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# On Evolution of Irreversible Systems: Computing Analysis Results.

Andrés León Baldelli & Sebastien Neukirch, \*

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What happens to the stability of critical equilibrium states, as time goes by? In evolutionary problems, structural effects play a key role. We study evolving systems, singularly perturbed and under constraints. Picture *fracture* as a process: as a phenomenon of natural evolution, it offers an instance of a particularly complex natural system showing pattern formation and jumps, *unknown* both in time and space.

From a mechanical perspective [1, 5] the structure of the incremental (quasistatic) evolution problem of fracture is cast in three statements: an irreversibility constraint (pointwise and stiff), an energetic stability criterion (a unilateral condition), and a power balance (in integral form).

Fully exploiting the stability statement provides a functional criterion of choice and a tool to correctly predict path bifurcations [4] and loss of stability of global structures under time-dependent loads, cf. Figure 1. The problem is reduced to a variational constrained eigenvalue problem involving a nonlinear operator which handles both structural elasticity and an internal dissipative softening (damaging) process identifying fractures. In the perspective of large scale numerical experiments, I shall discuss some remarks on its energetic formulation as a variational problem driven by inequalities, relying on its purely energetic considerations.

Presenting a numerical FEniCS (dolfinx) implementation for this singular nonlinear evolution problem with several examples of its discrete approximation, I share the motivation and interest that lays at the intersection between mathematical analysis, theoretical mechanics, and advanced numerics, addressing the understanding of general mechanisms of evolution of irreversible systems, to find stable continuation paths.

The continuous-stability of a multiscale system along its nontrivial paths in phase space is a property that is difficult to check: numerically, at large scales with several material lengths involved, and analytically, in the infinite-dimensional setting.

Exploiting a connection with an existence theorem [3] stated for compact operators in a convex cone, a rigorous implementation of a general energetic statement for a complex system provides the challenge to deploy a fully transparent numerical experimental platform to address the prediction of large scale fracture events.

Think of Antarctica.

<https://news.google.com/search?q=antarctica%20ice%20cracks>

## References

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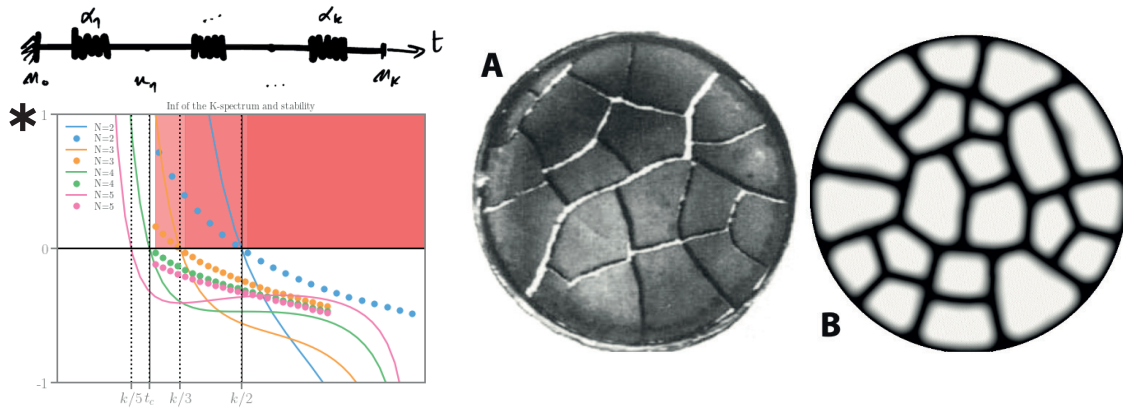


Figure 1: \*: Stability of a discrete chain of nonlinear softening (damaging) springs, matching of independent discrete (dotted) and analytic (continuous) stability criteria, as a function of the load ( $t$ , horizontal axis) and parametrically with respect to the number of springs  $N$ . **A**: fragmentation of a thin brittle layer, controlled experiment [2], **B**: two-dimensional numerical computation of a fracture pattern (in black) for a thin film over foundation subject to homogeneous loads. Symmetry-breaking is a consequence of loss of stability. cf. [4].

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