a set of available measures.

It should also be noted that there are some studies related to the design of pipelines networks for other fluid distribution under the discrete diameter approach that can be extrapolated to the study

of hydrogen. In literature related to water distribution Bragalli et al. [7] propose a non convex continuous NLP relaxation and a MINLP search for a problem of dimensioning over a fixed topology. Similarly, Shiono and Suzuki [17] propose a continuous relaxation for a sizing problem on a fixed topology. Among those who study natural gas pipeline networks, [11] presents a genetic algorithm to determine the optimum pipe size for networks, considering a fixed topology, and [6] presents a convex relaxation for gas expansion planning.

## 1.2 Statement of the Problem

The design and dimensioning problem of a hydrogen injection pipeline network consists of finding both a network topology, and the diameter dimensions of each pipe section for the distribution of hydrogen, together with the pressure at each node, and the flows at each pipeline. Formally, given a connected digraph  $G = (N, A)$ , where  $N$  is the set of all hydrogen supply and consumption nodes, we define a network topology as the connected subgraph  $G' = (N, A')$  induced by a set of connected arcs  $A' \subseteq A$  and the set of nodes  $N$ . Each node  $i \in N$  has a hydrogen demand  $d_i$  to satisfy, or a quantity  $s_i$  of hydrogen it can supply, assuming that  $\sum_{i \in N} s_i = \sum_{i \in N} d_i$ . The main objective is to minimize the costs of construction of the hydrogen distribution network, in such a way that the proposed pipeline network can meet customer demand, considering the fluid mechanics equation (1).

To address this problem we rewrite equation (1), without loss of generality, as:

$$(\pi_1 - \pi_2)D^5 = k'Q^2L \quad (2)$$

where  $\pi_1$  and  $\pi_2$  represent the square of the pressure at the inlet and outlet of a pipe, respectively,  $L$  is the length of the pipeline, and  $k' = \frac{\lambda Z_m T_m \delta}{C^2}$  is assumed constant. The cost associated with choosing a pipeline relates the diameter of the pipe to the investment costs by units of length. Its quadratic version is the most accepted [15], precisely,  $(a_0 + a_1D + a_2D^2)L$  where  $a_0$ ,  $a_1$  and  $a_2$  are prefixed constants. In the following, we use this expression as the optimization criterion.

## 1.3 MINLP Formulation Based on Continuous Diameters

In the literature, there are several formulations for the design and dimensioning of pipelines problem. We present the following MINLP model, within the continuous diameter approach, and inspired by the model proposed by André et al. [2]. The network topology is represented by the binary decision variable  $x_{ij}$  for  $(i, j) \in A$ , which is 1 if and only if it is decided to install a pipeline between nodes  $i$  and  $j$ . Variable  $D_{ij}$  is the diameter of the pipeline installed between nodes  $i$  and  $j$ , this is a continuous variable with a value between  $D_{min}$  and  $D_{max}$  if a pipeline is installed between those nodes, and 0 otherwise. The square of the pressure at each node is represented by variable  $\pi_i$ , and lies between  $\pi_{min}$  and  $\pi_{max}$ . Finally, variable  $Q_{ij}$  represents the flow within a pipeline from node  $i$  to  $j$ .

In this model, Constraints (3b) represent the fluid mechanics equation for the pressure drop in a pipeline. Equations (3c) represent the flow conservation at each node, and the set of Constraints (3d) establish the limits on the pipeline's diameter. Finally, equations (3e) set a limit in the required pressure of hydrogen at each node.

$$(CP) \left\{ \begin{array}{l} \min_{D, \pi, x, Q} \sum_{(i,j) \in A} L_{ij}(a_0 x_{ij} + a_1 D_{ij} + a_2 D_{ij}^2) \quad (3a) \\ (\pi_i - \pi_j) D_{ij}^5 = k' Q_{ij}^2 L_{ij} \quad (i, j) \in A \quad (3b) \\ s_i + \sum_{j|(j,i) \in A} Q_{ji} = \sum_{j|(i,j) \in A} Q_{ij} + d_i \quad i \in N \quad (3c) \\ D_{min} x_{ij} \leq D_{ij} \leq D_{max} x_{ij} \quad (i, j) \in A \quad (3d) \\ \pi_{min} \leq \pi_i \leq \pi_{max} \quad i \in N \quad (3e) \\ D_{ij}, Q_{ij} \geq 0 \quad (i, j) \in A \quad (3f) \\ x_{ij} \in \{0, 1\} \quad (i, j) \in A \quad (3g) \end{array} \right.$$

Consequently, (CP) model is a mixed integer polynomial optimization problem of degree six, with a non convex continuous relaxation.

### 1.4 Delta Change Heuristic ( $\Delta C$ )

A well-known algorithm in gas pipeline design is the Delta Change heuristic, which was proposed by Rothfarb et al. [16] for the optimal design of offshore gas pipelines, and proved to be suitable for (CP) by André et al. [2]. This heuristic splits the solution into two subroutines, finding a network topology, and dimensioning the pipelines.

To find a network topology, this heuristic relies on Theorem 1 from [5], which states that the optimal network of this problem has a tree structure. Therefore, we initialize the algorithm with the minimum spanning tree, and then we move to other spanning trees, looking for a better solution. To select the next tree the delta change heuristic explores a randomly selected subset of nodes  $S \subset N$ , where for each node  $i \in S$  it explores a subset of the closest nodes  $V \subset N$  not connected to  $i$ . Then, for each  $j \in V$ , it creates a cycle by connecting it to  $i$ , and removes a different arc from the created cycle, thus finding a new tree.

Now, it remains to solve the dimensioning problem. To this, let  $T \subset A$  be a subset of arcs such that  $G' = (N, T)$  is a tree. We can deduce  $Q_T$  by solving the linear system given by Equations (3c), which, if it has a solution, is unique. Consequently, we can write model (CP) over this tree, obtaining model ( $CP_T$ ).

Algorithm 1 shows the pseudocode for the delta change heuristic ( $\Delta C$ ).

$$(CP_T) \left\{ \begin{array}{l} \min_{D, \pi} \sum_{(i,j) \in T} L_{ij}(a_0 + a_1 D_{ij} + a_2 D_{ij}^2) \quad (4a) \\ (\pi_i - \pi_j) D_{ij}^5 = k' (Q_T)_{ij}^2 L_{ij} \quad (i, j) \in T \quad (4b) \\ D_{min} \leq D_{ij} \leq D_{max} \quad (i, j) \in T \quad (4c) \\ \pi_{min} \leq \pi_i \leq \pi_{max} \quad i \in N \quad (4d) \\ D_{ij} \geq 0 \quad (i, j) \in T \quad (4e) \end{array} \right.$$

### 1.5 Contribution

Considering the context presented above, our proposal seeks to complement existing studies, addressing the design and dimensioning of natural gas pipelines with hydrogen injection problem, considering discrete diameters. We show that, in this case, model (CP) can be reformulated as a mixed integer quadratic problem, considering fluid mechanics constraints in its formulation. We also

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#### Algorithm 1 Delta Change ( $\Delta C$ )

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**Input:** Instance, fixed formulation  $\overline{FF}$   
**Output:**  $D, \pi, T, Q$  (Feasible solution)

```

1: Initialize:  $G' = (N, T) = \text{minimalSpanningTree}(\text{instance})$ 
2: Compute  $Q^*(T)$ 
3: Solve ( $CP_T$ )  $\implies D^*(T), \pi^*(T), \text{Cost}(T)$ 
4:  $D = D^*(T), \pi = \pi^*(T), Q = Q^*(T)$ 
5: Randomly select  $S \subset N$ 
6: for  $i \in S$  do
7:   Select  $V \subset N$  such that:  $(i, v) \notin T, v \in V$ 
8:   for  $v \in V$  do
9:      $\overline{T} \leftarrow T \cup (i, v)$ 
10:    Determine the cycle  $C \subset \overline{T}$ 
11:    for  $a \in C$  such that  $a \neq (i, v)$  do
12:       $T_a = \overline{T} - a \implies T_a$  is now a tree
13:      Compute  $Q^*(T_a)$ 
14:      Solve ( $CP_{T_a}$ )
15:      if feasible &  $\text{Cost}(T_a) < \text{Cost}(T)$  then
16:        deduce  $D^*(T_a), \pi^*(T_a)$ 
17:         $T = T_a, D = D^*(T_a), \pi = \pi^*(T_a), Q = Q^*(T_a)$ 
18:      end if
19:    end for
20:  end for
21: end for
22: return  $D, \pi, T, Q$ 

```

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propose a mixed-integer quadratic convex relaxation. Finally, numerical results are presented to compare the performance of these models applied to a test example instance and a case study in France based on existing data from the literature.

## 2 MATHEMATICAL FORMULATIONS AND A HEURISTIC CONSIDERING DISCRETE DIAMETERS

A more realistic assumption to this problem is to discretize the pipe diameter, in order to choose the diameter to build a pipeline from a set of available values from the market. Based on this, in this section we start by deriving a MIQP model for the discrete diameters case.

### 2.1 MIQP Formulation Based on Discrete Diameters

Here, we propose to select the diameter of the pipelines from a set of available diameter-values  $H$ . Let  $dm_h$  be the value of diameter  $h \in H$ , and let  $v_{ij}^h$  be a binary variable indicating if diameter  $h$  is selected for the pipeline installed on arc  $(i, j)$ , where each diameter is modeled as  $D_{ij} = \sum_{h \in H} dm_h v_{ij}^h$  for  $(i, j) \in A$ . To simplify the notation of the different models, let  $B = \{(i, j, h) : (i, j) \in A, h \in H\}$ .

$$\begin{cases}
\min_{\pi, Q, x, v} \sum_{(i,j) \in A} L_{ij} (a_0 x_{ij} + \sum_{h \in H} (a_1 dm_h v_{ij}^h + a_2 dm_h^2 v_{ij}^h)) & (5a) \\
(\pi_i - \pi_j) \sum_{h \in H} dm_h^5 v_{ij}^h = k' Q_{ij}^2 L_{ij} & (i, j) \in A \quad (5b) \\
s_i + \sum_{j|(j,i) \in A} Q_{ji} = \sum_{j|(i,j) \in A} Q_{ij} + d_i & i \in N \quad (5c) \\
\pi_{min} \leq \pi_i \leq \pi_{max} & i \in N \quad (5d) \\
Q_{ij} \leq Mx_{ij} & (i, j) \in A \quad (5e) \\
\sum_{h \in H} v_{ij}^h = x_{ij} & (i, j) \in A \quad (5f) \\
Q_{ij} \geq 0 & (i, j) \in A \quad (5g) \\
x_{ij} \in \{0, 1\} & (i, j) \in A \quad (5h) \\
v_{ij}^h \in \{0, 1\} & (i, j, h) \in B \quad (5i)
\end{cases} \quad (DQP)$$

Since variables  $v_{ij}^h$  are binary and at most one diameter is selected for a pipeline on  $(i, j) \in A$ , it holds for any positive integer  $p$  that

$$D_{ij}^p = \left( \sum_{h \in H} dm_h v_{ij}^h \right)^p = \sum_{h \in H} dm_h^p (v_{ij}^h)^p = \sum_{h \in H} dm_h^p v_{ij}^h$$

therefore, any power of variable  $D_{ij}$  can be reformulated by a linear expression in  $v_{ij}^h$ . Thus, equation (5a) represents the cost associated to the diameters and the length of each pipeline, which becomes a linear equation when we move to the case of discrete diameters. Constraints (5b) represent the drop pressure equation, which can now be written as an equation of degree 2 due to the discretization. Constraint (5e) is a valid inequality, which states that a flow between two nodes can only exist if there is a pipe between those two nodes, and equation (5f) indicates that if there is a pipe between two nodes, we must choose a single pipe diameter.

## 2.2 Relaxation into a MIQCP

Model (DQP) is quadratic but even its continuous relaxation is not convex. Non-convexity comes from the quadratic equation (5b) and from the integrality of variables  $v$ . We below present a relaxation of this problem, whose continuous relaxation is convex. First we linearize the product of variables  $\pi$  by variables  $v$ , which represents a reformulation of (DQP). To this, we introduce variables  $\beta$  and  $\gamma$ , where  $\beta_{ij}^h = \pi_i v_{ij}^h$  and  $\gamma_{ij}^h = \pi_j v_{ij}^h$ . Then we replace the new version of equation (5b) with a quadratic convex inequality.

$$\begin{cases}
\min_{Q, x, v, \pi, \beta, \gamma} \sum_{(i,j) \in A} L_{ij} (a_0 x_{ij} + \sum_{h \in H} (a_1 dm_h v_{ij}^h + a_2 dm_h^2 v_{ij}^h)) & (6a) \\
\text{s.t.: (5c) - (5i)} \\
\sum_{h \in H} dm_h^5 (\beta_{ij}^h - \gamma_{ij}^h) \geq k' Q_{ij}^2 L_{ij} & (i, j) \in A \quad (6b) \\
\pi_{min} v_{ij}^h \leq \beta_{ij}^h \leq \pi_{max} v_{ij}^h & (i, j, h) \in B \quad (6c) \\
\beta_{ij}^h \geq \pi_i - \pi_{max} (1 - v_{ij}^h) & (i, j, h) \in B \quad (6d) \\
\beta_{ij}^h \leq \pi_i - \pi_{min} (1 - v_{ij}^h) & (i, j, h) \in B \quad (6e) \\
\pi_{min} v_{ij}^h \leq \gamma_{ij}^h \leq \pi_{max} v_{ij}^h & (i, j, h) \in B \quad (6f) \\
\gamma_{ij}^h \geq \pi_j - \pi_{max} (1 - v_{ij}^h) & (i, j, h) \in B \quad (6g) \\
\gamma_{ij}^h \leq \pi_j - \pi_{min} (1 - v_{ij}^h) & (i, j, h) \in B \quad (6h) \\
\beta_{ij}^h, \gamma_{ij}^h \geq 0 & (i, j, h) \in B \quad (6i)
\end{cases} \quad (DQPR)$$

## 2.3 Delta Change Heuristic for Discrete Diameters Approach (DΔC)

In the same way that we can adapt the (CP) model into a NLP model over a fixed topology network, we can adapt (DQP) into a MILP model over a fixed topology. First, we linearize the product of variables  $\pi$  by variables  $v$ , as before. However, now we keep the equality constraint which becomes linear in this case. Then, considering a tree topology  $G' = (N, T)$  with  $T \subset A$ , (DQP) for tree  $T$  is simplified into  $(DQP_T)$ , which seeks to determine the diameter of the pipes and the pressure at the nodes, considering that the flows between nodes are fixed by the tree topology of the network.

We are aware that restricting the network topology to trees may be a drawback of this heuristic. Indeed, by using discrete diameters, we lose the structural property of the continuous case, i.e., the optimal solution of our problem (DQP) may not be a tree. This fact will be illustrated in the test example of Subsection 3.1.

$$\begin{cases}
\min_{v, \pi, \beta, \gamma} \sum_{(i,j) \in A} L_{ij} (a_0 x_{ij} + \sum_{h \in H} (a_1 dm_h v_{ij}^h + a_2 dm_h^2 v_{ij}^h)) & (7a) \\
\text{s.t.: (5d), (6c) - (6h)} \\
\sum_{h \in H} dm_h^5 (\beta_{ij}^h - \gamma_{ij}^h) = k' (Q_T)_{ij}^2 L_{ij} & (i, j) \in T \quad (7b) \\
\sum_{h \in H} v_{ij}^h = 1 & (i, j) \in T \quad (7c) \\
\beta_{ij}^h, \gamma_{ij}^h \geq 0 & (i, j) \in T, h \in H \quad (7d) \\
v_{ij}^h \in \{0, 1\} & (i, j) \in T, h \in H \quad (7e)
\end{cases} \quad (DQP_T)$$

We observe that problem  $(DQP_T)$  is a mixed integer linear programming (MILP) model, whereas  $(CP_T)$  is a non-linear programming model.

## 3 NUMERICAL EXPERIMENTS

In this section we illustrate the solution methods presented in this work, first in a test example, and we compare their solutions and the performance of each method. Then, we apply them to a real case study based in a future French national hydrogen network obtained from [2].

We implemented all our models and resolution methods in Julia 1.8.1, using Ipopt 3.14.4 for NLP models and Gurobi 10.0 for MINLP models.

### 3.1 Test Example

We generate a small instance to show the strengths and weakness of each proposed method. This instance considers 9 consumption nodes, and a single source node with a supply capacity equal to the sum of the demand of the client nodes, likewise, the nodes considered as clients, present a random demand between 15 and 50 units each, the coefficients for the cost functions are  $a_0 = 2$ ,  $a_1 = 3$  and  $a_2 = 4$ ,  $\pi_{min} = 1.2$  and  $\pi_{max} = 1.8$ . The nodes were randomly positioned in a squared plane of size  $200 \times 200$ , identified by their coordinates  $(x, y)$ , and the distance between them was calculated as the Euclidean distance. For the discrete diameter approach we consider two measures,  $\{2.5, 10\}$ , and for the continuous approach we set  $D_{min} = 2.5$ , and  $D_{max} = 10$ .

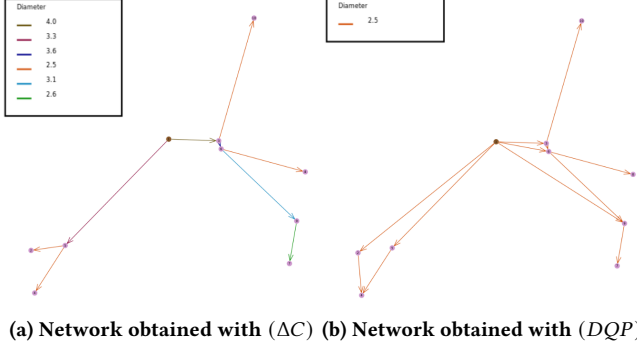
For the continuous diameter approach we solved  $(CP_{\bar{T}})$ , where  $\bar{T}$  corresponds to the minimum spanning tree, and  $(\Delta C)$ ; and for the

discrete diameters approach, we solved  $(DQP_{\bar{T}})$ ,  $(D\Delta C)$ ,  $(DQP)$ , and  $(DQPR)$ . For the delta change heuristics we decided to explore the 50% of the nodes, and two neighbors of each explored node, i.e.,  $|S| = 0.5 N$  and  $|V| = 2$ . Table 1 shows the optimal value and resolution time, obtained with each of these methods. For the continuous diameter approach we can see how  $(\Delta C)$  improves the solution with respect to the minimum spanning tree, however, this is not the case for the discrete diameter approach. In Figure 1 we present the network of the best solution obtained with each diameter approach, in Figure 1a the network uses 6 different diameters for 9 arcs, with small differences between them. Whereas, in Figure 1b we only use one diameter, but the network obtained contains cycles, i.e., is not a tree.

**Table 1: Summary of results for test example**

Method	Time(s)	Objective
$(CP_{\bar{T}})$	0.10	22268.3
$(\Delta C)$	26.41	21628.9
$(DQP_{\bar{T}})$	0.01	100972.5
$(D\Delta C)$	2.73	100972.5
$(DQP)$	19.23	26254.5
$(DQPR)^*$	2.39	25806.0

\* Non feasible solution



**Figure 1: Network of the best solution obtained for the test example according to each diameter approach**

### 3.2 Case Study: Hydrogen Network in France

The future french national hydrogen network from André et al. [2] will consider 79 nodes, corresponding to the 78 urban areas with more than 100,000 inhabitants and a source near Paris. The source node will have a supply capacity equal to  $105348395 m^3/day$ , and the client nodes will have a demand between 298271 and  $33146708 m^3/day$ , according to predictions for 2050, the coefficients for the cost functions are  $a_0 = 236663, 6385$ ,  $a_1 = 210, 4168253$  and  $a_2 = 0, 949507363$ ,  $\pi_{min} = 1225 [bar^2]$  and  $\pi_{max} = 5041 [bar^2]$ , and the nodes are identified by their geographic coordinates. The pressure loss constant is  $k' = 165.778$ .

For this case study we implemented the same six solution approaches used for the test instance,  $(CP_{\bar{T}})$ , where  $\bar{T}$  corresponds to the minimum spanning tree, and  $(\Delta C)$  for the continuous diameters approach; and for the discrete diameters approach, we solved  $(DQP_{\bar{T}})$ ,  $(D\Delta C)$ ,  $(DQP)$ , and  $(DQPR)$ . We set a time limit of 1 day.

For the continuous diameters approach we set  $D_{min} = 10$  and a  $D_{max} = 2000$ . While, for the discrete diameters approach we consider 5 diameter-values: 100, 200, 400, 600 and 700.

Table 2 shows the results of the different implementations carried out. In the continuous diameter approach, similarly to the results obtained for the test example, the  $(\Delta C)$  heuristic slightly improves the solution obtained over a minimum spanning tree. Furthermore, both methods select a 78 different diameters, i.e., a different diameter for each arc. In the discrete diameter approach, we can see that  $(D\Delta C)$  also improves slightly the solution obtained over a minimum spanning tree, moreover, this solution is the most expensive one.

By analyzing these four heuristics we can see the cost, in terms of time, of implementing the delta change heuristic, which solve a NLP problem (MILP for discrete diameter approach), for each arc of the cycle created for each neighbor of each explored node, i.e., if it is decided to explore 10% of the nodes and two neighbors of each explored node, at least  $0.2 * N$  problems must be solved.

**Table 2: Summary of results using different approaches. (TL= Time limit of 1 day)**

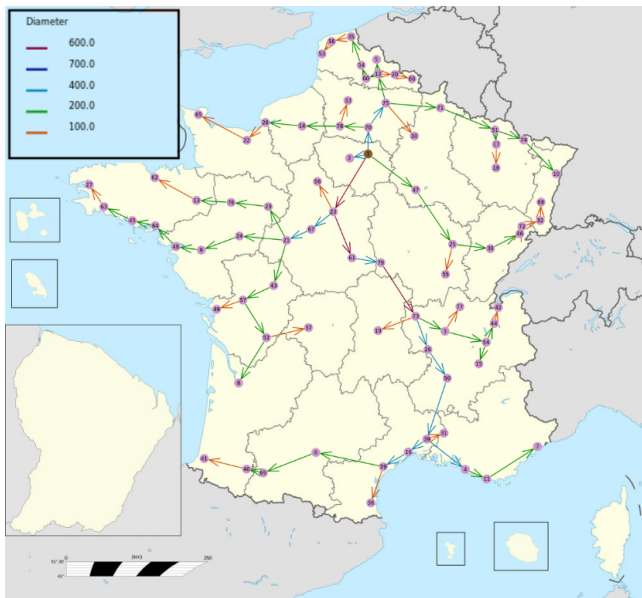
Method	Time(s)	Objective ( $10^6$ )	Final GAP
$(CP_{\bar{T}})$	422	2079.05	-
$(\Delta C)$	TL	2069.11	-
$(DQP_{\bar{T}})$	124	2195.49	-
$(D\Delta C)$	7076	2181.51	-
$(DQP)$	TL	-	-
$(DQPR)$	TL	1894.45	0.12

It can also be seen that the cheapest network is obtained by solving the  $DQPR$  problem. Furthermore, it presents savings of 13% compared to the most expensive one. In Figure 2 we illustrate this solution, which reached the time limit with a final optimality gap of 12%. In this case, even if this model is a relaxation, the solution obtained satisfy the fluid mechanics equations, therefore, is a feasible solution of  $(DQP)$  and an upper bound to the optimal solution of  $(DQP)$ .

We also remark that Ipopt, the solver used for NLP problems, can only ensure local optimality of the solution, and that  $(DQP)$  is not able to find a feasible solution in the allotted time.

## 4 CONCLUSION AND PERSPECTIVES

This work addressed the problem of the design and dimensioning of natural gas pipelines with hydrogen injection. We presented a mixed integer polynomial programming model of degree six for this problem under a continuous diameter approach, which includes the fluid mechanics equation that describes the flow within a pipe. We proposed a quadratic mixed integer programming model for the discrete diameters approach, considering also the fluid mechanics



**Figure 2: Hydrogen network for France obtained using (DQPR) model**

equation. We also derive a mixed-integer quadratic convex relaxation. Finally, we implement a Delta Change heuristic for both cases.

To illustrate our proposals, we used a real case study in France, considering estimated demands for 2050, where the best solution is obtained using the MIQP convex relaxation, which, in this case, is a feasible solution given that satisfies the fluid mechanics equation. Furthermore, we showed the advantages of using a discrete diameters approach, and how Delta change heuristic is not suitable for our problem.

Following this work, a future direction is to evaluate the quality of the convex relaxation, as well as its use in an exact resolution approach to this problem. Moreover, a future work is to improve the heuristic, considering another local search heuristic.

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